Modern Optics
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Lecture – 18
Wave in guided structures and modes

We have discussed the reflection and refraction properties of electromagnetic waves at interfaces. And now, with that background, we will switch over to the electromagnetic waves in guiding structures and wave guides. We have also made some introduction about the concept of wave guiding 3 interfaces.

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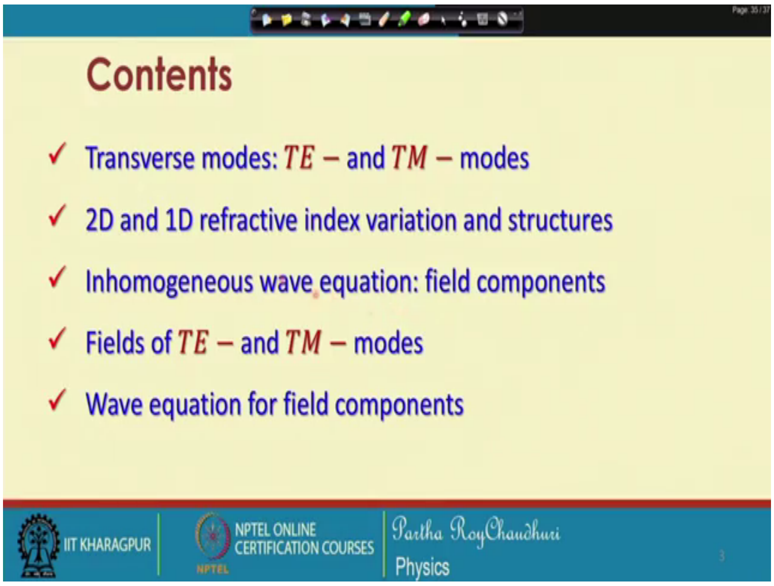

So, with that understanding in mind, we have organized the contents of the discussion. Today, the transverse modes that is T E and T M modes in wave guides, then to do that, we will talk about the 2 dimensional and one dimensional refractive index variation along the structure and for that we will try to write the equation of the waves which will be propagating through that structure.

In the process, we will derive this inhomogeneous wave equation and the related field components, then the fields of the T E and T M modes in terms of the electric fields and magnetic fields a set of electric and magnetic fields put together will constitute this T E modes. And similarly, for T M modes also a set of electric and magnetic fields out of the 6 components of both electric and magnetic fields will be put together to represent this transverse magnetic modes. Then we will write down this wave equation for the field components ok.

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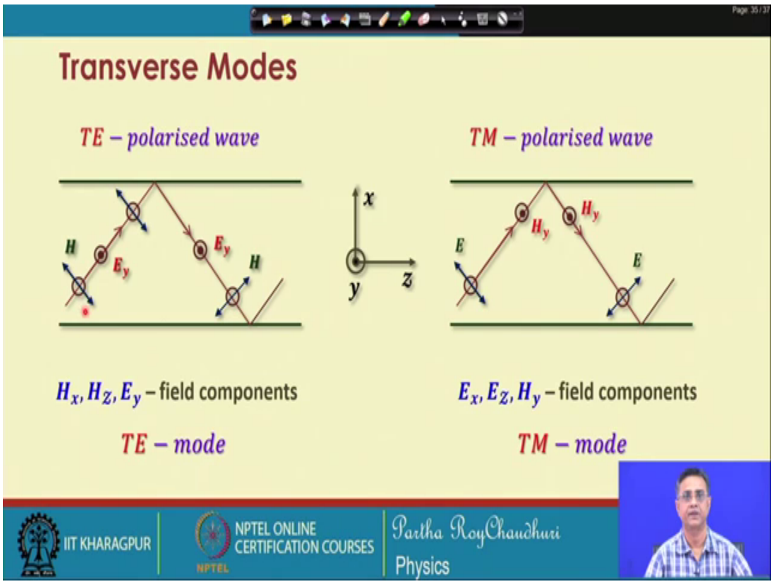

So, we have already seen that the T E polarized wave confirms this configuration; that is your electric field is perpendicular to the plane of the board and magnetic field is lying in the x z plane and this is your z direction, this is your z direction and there is a there is a change in the in the refractive indices across these 2 interfaces.

So, the interface which is embedding this medium that is n 1 n 2 or n 3. So, for such a structure, we can see that the electric field is purely tangential. So, that is represented by E y whereas, the magnetic fields will have 2 components H x and H z for this T E polarized wave and with respect to this structure, when the waves will be guided in this structure, we will call this is a T E mod. And, in the same way, when we will look at this configuration that they just the opposite; that is the electric field is now lying in the x z plane.

But the magnetic field has only the tangential component; that is H y and such a configuration will correspond to this T M polarized wave and the mode which will be supported by this structure will be called T M modes.

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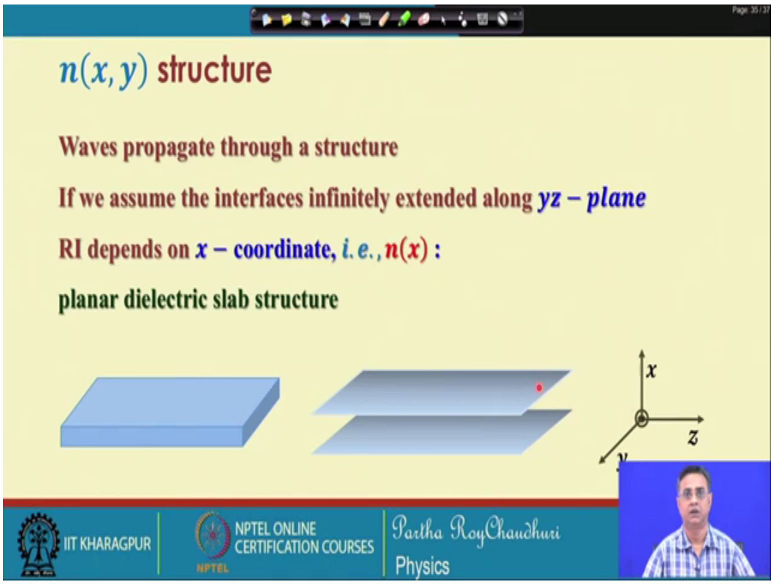

Now, we have seen that the wave propagates through a structure which is between 2 interfaces and this tells you that we assume the interfaces, which is infinitely extended along the y direction. In such a case, that is along the y z plane and in that case, you have refractive index variation only along the x direction.

And so, this refractive index depends only on the x coordinate and you can represent this index variation. The index term as n of x only and there is no dependence on the y and z direction. Because, along the y direction, we assume that the structure is infinite and along the z direction, it is invariant. The structure is does not vary. So, the example of such a situation is a planar dielectric slab, you can see from this figure is a small portion of the planar dielectric slab, which are very often used in integrated optic devices.

