Modern Optics
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Lecture – 15
Wave propagation in layered structures (Contd.)

We have seen electromagnetic waves, optical waves at the interfaces particularly in terms of the p polarised light that is when the magnetic field lies in the interface plane. We have seen in terms of the reflection, transmission amplitudes, and energy reflection and transmission coefficients. And, now with that understanding we will proceed to continue our discussion about the s polarised wave.

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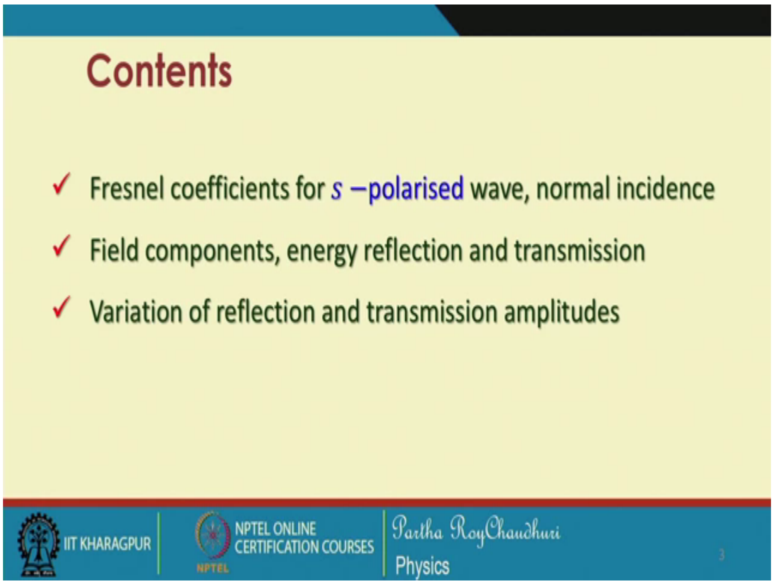

S polarised wave we characterized by the wave when the configuration is like the electric field is in the plane of the lying in the plane of the interface and the magnetic fields are in the x z plane. So, in this discussion we will be talking about the s polarised polarised wave and some cases like normal incidence we will try to compare the normal incidence for the s polarised wave as well as for the p polarised wave. And, we will see that they will reduce to the same equation under the condition of normal incidence.

Then we will continue our discussion with the various field components, the magnetic field components, electric field components in the transmitted and reflected wave. Then the energy reflection and transmission of this s polarised wave. Then we will make a comparison of the variation of the reflection and transmission amplitudes as a function of the incident angle.

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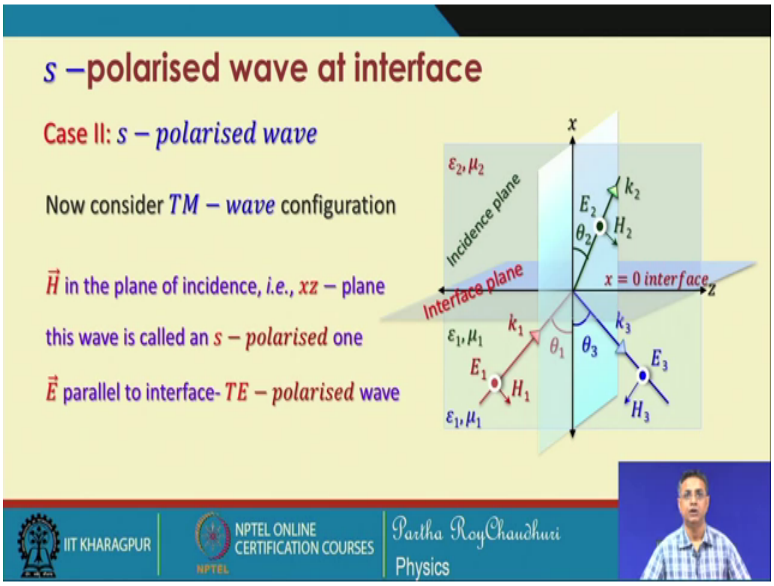

So, this s polarised wave we will call this particular case as case II because, we have considered case I as the p polarised wave. Now, this TM configuration we also call this configuration is transverse magnetic because, the sorry this will be transverse electric configuration. H is in the plane of incidence this wave will be called the s polarised wave. E is parallel to the interface plane so, then it is called TE polarised wave, s polarised TE polarised wave ok.

