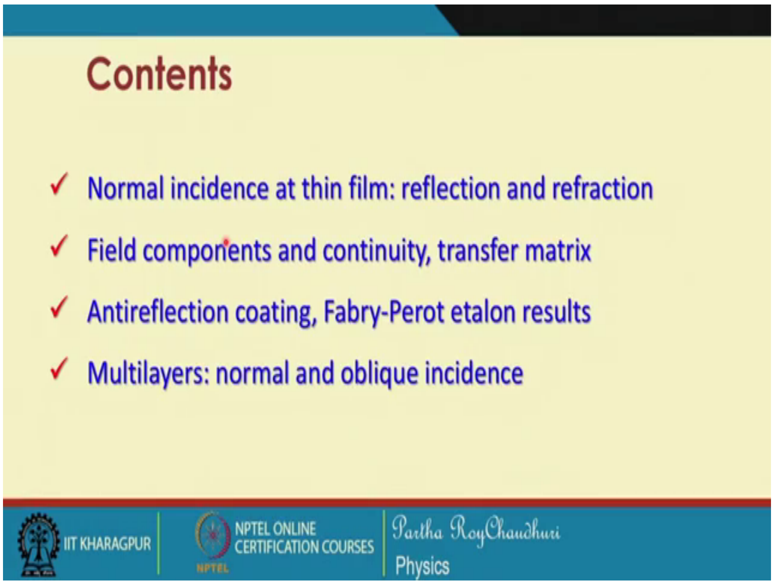
Modern Optics
Prof. Partha Roy Chaudhuri
Department of Physics
Indian Institute of Technology, Kharagpur

Lecture – 17
Wave propagation in layered structures (Contd.)

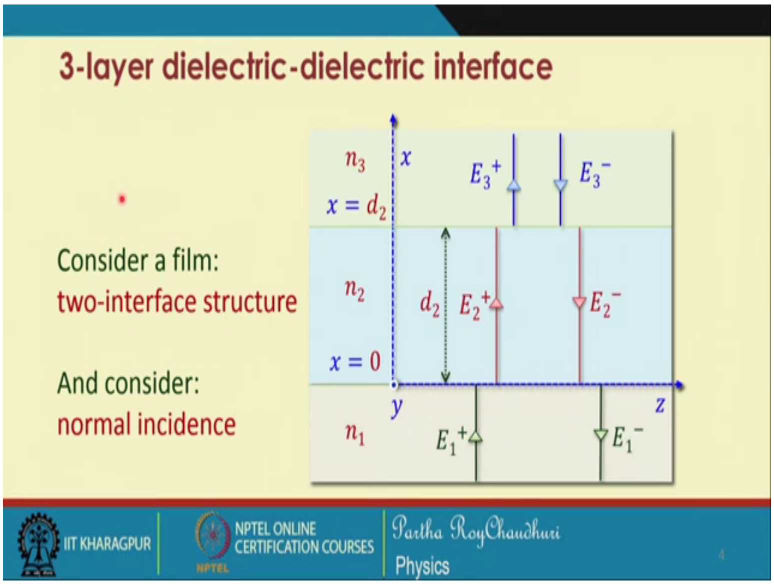
So, we have seen the electromagnetic wave propagation in 2 layered structures; particularly, in terms of the reflection and transmission, amplitude reflection coefficients, energy transmission and reflection coefficients. Now, we will consider the reflection and transmission particularly the general propagation of electromagnetic waves through layered structure a 3 layer and multi layer.

(Refer Slide Time: 00:50)



So, we will organize this discussion in this way that, the normal incidence at a thin film we will consider reflection and refraction. We will consider the field components and impose the continuity conditions. And we will see that how we can devise a transfer matrix which will help us to determine the electric fields at successive layers by knowing the information of the electric and magnetic fields in the first layer or the previous layer. And then we will consider few applications, some anti reflection coating, will we will discuss the Fabry Perot etalon results which we have seen earlier. Then we will switch over to the multi layers, normal and oblique incidence, interference filter, Bragg reflection and so on.

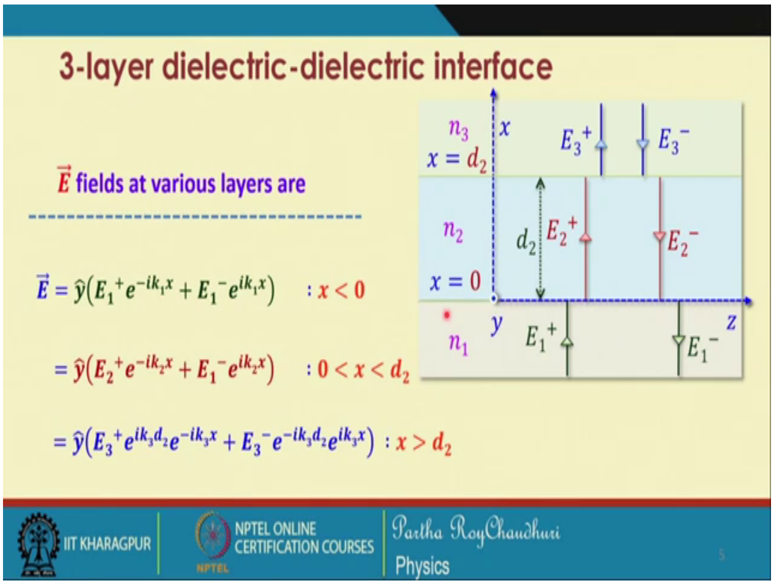
(Refer Slide Time: 01:56)



So, for a 3 layer dielectric dialect interface consider a film, the refractive index of the film is n 2 which is surrounded by the by the layers of refractive index n 1and n 3. And let us consider the normal disc incidents so, in this case, but again we will switch over to oblique incidence, but first for the case of simplicity we will consider normal reflection.

Let us consider that the electric field is incident along this direction and we designate this electric field by E 1 plus. And from this interface part of the wave will be reflected, the electric field reflected from this interface will designate that E 1 minus. And, similarly for the second layer that is the film this will be designated by E 2 plus and the one which will be reflected from this interface will be designated as E 2 minus then similarly E 3 plus and E 3 minus. The thickness of the film we consider as d 2 because this is the second layer.

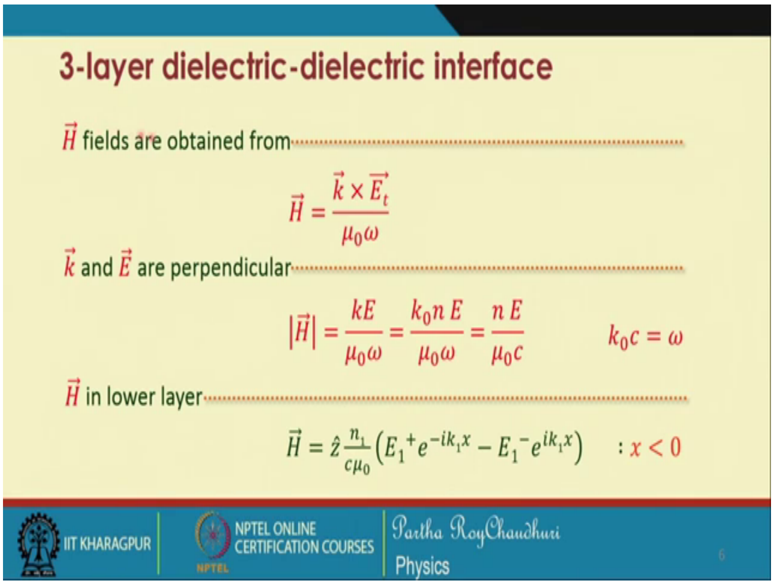
(Refer Slide Time: 03:13)



Then we can write down the fields, electric fields and also the magnetic fields we will see later. In the different layers for example, E when x equal to 0, we have placed the coordinate system in this way this is x along which there is a variation in the refractive index. And this is y, which is along the direction of the along the interface plain. Therefore, the electric field in the in this layer which is x less than 0 is equal to E 1 plus a forward propagating wave i k 1 x with a minus sign.

