Modern Optics
Prof. Partha Roy Chaudhuri
Department of physics
Indian Institute of Technology, Kharagpur

Lecture -16
Wave propagation in layered structures (Contd.)

So, now we will continue our discussion of the interface properties as regards the optical waves for the dielectric interface. We have seen the p polarized wave and s polarized wave, their properties. The energy reflection coefficient, amplitude reflection coefficients, amplitude transmission coefficients, energy transmission coefficients, also we have seen the Brewster angle normal incidence.

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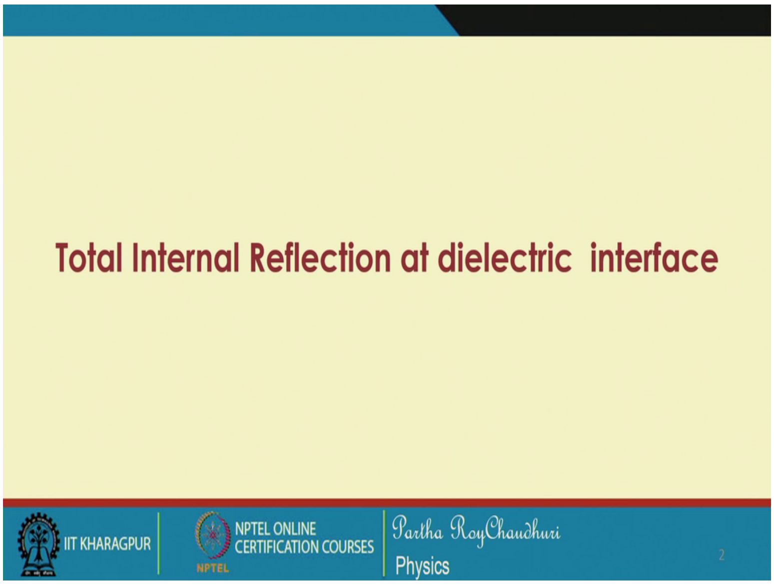

Now, there is an interesting aspect of this inter reflection at interface and which appears in as a common fact and known to almost every one of us, this is what we call the total internal reflection, under certain angle of incident condition.

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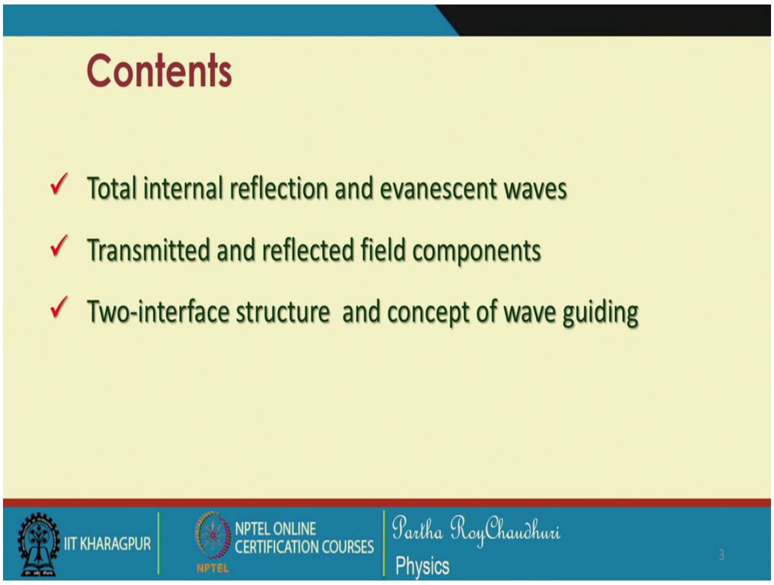

So, with that background now we will we will proceed with total internal reflection and evanescent waves. Then we will talk about the transmitted and reflected electric and magnetic field components in this particular case from where we will try to understand whether the energy flows across the interface or along the interface. Then then we will bring two such interfaces together to see that whether the waves can be confined between the interfaces which will give rise to the concept of wave guiding.

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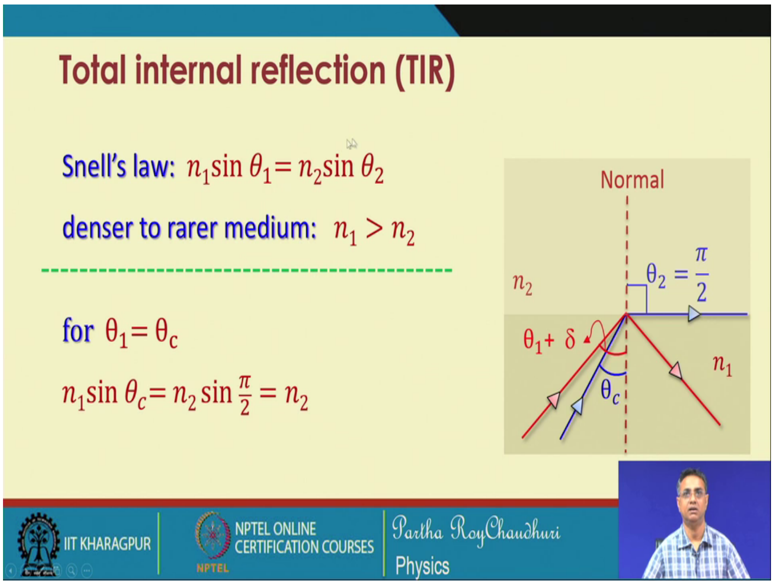

So, let us start our discussion with the picture of this total internal reflection when you have angular or the a angle of incidence, which is equal to the critical angle then the wave will be emerging out along the interface and if it is more than that then, there will be a total internal reflection. So, this is a critical situation when the reflected the transmitted wave will be along the interface.

So, that condition so the Snell's law gives you n 1 sin theta 1 equal to n 2 sin theta 2 is equal to constant which is the invariant of the structure. Now for the denser to the rarer medium if we consider this case that is n 1 is greater than n 2, then for theta 1 equal to theta c, you have theta c the critical angle of incidence for which you will get a grazing angle of refraction. So, we can write for theta 1 equal to theta c equal to n 1 sin theta c which will be equal to n 2 sin pi by 2 because this angle of refraction is now pi by 2. So, therefore, n 1 sin theta c must be equal to n 2 under the condition of total under the condition of critical angle of incidence.

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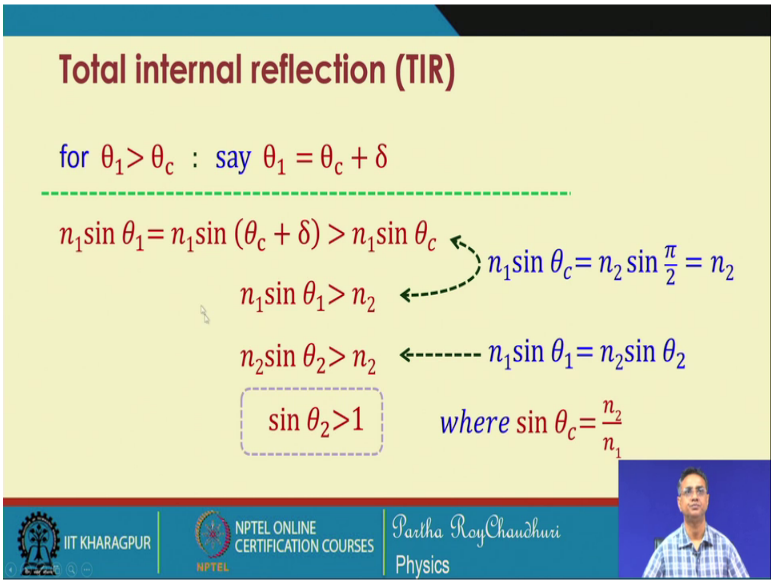

So, for theta 1 so this is for exactly when it is theta c the angle of incidence you have n 2, but if I increase the value of theta c, if I increase the value of theta c by a small amount delta then; what happens? I am increasing the left hand side quantity that is theta 1 is now greater than theta c. So, n 1 sin theta 1 this quantity is now more than n 1 sin theta c n 1 sin theta 1 is greater than n 1 sin theta c.