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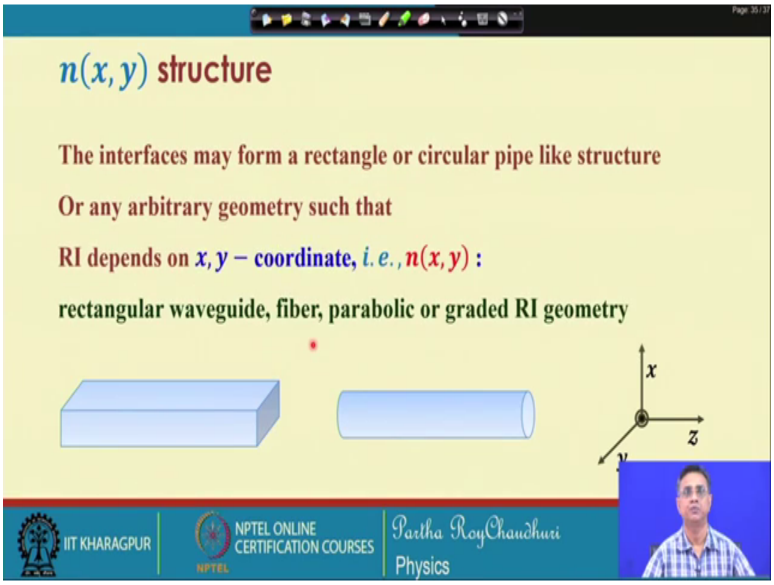

Now, in that cases, the interfaces may form. It is also possible that the interfaces interfaces may form a rectangle or a circular pipe like structure like this; that is, you have a 2 dimensional confinement not only the interfaces are along these direction, but also along these direction. So, in that case, the wave will be confined in 2 dimension.

In this case, there is a cylindrically symmetric structure, a pipe like structure. An example of this is an optical fiber and this is a rectangular waveguide and for such a situation the refractive index depends on both x and y coordinate. So, you can write that refractive index variation in the form of n of x y.

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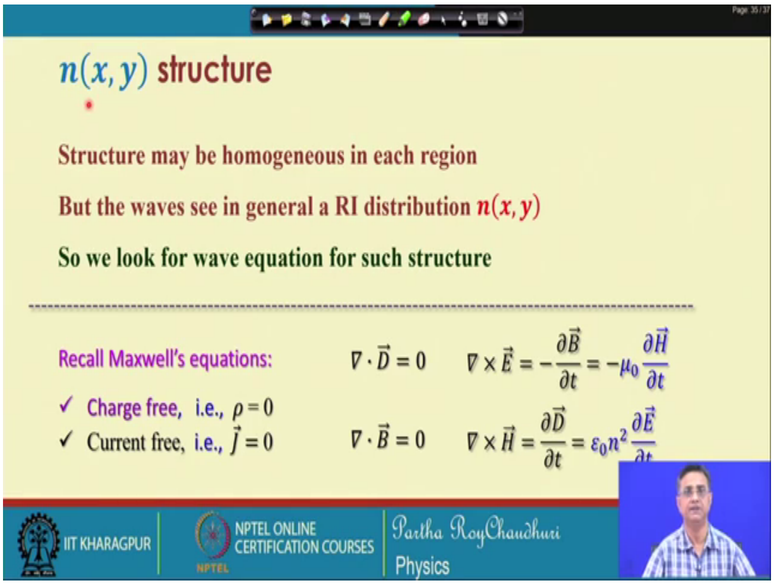

So, when the structure is x y dependent, that is you have refractive index variation along both x and y direction. But, separately each of the each of the region is homogeneous even though each of the region is homogeneous.

But the refractive index of each of the region, region is different from the remaining others. So, in such a situation, the wave sees in general a refractive index distribution which is n of x y we have understood that. So, we look for the wave equation for such a structure and to do that, we will recall that Maxwell's equation. Maxwell's equation for a charge free and current free region. So, rho equal to 0 j equal to 0 and you have a set of 4 Maxwell's equation del dot D equal to 0 del dot B equal to 0 so on and so forth.

So, del this curl equation, you can write in terms of we assume that the media are non magnetic all the media involved are non magnetic. So, you can write in place of mu, mu naught which is the free space permeability and del H del t and for del cross H. I have this epsilon; that is the permittivity of the medium of each of the layers and you can write this as epsilon naught n square square of the refractive index. So, del E del t. So, this is how we start with the 4 Maxwell's equation and if you take the curl of the first equation, that is del cross del cross E.

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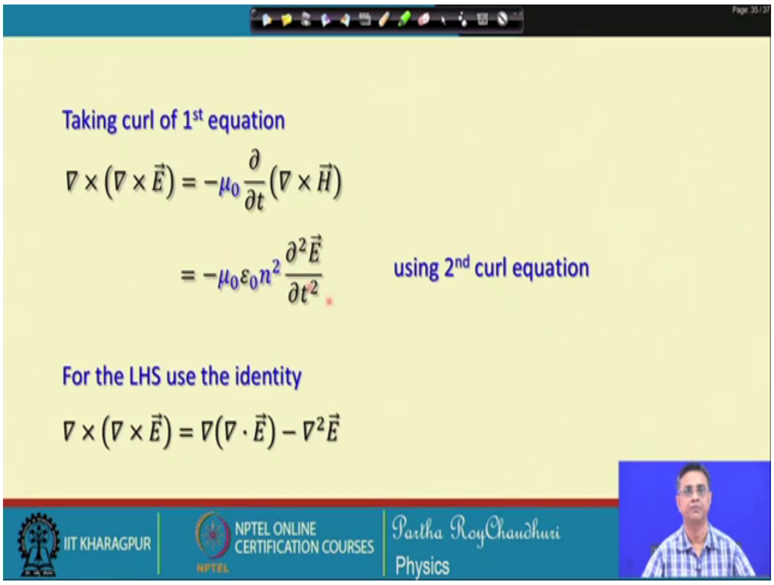

So, you can write this curl of curl of E is equal to minus mu naught del, del t of del cross H. But, this del cross H is again given by the second curl equation that is equal to epsilon n square del E del t. So, if I substitute the expression for del cross E, we can write down that del cross del cross E is equal to minus mu naught epsilon naught n square del square E del t square.

So, for the L H S part of this equation, we use this well-known identity curl of curl of E is equal to del of del dot E minus del square E. Now, this time, this del dot E is not 0, but del dot D equal to 0. Because, this we have seen del dot D equal to 0 that comes directly from the Maxwell's equation. So, del dot D equal to 0. So, from this equation, that is the one which we have been able to write now that the right hand side is this and the left hand side of this equation is this.

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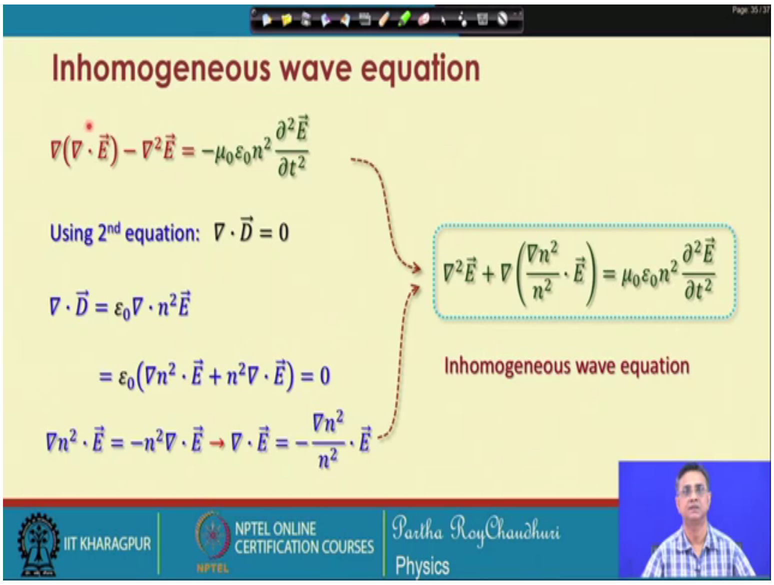

So, I write this equal to this and then del dot E, this quantity 2 to see what it is, what it represents del dot D equal to 0. So, you can write the del dot D equal to epsilon del dot n square E n depends on x y. And therefore, epsilon del n square dot E simply use this identity and you can write in this form, but this del dot D equal to 0. So, you can write this expression as equal to 0. So, this quantity must be equal to minus of this. That is what is written here; del n square dot equal to minus n square del dot E that gives you the value of del dot E that is del dot E equal to minus del n square by n square dot E.