Because, this is moving in the positive of x direction and this is the reflected one is E 1 minus and because it is propagating in the negative x direction therefore, there is a plus sign. For the middle layer that is x greater than 0, but less than d 2 we can write this field equation in this way where E 2 plus and E 1 minus they are the forward propagating and reflected wave. And similarly, for the third layer we can write down this equation in this form because, now this wave has already traveled a distance d 2. So, we can write this equation as e to the power of i k 3 d 2 into this exponential, this oscillatory variation of the field e to the power of minus i k 3 x and so on for the transmitted for the reflected wave.

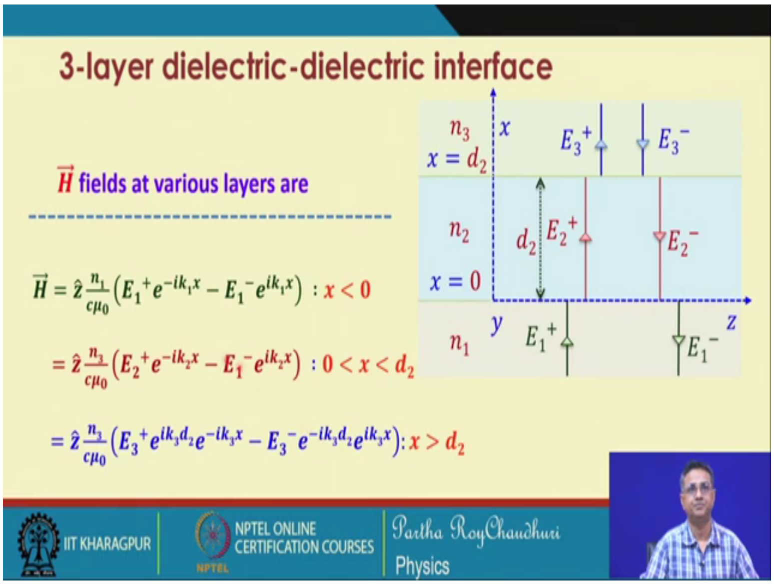
(Refer Slide Time: 05:07)



So, having known this electric field components in the various layers we can now find out the magnetic fields by using this equation that is H equal to k cross E by mu naught omega. Now, since k and E are perpendicular to each other we can write that k cross E is simply the product k into E. And so, the magnitude of H will be represented by this k naught n E by mu naught omega using k naught c is equal to omega we can simplify this magnitude of the magnetic field in terms of the electric field magnitude as n by mu naught c times E.

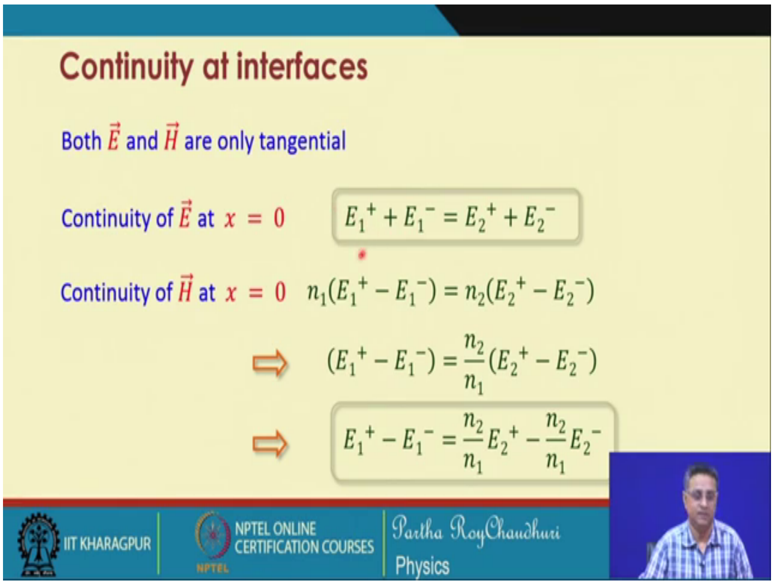
So, H in the lower layer, first layer we can write down in this form because from this equation we can see that H will have z component. So, n 1 by c mu naught e to the power E plus e to the power of minus i k 1 k 1 is the propagation vector in the first layer therefore, we can write in this equation.

(Refer Slide Time: 06:24)



And similarly the fields at various layers will be of this form for x less than x less than 0, we can write this magnetic field in this layer in this form, but it is again in terms of the electric field components. Similarly for the middle layer that is x greater than 0, but less than d 2. We can write the field magnetic field in this way you can also calculate the magnetic field in the third layer using the same relationship. So, by now we have electric field and magnetic field components in all the 3 layers.

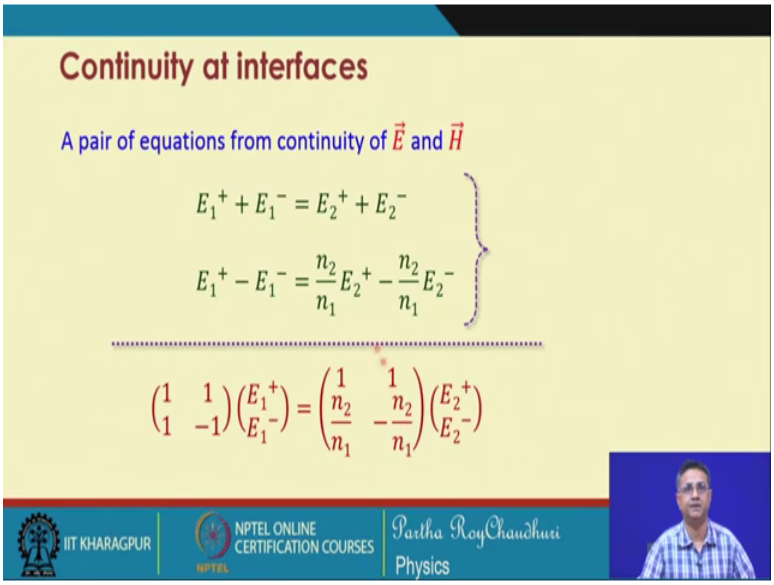
(Refer Slide Time: 07:09)



So, having known this thing, we will now utilize the continuity condition for both E and H. They are in this situation this particular case both of them are tangential component, electric field vectors and magnetic field vectors both lying in the interface plane. So, k and E are perpendicular which has given me this relation which have seen and then H in the lower lyceum, this also have calculated ok. Now using the continuity condition we can write the first equation at x equal to 0 at x equal to 0 you will have E 1 plus plus E 1 minus.

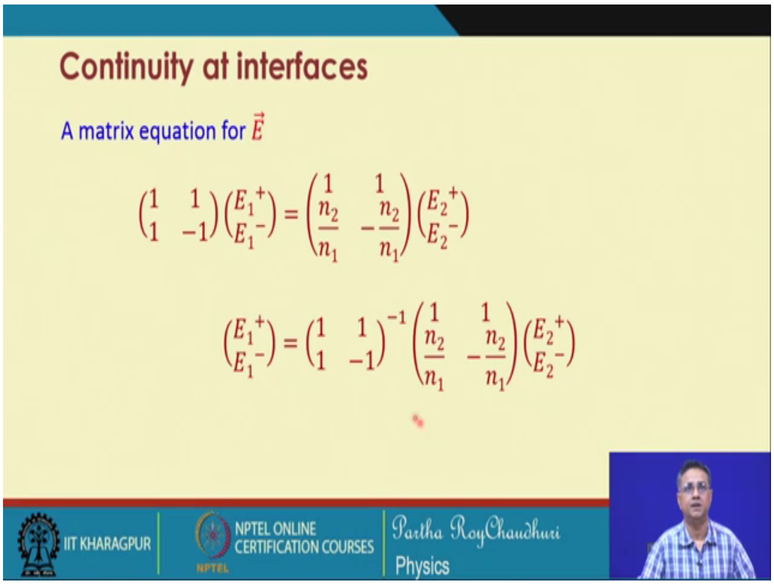
And for H field at x equal to 0, we will have this relation at x equal to 0 that is at this interface this quantity will become E 1 plus minus E 1 minus multiplied by n 1 upon c mu naught. So, that is what we have written here so, and from here if I transpose if I if I take this n 1 to this side, then I can write down this equation which is the consequence of continuity of the magnetic field at x equal to 0. So, I have a pair of equations 1 is plus between E 1 plus and E 1 minus another is E 1 plus and E 1 minus with a minus sign between them.

