Modern Optics
Prof. Partha Roy Chaudhuri
Department of physics
Indian Institute of Technology, Kharagpur

Lecture – 19
Waves in guided structures and modes (Contd.)

So, we have seen the T E polarization and T M polarization waves propagating through the interface structures and we have seen that how they give rise to the electromagnetic waves as guided waves in the structure.

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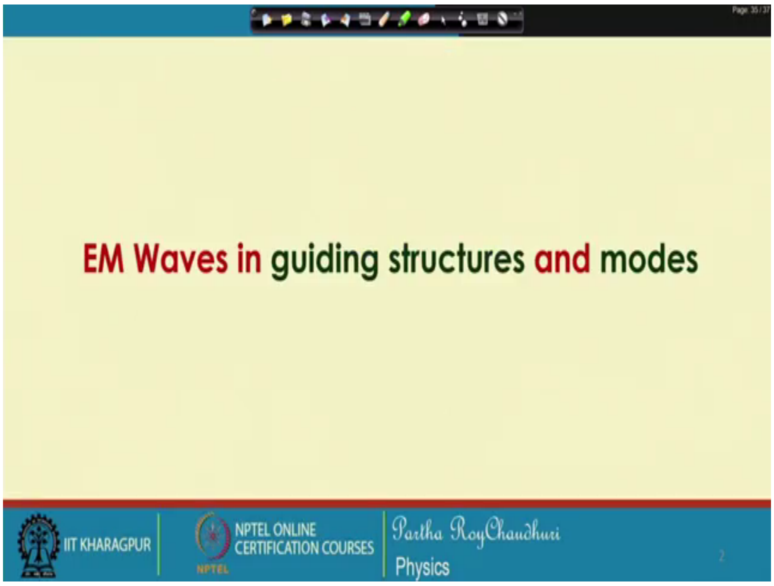

Now, as a first example, we will discuss a metallic waveguide electromagnetic waves in guiding structures and modes. We will look at the mode properties.

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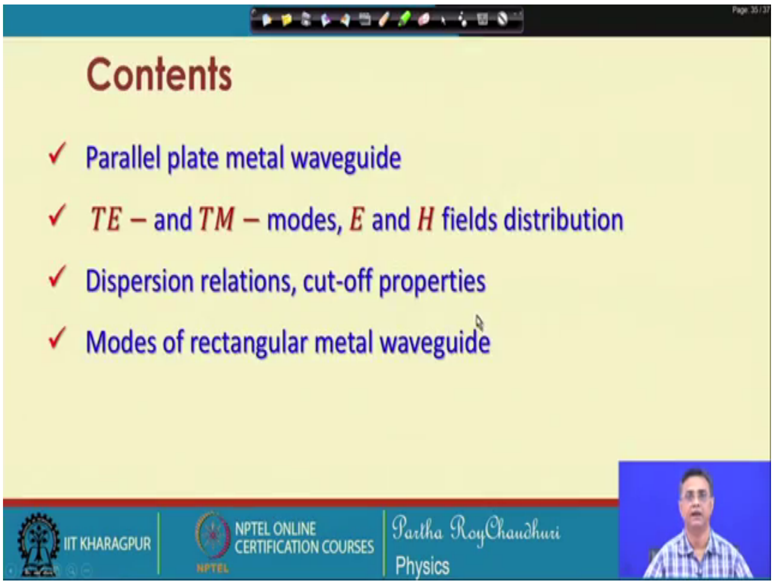

And so, the. So, we have organized this part like we first consider a parallel plate metal waveguide which is the simplest structure then and we will look at the T E and T M modes of this structures and to do that, we will write down the electric and magnetic field distributions field components and once we have these modes, then we will be able to depict the electric and magnetic field distribution across the web guide.

Then, we will we will discuss the dispersion relations and cutoff properties that till where till which frequency the wave will be supported and the wave will not be supported modes of the rectangular metal waveguide will be discussed as an example of 2 dimensional confinement of the waveguide.

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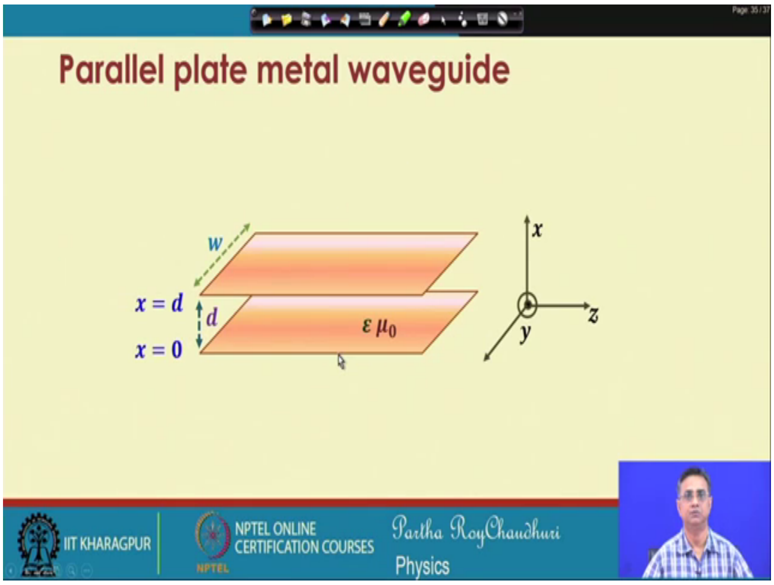

So, we first consider a parallel plate metal waveguide the structure like it may be infinitely extended along this y direction and along this y direction and there is a variation of the medium along the x direction that is the 2 plates are the metal plates. So, and let us suppose that the separation between the 2 plates are x equal to d. So, is equal to d. So, this is located at this is placed at x equal to d and the bottom one is that x equal to 0 inside medium I represented by epsilon mu naught and this is a coordinate system.

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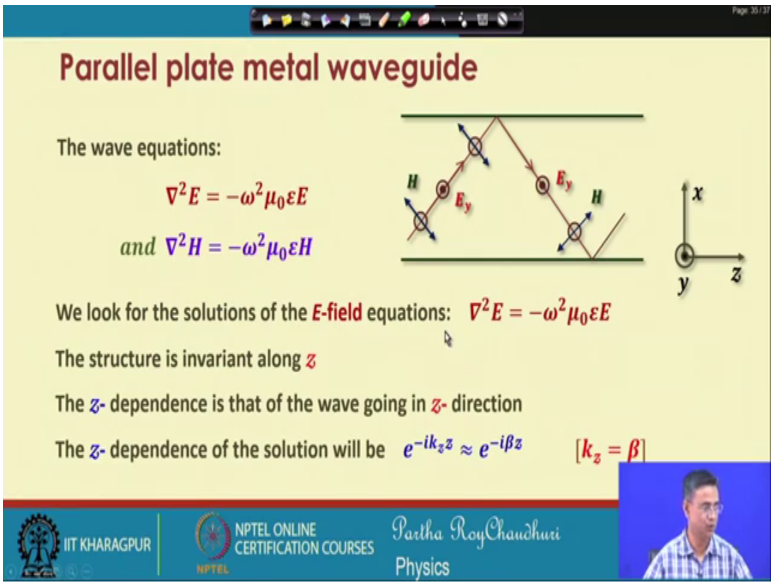

So, we will again refer to our configuration of this is very useful and a nice way of understanding the modes in the structure you have a possibility that you have a possibility that the electric fields are tangential components.

Whereas, the magnetic fields are lying on the x z plane, which will constitute a T M mode T E mode and the other one that is when the electric magnetic field is tangential to the to the interface plane and the this magnetic the electric field will have 2 components they are lying in the x z plane. So, that is what is the T M mode.

So, these 2 we have been able to very clearly dissociate in terms of the electric field and magnetic field groups, which will be supported by the structure. So, the wave equation for such a configuration we can start with this del square E equal to minus omega square mu naught epsilon E and for the magnetic field we have a similar expression. Now, we are looking for the solutions of the electric field from this equation and you can see that this structure is invariant. This is our assumption that it is it does not vary along this z direction and because of this the solution should be independent of z.

So, the z dependence. So, this the z dependence should be represented separately, the z dependence of the solution. We have seen earlier can be represented by e to the power of minus i k z z, but k z we have called is equal to beta which is the z component of the propagation vector.

So, one part is clear that, if the wave is traveling along z direction and the structure is invariant along this z, you can always represent that a electric field with a with a factor e to the power of minus beta z.