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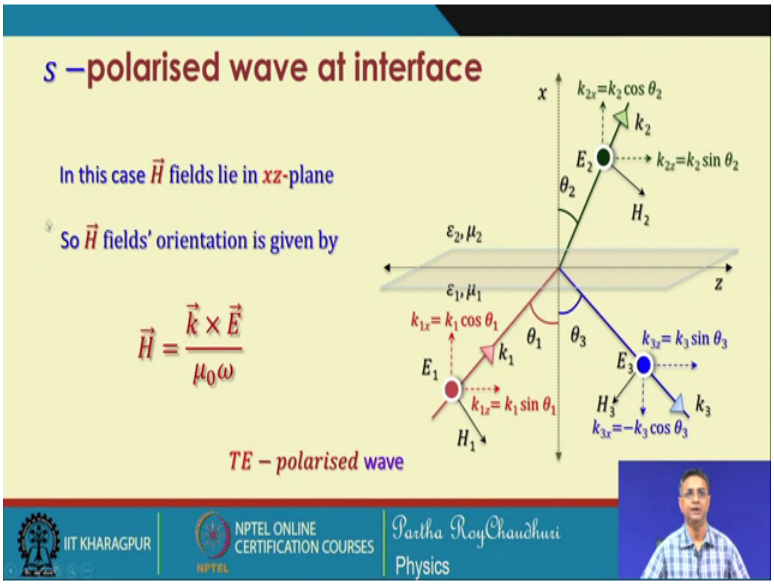

So, in this case the H fields this is the configuration where the electric field is parallel to this interface plane in the incident wave, in the refracted wave as well as in the reflected wave. And, the corresponding magnetic fields can be determined from this electric field, magnetic field, relation equation that is H is equal to k cross E by mu naught omega. So, using this we will find out the magnetic field components all in the reflected and transmitted optical wave for this polarization ok.

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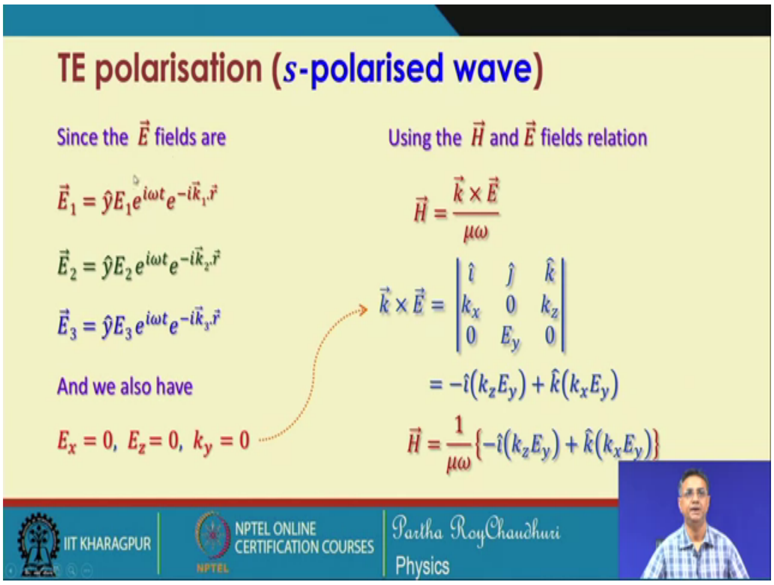

Since, E fields are now only directed along the y direction according to our configuration this perpendicular to the plane of this board is the y direction therefore, electric fields are all in the y direction. So, the incident electric field can be written in this form that with the amplitude E1 e to the power of i omega t e to the power of minus i k 1 dot r. Similarly, for the transmitted electric field we can write this equation only difference is now k 2 r k 2 dot r and for the reflected part of the wave, we can write this electric field equation in this form. And, additionally we also have that because E x and E z are 0 we can make out from here there is no x or z component of electric field.

It is exclusively in the y direction, but k y because this k vector is lying in the x z plane therefore; there is no k y component. So, if we put these three values in the connection equation H and E. Then we will see that k x, k z and E y they will constitute the determinant and as a result you will get H field will be a will be due to the E y and k z, k x and E y combination. So, we can now evaluate the various magnetic field components from this expression.