(Refer Slide Time: 08:53)



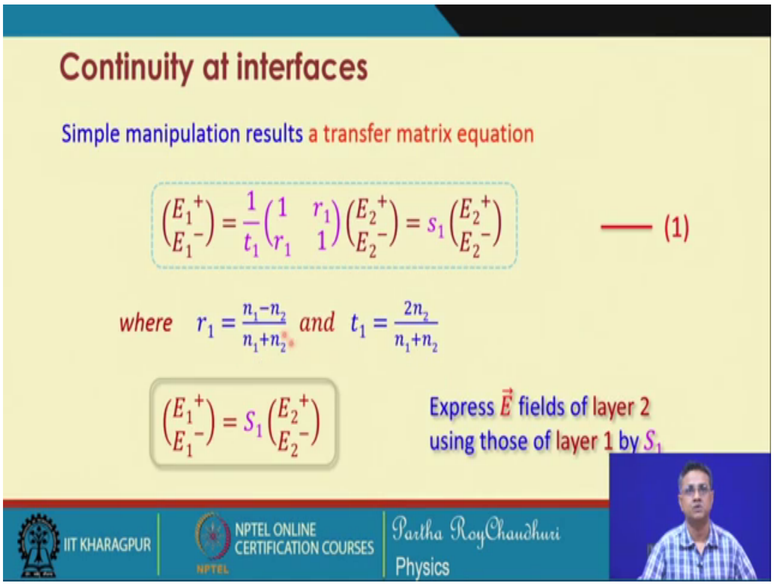
So, these two equations, if we write together then we can form a matrix equation. So, we can write this pair of equations in this form E 1 plus E 1 minus and a coefficient matrix which is given by this and the electric fields of the second layer.

(Refer Slide Time: 09:15)



Therefore, this matrix equation this matrix equation can be rearranged in this way to write to represent the electric fields of the first layer in terms of the second layer.

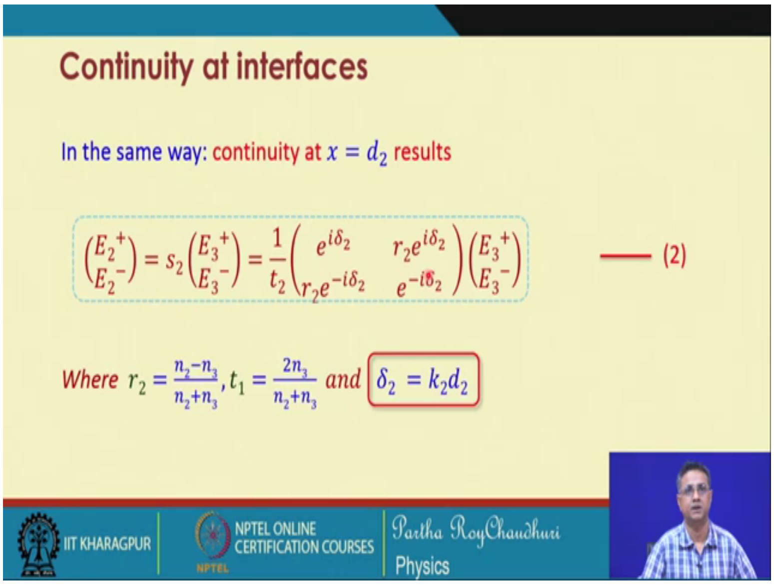
(Refer Slide Time: 09:37)



And this if we put together these two matrices, then we can write E 1 plus E 1 minus is equal to this coefficient matrix into this electric fields of the second layer. This matrix can be called as S 1 because; it transfers the information of the electric field between E 1 and E 2.

So, this S 1 matrix which contains r 1 and t 1, which are the reflectivity and trans reflection amplitude reflection coefficient and amplitude transmission coefficient. So, in terms of this I can write S 1 equal to this equation. And this tells you that electric fields of layer 2 using those of layer 1 can be expressed by a by a matrix multiplier S 1.

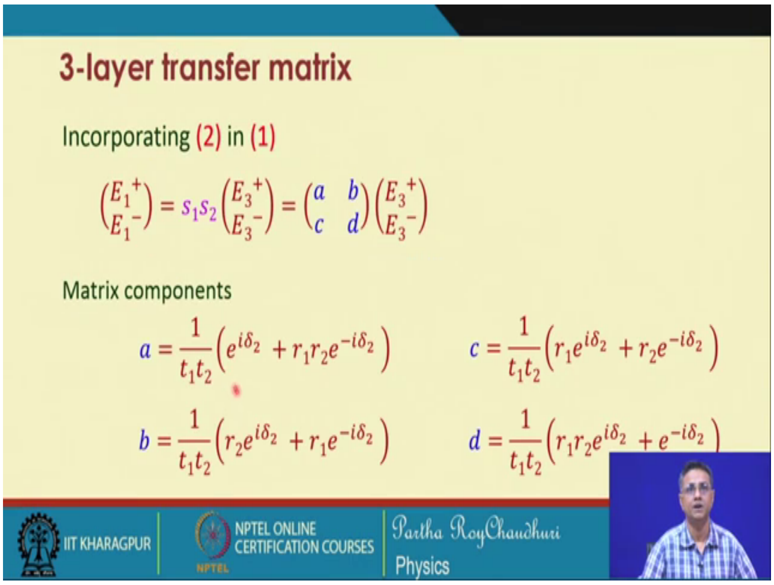
(Refer Slide Time: 10:35)



And this process can be continued for other layers as well for example, if I have this electric field information in the third layer I can connect to the second layer and vice versa. And in that case, the coefficient matrix will look like this 1 upon t 2 then e to the power of i delta 2 r 2 e to the power of i delta 2 and so on.

Where this delta 2 is equal to k 2 d 2, d 2 is the thickness of the layer and this k 2 twice pi upon lambda for the second layer. So, and again r 2 the amplitude reflection coefficient and amplitude transmission coefficients corresponding to the second and third layers will appear here in this form this would be r t 2 this should be t 2.