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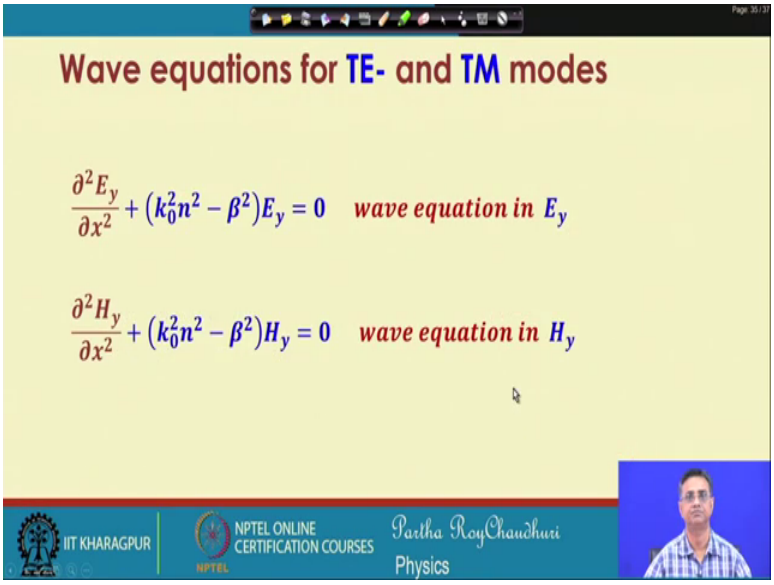

The wave equation for T E and T M modes for the by now, it is very clear if it has to represent a T E mode. Then, it is the electric field component which will satisfy this equation and for the T M mode, it is the magnetic field component which will be satisfying this component. This equation and this are these are these equations are valid for the homogeneous medium each of the homogeneous layers which will constitute the structure of the medium.

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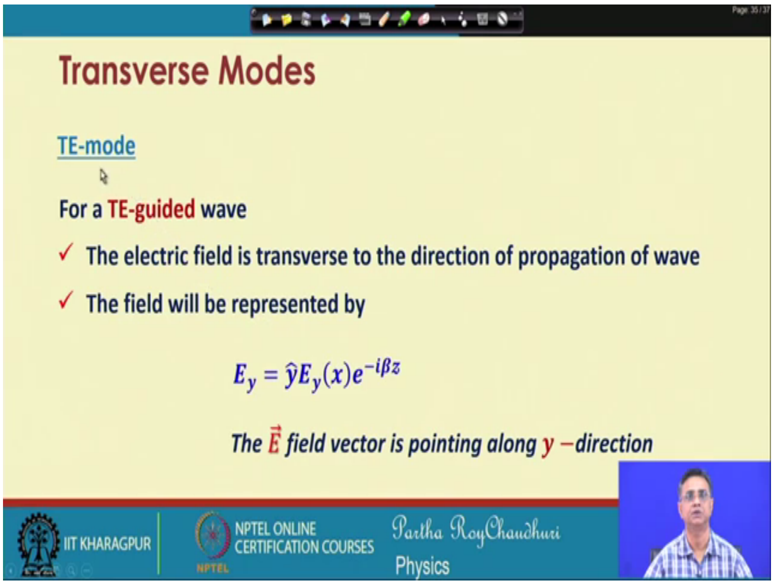

So, for T E guided mode the electric field is transverse to the direction of propagation of the wave the field will be represented by this. We have just now worked out and you have seen that the field, the amplitude should depend only along only on the x coordinate and it the electric field vectors are directed along the y direction.

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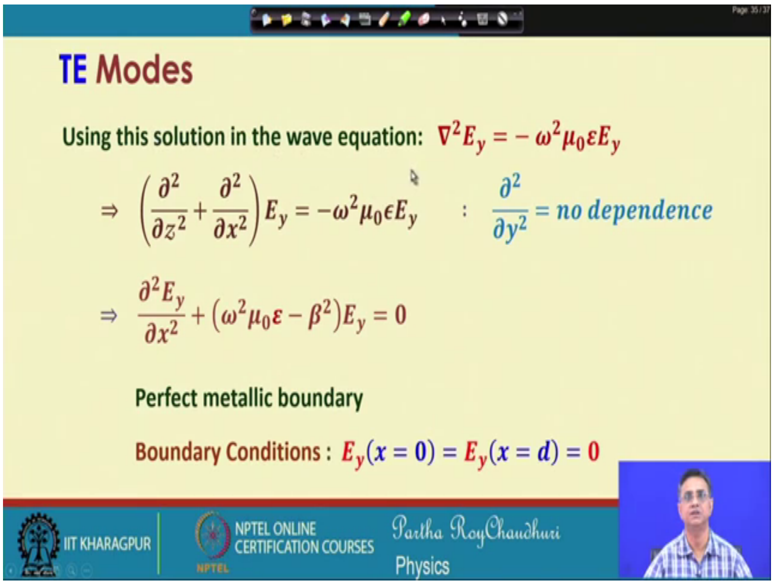

So, using the solution for this equation that e to the power of i beta z and e to the power of i omega t from like this equation, this solution we can write this equation in this form. This is the transverse, this is the z and x component of this del square operator multiplied upon this and del square of oh. So, because there is no y dependence because there is no y dependence.

So, this form reduces to this form only and further because del square del z square can be represented as i beta square. So, that comes on to this side as minus beta square. So, you have the wave equation for the E y field in the T E mode which is represented by this equation.

And now, we will apply the perfect bound metallic boundary condition by saying that if this problem is a one dimensional rigid wall, quantum mechanical problem, that is the fields will be 0 exactly at the boundaries because they are metal boundary. So, at the metallic boundary at x equal to 0, the boundaries are placed at x equal to 0 and x equal to d. So, that the boundaries E y will be equal to 0 and at x equal to d; that is another boundary the field will be equal to 0.

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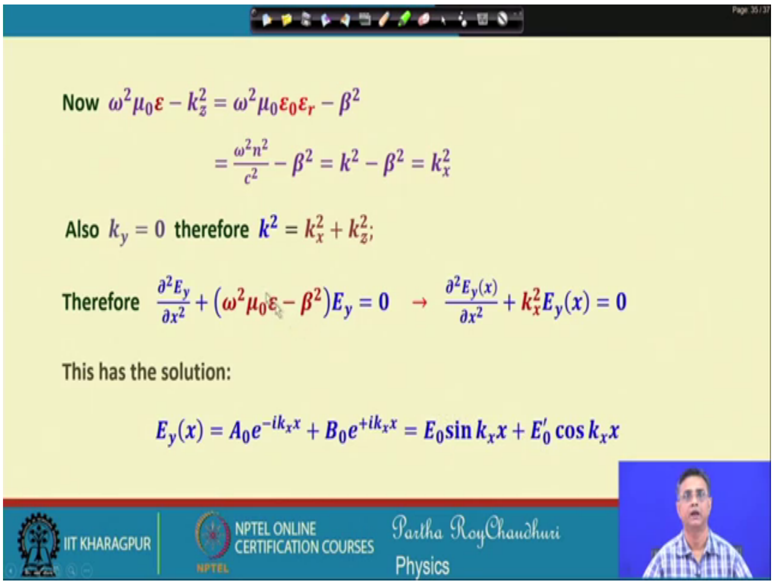

So, and now also that this quantity that is omega square mu naught and epsilon minus k z square which is represented by this equation just transforming into this part that is epsilon will be equal to epsilon naught epsilon r that is the relative or permittivity of the medium which will be equal to n square and k z i write by beta square.

So, I can rewrite this as that transverse component that is the x component of the propagation vector because k y is equal to 0. So, this total k square proper the mod of the k vector square will be equal to k x square plus k z square, but this is equal to beta square. That is why I have got this form this connection.

So, therefore, I can write this equation in this form because this quantity this quantity is nothing but k x square this quantity this is your total k square minus z component y component is not appearing. So, therefore, it is equal to k x square equal to 0. So, this has the solution of this form this equation is well known in physics and I can write down this equation as the oscillatory waves that A e to the power of minus and plus i k x and this can also be written in this form as a sine cosine sinusoidal variation of the electric field. So, E 0 and E 0 dash are the amplitudes.

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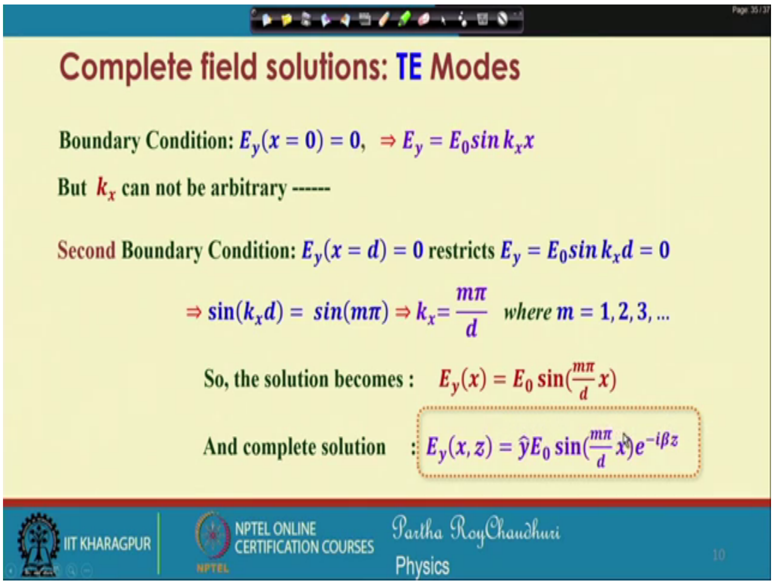

Now, we will use the boundary condition that is E y equal to E y equal to 0 at x equal to 0 which will immediately give you that if I put into this equation this is equal to 0. So, I will get that E dash must be equal to 0. So, I get this equation E 0 sine k x, but k x cannot be 0. It represents the waveguide structure and the second boundary condition. If we impose that at x equal to d, again E y will be equal to 0.