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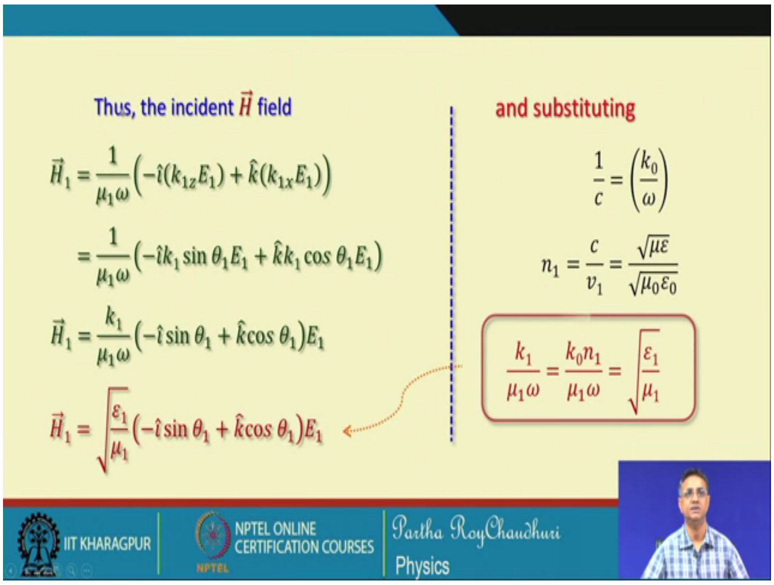

Thus in the elect incident magnetic field we can represent that H 1 is equal to 1 upon this mu 1 omega, then this x component of this combination y z component of this combination. So, if you simplify because k 1 z is k 1 z is k 1 sin theta 1 k 1 x is k 1 cosine theta 1 so, therefore, and likewise for the reflected field k 3 cosine theta 3 and k 3 sin theta 3. So, we will use the two resolved components for the incident, for the reflected wave as well as for the transmitted wave to constitute the individual electric field components. And, in this way then we can write the incident electric field as i k 1 sin theta 1 and k 1cosine theta 1 into E the z component.

So, we can write in this compact form epsilon naught by mu 1 under root of that into minus i sin theta 1 plus k cosine theta 1 into the incident electric field amplitude. And, this relation E 1 by epsilon naught has come from this equation we have seen it previously also, k 1 by epsilon mu 1 omega can be written as k 1 is k 0 n 1 which will include the refractive index of the medium and mu 1 omega mu 1 anyway is mu 0 in this case. So, for each of the medium we can write this 1 by mu 1 omega is equal to epsilon by mu 1 epsilon 1 by mu 1 under root of that. So, we will use this value for each of the magnetic field components.

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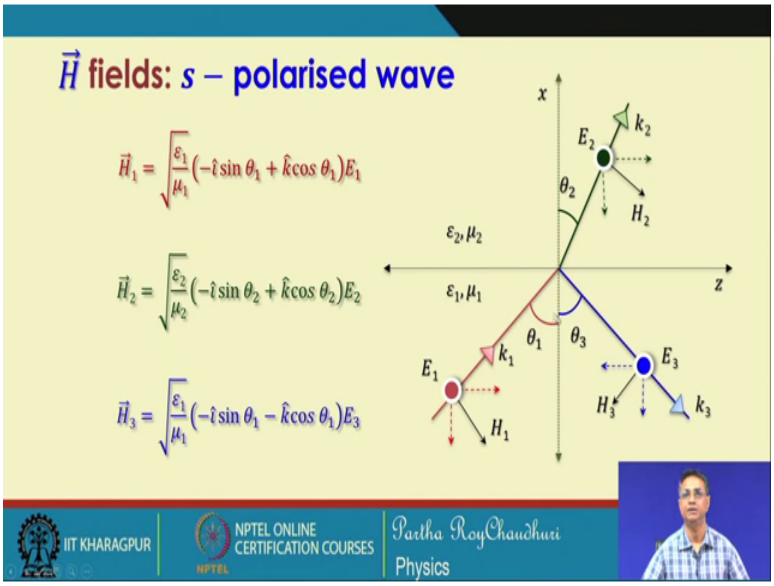

So, doing that we can write that the incident magnetic field you can very easily construct these three components of the magnetic field just by looking at this diagram, you can see that the electric field is here. So, k cross E will be the magnetic field along these direction and if you resolve these magnetic field into the two components you will have a z component which is positive, but x component is negative and it has appeared in that way exactly in that way and for the transmitted field transmitted magnetic field you have k cross E.