So, this time, it is not 0 which was 0 in the case of homogeneous medium del dot E was 0. But this time, it is not. So, it appears in the wave equation because, I will replace this del dot E del of del dot E by this quantity. So, I get the wave equation for this inhomogeneous structure that del square E equal to del of del n square dot E and then right hand side as it is. So, this equation represents the electric field for inhomogeneous wave equation.

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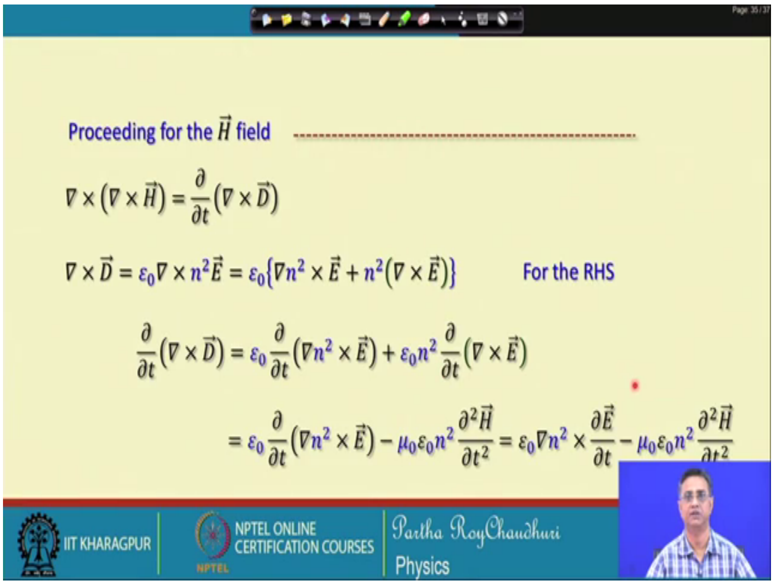

Proceeding for the magnetic field, we can similarly write down the left hand side that is curl of curl of H which is equal to del t of curl of D, but this curl of D. You can again write in this form epsilon naught curl of n square E. So, using this identity for del cross n square E. Here, for the R H S part, we can write down this equation del n square cross E plus del del t of del cross E.

Now, you can write this equation this one is equal to minus of because del cross E is again known to us before hand is equal to mu naught epsilon naught epsilon naught n square mu naught del H del t with a minus sign. So, altogether it becomes a minus sign mu naught epsilon naught n square del square H by del t square which is the right hand side of this equation.

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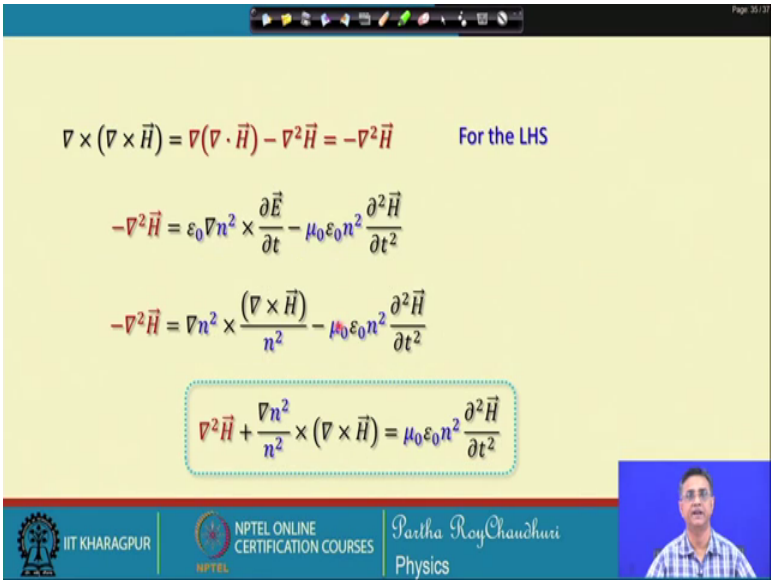

And for the left hand side, again we will use this identity and, but this time, del dot H equal to 0. The medium is non magnetic and del dot B equal to 0 means del dot H is also equal to 0. So, the left hand side gives you minus del square of H.

So, the equation complete equation becomes minus del square of H equal to this quantity minus mu naught epsilon naught n square del square H by del t square. So, as a result, we can reduce this equation into this form and it is somewhat similar to the one for the electric field, but there is a difference and we will see that for the T E mode the scalar equation retains whereas, for T M mode this equation becomes valid.

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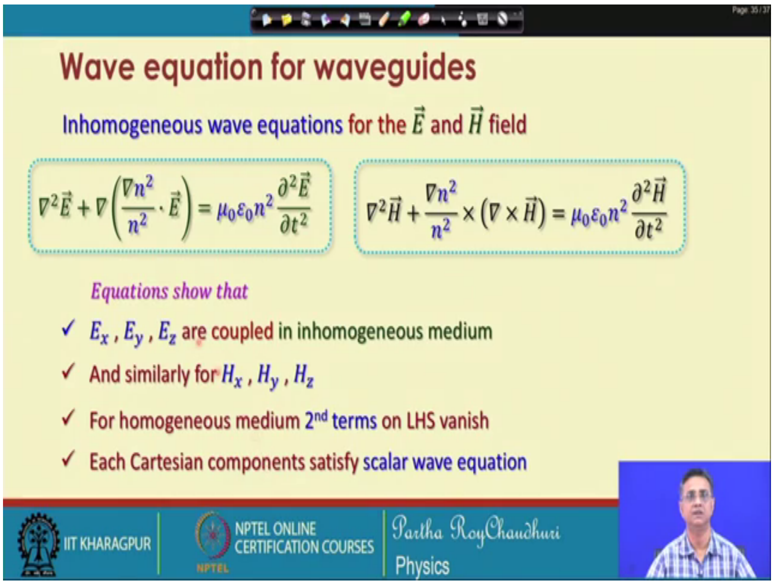

We will see. So, the wave equation for the wave guides inhomogeneous wave equations for the E and H fields, we can write side by side just to compare that there is a there is a difference in the middle term, which has appeared here in the case of inhomogeneous medium. But, this term as a check point. You can check that if there is no if the medium is homogeneous; that is if there is no del n square, then this quantity becomes 0 and this equation reduces to the form of homogeneous wave equation and likewise, for this quantity also for the magnetic field part.

If del n square itself is 0 for homogeneous medium, the equation simply reduces to del square a is equal to mu naught epsilon naught n square which is the same old equation for the homogeneous medium. But, from these 2 equations, these are the observations we can see that E x E y E z are coupled in inhomogeneous medium you have E x E y E z. They are coupled and similarly for H x H y H z.