So, E 0 sine k x d equal to 0 that gives you that sine k x d is equal to sine m pi and this gives you the discrete values of this k x; that is the x component of the propagation the transverse component of the propagation vector which are discrete because this m can assume 1 2 3 all integral values. So, the solution becomes E of y equal to E 0 sine m pi by d x and the complete solution because the fields are now directed along the y direction y axis.

So, y cap E 0 sine m pi x and this is the phase vector and there should be one more e to the power of i omega t for the complete solution, but this is the wave.

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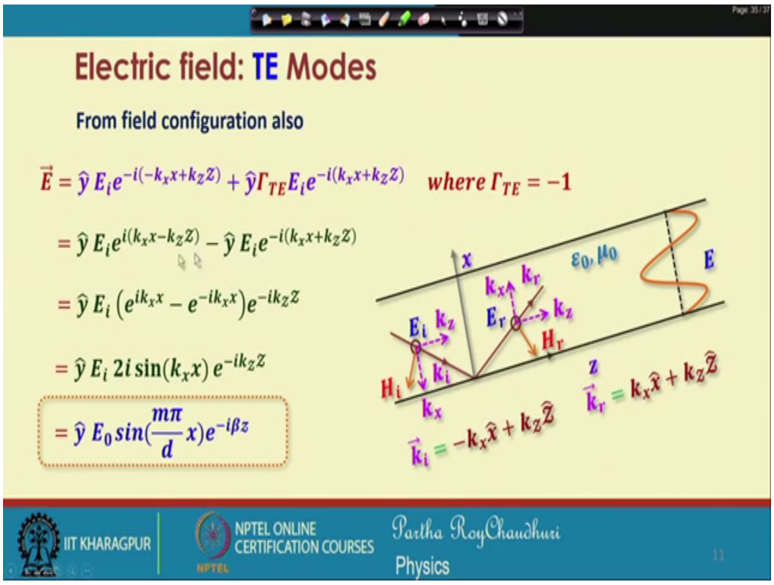

So, from the field configuration, we can also make out this solution you can also bring out, this solution not by doing all these calculations. All these following this wave equation simply by just looking at the configuration that that you have your E field like this which is incident and E field, which is reflected with a reflection factor of this equal to minus 1 just because of this is a metallic boundary and as a result this quantity will become minus.

So, this is a wave which is propagating along the positive x axis. This is this is along the positive x axis and this is the one which is no. So, this will be minus because there is a minus sign and this is the wave which will be the reflected wave, which is propagating along the positive x axis. So, I take the superposition of these 2 waves which you can put together in this form because e to the power of i k z Z will be taken out and you are left with this part inside.

Now, if you multiply the numerator and denominator by twice i, this equation this expression gives you that sine of k x x. So, and k x is already known that m pi by d. So, we can again arrive at the same expression for the electric field y equal to E 0 sine m pi d by x.

So, just looking at this, so, they are very consistent and is a very good way of understanding how the electric fields are oriented in the structure; particularly in the case of this metallic boundary condition.

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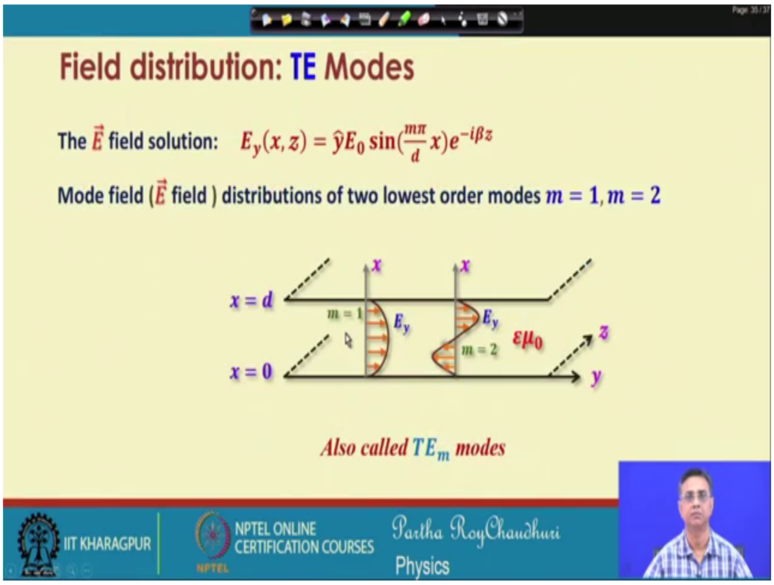

Now, the electric field solution for this we can write in this form and then it should look like this is m equal to 1. If you put m equal to 1, then this will be equal to just pi by d and depending on the value of the d, you have a distribution of the field, which will look like this x and d when x equal to 0. This quantity is 0, this quantity is 0. When x equal to when x equal to 0, this quantity is 0. When x equal to d this quantity is equal to pi.

So, again it is 0. Similarly, for 2 m equal to 2 if x equal to 0 this quantity is anyway 0, but when x equal to when x equal to d this quantity equal to twice pi. So, sine twice pi is again 0. So, you have this distribution for various distribution of the electric fields for various possible values of m equal to 1 2 3 etcetera. So, this is how we can look at the field distribution in such a metallic waveguide with 2 plates.

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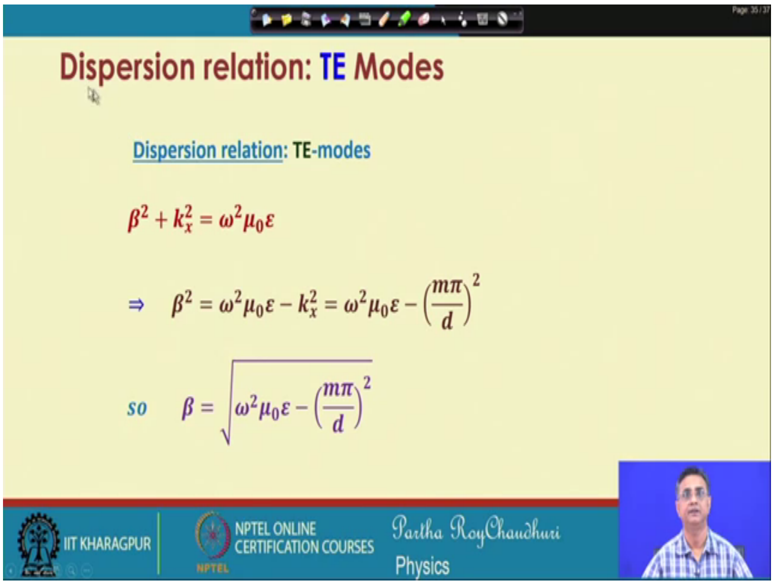

And then, the dispersion relation will come from this equation that this beta square plus k x square will be equal to omega square mu naught epsilon naught. And this tells you that the allowed values of the propagation constant. This should be will include the value of m equal to 1 2 3 etcetera and this gives you beta equal to under root of omega square mu naught epsilon minus m pi by d and m can assume 1 2 3 etcetera all these values.

So, this is the dispersion relation for the waves which will be the modes which will be supported by the structure. So, you can knowing the value of m knowing the value of this waveguide parameter d. The property and this omega the electric field we can calculate the propagation constant which will give you the solution for the waves which are travelling in the medium bound by the 2 plates that is this will appear in the wave equation as e to the power of i beta z ok.