So, the field will be directed in this direction and if you resolve these two components along the z and x direction x will be anyway negative, but z is positive that is how it has appeared. And, similarly for the reflected wave if I decompose k cross k cross E will give you the magnetic field along these direction. If you decompose this H 3 field into two components the x and z components you can write because, you see that x component is negative and z component is also negative therefore, you have both minus signs for x and z components. So, that is how we could constitute the magnetic field components in the incident reflected and transmitted optical wave when the polarization is s polarized.

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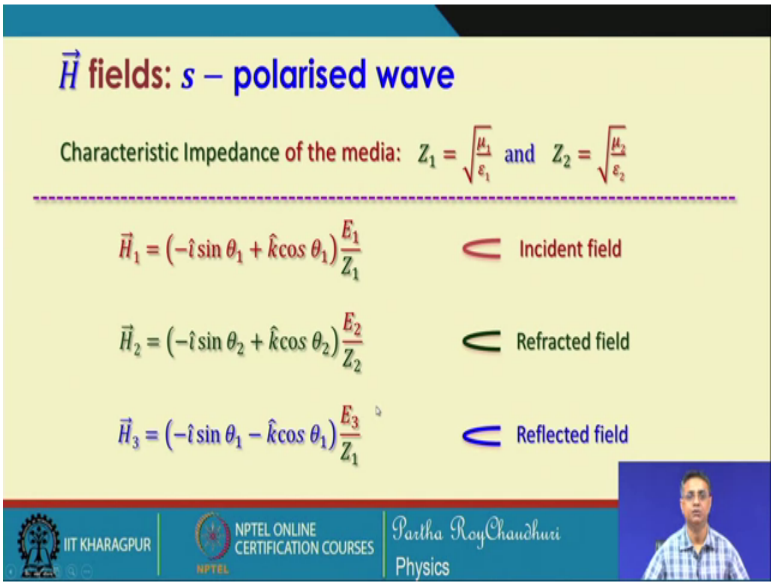

Now, this Z 1 we use for mu 1 by epsilon 1 under root of that this is what is called the characteristic impedance of the medium. This is the property of the medium which includes by this quantity Z 1 and Z 2 similarly this. So, if I substitute these values E 1 in the in these equation that is epsilon 2 by mu 2 epsilon 1 by mu 1 and so on.

So, we can write each of them like E 1 by Z 1, E2 by Z 2 and this is also E 3 by Z 3 will be equal to E 3 by Z 1 because, Z 1 and Z 3 they are the same medium. So, I have been able to write the incident wave, refracted wave, and reflected wave the magnetic field in this three equations where I have used Z 1, Z 2 and Z 3 equal to Z 1 as the characteristic impedance.

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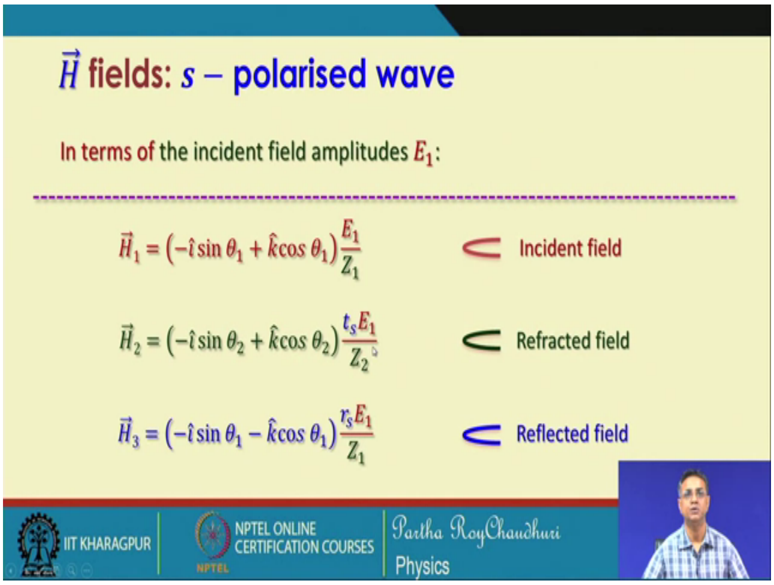

So, now if I include the reflection amplitude and transmission amplitude in the magnetic field components. Then we can write down this equation in this form all in terms of the incident electric field amplitude, for the incident wave we have this equation as it is. But, for the transmitted wave there will be a transmission amplitude of the electric field E 1 and for the reflected wave you have a reflection amplitude multiplied by the incident electric field.