So, that is what I have written that this theta c plus delta must be greater than n 1 sin theta c, but n 1 sin theta 1 n 1 sin theta c itself is equal to n 2. So, this quantity this quantity must be greater than n 2, that is what I have written. So, this n 2 sin theta 2 must be greater than n 2 because n 1 sin theta 1 is equal to n 2 sin theta 2. So, that tells you have n 2 on either side; that means, if you remove n 2 which is a non-zero quantity sin theta 2 must be greater than 1, but that is a situation which never happens so; that means, sin theta c n 2 by n 1 I have used this relation. So, sin theta 2 is now greater than n 1.

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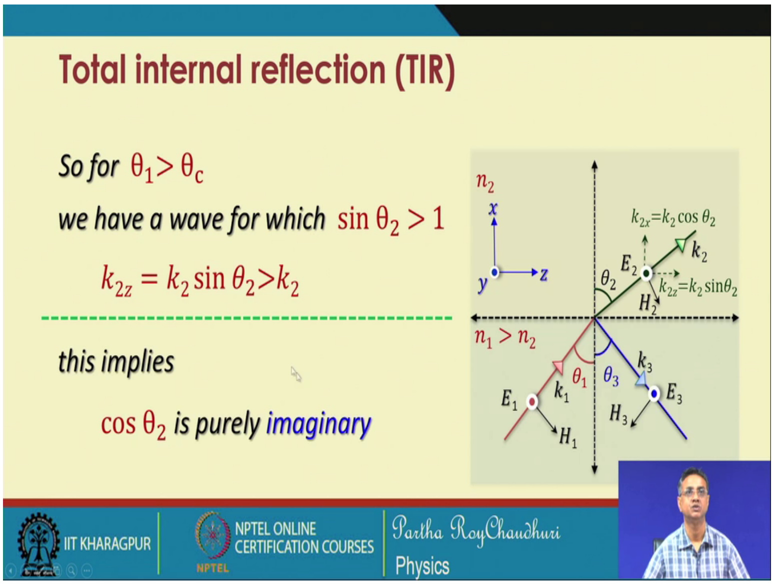

So, for theta 1 greater than theta c, we have a wave for which sin theta 2 is greater than 1 unity; that means, k 2 z k 2 z this quantity equal to k 2 sin theta 2 which must be greater than k 2 this simply means that cosine theta 2 must be purely imaginary.

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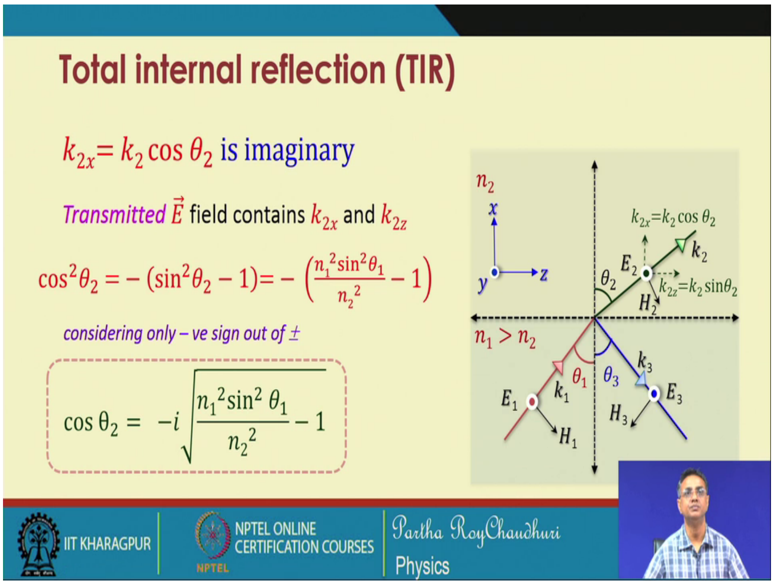

So, if k 2 x is equal to k 2 cosine theta 2 as given by this picture k 2 x equal to k 2 cosine theta 2 is imaginary. Then transmitted electric felid contains k 2 x and k 2 z this we have to evaluate cosine theta 2, we have to evaluate to find that cosine square theta is equal to minus sin square theta 2 minus 1 because this is 1 minus sin square theta. I have written in this form for some particular interest that will be very clear re soon that cosine square theta 2 will be equal to minus.

Because sin square theta 2 equals to n 1 sin square theta 1 by n 2 square this comes from the Snell's law. So, if I express this cosine theta 2 it should be plus minus I of this under root of this quantity, but we have taken only the negative quantity negative value of this root of cosine theta cosine square theta the meaning is very clear because if you take positive root we will see that this will appear in the exponent as a positive quantity which will grow exponentially wit H x and that is not physically admissible. So, for absorption of this wave, we will see shortly that it is important to written this minus sign here ok.

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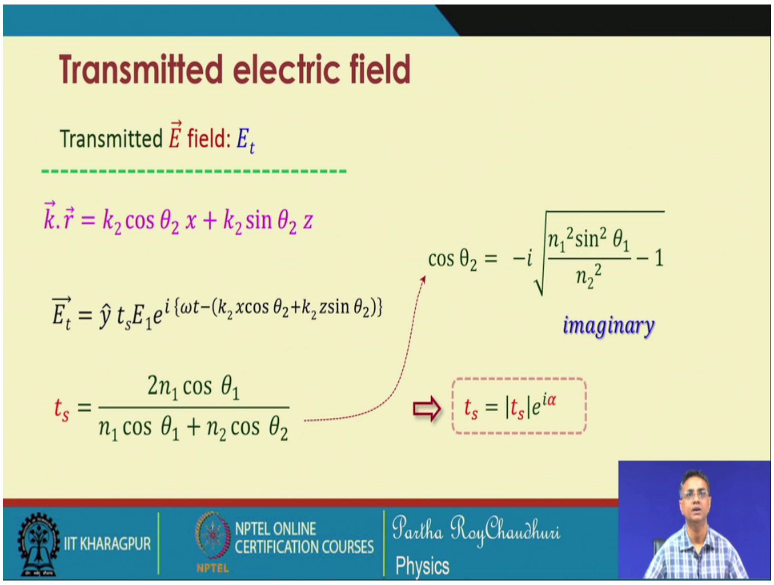

So, in the transmitted field you have this k dot r is equal to k 2 cosine theta 2 x we have seen this, and k 2 sin theta 2 z these are the x component and z component of the k vector. So, in the transmitted field we can write the electric field of the transmitted wave as the transmission amplitude multiplied by the electric incident electric field amplitude and this phase factor. So, omega t minus k dot r, I have written in this form because there is no k 2 y it is only k x and k y k z they are involved.