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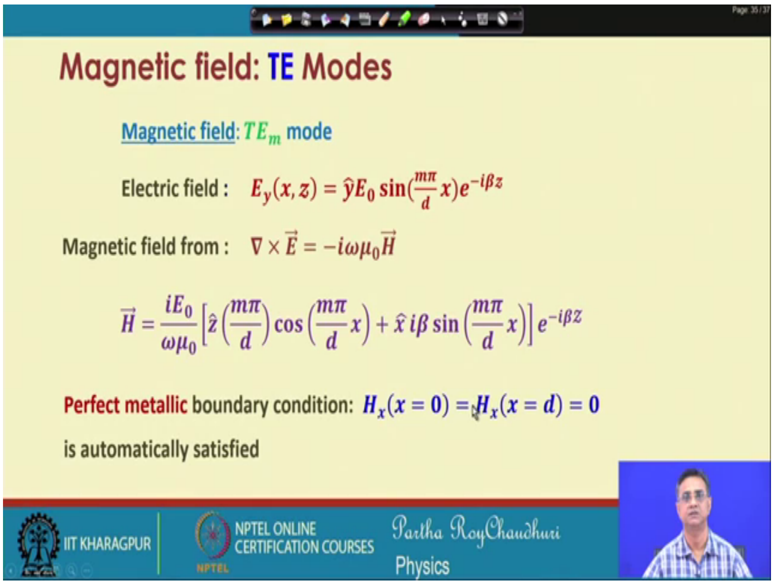

So, the cut off property that which of the which of the frequencies will be supported if you look at the dispersion relation for T M modes we call this modes to be T M modes because it has a general index m which can take up a 1 2 3 4 etcetera. So, these modes will be T E m T E 1 T E 2 etcetera.

So, beta is we have seen this is the dispersion relation for this T E m mode if the frequency omega is less than this value; that is if you put the value of this omega equal to this, then this quantity this quantity becomes purely imaginary. Because, this is now this is now more than this quantity. If I put the value of omega square omega equal to this, omega square will be equal to this m pi square by d by epsilon naught. So, that means, this quantity will be and which is less. So, this quantity will be will be more and this quantity will be less for the value of omega which is equal to this.

That means, the value of beta is now purely imaginary and the mode does not propagate, but decays exponentially. We have seen in the evaluation field that if the beta value if the exponent becomes negative, then that it becomes beta or k x. These things becomes imaginary, then the overall field will decay exponentially with the distance.

So, that is why the electric field, will this will decay exponentially through the structure and that means, the then that the cutoff frequency is determined because these wave this modes will not be able to propagate through the structure. And therefore, we set this cutoff frequency as this that at this up to this frequency given by this we have the waves which will be propagating. So, the cutoff frequency is given by this equation this expression. Now, the magnetic field we have talked about the electric field components various components of the electric field.

Now, from there we can calculate the magnetic field simply by using this connection that is curl of E is equal to some i omega mu naught times this magnetic field. So, H can be calculated by knowing the electric field components E. So, H in this case we can write in this form, you have seen that H will have the z component and x component and both of them will take up the sign depending on whether it is moving up or whether it is moving down.

Now, we can impose the perfect metallic boundary condition that H x at x equal to 0 is 0 and H x at x equal to d is also 0 and that condition is automatically satisfied here. If you plug in this x equal to 0 x equal to 0 here and here, then this H x component of the field will become automatically equal to this is the x component of the field and this is the z component of the field, but x component of the field which is the phase a which is not the tangential component that will be equal to 0.

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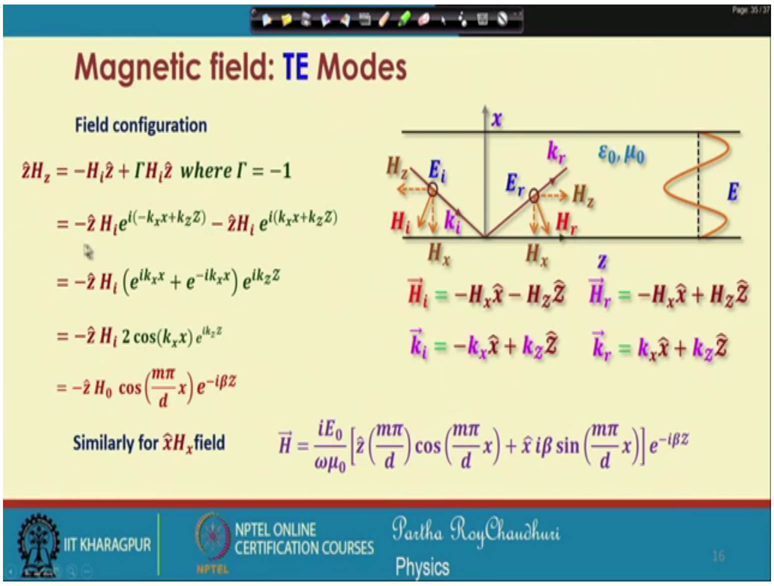

Now, now we will look at the configuration like the way we have seen the electric fields from this configuration we can also make out the field dependence the z field dependence; that is in this case tau is again equal to minus 1. So, this is the reflection amplitude coefficient if I take that tau equal to minus 1 and put together in this form. We can write down this equation and which will be a cosine function.

Therefore, for the z component we have a cosine factor appearing here and if we work out for the x component as well, then you can see that this will appear as a sign component. So, from this configuration also that is z and H z and H x and E y is actually perpendicular to the plane of this board. So, it all becomes consistent with our formulation that we have done previously.

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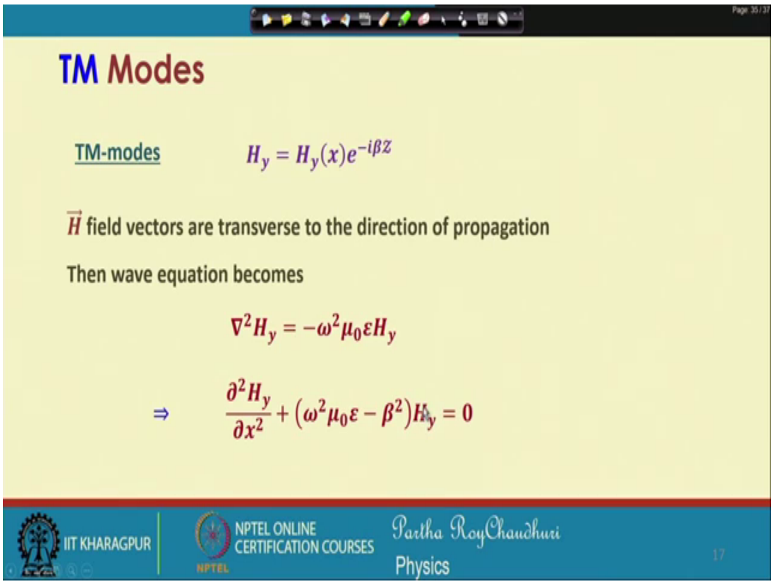

Now, we will discuss the T M modes the H field vectors are transverse in this case tangential component. So, the wave equation which will be satisfied by this H y field component is this. And you can write in the similar way this equation which is exactly the same as it was in the case of the electric field.

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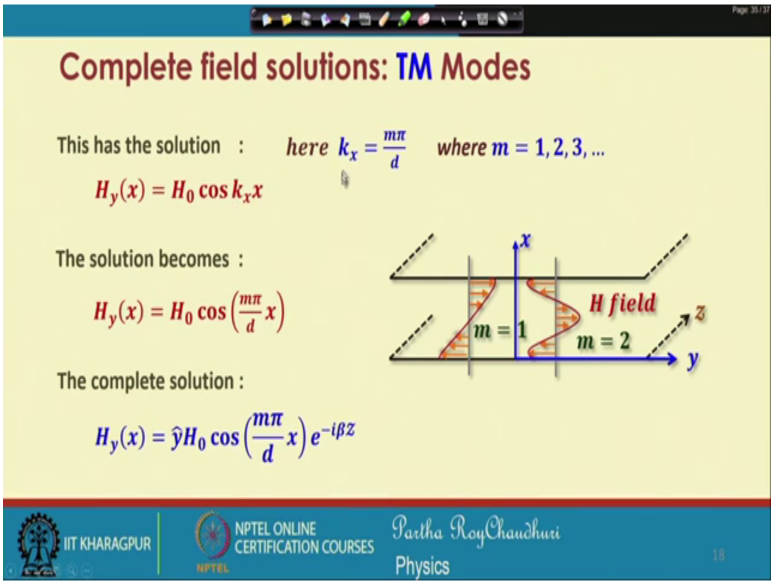

And the this equation will have a solution of H y equal to H 0 cosine k x and in this case in this case k x equal to m pi by d where m equal to 1 2 3 all the integral values the solution. Now, becomes imposing the second boundary condition that at x equal to d you have this condition to be satisfied. We have the discrete values of this k x, which gets into this equation to get you give you the solution of the magnetic field that is H y is equal to this.

The complete solution for this, then can be written in terms of this cosine function and you can see that at x equal to 0 x equal to 0 the field is maximum at x equal to d the field is also maximum and that is what happens in both the cases for m equal to 1 2. And for all the integral values of m, we can find out and depict the field distribution across the waveguide.