They are also coupled for homogeneous medium the second term this is just we have discussed that for homogeneous medium. The second term of the L H S, it becomes 0 because there is no del n square and also for this part and it reduces to it reduces to the wave equation for homogeneous medium. But, each Cartesian components of this equation will satisfy the scalar wave equation each of the components for example, E x E y E z H x H y and H z. All of them will satisfy the scalar wave equation.

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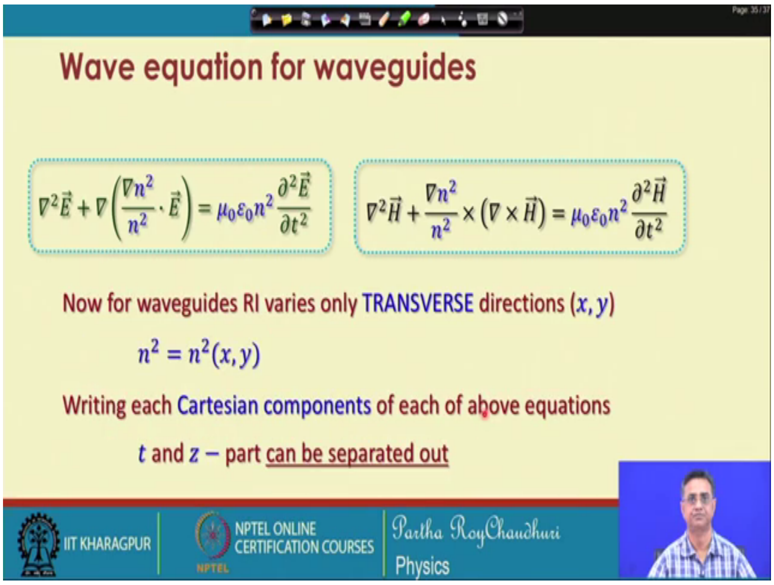

Now, for the wave guides where refractive index varies only in the transverse direction, that is a along x y that is n of x y.

So, we can write this n square equal to n square of x y for these equation. Wherever I have n square and writing each Cartesian components of each of the above equations , you can see that t and z part can be separated out; t part can be separated out from here and z part because, it has a transverse dependence. But there is no longitudinal dependence; that is there is no dependence on the z direction. So, these 2 parts can be separated out. We can just simply work out take the component of each of the field components and then we can see that t and z can be separated out.

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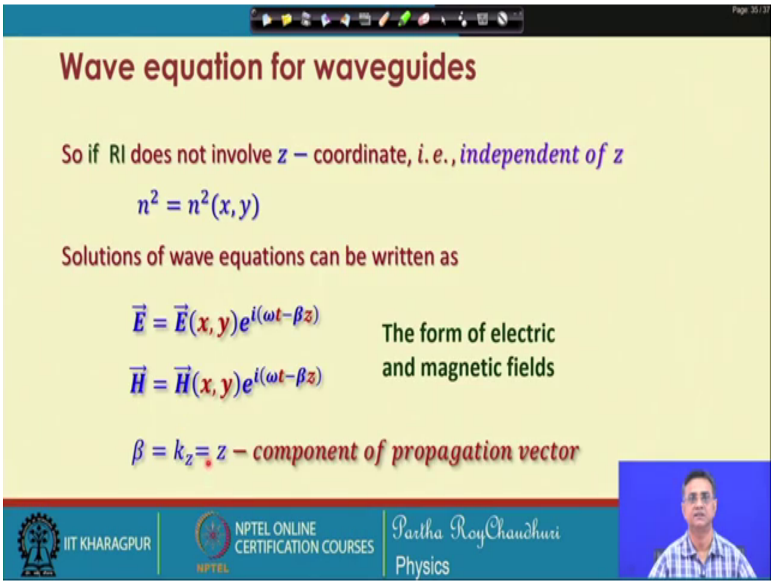

So, if refractive index does not involve z coordinate, that is if it is independent of z that is to say that the refractive index is a function of only x and y. But, there is it is not a function of z. In that case, the solutions of the wave equations can be written in this form because we have been able to isolate the z part z dependence and we have been also able to isolate the time dependence.

So, they appear in this form, but x and y part remains together in the amplitude of the electric field. So, it is the amplitude of the electric field which is a consequence of the of the refractive index distribution across the waveguide. So, this is the form of the electric and magnetic field. This these 2 forms will be used; we can further simplify. We will see later that if it depends on only x or only y, then it will be even simpler. It will only depend on x and y. So, and here what we have used that beta is the z component of the propagation vector. So, that is a general notation. We have already introduced that beta equal to k z because, it is the wave propagates along the z direction.

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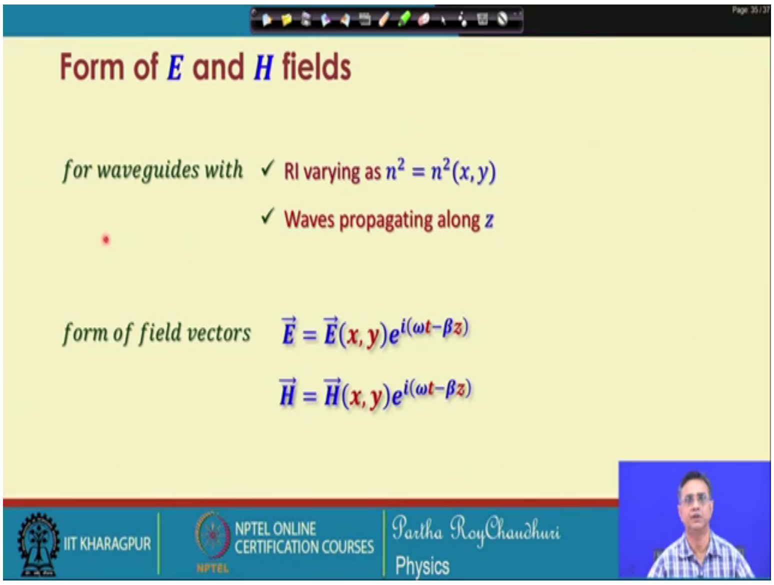

Now, for the wave guides with refractive index 2 dimensional variation of the refractive index that is n square equal to n square of x y. The waves propagating along z the form of the field vectors are also these are by now all established.

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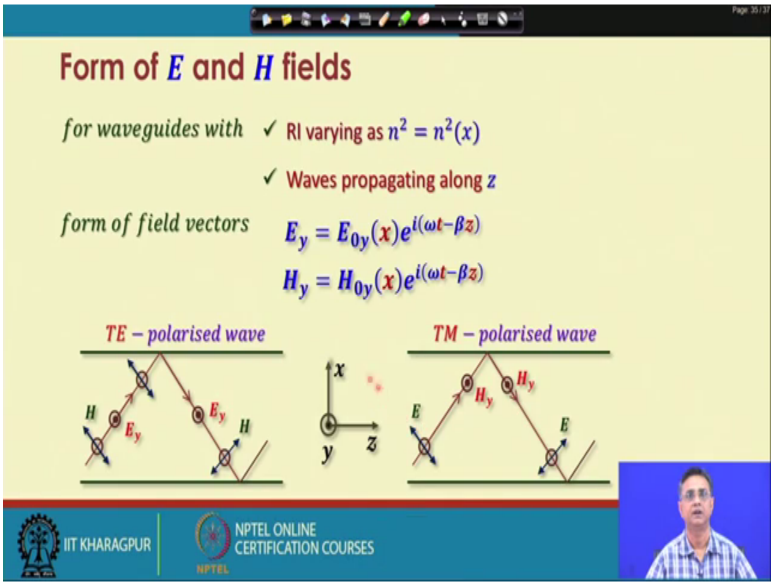

And now, we will switch over to a simpler form that where the refractive index varies only along the x direction; that is an one dimensional refractive index structure, you have 2 interfaces one at the top, one at the bottom and the wave will be wave will be confined between these 2 interfaces. So, the wave and will be propagating along the z direction, we assume that the along the y direction, the structure is invariant and also along z direction the form of field vectors. In this case, as we have mentioned because, it does not depend on the y coordinate.