So, they will be the most reduced form of the magnetic field components in the three incident field, the refracted field, and the reflected field. So, we have all the three electric field components and the three magnetic field components. Now, we can derive, we can calculate all that is needed for the phenomenon of reflection and transmission through the interface.

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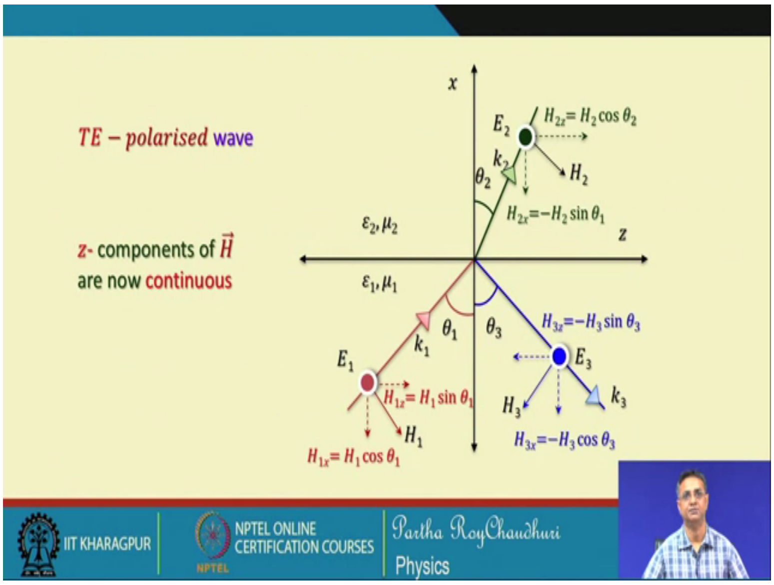

So, for TE polarised wave z components of H these are now continuous this is the boundary condition, z component of H means this is the z component, this is the z component, and this is the z component. So, these two put together must be equal to this. So, that is the continuity condition for the magnetic field.

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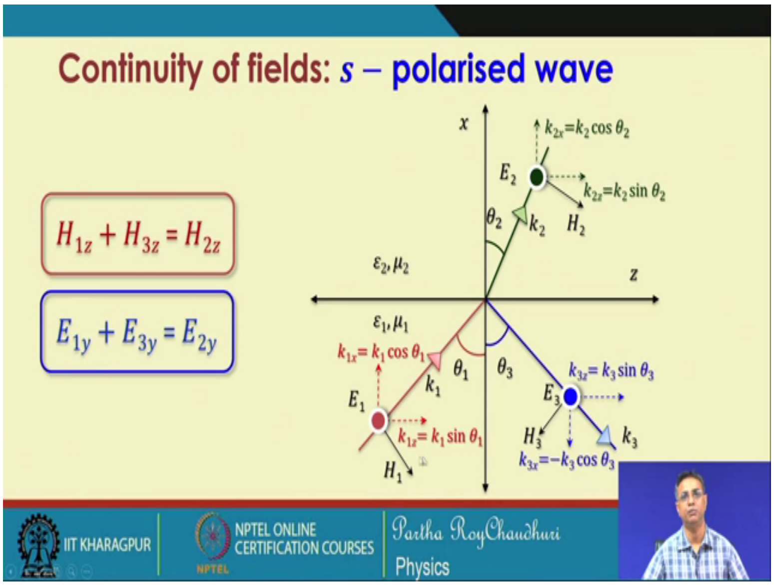

So, if I use both the continuity condition that is the y component they are already continuous because they are along y direction they are along the interface plane. So, electric field E 1 y plus E 3 y is equal to E 2 y they are already continuous, but for the magnetic field we will have to take the z component that is E 1 z, E 1 z plus E 3 z must be equal to E 2 z. So, we will use these two continuity conditions to conclude, to derive the reflection and transmission amplitudes and energy and reflection and transmission coefficients.