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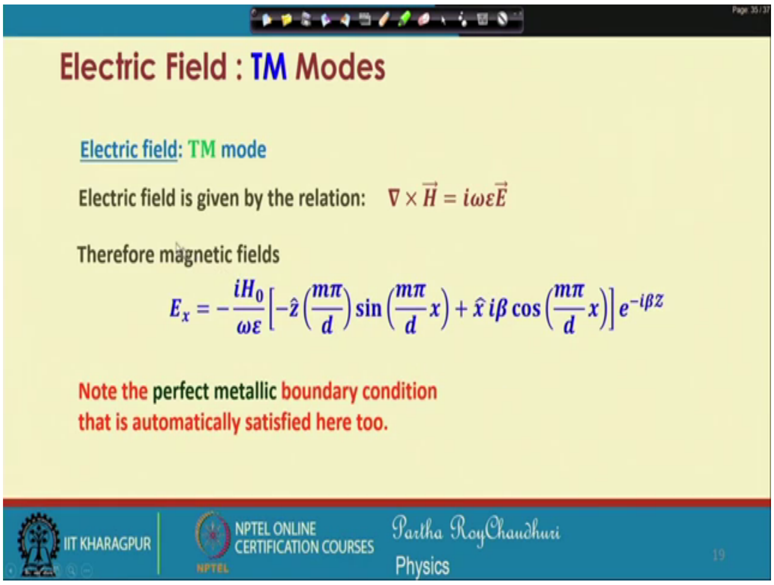

So, electric field of this T M mode again will come from this relation del cross H equal to this. Therefore, the magnetic fields can be the electric fields can be written in terms of this. So, you have a z component of the electric field and you have an x component of the of the electric field in this case and again this perfect metallic boundary condition that at x equal to 0 and x equal to d, the fields will vanish. They are again satisfied here and just check this.

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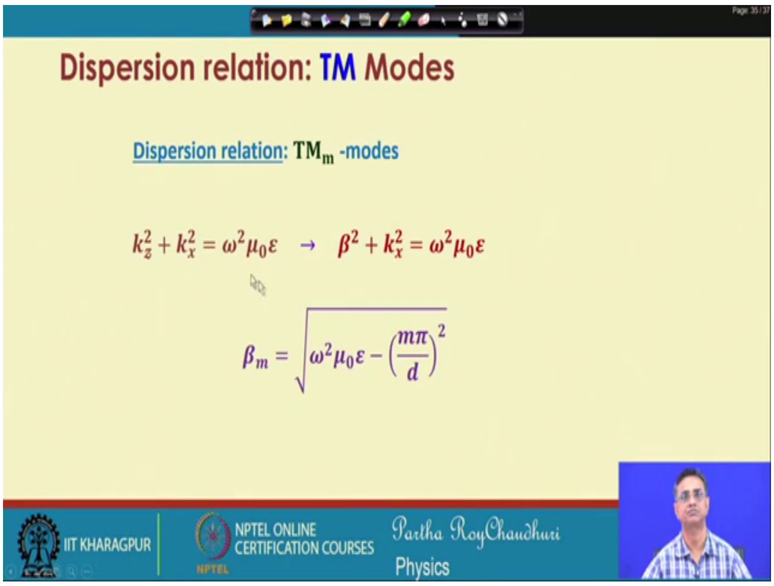

So, the dispersion relation in this case is in the same way that beta square plus the transverse component of the propagation vector square will be equal to the total propagation constant propagation vector mod square. So, that gives you this quantity beta square plus k x square equal to this. And therefore, beta m can be represented because k x is known beforehand m pi by d. So, beta m is equal to this quantity which is which from here we can again bring out the cutoff condition.

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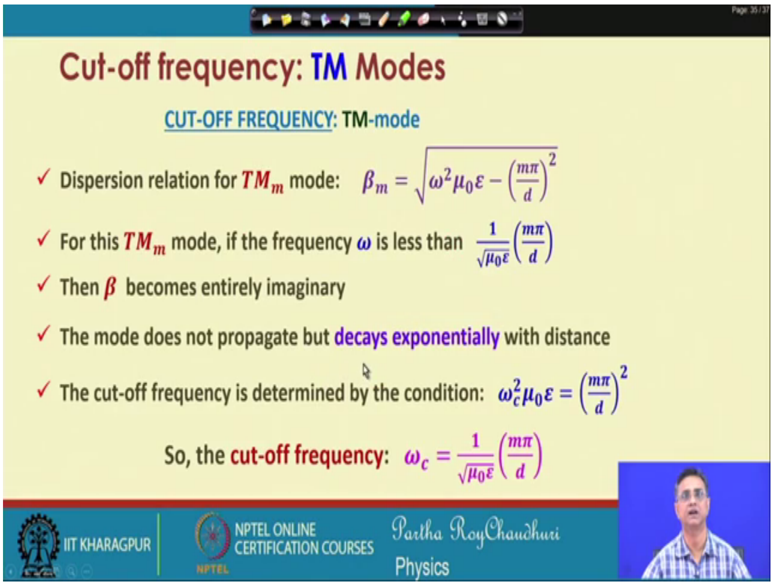

So, the dispersion relation for this is given by this equation and for this T M mode. We will call this modes to be T M m can have 1 2 3 all integral values. If the frequency omega is again less than this, then this quantity becomes in imaginary in the same way and the wave decays exponentially with the distance and you know that that imaginary quantity in the exponent with the oscillatory function will give you an exponentially decaying field.

And as a result, this mode will not be able to propagate and we can then set up the cutoff condition which is decided by which is determined by this expression. And therefore, omega c will be this. So, that gives you that till up to which frequency the structure will support for propagation through the waveguide.

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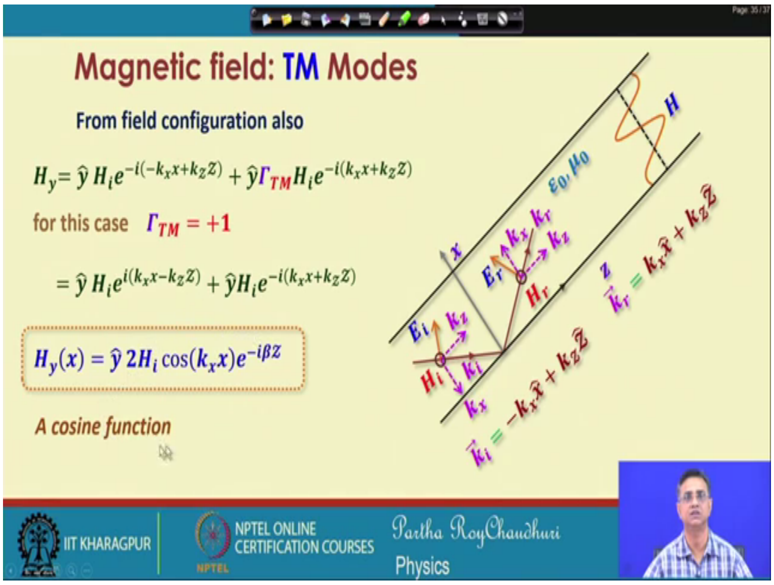

Now, magnetic field components we have seen the electric field magnetic field components electric fields. So, from here you can again organize looking at this configuration you have electric field along you have you have this k x and k z along this direction. So, that is why I have written this k x and k z as the phase factor, which is the propagation k dot are basically represented by this 2 quantities; one is forward propagating that is x positive, another is x negative and then in this case for magnetic field this tau T M is equal to plus 1.

So, if I plug in this value of this tau, then I can rewrite this equation in this form which will lead to this form of the equation. So, this is again a cosine function for H x. So, that is what we have seen earlier also that it is a cosine function of the H x field. This field H x field is a cosine function ok.

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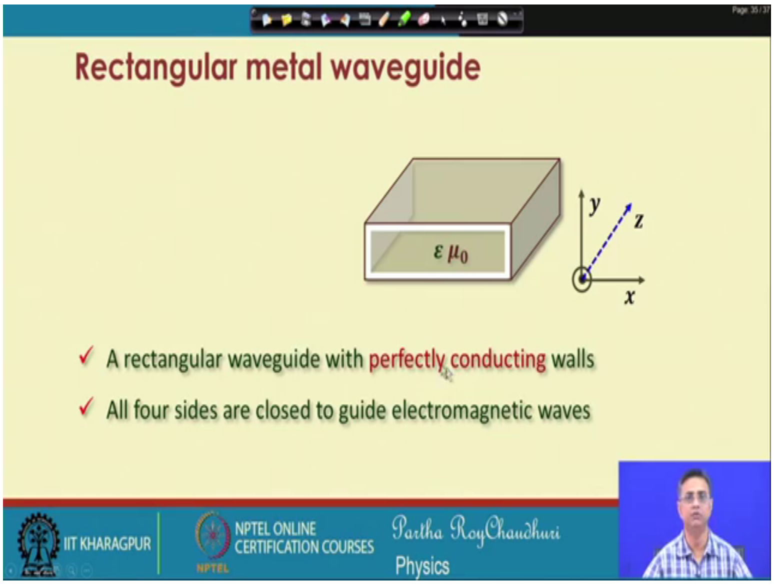

So now, the as a last example in the case of metallic waveguide, in this discussion we consider a hollow metallic rectangular waveguide and this wave guide is having a perfectly conducting walls metallic sheet or metallic wall all 4 sides are now closed to guide the electromagnetic waves. And let us consider this configuration that this is the propagation direction this is x and this is y. So, z. So, x y z. So, this is how we will we will put the coordinate axis.