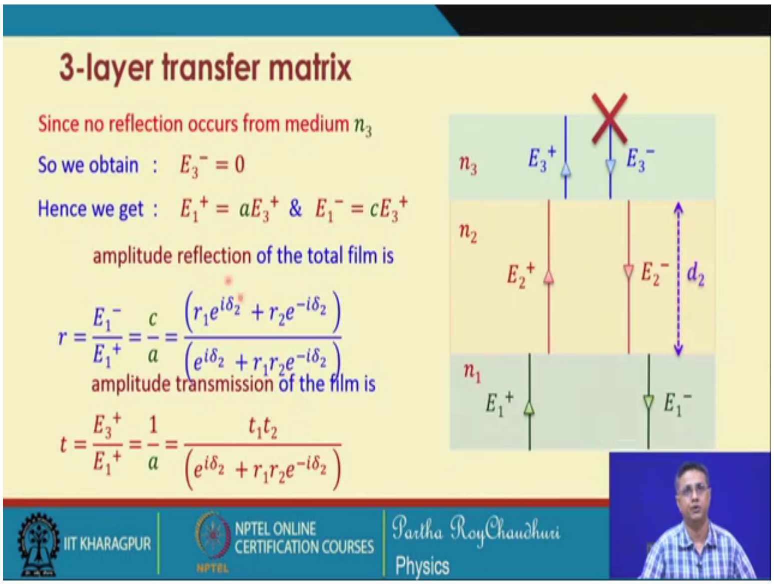
(Refer Slide Time: 11:31)



Therefore, if I connect both the layers using these two equations this 1 and this equation that is I write in place of E 1 plus E 2 minus E 2 plus E 2 minus column vector as this. Then we can write this equation in this form that is E 1 plus E 1 minus can be related to E 3 plus and E 3 minus through a multiplication of these 2 matrices S 1 and S 2.

And if we calculate this product of S 1 and S 2 because they are 2 by 2 matrices the result will be another 2 by 2 matrices square matrix at the coefficients a b c d can be calculated as this for a b for c and d. So, they will contain t 1 t 2 r 1 r 2 and delta 2 so, these all these parameters are known.

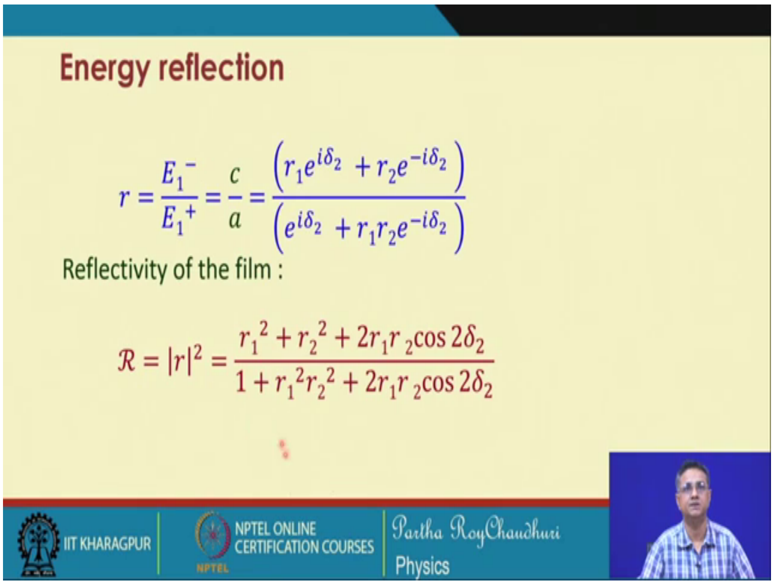
(Refer Slide Time: 12:38)



Therefore, we can now look at the film and consider the situation that because this layer n 3 is infinite and there is no reflected wave in these direction therefore, E 3 will be equal to E 3 minus will be equal to 0 whereas, E 3 plus is a quantity which is non-zero. So, if I use this E 3 minus equal to 0, in this equation in this equation. Then this part will become 0 and we readily get that E 1 plus is equal to a into E 3 plus look at this E 1 plus will be equal to will be equal to a into E 3 plus and E 1 minus will be equal to E 1 minus will be equal to c into E 3 plus. Therefore, the amplitude reflection of the total film can now be calculated for the total field for the total system can be calculated as r equal to E 1 minus by E 1 plus.

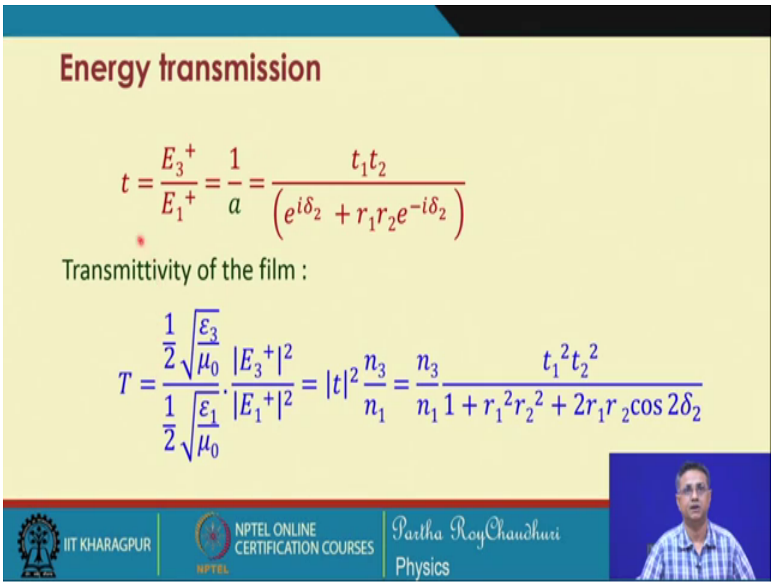
Which will be the reflected wave from the overall film and this comes from this E 1 plus E 3 plus ratio that is c by E 1 plus and E 3 plus which will be equal to c by a because, I have to divide this equation by this which will give me this relation. Then amplitude transmission of the film can be in the same way can be represented by 1 by a that is I just take this equation. So, E 3 plus by E 3 E 1 plus will give me 1 upon a which is equal to this is very interesting and from here we can find some old relations.

(Refer Slide Time: 14:50)



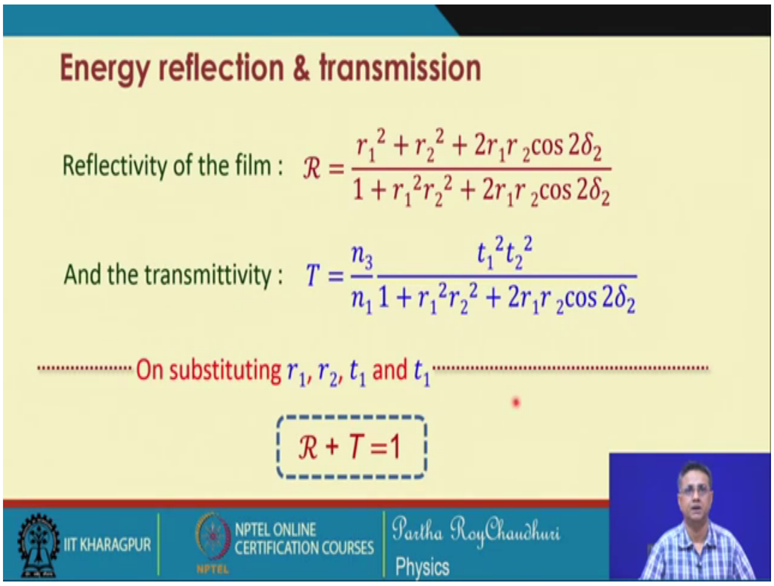
So, this is the this equation will give me the energy reflection that is the reflectivity which will be the mod of the square mod of this amplitude reflection coefficient. And this can be written in this form the square of this quantity that is mod of this quantity mod square will give me r 1 square plus r 2 square mod square plus twice r 1 r 2. And the phase factor that is cosine of twice delta in the denominator this would be 1 plus r 1 r 2 and then twice r 1 r 2 cosine 2 delta.