So, the electric field will be represented by E 0 of x only and this phase factor will remain the same. And similarly, for the magnetic field, it will be E 0 of y function of x there will be a variation of the amplitude of the magnetic field and along the x direction and also for the electric field. So, look at this T E polarized wave for this one dimensional structure you have an electric field which is directed along y.

Which is directed along y in this is the structure in which electric field is directed. So, this is E y whereas, the magnetic fields will have 2 components that is x and z in the T M polarized wave. We have this magnetic field which are the tangential component and will be directed along y direction and the electric field will have 2 components.

So, for T E polarized wave we will have 3 components of the field vectors E y H x and H z. Similarly, for the T M polarized wave you have 3 components of the field vectors H y E x and E z. So, these are the group of 3 field vectors, which will constitute individually this transverse electric transverse electric polarized wave and transverse magnetic field polarized waves which so, from the Maxwell's equation, now we would be interested to look at the individual dependence of the field components.

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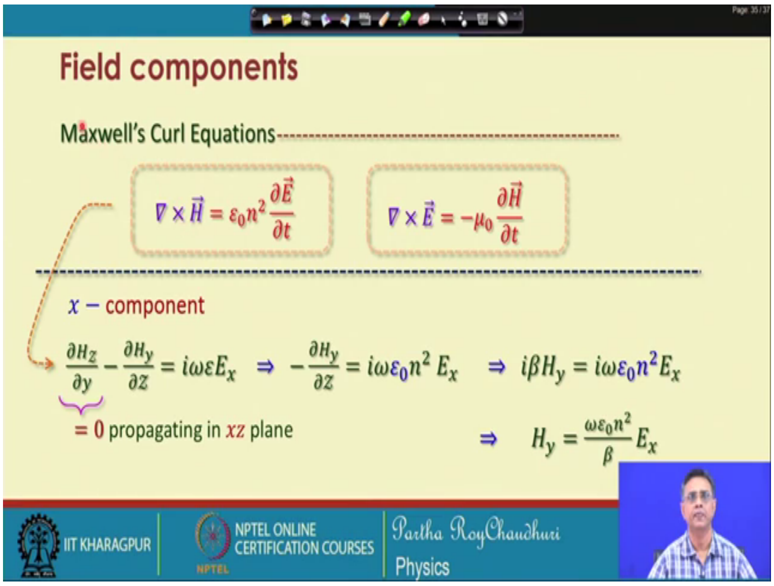

So, if I concentrate on this del cross H and look at the x component y component and z component of this equation, then it should give us how they are related. So, if you take the x component of this curl equation, we can write del H z del y minus del H by del z equal to this. But, since there is no y dependence of the wave for one dimensional structure, we have this equal to 0; that is wave is propagating along the x z plane.

So, this reduces this equation in this form and finally, we can we can write this equation as because we also know the time dependence of the field. So, you can write this in terms of i omega will be detached for the del t operator. So, this equation represents the x component of this equation. So, H y you can write in this form omega epsilon naught n square by beta into E x. So, H y and E x, they are connected in this way.

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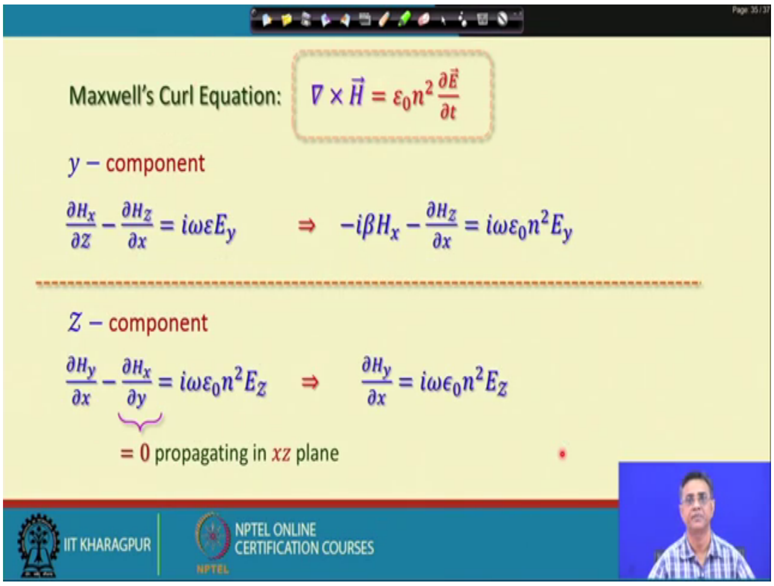

Similarly, if you take the y component of the field, you can represent the field in this form and which will reduce to this form and for the z component again you will have del H x del y because, there is no dependence along the y direction. So, we can write this putting this quantity equal to 0. We can write the relation between H y and E z in this form.

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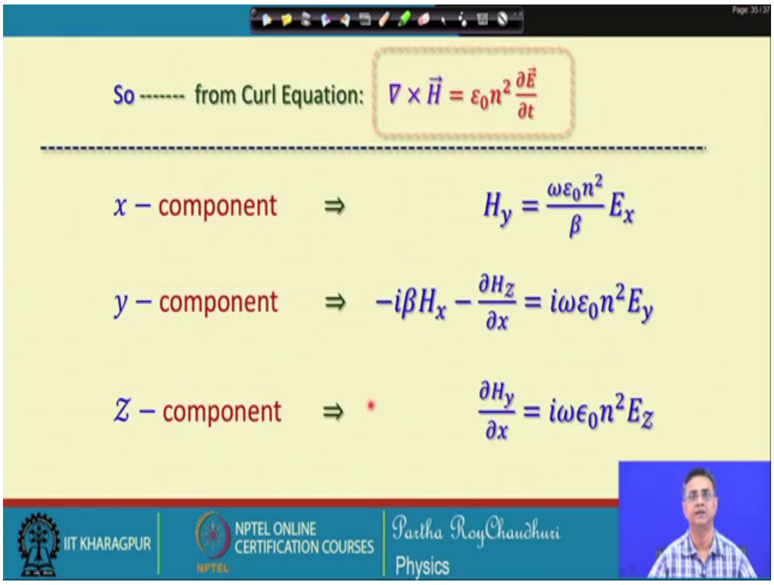

Now, we will take the curl equation for the so, from here, if we summarize all the 3 equations, that is this one this one and this one if you put together. So, you have x component of the field has given you the connection between E x and H y; y component of the field H x H z is connected to a E y and for z component you have a connection between H y and E z.