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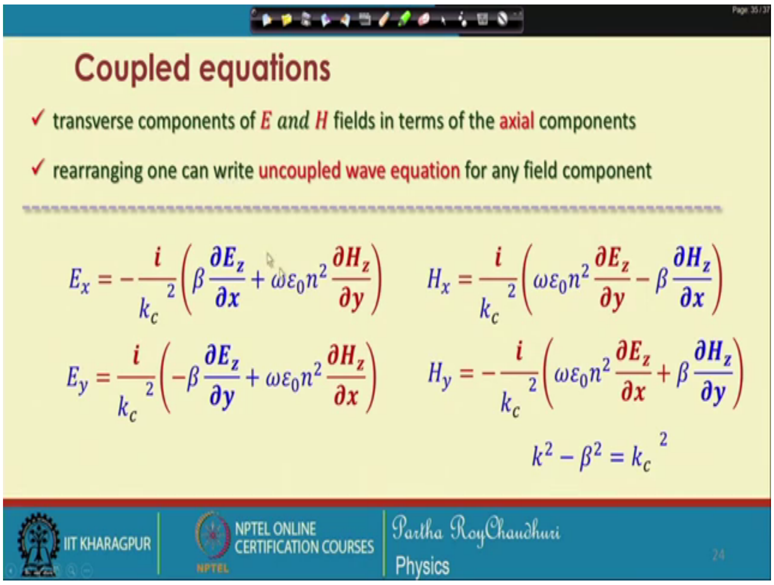

Now, let us remember that we have seen that all the transverse component of the electric and magnetic fields can be represented in terms of the longitudinal components of the of the electric and magnetic fields the same equation the 4 the set of 4 equations where k c is the transverse component, that is the total propagation vector this is z component of the propagation vector. So, this is the transverse component of the propagation vector.

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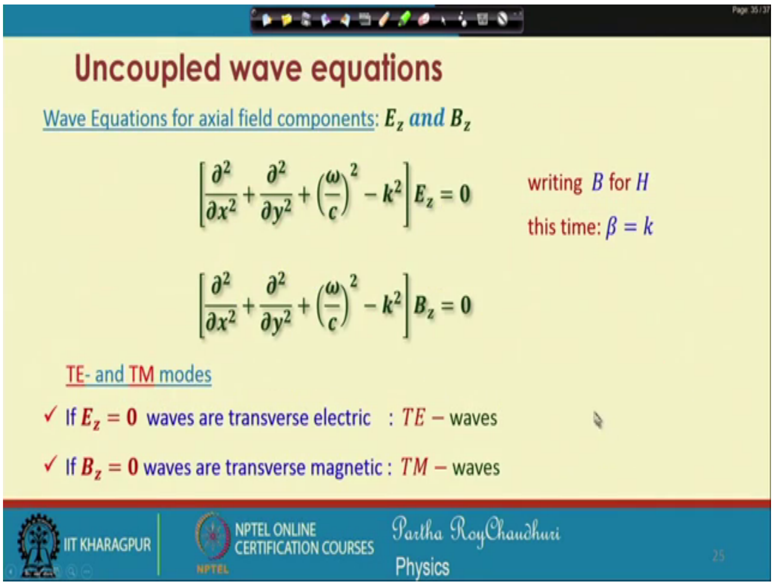

Therefore, the wave equations for axial field components E z and B z we can write in this form omega square omega by c minus k square. Therefore, this equation this is for writing B for H that is if I earlier it was because mu naught is the free space. And therefore, we can write that this H equal to B H equal to a even place of H we can write b. So, T E and T M modes are now represented by this condition that where E z equal to 0 you have T E waves and if B z equal to 0 you have T M waves.

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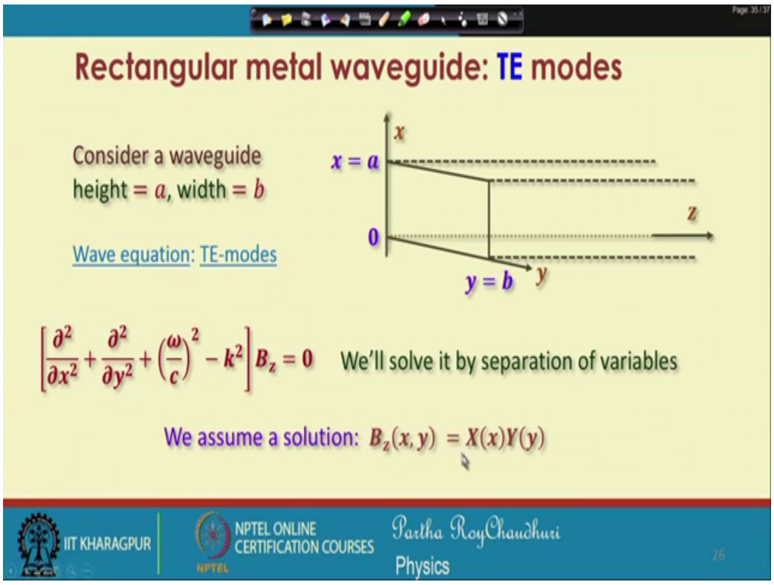

So, consider this waveguide with these structures and these parameters at x equal to a you have a boundary a metallic boundary at a y equal to b you have another metallic boundary and so on. So, it forms a rectangular pipe wave equation for this T E modes can be represented by this. You have seen that for T E modes, it is B z equal to B z with this equal to 0. We will solve it by separation of variables we assume that B z equal to this is a product of this x dependent and y dependent functions.

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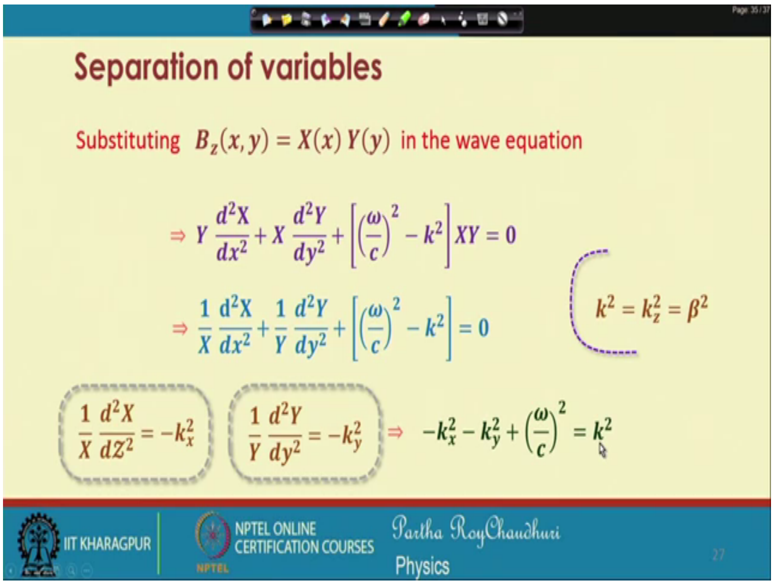

Then, we can use substitute this into the wave equation and separate them. This is again the very well-known technique of separating the variables. This quantity is now completely a function entirely a function of x and this quantity is only a function of y. So now, putting these 2 things together, which is equal to minus k x square minus k y square plus omega by c whole square, this will be equal to k square.