(Refer Slide Time: 15:39)



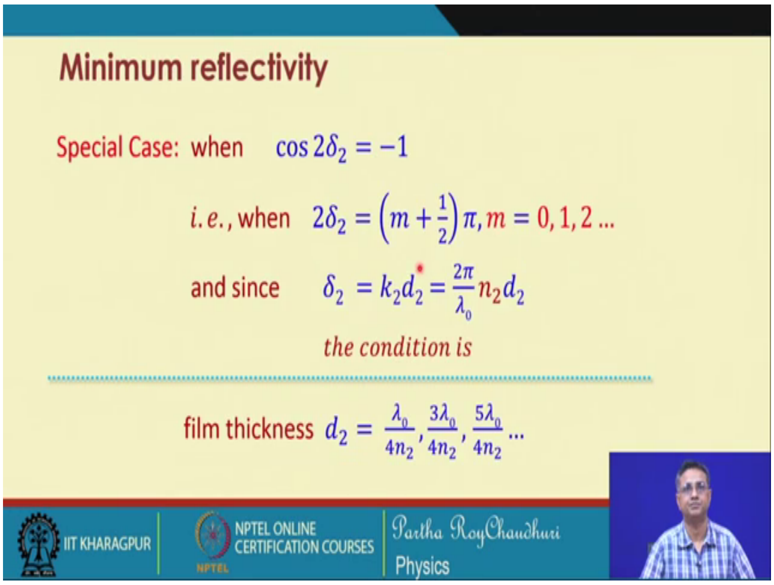
So, this is now complete and we have that energy reflection from the film in a compact form. And similarly for the transmitted wave we can also calculate the transmittivity using the same principle that 1 upon a if I take the mod of that. That is t square will appear, but because it involves 2 layers that is E 1 and E 3. Then this will come with this coefficient n 3 upon n 1 and this give me this will give me this expression.

(Refer Slide Time: 16:08)



So, we have this reflectivity of the film as this and the transmittivity of the film as this. These two put together if we take the sum of these two we can again see the conservation of energy that is the energy reflected and transmitted will be equal to 1 which is very simple to check.

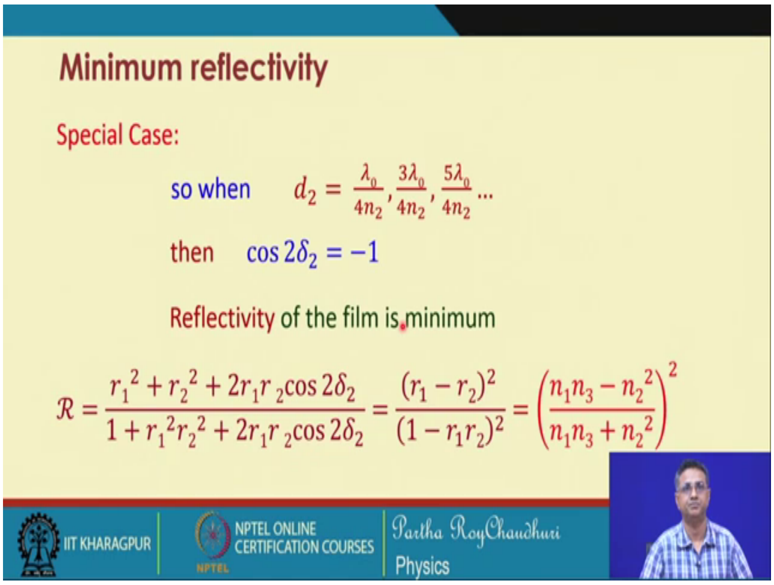
(Refer Slide Time: 16:32)



Now, we look at the condition for minimum reflectivity if I consider this expression this expression, this quantity will be minimum when this cosine 2 delta 2 delta 2 will be equal to minus 1. Because in that case, I will get an expression r 1 difference r 2 mod square of that and in the denominator I will get 1 difference r 1 r 2 mod square of that. Because this quantity will become minus twice r 1 r 2 here also it will become minus twice r 1 r 2. Therefore, this when cosine 2 delta 2 equal to minus 1 then, the condition for this to appear is twice delta 2 will be equal to m plus half pi where m equal to 0 1 2 all integers and since delta 2 is equal to this.

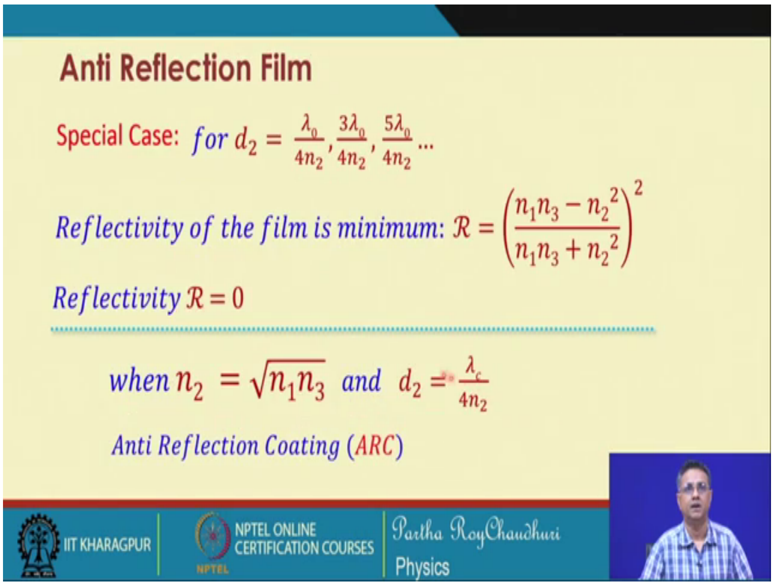
So, we can write delta 2 in terms of this. This gives me the condition for this quantity that is cosine twice delta 2 equal to minus 1. As the film thickness d 2 should be lambda by 4 times n 2 3 lambda 0 by 4 times n 2 5 all odd integers of the factor lambda by n 2 4 4 n 2. So, I can retrieve the condition for the film thickness for which the reflection will be minimum.

(Refer Slide Time: 18:10)



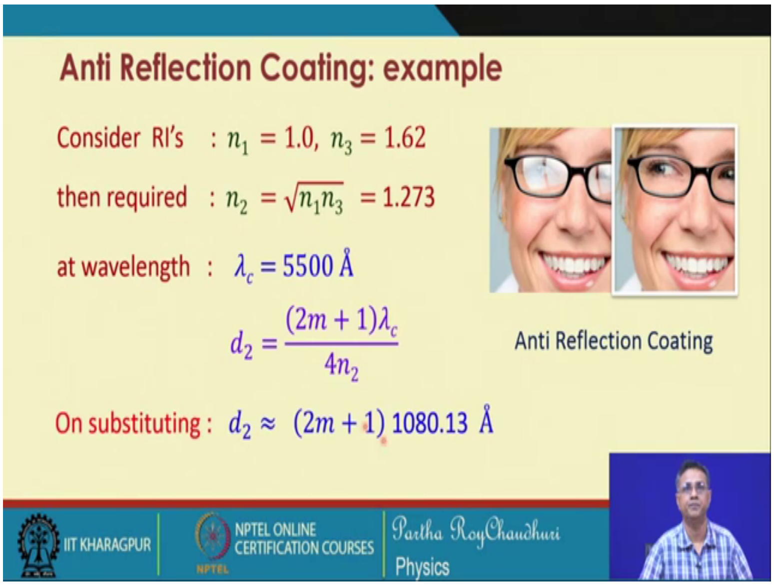
A special case is that under this condition d 2 equal to this we find that cosine 2 delta 2 will be equal to minus 1. And the reflectivity of the film is in that case, will be the minimum. That is this we have already seen r 1 minus r 2 square of that and 1 minus so this is the minimum reflectivity condition.