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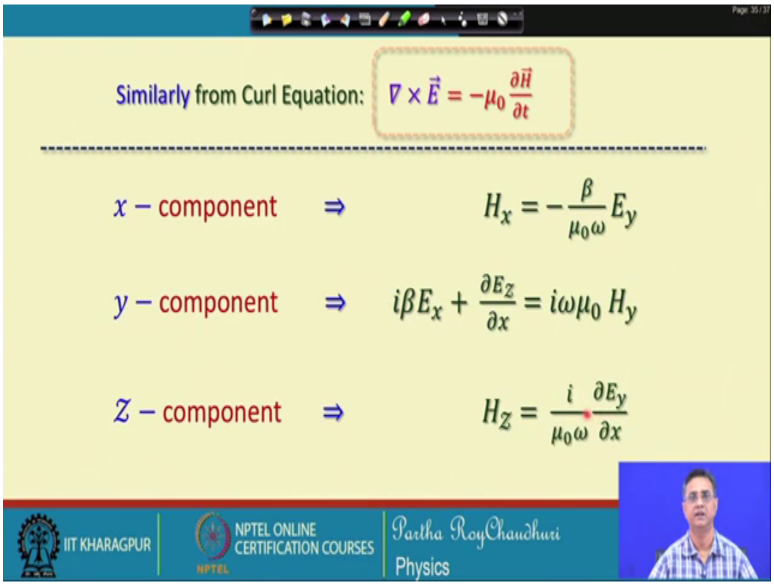

And in the same way, if we look at the components of the of the curl equation like x component y component and z component and we do the same operation we can. So, that

H x will depend on E y. In this way, E x E z and H y will have a dependence, in this way H z and E y. So, these are again 3 set of 3 equation set of 3 equations from obtained from this curl equation del cross E.

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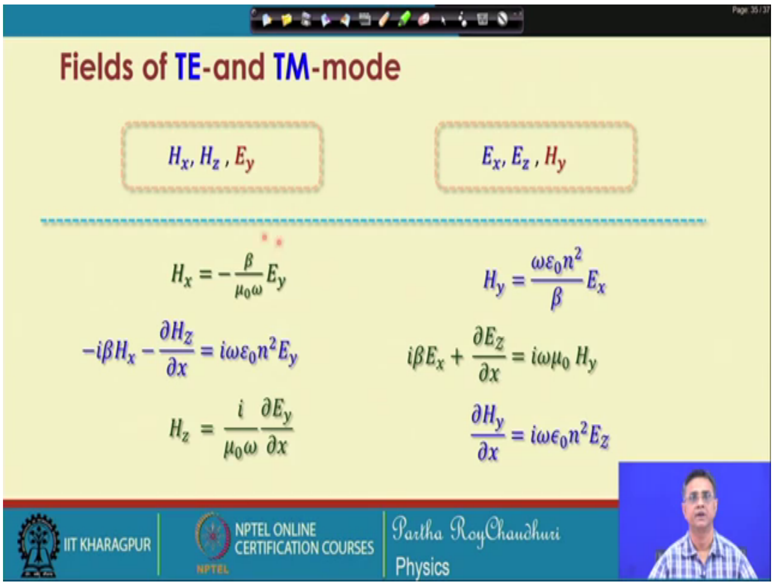

So, you have for T E modes H x H z E y we have seen that because in the case of T E mode, you have electric field which is tangential. But, this will also associate itself with H x and H z. So, put together all these 3 field vectors will constitute the T E mode and you can see how we have organized that this and this they have come from the curl equation of H curl equation, and this have come from the curl equation of E. So, these are the 3 equations to represent the T E waves and these are the 3 equations to represent the T M waves. So, we have been able to separate the T E and T M fields.

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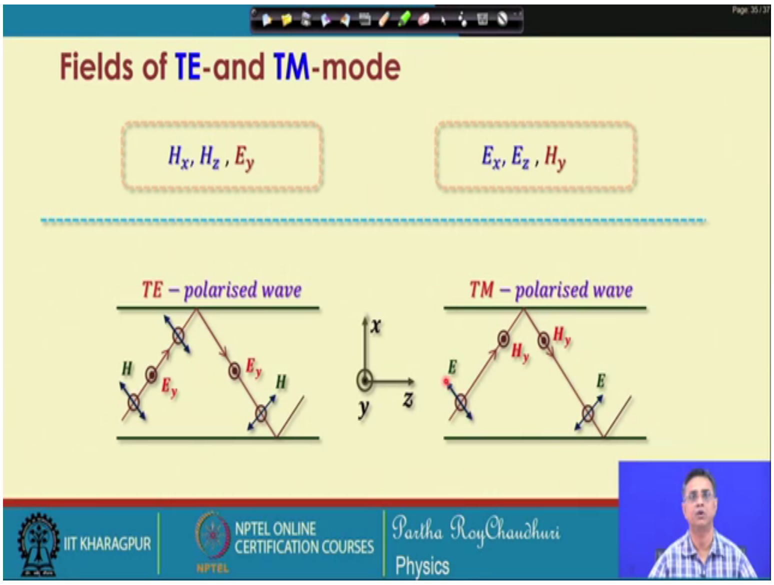

Now, look at this. This I have already mentioned that H will have 2 components; E has only the tangential component. So, this is a T E polarized wave and similarly, this one is the T M polarized wave where you have H y is the only tangential component, but E x and E z are also associated with this wave propagating in this structure.

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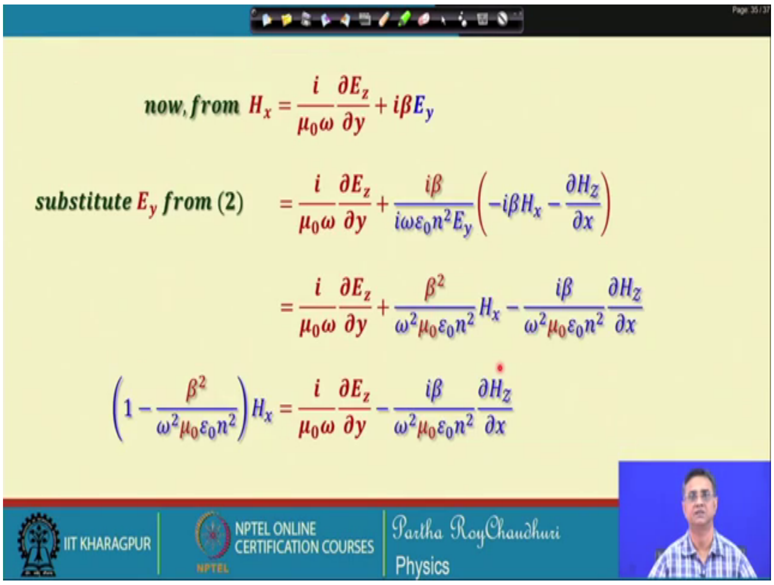

Now, from H x equal to this equation, the equation which has which has been mentioned H x equal to this. So, from this equation, if you substitute the value of E y from equation

2 that is the other equation that is you really have to play with the all available 6 equations.

So, if I substitute the value of E y from this equation from equation 2, then you can write that this is the expression for E y and multiplied by i beta has given you this equation. So, if you open up this bracket, then you can write this equation in this form. As a result, you can write this equation in this form because you have you have H x H z E z. So, E z H z and here you have H x. There you have H x on the left hand side.