So, we can write this and this is also known that t s the value of t s is twice n 1 cosine theta 1 by n 1 cosine theta 1 plus n 2 cosine theta 2 this value if we substitute for these then cosine theta 2 will come out to be because this is the value that we have assumed ok. So, from in this expression if I use the cosine theta 2 value here then because cosine theta 2 itself is a imaginary this t s becomes an imaginary quantity and that adds to a phase additional phase.

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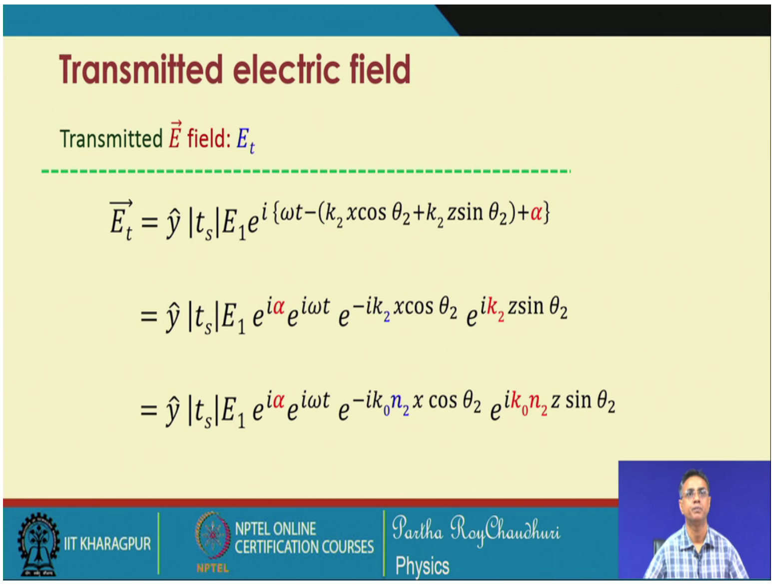

Therefore we can write this imaginary quantity for t s as t s equal to mode of transmission amplitude into e to the power of i alpha where alpha is the phase factor which is the addition phase. So, it means that the transmission amplitude coefficient includes the phase factor so due because of transmission there is a change in the t s also therefore, the transmitted electric field can be written in this form that the because electric field is parallel to the interface plane and we can write that equal to y cap t s E 1 e to the power of i omega t plus alpha which is the additional phase arising out of the t s that is the transmission amplitude coefficient.

So, rewriting this equation by just by separating all the individual phase components that is e to the power of i alpha, e to the power of i omega t, e to the power of i k 2 x cosine theta and so on. We can show that this quantity all of them are positive this is the only quantity which is negative. Now if I use negative with eta i if I use this cosine theta 2 the value of cosine theta 2 the value of cosine theta 2 as we have already determined we have already calculated.

So, if I use this value of i here, then i into i and minus and minus. So, it will be simply minus e to the power of some minus of this quantity.

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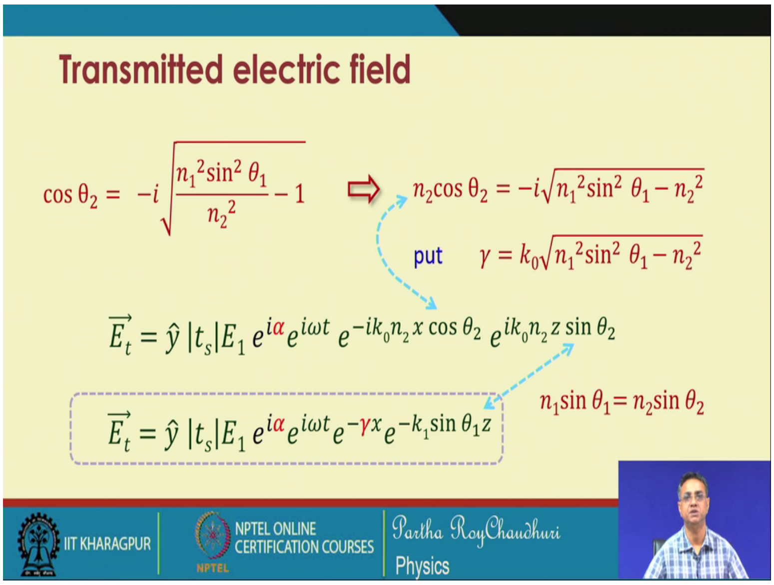

This is exactly what we have shown here n 2 cosine theta 2 just multiply on either side and for this quantity if you multiply both sides by k 0; then this quantity can be written as gamma. So, k 0 n 2 put together we write gamma with minus i of course.

Then this transmitted electric field amplitude electric field can be all in terms of the phase and amplitude can be completely written in this form and if I use this gamma then the equation for the transmitted electric field can be written in this form that is the transmission amplitude in terms of the electric field incident electric field the additional phase the time varying components and this is the amplitude attenuation factor and this is the z component there should be 1 i missing here there should be 1 i here

So, this component this quantity e to the power of i gamma x is now attached to the amplitude; that means, in the transmitted wave decays the exponentially with x.

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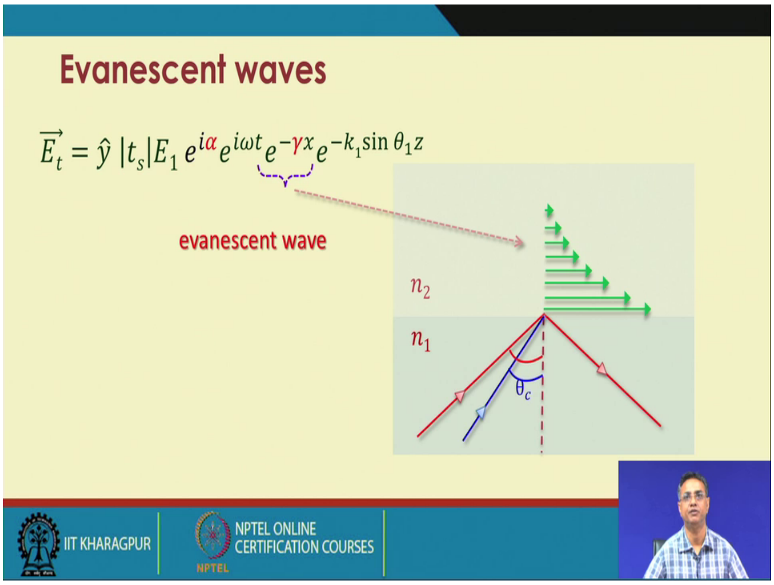

So, there is a wave after transmission from the interface that decays exponentially the amplitude of the wave decays exponentially and that is what is called the evanescent wave. So, it appears as the as the attenuation of the wave by a factor minus gamma there should be 1 i here because this will add to a phase factor.