(Refer Slide Time: 18:33)



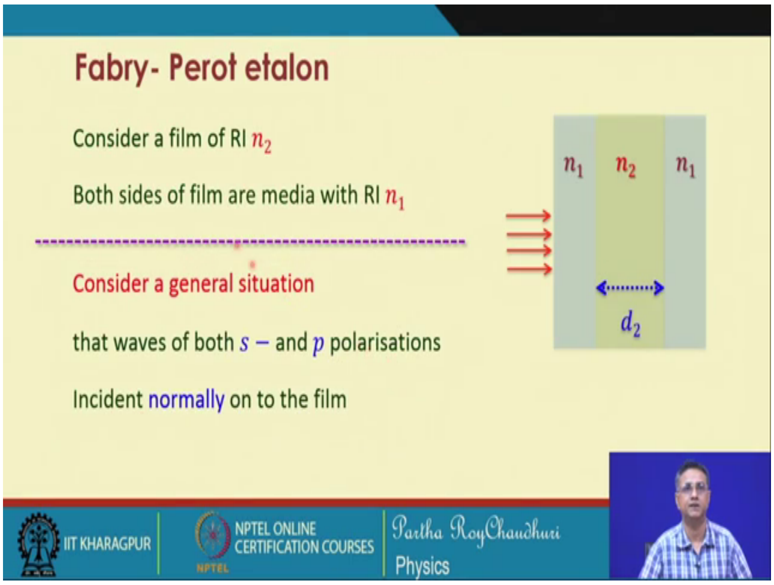
Now, under this condition minimum reflectivity is this, but the reflectivity will become 0 R equal to 0 will give me a condition that n 2 will be equal to under root of n 1 n 3. So, this is the condition if we can choose the values of n 1 and n 3 in such a way that the product of these will be equal to n 2 square. Then we will get back this condition for a thickness d 2 we will get R equal to the reflectivity will be equal to 0. So, that is if I choose these values n 2 n 1 and n 3 according to this equation this expression then there will be no reflection from the film. And that is what is called the anti reflection and by choosing these refractive index materials, we have this anti reflection coating.

(Refer Slide Time: 19:34)



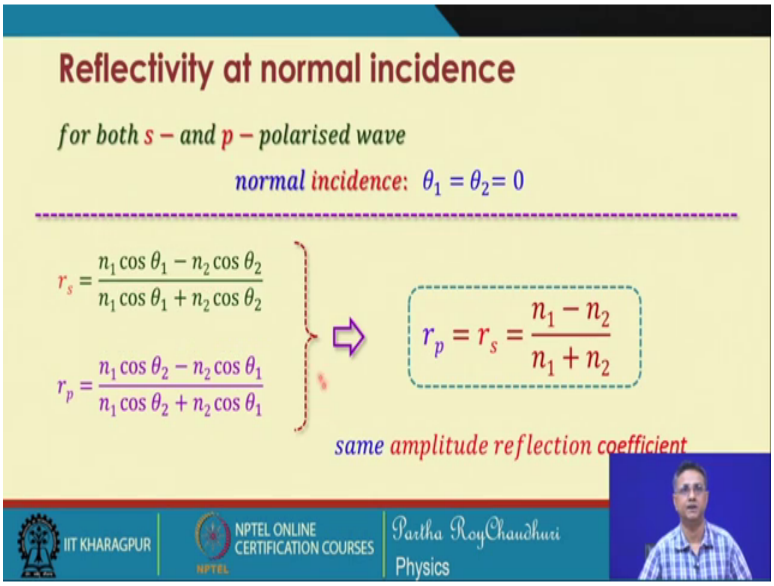
Let us look at this values for example, which are very practical numbers n 1 for air it is 1 n 3 we choose as this and n 2 equal to this. So, at a wavelength lambda equal to 5500 angstrom the value of d 2 which is equal to an integral or integral multiple of 1018 angstrom we can make an anti reflection coating. See the example, when there is no coating there is a reflection from this spectacled glass, but after the coating there is no reflection. So, this is a very beautiful example of anti reflection application of anti reflection coating.

(Refer Slide Time: 20:18)



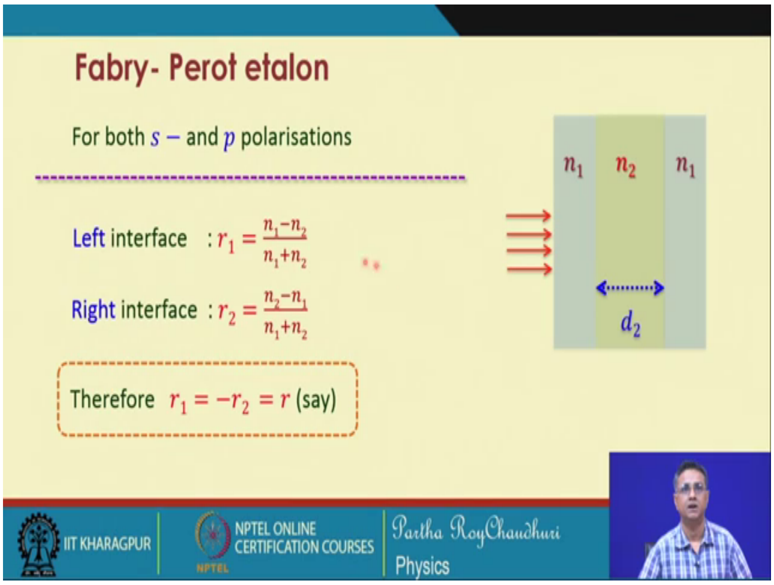
Now we will look at our old notion about this Fabry Perot, but this time we will use this matrix equation consider a film Fabry Perot etalon, which is which has a thickness of d 2 and refractive index n 2 surrounded by n 1 and n 1 consider a general situation for both s and p polarized light and the light is incident normally.

(Refer Slide Time: 20:51)



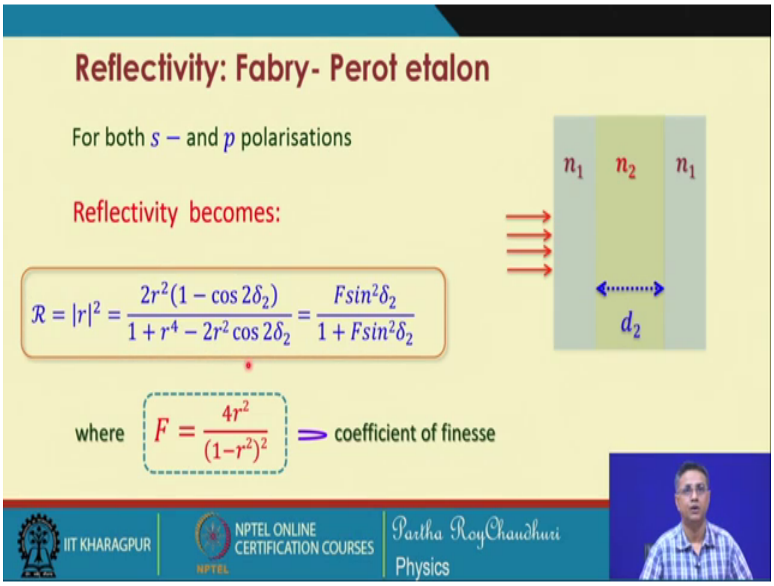
In that case we have theta 1 equal to theta 2 equal to 0 for normal incidence and both r s and r p will give me this value r s equal to r p equal to this for both the polarizations same amplitude reflection coefficient.

(Refer Slide Time: 21:08)



Now, whether it is p or s polarized light from the left from the left interface from here, the reflection coefficient is r 1 will be this. From the right side it is just the reverse in terms of the sine n 2 minus n 1 by n 1 plus n 2. And therefore, we can write r 1 equal to minus r 2 is equal to r let us suppose this value is equal to r.