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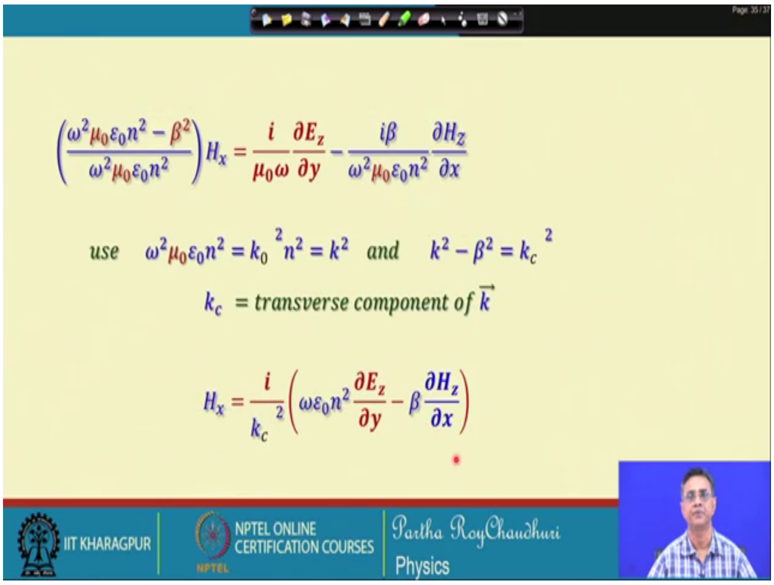

So, this equation will give you this form that H x can be written in this form and here if I if I write this equation, this quantity epsilon mu naught epsilon naught n square, you can you can make out what it is epsilon naught mu naught is equal to 1 by c square.

So, omega square by c square is k 0 square into n square that is equal to k square and k square minus beta square. This quantity is now equal to k square k square minus beta square is equal to k c square we write simply here. So, this k c square this k c actually is the transverse component of the propagation vector because this is beta is the z component. This is the total which is equal to k x square plus k y square plus k z square, but this itself equal to k z square.

So, k c square is nothing but k x square plus k y square. So, that is the transverse component of the propagation vector. So, we can use this H x. From here we can write down that H x equal to this quantity will go to the denominator on the right hand side. So, I can write i by k c square omega naught omega epsilon naught n square del E del E z del y and this. So, this will now become the H x field.

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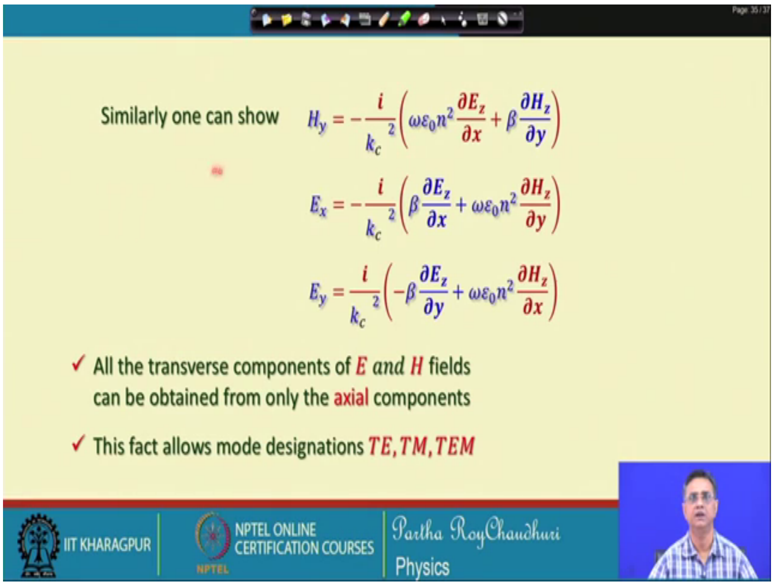

Similarly, one can show that H y equal to this quantity and E x equal to this quantity E y equal to this quantity. In the same way, if I just proceed and play with the all available 6 equation, right now one interesting point here is that by doing this, we have been able to express the transverse component of the electric and magnetic fields; that is H x H y E x E y. All these 4 transverse components of the electric and magnetic fields are now represented individually by only the longitudinal components that is E z and H z in some form and here also H y is represented as a function of E z and H z E x.

Similarly, E z H y E y in the same way E z H z. So, this is a beautiful outcome that we have been able to express the transverse field components in terms of the longitudinal field components. So, that means, all the transverse components of electric and H fields can be obtained from only the axial components. That means, if you know E z and H z, then you can calculate H y, you can calculate E x, you can calculate E y and all the transverse components can come only from the knowledge of E z and H z.

And, it may so happen that E z is equal to 0 in some situation in some other situation H z equal to 0, then it becomes even simpler. Because, your field component will depend directly through this relation to the individual transverse components, this fact allows more designation T E T M and T M because, whether you have a transverse electric field whether you have a transverse magnetic field and so on whether you have both transverse electric and magnetic field.

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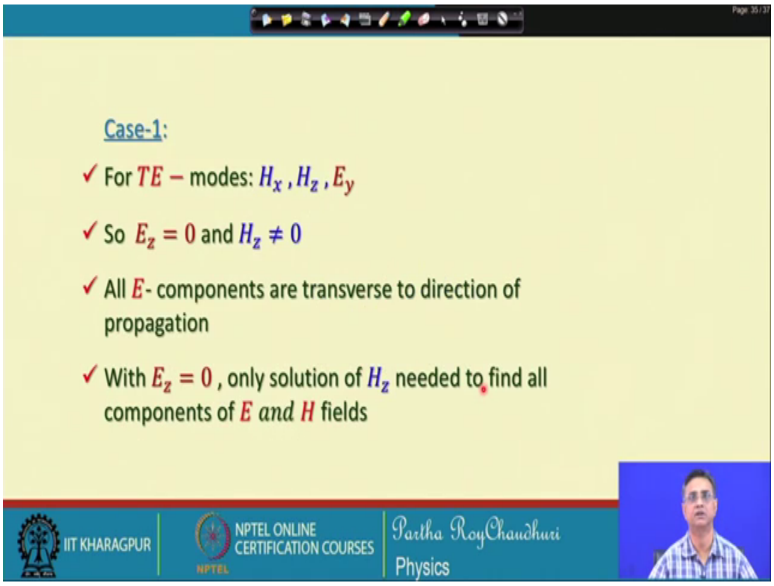

So, this is the fact which tells you how. So, case one that for T E modes you have H x H z E y equal to 0. So, E z equal to 0 E z equal to 0 you can look at this. This components which constitute the T E modes, here E z is missing; that means, E z equal to 0. But, H z is not equal to 0; that means, all E components are transverse to the direction of the propagation. So, just from the knowledge of H z you can calculate H. H x H z and E y with E z equal to 0 only solutions of H z needed to find all the components of E and H fields; that is what just I have mentioned for case 2.