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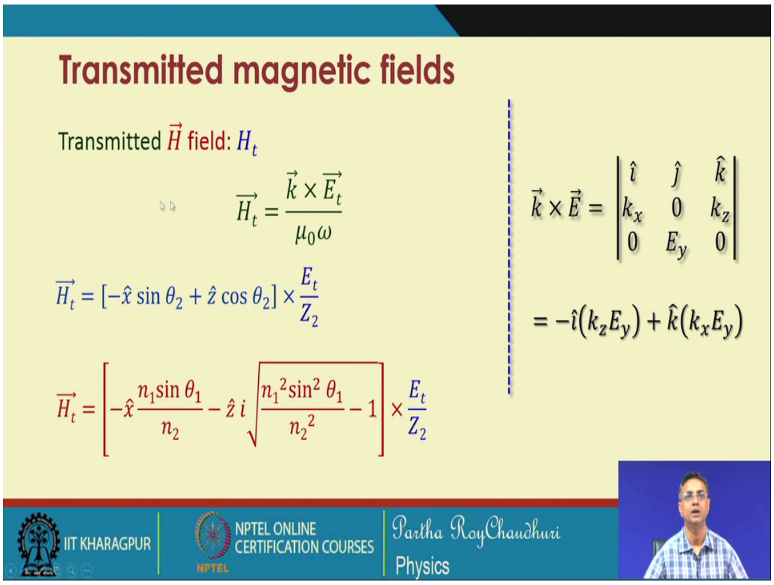

So, by knowing that electric field the incident electric field and the transmitted electric field we can now calculate the magnetic field components magnetic fields we have seen related in this way this also we have seen that k x e y and k z they are the only surviving quantities which will give you this k z E y and k x E y of the z components for z component. So, h t you can write in terms of the characteristic impedance of the second medium. So, transmitted magnetic field can be written in this form.

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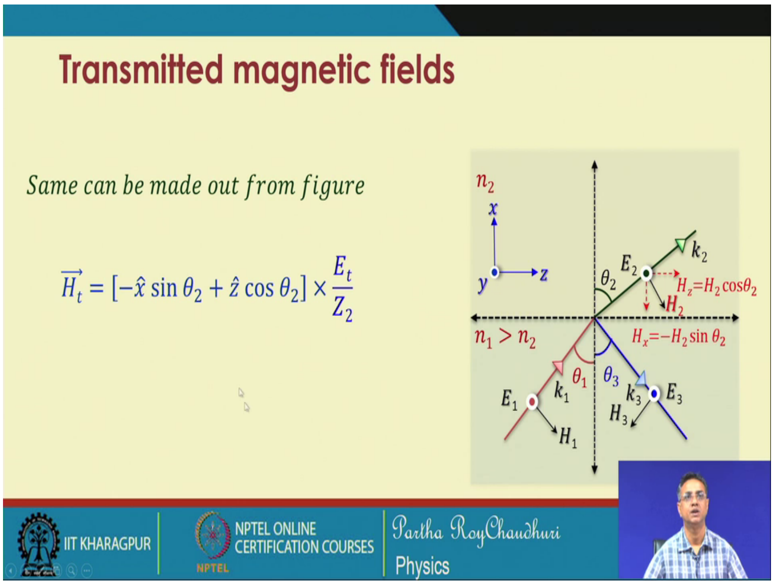

Now the same can be made out in this figure also because if you have a we are looking at the transmitted a magnetic field amplitudes. We have a z components of them which is positive z 2 cosine theta 2 which comes appears here and you have you have a x component of the magnetic field which is negative. So, you have minus sin theta 2 and then because it is in terms of the transmitted electric field. So, transmitted electric field by the characteristic impedance we will represent this h 2 that is the magnetic field in the transmitted wave.

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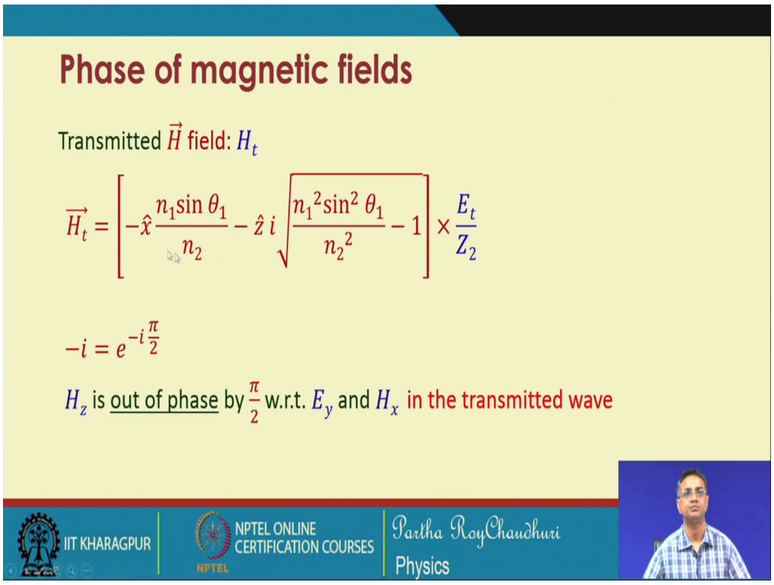

So, the transmitted field can also be written in this form because you have a cosine theta cosine theta 2 and this cosine theta 2 for this case that is from the denser to the rarer medium are interfaced this becomes imaginary.

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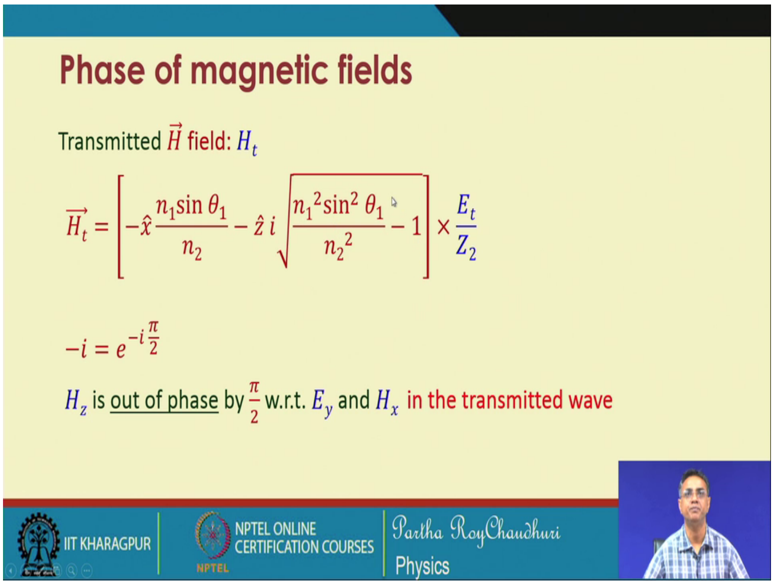

And if I substitute the value of cosine theta 2 I can write in this form therefore, this h t involves an i in the z component of the of the transmitted magnetic field so, but minus i minus i can be written in terms of e to the power of minus i pi by 2.

So, it tells you that the z component of the magnetic field this is the x component of the magnetic field in the transmitted wave and this is the z component of the magnetic field in the transmitted wave z component means along the along the interface direction. So, the component of the magnetic field which is along the interface direction is out of phase by pi by 2 because of the attachment of this i with this amplitude of the z component of the magnetic field with respect to this phase out of phase is with respect to E y and H x this is H x and this is H x and it is also with respect to E y in the transmitted wave. So, this is our interesting finding that the z component of the of the magnetic field in the transmitted wave is out of phase with respect to the E y electric field and the x component of the magnetic fields.