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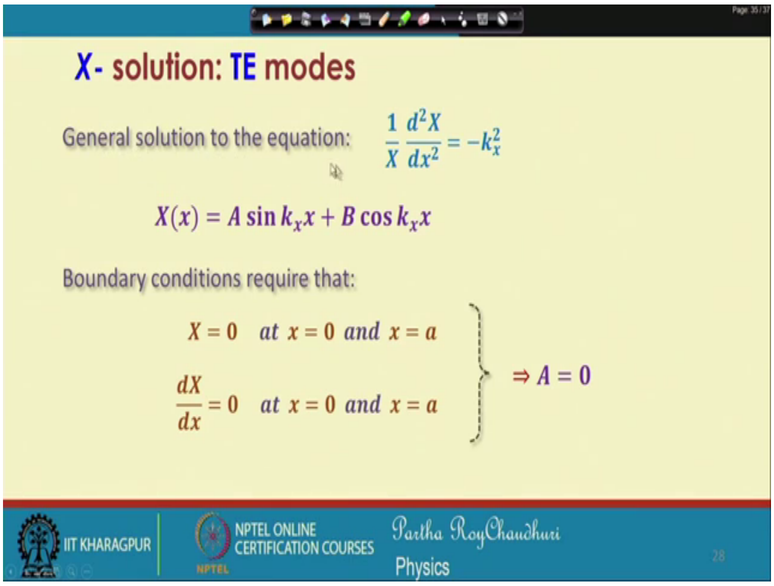

So, we can write down the solution for this equation general solution of the, this first part this one will give you this is again a very well-known equation in physics. So, d 2 X d x square plus k x square into x equal to 0, which will give you this solution sinusoidal sine and cosine function. Boundary condition if we impose that at X equal to 0 at x equal to 0 and x equal to a your x. This field will be will vanish because of the metallic boundary and the continuity of the field that is the derivative of the of x will also become 0 at these 2 points at these 2 boundaries. And if I use these conditions, then we can write a must be equal 0 because this thing equal to 0.

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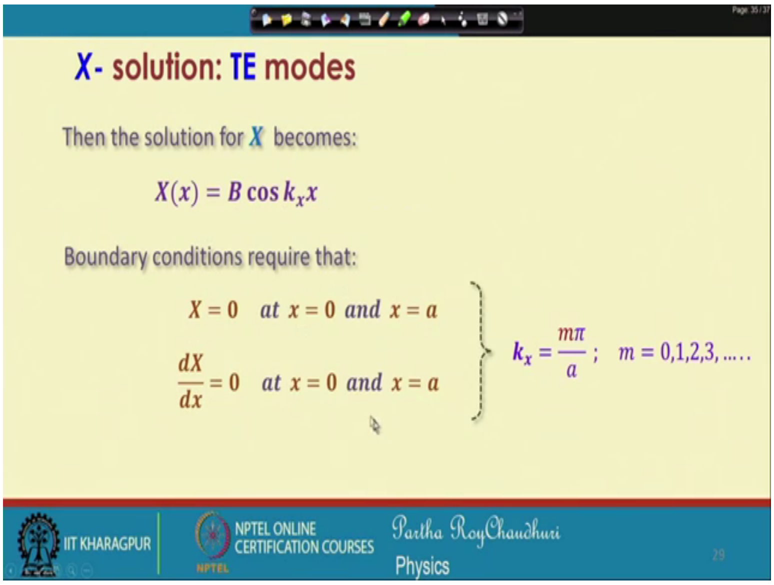

And therefore, if I use this condition the second condition then, so, the solution becomes this. Using the second boundary condition, we can again write that k x must be equal to m pi by a the exactly in the same way, we did it for the parallel plate metals, but in this case you have 2 such cases and they appear in the form of product.

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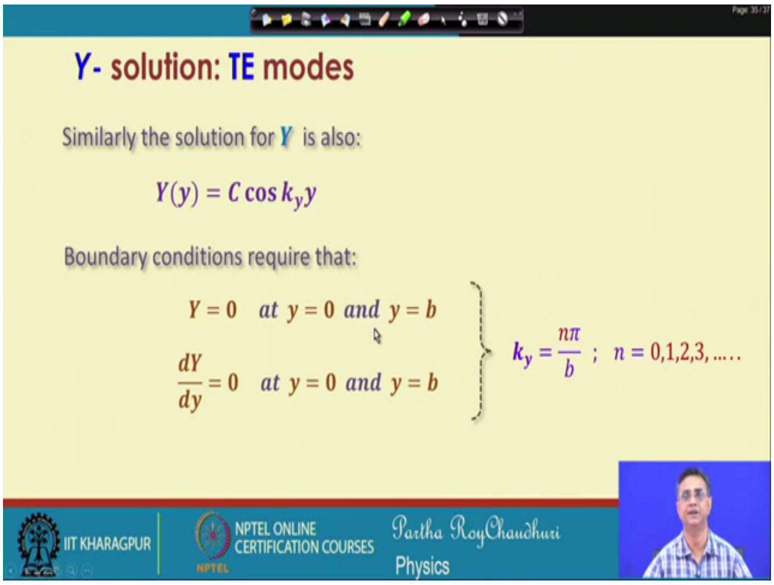

So, similarly the solution for y is also like in this form this was the solution for the x field and this is the solution for the y part of the wave equation of the solution and this again if you put the boundary condition. We can write that k y equal to n pi by b where n equal to 0 we can take up all integral values.

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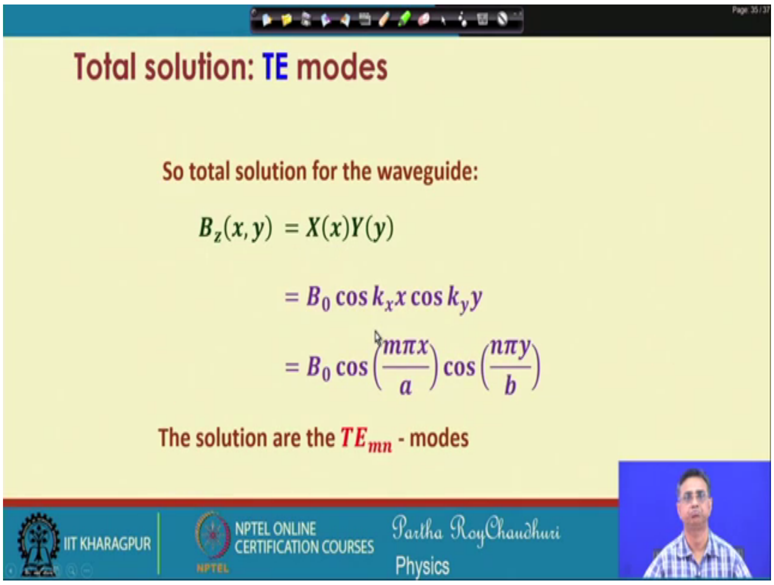

So, the total solution for this waveguide can be represented in terms of this, this is due to the x part of the solution and this one is for the y part. So, put together, but a k x and k y are also known from the boundary conditions. So, m and n are the integers and the combination of the m and n values will give you all possible modes for example, m equal to 1 n equal to 1 m equal to 1 n equal to 2. So, you get all the T E m n modes 1 2 2 2 2 3 etcetera. So, these are the various modes which can be found out simply by plotting this function as a function of x and y.

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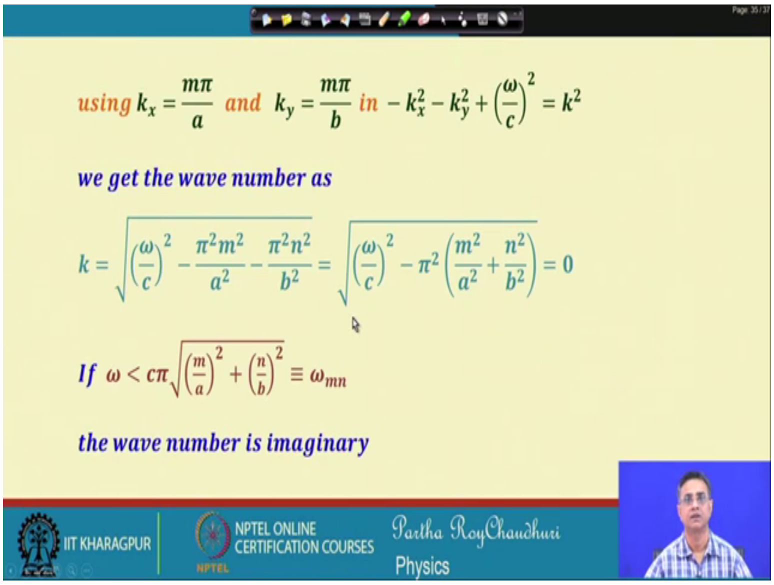

So, using this condition, once again we can look at the dispersion relation and the cutoff conditions. We get the wave number in this case as omega square minus k x square minus k y square. So, square under root of that will give you this k which is the wave number. Therefore, from this, this is equal to 0. This condition will give you omega this must be less than this quantity omega must be less than this quantity. So, this is the omega m n because in that case the wave number is imaginary.