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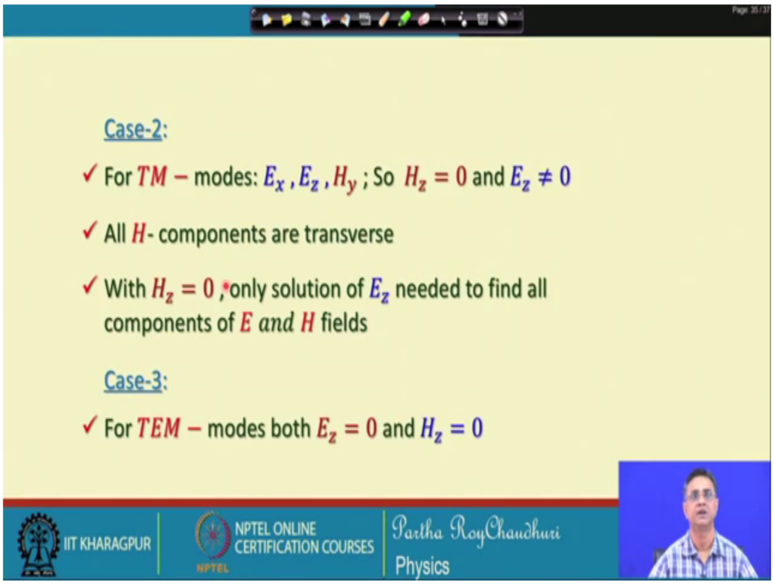

If T M equal if you consider the T M modes in that case, you have these are the these are the field components which constitute the T M modes E x E z and H y. So, H z is equal to 0. Obviously, because it is not appearing in this the this equal in this electric field components which constitute this T M modes. And therefore, E z is not equal to 0.

So, all H components are now transverse because H z equal to 0 only solution of E z is needed to find all the components of E and H fields. This is what I have just now explained that it is only the longitudinal components that can constitute all the transverse components. But eventually, in this case in this case your H z is 0.

So, it is only the E z component of the electric field. If I have the solution for that, this can give me all the transverse components because, in this case the transverse components are only E x and E z now and for similarly for T M mode, both E z and H z both of them are 0 and in that case we can actually calculate all the connections.

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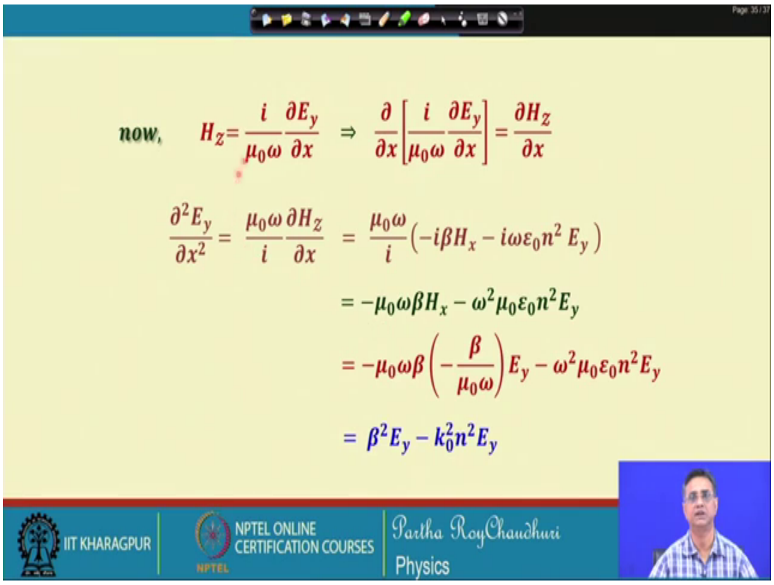

Now, looking at this expression, this is one of the field expression one of the connection out of there. So, 6 equations if you if you now rearrange this equation that if you replace this H z by a connection with E y H z a connection with E y this quantity, then you can write this equation as this. And again, because it involves H x, I will have to replace this H x by E y again. So, that the entire expression is now in terms of E y and you have E y on the left hand side. So, this gives you an expression which is completely depending on the E y and that forms the wave equation.

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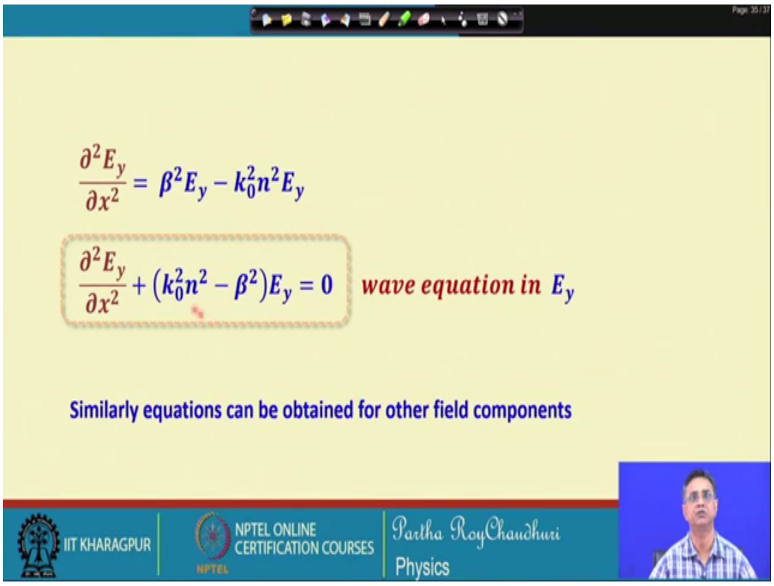

For the E y component of the electric field, similarly equations, so this is the wave equation for E y component of the electric field E y you can actually organize those equations substitutions and arrangement and algebraic manipulations to arrive at all other components of the wave equations by playing with that 6 electromagnetic field equations.

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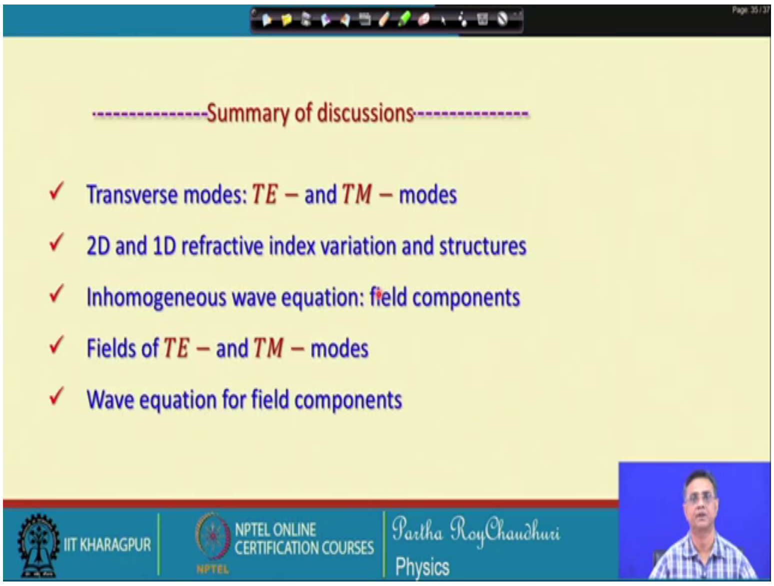

So, today in this discussion, we will continue with this for the wave guides and we will use this wave equations for solving the wave guide modes. So, but in this discussion to just begin with, we have talked about the T E and T M modes and how they are related to the T E and T M polarized waves 2 dimensional and 1 dimensional refractive index variation and of the and the structures involved in.

And some examples also in homogeneous wave equation and then the field components related to that the fields of T E and T M modes. And we have also shown how each of the field component can be represented to an wave equation which will be uncoupled which will just represent one component of the field to show the a wave in the structure.

Thank you.