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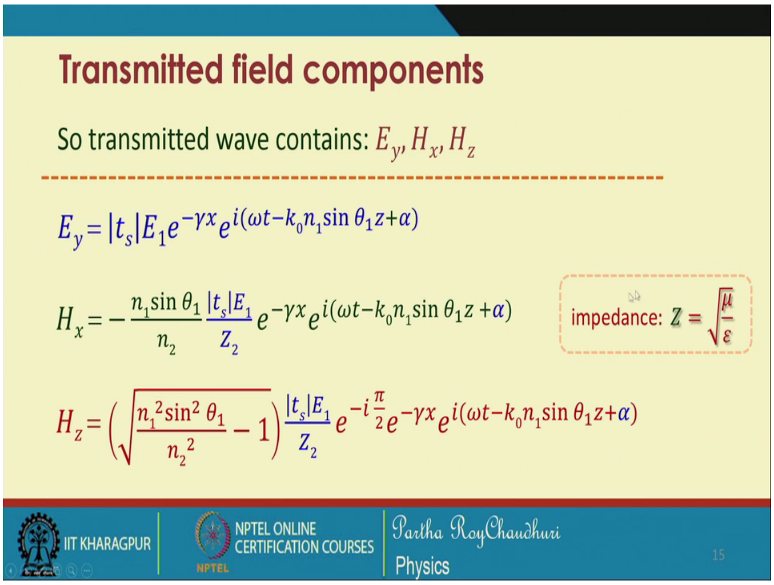

So, for the transmitted wave the components that is to describe the transmitted wave are E y H x and H z. So, E y is we have seen can be written in this form which includes a phase factor of alpha and H x is also includes the same phase factor alpha and all of the, but additionally there is an there is an extra phase which is due to the z component of the z component of the magnetic field in the transmitted wave.

So, this will make this quantity if take the real part this will make this quantity sin, but these 2 will be still staying back as cosine.

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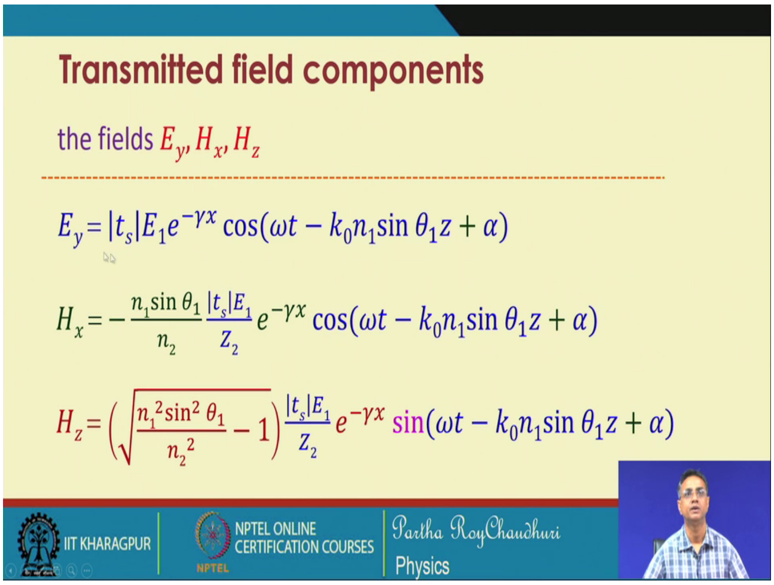

So, we can write E y in this form which will be if I take the real part of this of the individual filled components we can write these as cosine theta. This will also remain cosine theta, but this will become sin theta and there will be a phase attenuating part in the transmitted wave this is because of the electric field attenuation.

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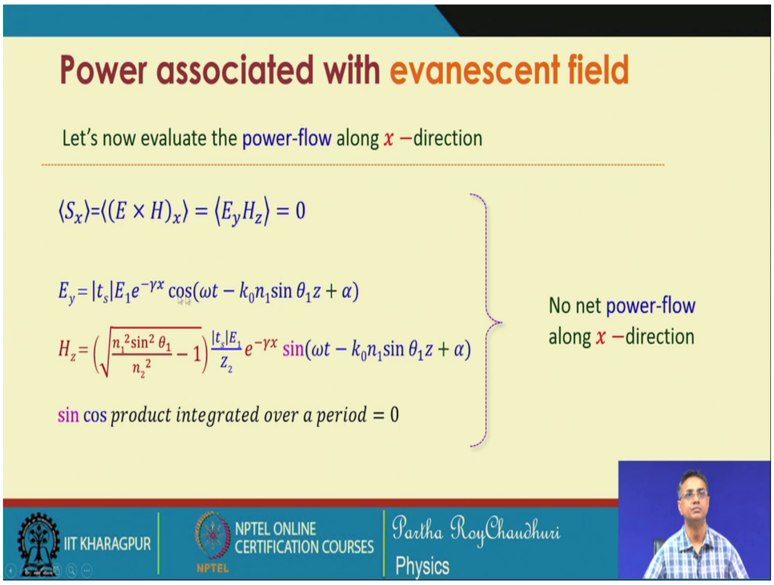

So, let us now evaluate the power flow across the x direction. So, S x is equal to E cross H x component we have seen now E y and H z this component will give you E y and H z which will be equal to 0 why because E y will contain a cosine component cosine factor whereas H z contains the sin factor. So, you know the time average over a period for sin and cosine product will give you always 0 so; that means, there is no net power flow along the x direction in the transmitted wave.

So, there is an evanescent wave, but the wave is moving along the interface, there is no net energy flow out of the interface along the x direction.

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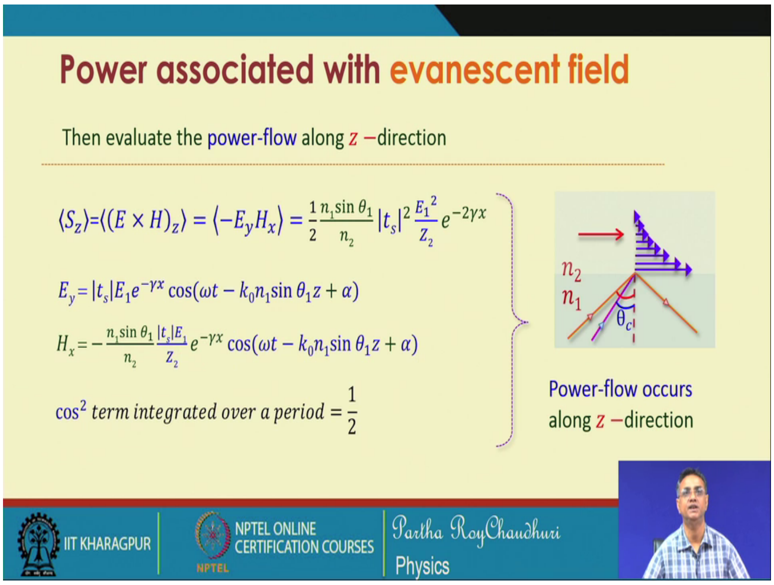

So, this is an interesting finding. So, what we find that there is no power flowing along this line, there is no power flowing along this line, but there is a power flow which is along the z direction.

So, let us calculate the power flow at the z direction that is S z. So, we just calculate the z component of the pointing vector and this will give you minus E y H x if you simply open up the cross product and then take the z component, but this time the value is non-zero and this quantity comes from that E y you know that this is a cosine factor involved with this E yand for H x there is another cosine factor. Now the time average of the cosine square term integrated over a period will be equal to half.