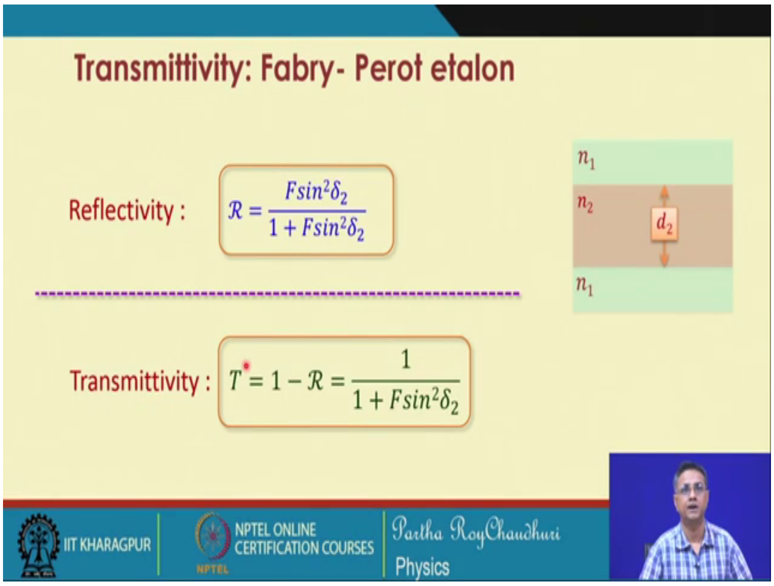
(Refer Slide Time: 21:32)



Then the reflectivity becomes mod r square and if I put in the equation of the reflectivity then I end up with this expression. And you know that this you can rearrange this equation to write in this form F sine square delta 2 by 1 plus F sine square delta 2.

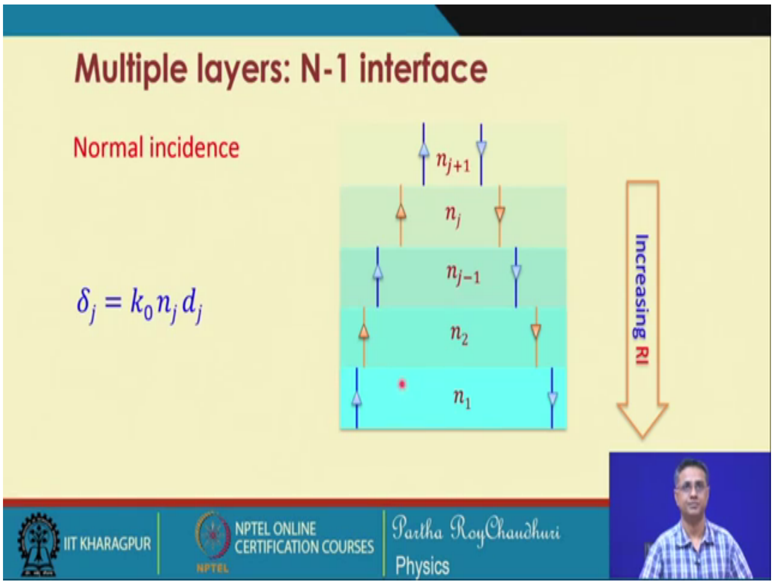
Which is our well known energy reflection for the Fabry Perot etalon and this F which is equal to 4 r square by 1 minus r square whole square is called the coefficient of finesse.

(Refer Slide Time: 22:17)



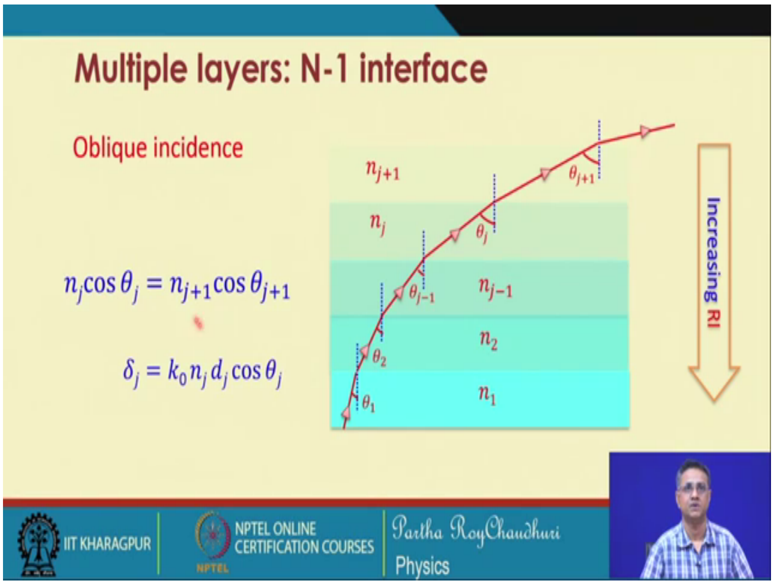
This is our old result and using this expression, we can also calculate the transmittivity that is T equal to 1 minus R which will give me 1 by 1 plus F sin square delta 2. So, these are these are the reflection reflectivity and transmittivity for a Fabry Perot etalon whose thickness is given n 1 n 2 are given right.

(Refer Slide Time: 22:37)



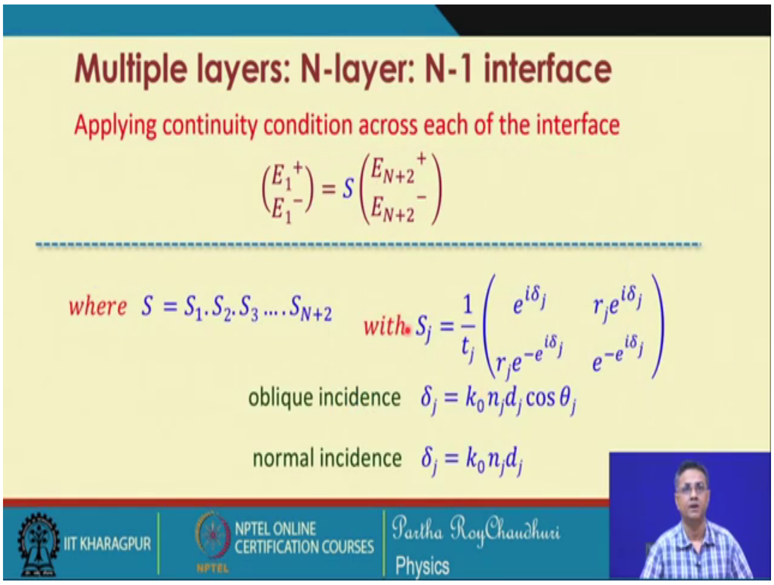
Now, we will consider multi layers, that you have successive layers of increasing or decreasing refractive indices. And let us suppose that the wave is incident at this interface part of the wave will be reflected part of the wave will be transmitted from this interface there will be reflection and so on and so forth.

(Refer Slide Time: 23:06)



In this case because it is normal incidence so, delta j will be k 0 n j d j. For oblique incidence delta j will be k 0 n j d j cosine theta j, j represents any intermediate layer. And this has come from this connection which is basically the Snell's law n j cosine theta j is equal to n j plus 1 cosine the local connection of the refractive indices in terms of the cosine of the angle.

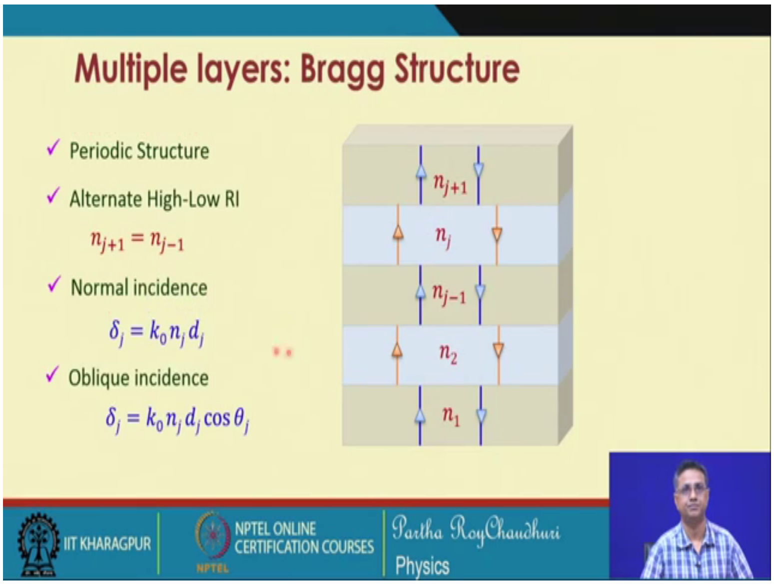
(Refer Slide Time: 23:33)



So, applying the continuity condition across each of the interfaces we can for this particular stack of the layers of increasing or decreasing refractive indices; we can write down this equation the electric field in the first layer will be connected to are E N plus 2 th layer where this S is a compact matrix representing the product of S 1 S 2 S 3 S N plus 2; which is for any general layer S 1 or S 2 we can write S j equal to 1 by t j this we have seen and we can write the corresponding coefficient.