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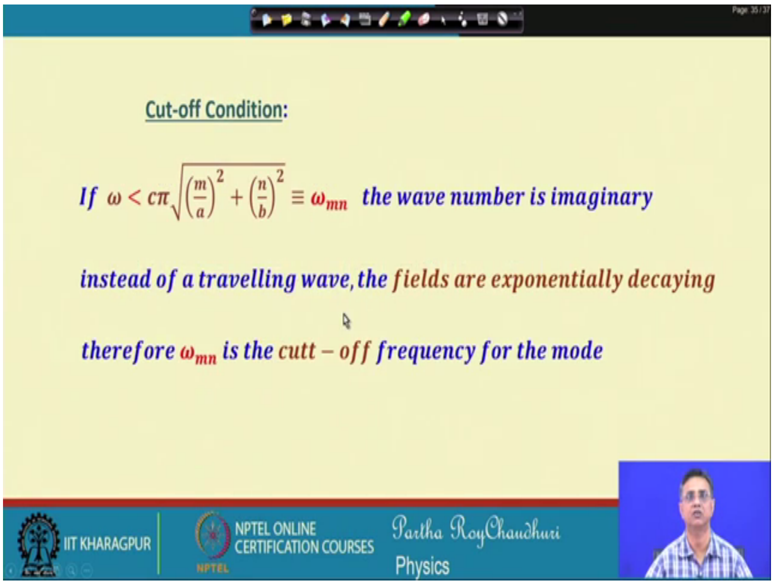

So, that is what is going to decide the cutoff frequency. So, we can we can set the cutoff frequency for the parameters given as a and b as the dimension of the waveguide and a m and n being the solutions coming by the integral numbers 1 2 3 etcetera for m and n. So, the wave number for this will be and instead of a travelling wave the fields are again we have explained this exponentially decaying with distance. Therefore, omega m n is the cutoff frequency for this modes.

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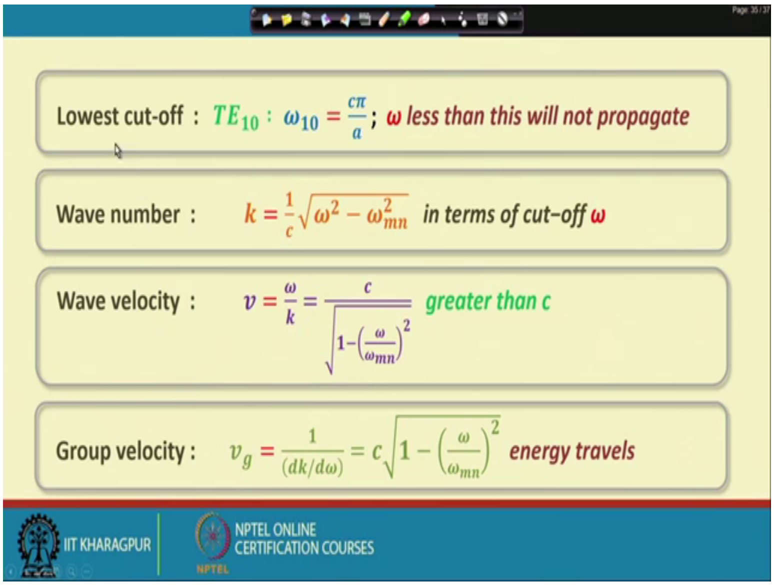

Now, we have tabulated the all the all our findings that the lowest cutoff frequency that is for T E 1 0 can be written as this, this you can find out for putting one and this is equal to 0. So, omega 1 0 this is the and omega less frequent any frequency less than this will not propagate in the structure and for that, this the wave number will be this where you can substitute the value of m and n. So, in terms of the cutoff frequency.

So, you can represent the wave number of the wave which is propagating where you know the integers m and n, then you can calculate this. So, the wave velocity that is the velocity with which the wave travels in the structure that is v equal to omega by k and this gives you the wave velocity for the wave for which I know the value of m and n. For example, I look at the T E 1 0 wave then this omega will be omega 1 0 and if I substitute this value, I know that the wave velocity will be this.

So, for various modes the propagation velocity will be different and as they propagate along the structure, they will develop a phase difference. We will utilize this property in some device applications group velocity for this the is given by d omega d k which is this. And in that case again, also we can find out the group velocity that is the velocity with which the energy travels in the medium by selecting the values of for looking at the individual modes. For example, T E 1 0 I have to substitute omega one 0 for this and then we can calculate the velocity wave velocity of propagation of this wave.

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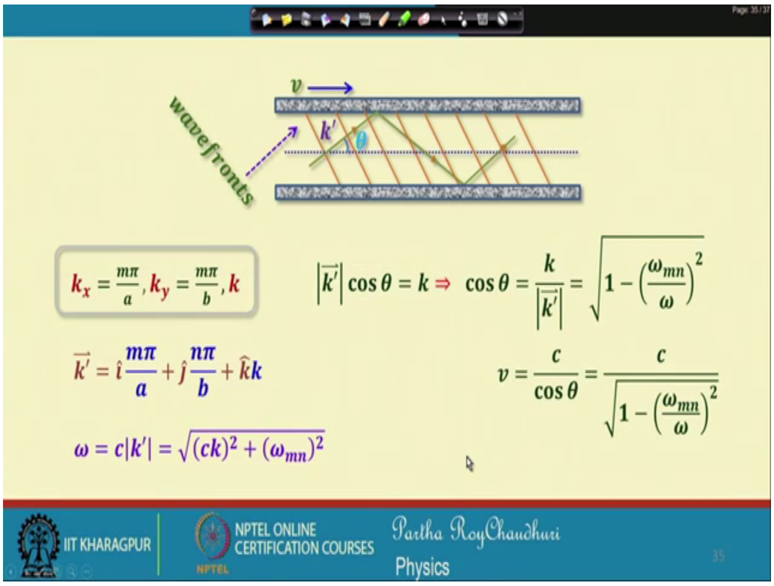

Now, I look at this structure this is the wave front these are the yellow lines are the wave fronts at regular intervals of the wave length and the wave is traveling in this direction getting reflected back and forth from the upper and lower interface and you have also the similar thing on side wise. So, the wave moves this wave moves the z component of the wave that gives you the motion of the, that is the travel of the traveling of the wave along this direction. That is with the velocity v and this k dash the propagation constant will come from here.

You can look at this; this k will be decomposed into because you have k x k y and k z. All 3 of them are present the value of k x is this the value of k y is this and the value of k z we have called this is equal to beta. So, all of them put together will give you the total propagation vector of the wave which is traveling within the structure. So, omega we can calculate from here. So, c by omega will be k dash. So, that gives you this value. Now, again for the different modes m and n, we can use the numbers and calculate the frequency.

So, cosine k dash cosine theta equal to k. This you can look at this cosine this is your k direction and this is your z component of k. This is your k prime direction k dash direction and this is the z component of k. So, k dash cosine theta will be the value of k. So, that is what it is written here. So, we can calculate the cosine of the angle that it makes with this the wave makes with this, this incident and reflected waves.

So, which we can calculate that is v equal to c by cosine omega and this velocity with which it actually the energy travels in this direction right. So, this v g and v energy travel group velocity whereas, v is the phase velocity of the wave which is along this direction, ok.

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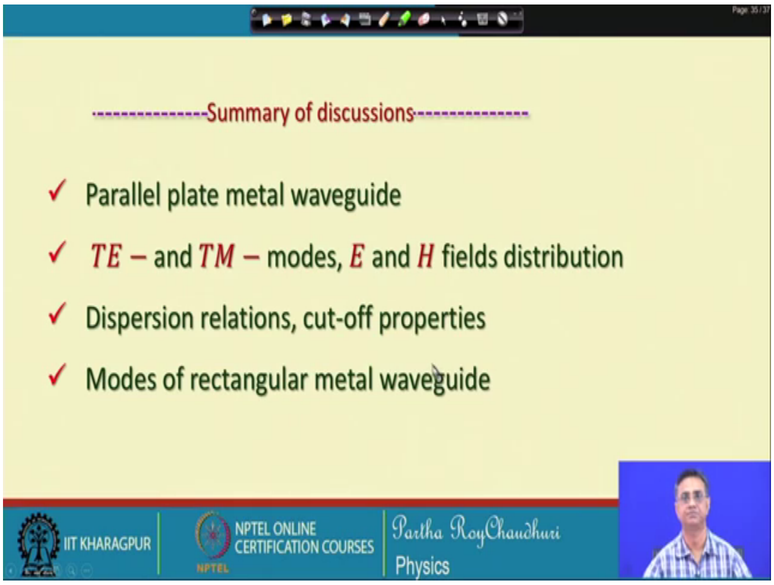

So, we have discussed the starting with a parallel plate metal waveguide there a T E and T M polarization orientation of the wave and that that give rise to the T E and T M modes. And, we have also seen the electric and magnetic field distribution across the waveguide for various modes. They are dispersion relation cut off properties.

We will also consider these T M modes in terms of the electric field and magnetic field configuration, they are cut off properties and all and also we have discussed a very special case of a rectangular metallic waveguide which is very, very common in the microwave technology.

And, we have seen the cut off properties the field distribution and in terms of electric and magnetic fields within the waveguide and this background. We will now switch over to we will now look at the dielectric wave guides, which are presently more interesting enough sort of use in the form of devices.

Thank you.