So, there is a factor half coming here and on if you simplify the product of these two, then you will get this equation e to the power of gamma x and e to the power of gamma x here both put together will become e to the power of minus twice gamma x the electric field amplitude e 1 e 1 in both the places will make it e 1 square t s mod and t s mod will make it t s mod square and all other things are quite evident from here. So, we can see that there is a net power flow along the interface outside the outside the interface along the along the z direction the direction along which it is moving.

So, this evanescent field have the characteristic nature that there is no the field is exponentially decaying as you move away from the as you move away from the interface along the x direction, but there is no net power flow there is no net energy flow out of the interface along this direction, but the energy flows only along the interface only along the z direction along these direction.

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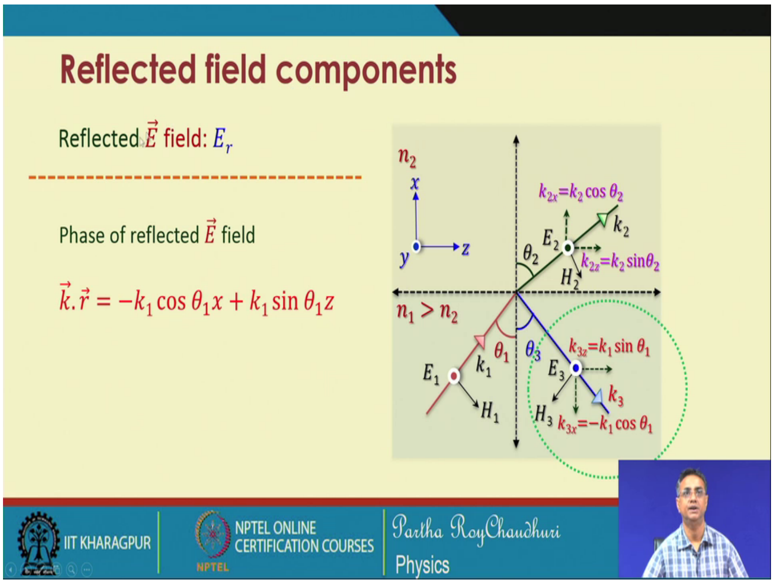

So, in the reflected field for the completeness, we have this phase which will involve k 3 z and k 3 x. So, that is why k dot r 1 of them is positive z, but x component is negative. So, we write this k hat dot r in this form.

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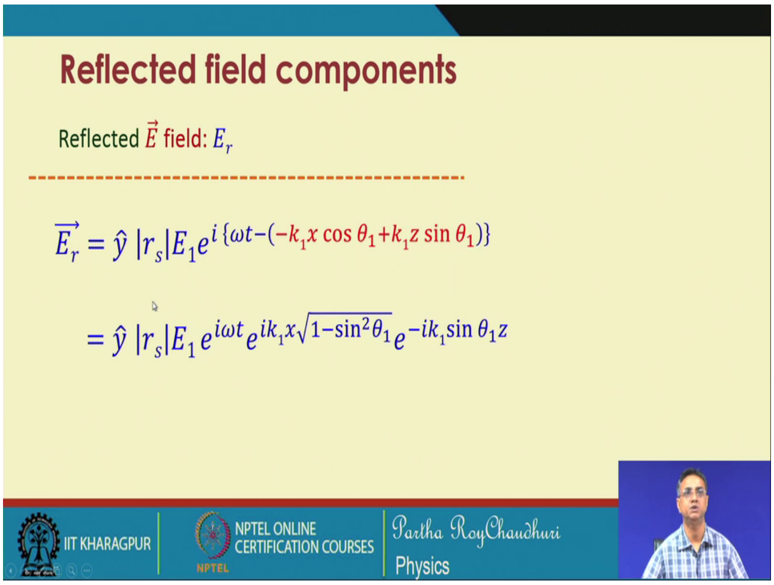

And that gives you the reflected wave amplitude of the electric field as like this and this if you just open up the bracket, you can write the individual components of the phase due to x due to time and due to z.

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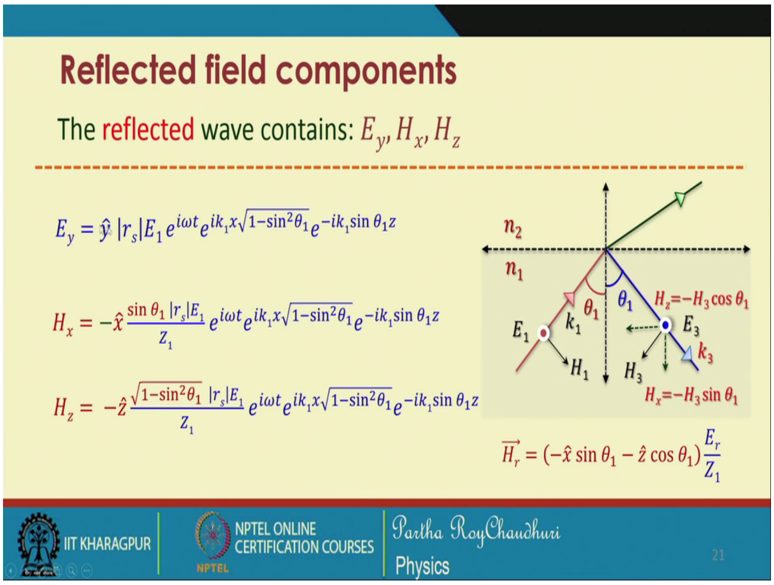

And in this way you can complete all the components of the electric field and magnetic field in the reflected wave, E y will be this we have seen in the reflected wave.

Because it is simply by a factor of r s modified the incident electric field and for H x you have seen this one and for H z, this again come from here you have this h field you have a negative value of the x component of the magnetic field and the negative value of the z component of the magnetic field. That is why your both H x and h z both of them are negative and it appears with the phase factor of the electric field multiplied by the reflection amplitude coefficient in terms of the incident electric field amplitudes.

So, these are complete e expressions for E y H x and H z which constitutes this s polarized wave or the t polarization wave at the interface. So, this is complete.

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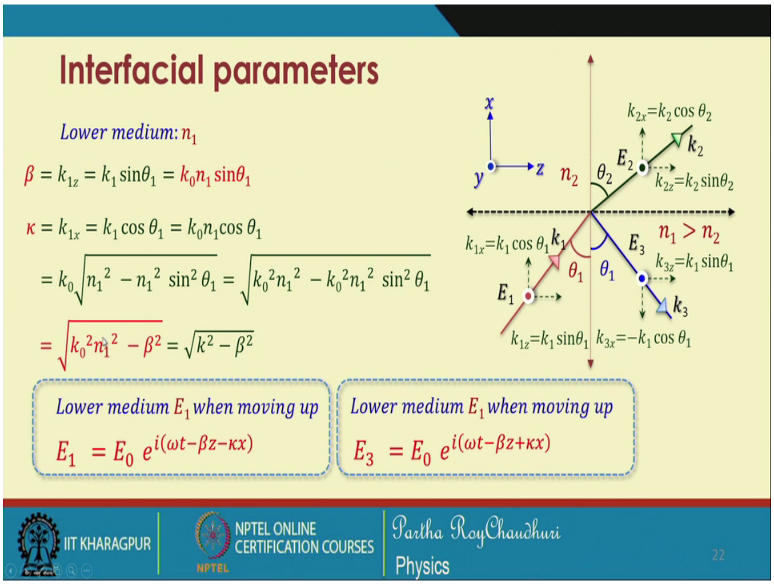

And now for the lower medium you have you can see that this beta, I write for k 0 n 1 sin theta 1 for the lower medium k 1 z; k 1 z is this component k 1 z this quantity. So, that is a z component of the propagation vector which can be written as k 0 n 1 sin theta 1 and I call this quantity as beta which is the z component of the propagation vector and the x component of the propagation vector k 1 x this is for the incident wave k 1 x equals to k 1 cosine theta equal to k 0 n 1 cosine theta this.