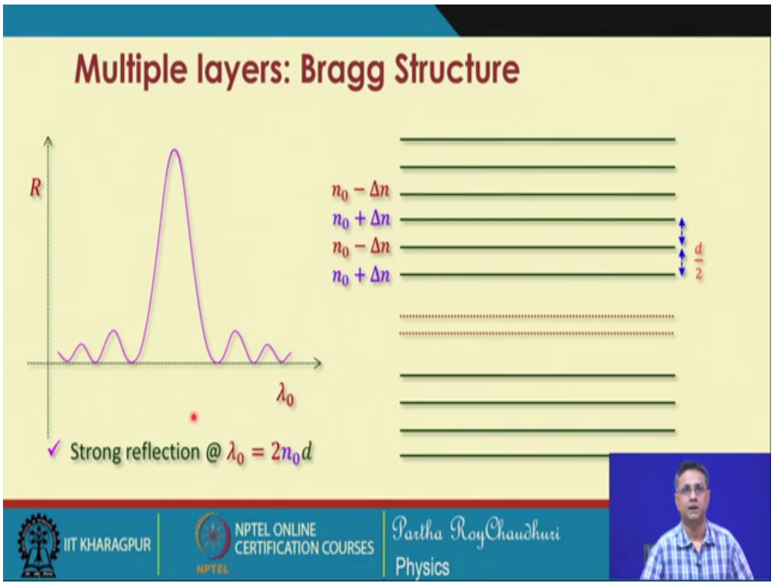
For oblique incidence this delta j in this case will be will be having a cosine theta j, but for normal incidence this will be only k 0 n j d j. So, this is very simple to formulate and this is well known matrix formulation for multi layer propagation of electromagnetic waves.

(Refer Slide Time: 24:40)



Now, we consider the Bragg structure, where we have a periodic structure of alternate high and low. And, the refractive indices are connected by this to it each layer to each alternate layer will have a connection like this. For normal incidence again we can write this and for oblique incidence we can write in this way.

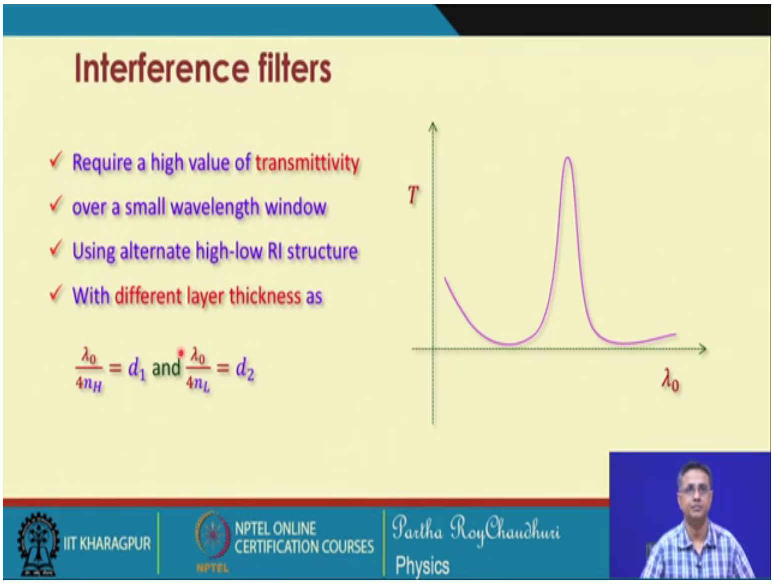
(Refer Slide Time: 25:02)



And, if you look at this structure for Bragg structure you have alternate high n low refractive indices. And you can represent this high as n 0 some base refractive index modified by delta n and this is less by minus less by delta n. So, you have the refractive indices in all the consecutive layers high and low. And if you look at the reflection, if you plot the reflection that is energy reflection R as a function of the wavelength; which is to represent the electromagnetic waves propagating through the structure.

Then you have a plot which is a well known plot of Bragg reflection you have side lobes. And you will get a very strong reflection at this wavelength center wavelength for which this lambda 0 will be equal to twice n 0, d this is again for the normal incidence of the Bragg deflection.

(Refer Slide Time: 26:13)

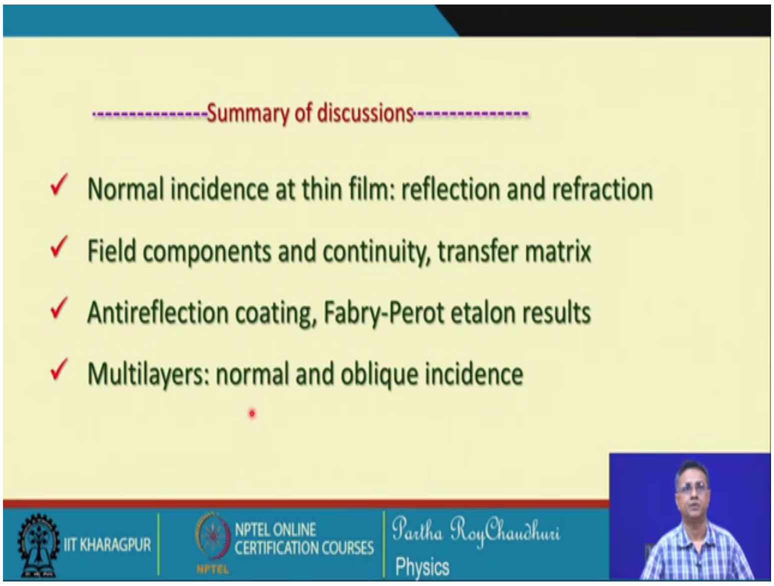


Interference filter so, this is another example, that if I choose the stacks the one which I have shown that these are not same because in this case, all the all the stacks will have the same thickness. So, that d by 2 represents the and the total one high and one low this distance is d. But if we have alternate layers having different thickness, but they are same for all the alternate layers which are given by d 1 and d 2.

So, for low refractive index region I have this thickness d 1 and for high refractive index layer I have this thickness d 2. In that case, if I plot the wave length and the transmittivity of the stack of the structure. Then we will see that this transmission function will look like this; that means, you have a strong transmission at a wavelength around this point and which is which is used for which is used for allowing for passing a small wavelength window. And used in many experiments and applications.

So, it when we require a system a device for high value of transmittivity and that to over a small wavelength window then, we will use this system which is called an interference filter. Using alternate high and low refractive index structure and using this condition we can get a strong transmission at certain small wavelength region. And this is a very useful device for selecting a small window of wavelengths which will be used for some application some experiment ok.

(Refer Slide Time: 28:24)



So, today all that we have discussed is that for a thin film we have considered and we looked at the reflection and refraction from the film. Then we wrote down the electric field components, then device the transfer matrix to relate the electric field components for the consecutive layers. From there we showed the principle of anti reflection coating, an old result of Fabry Perot etalon in terms of the finesse constant for transmittivity and for reflectivity. We also extended our discussion for multi layer structures for normal incidence as well as for oblique incidence. Thank you.

Thank you.