So, put together if I use this k 1 n 1 cosine theta you can write this equation and just by translating this k 0 you multiply both side bracketed the k 0 square inside. Then you can write k 0 square minus beta square which will be the x component of the propagation constant for the lower medium e 1 when moving up that is for this field for this optical field we have e 1 equals to e 0 e to the power of i omega t minus beta z minus k x. These are the factor which are appearing you can look back this electric field component you have this k 1 x into this cosine theta and k 1 z into sin theta 1.

So, I have written for k 1 x k 1 x for this I have written this quantity that is kappa for k 1 x. I have written k 1 x kappa into x and for the z component that is the second part of this equation i k 1 sin theta 1 z I have written beta. So, using this kappa and beta notation which are the x and z components of the propagation vectors, I can write this e 1 equals to e 0 e to the power of i omega t minus beta z minus kappa x, for the wave which is moving up towards. The interface and for the wave which is coming down from the interface the reflected wave everything remains same expect that this is now the travelling in the negative x direction effectively the propagation constant is along the negative x direction.

So, k 3 x is now minus k 1 x cosine theta which is the same, but with a negative sign k 1 x and k 3 x the magnitudes are the same, but it contains a negative sign that is why the kappa has now become plus kappa into x. So, these are the nit representation of the optical waves when they are moving up and moving down.

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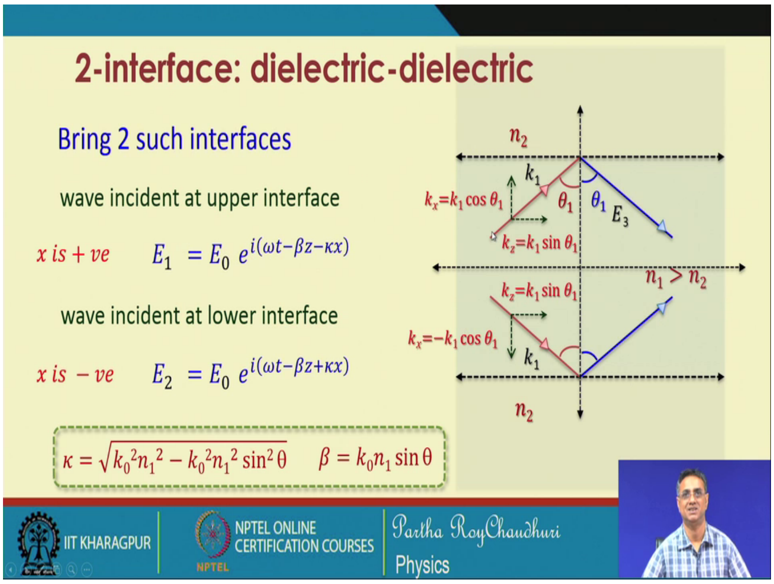

So, now with this knowledge we will bring in 2 such interfaces that is there is 1 interface which is binding this the upper and lower media essentially this n 1 is higher refractive index medium n 2 is lower. Similarly here also they are identical interfaces I assumed that and the medium inside is continuous. So, I place my coordinate axis in this along the axis and if the reflection takes place here I know that x for the wave incident at the upper interface. I can write this e 1 equals to this and for the wave which is incident at the lower interface I can write this equation.

This we have seen just now here I can write in this form both of them I assume that have the same amplitude of the electric fields. Therefore, and this is also known we have used this relation k equal to kappa equals to this and beta equals to this is the x component of the propagation vector and this is the z component of the propagation vector.

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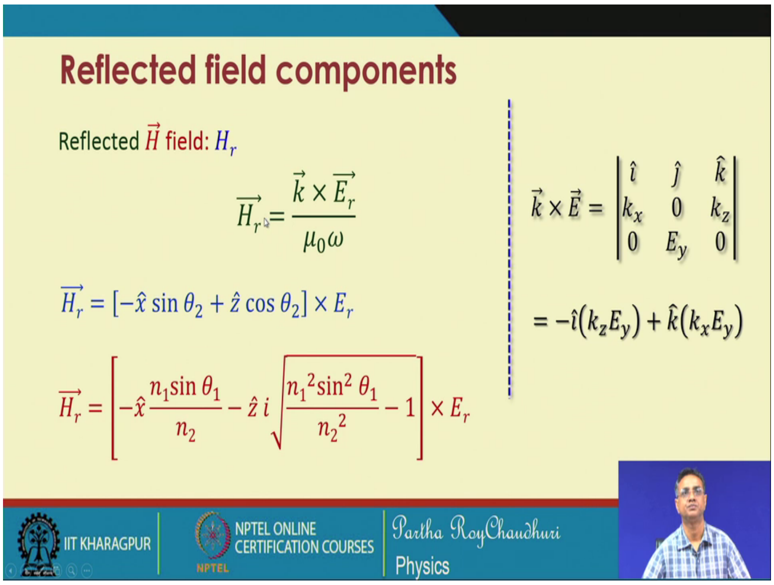

So, these 2 waves now on super position and the magnetic field components are in this case will be given by this equation.

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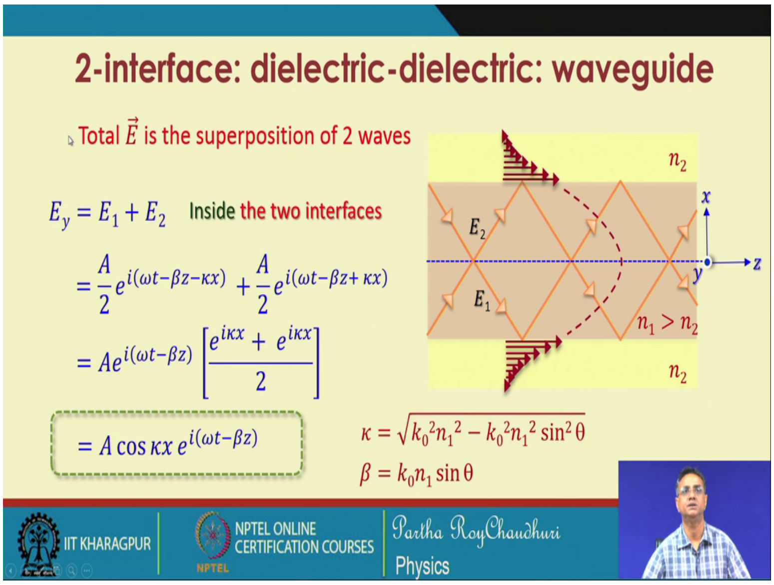

Now the total electric field is the super position of these 2 waves that is e 1 and e two. So, if I take the super position very simple manipulations shows that this can be written in terms of e to the power of plus i kappa x plus e to the power of there should be 1 minus sign here minus i kappa x. So, this quantity is nothing, but the cosine of kappa x. So, this quantity is a. So, you can see that within the within the medium of a refractive index n 1 which is the denser medium the wave that varies along the x direction with a function as a nature as k cosine of kappa x and this is the z component of the propagation vector which adds to phase.

So, you can see that there is a sinusoidal variation of the power the cosine variation of the power within the medium bound by 2 interfaces which gives you that the wave is confined here and you have a evanescent tail evanescent wave which is outside the medium that also we have seen. So, if we look at this structure and if we look at these superposed waves of the 2 waves which are simultaneously being reflected from the 2 interfaces, they will add up and there are many more waves. So, put together we can get an equation for the for the electric field which is a cosine function and that tells you that the wave will be confined within these and a part of the wave will be evanescently decaying along these.

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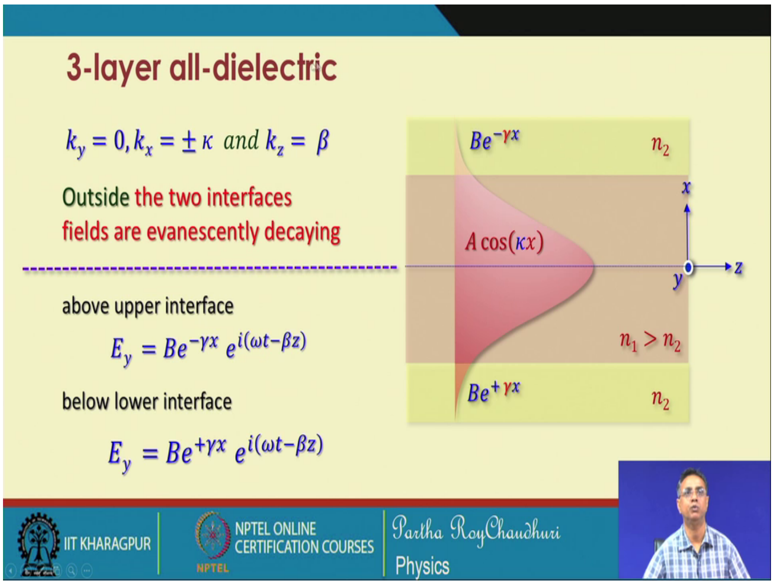

So, this gives you the concept of wave guiding. So, outside the 2 interfaces, you have a decaying field and within the medium that is within the high refractive index medium you have a wave which is a cosine variation depending on the phase. It could be a sin variation or it could be a super position of sin and cosine variation that all depends on the waves which are the phase of the waves which are reflected and from this and this interface.

Let us suppose the wave which is reflected from here and the wave which is reflected from here they are out of phase by pi by 2, then this cosine theta cosine of kappa x will now become sin of kappa x and if since all such wave are possible. So, it will be in generally a super position of k cosine kappa x a cosine kappa x plus b sin kappa x. So, inside you will get all possible variation of this is what we call the mode of the structure.

So, the above the upper interface you have an electric field which is given by this because if x is positive and the field is exponentially decaying and below the interface x is negative the field is exponentially decaying. So, we have these 2 expressions for these 2 regions that is the upper region and the lower region and for the in between we have that equation.

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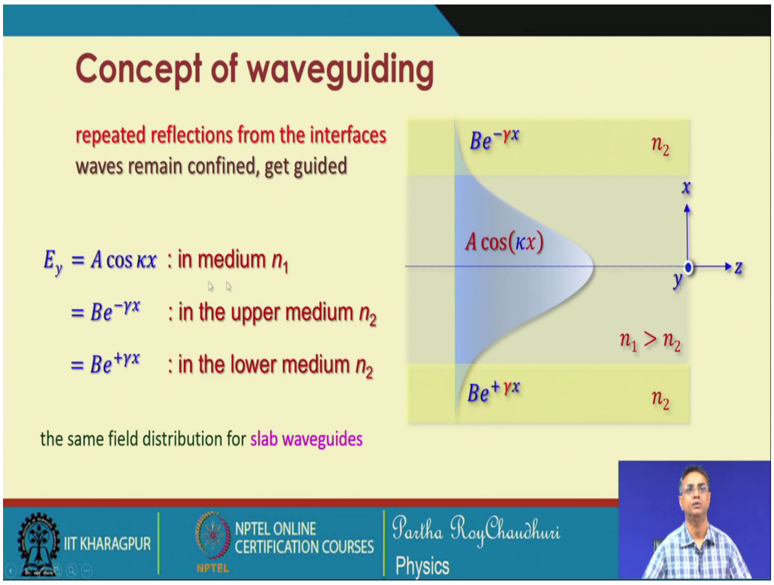

So, if I put all the all the electric field components across the region, we can write E y equal to a cosine, that is for this intermediate region that is for the region in which the wave is mostly confined and if B e to the power of minus kappa x gamma x will be for this region of course. We need a coordinate transfer translation for this because it should take x equal to 0 here, then only you can represent by this equation. Similarly if you take x equals to 0, then only you can represent by these, but essentially the field has the amplitude B e and B e they will be taking care of how much part of this field is already decayed.

So, this gives you a very clear understanding of how an optical wave can be confined by successive reflections from the 2 interfaces whether they are identical interface or non identical that will only effect the kind of wave that is propagating through the medium and this gives you the concept of how a wave can be guided by a structure which has at least 2 interfaces. And this the same expressions will also appear when we will discuss the slab wave guide slab optical wave guides the basic optical wave guides, but we will derive this equation from the wave equation with a more rigorous analysis and then we will continue with other aspects of the discussions.

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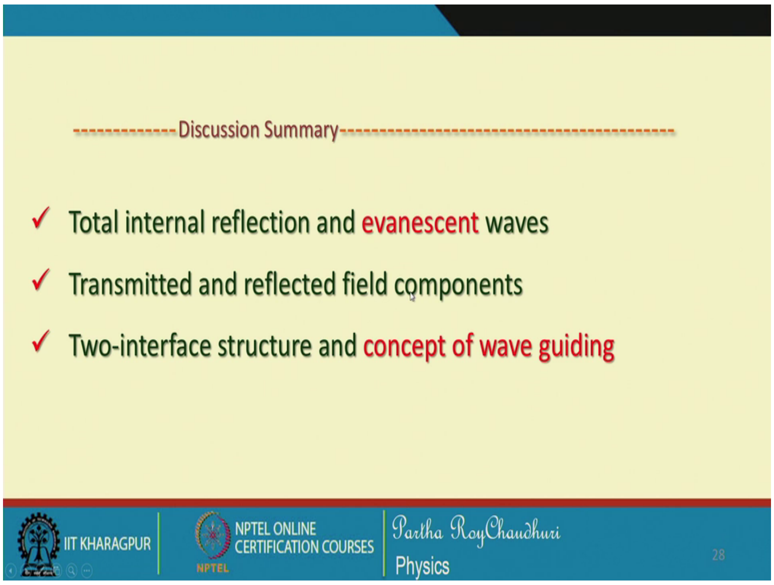

So, in this discussion we have mainly focused on the total internal reflection and evanescent wave that moves with the wave along the interface in the direction of propagation and the transmitted and reflected components all the components of the electric field and magnetic field in details with complete picture.

Then we have brought in 2 such interface structures together, we evaluated the field which will be confined within the 2 interfaces and also the field which will be outside the interfaces and all of them put together will give you a wave which will be guided through the structure by the 2 interfaces.

So, that gives rise to the concept of how the wave optical waves can be guided through such interfaces. We will continue our discussion with these guided wave optical waves in the next occasions.

Thank you.