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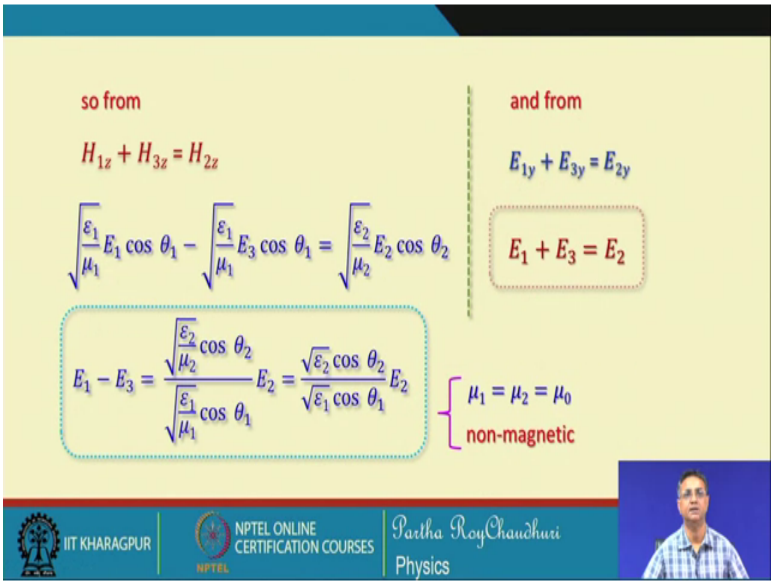

So, from this first boundary condition that is the tangential component of the magnetic field will be continuous, we can write this magnetic field in terms of this because anyway epsilon 1 by mu 1 E 1 cosine theta 1 will represent this H 1 z and we have seen this will be for H 3 z and for H 2 z. So, this is the equation that gives you the continuity condition for the magnetic field and if we rearrange this equation in terms of E 1 and E 3 in the left hand side on the right hand side you will have this E 2 multiplied by this factor.

So, E 1 minus E 3 is equal to this quantity which includes the permittivity constants of the medium multiplied by E 2 and from the electric field continuity condition H component of the electric field they are already continuous. So, E 1 plus E 3 equal to E 2 now we have a pair of equations where the left hand side is E 1 minus E 3 equal to some quantity into E 2 and on the right hand side you have E 1 plus E 3 equal to E 2. So, we can do some algebraic manipulation some calculation to arrive at the connection between E 1 and E 2 and E 1 and E 3.

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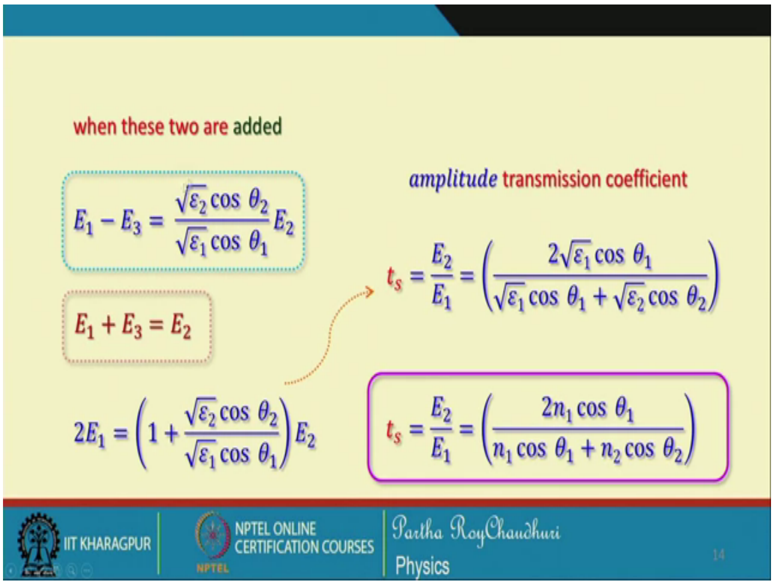

So, let us do that E 1 minus E 2 is this equation which is due to the continuity of the tangential component of the magnetic field and this is the continuity of the electric field across the interface. So, if we if we if these two are added up then we will be able to eliminate E 3. So, twice E 1 will be simply 1 plus this quantity multiplied by E 2 therefore, from here we can we can write this E 2 by E 1 is equal to is equal to your twice of under root epsilon naught cosine theta by under root epsilon 1 cosine theta 1 plus epsilon 2 cosine theta 2. So, in this form we can arrive at for the ratio of the electric field amplitudes of the incident to the refracted wave.

So, t s that is the amplitude transmission coefficient can be written in this form if I use the property that epsilon 1 is equal to n 1 square epsilon 2 equal to n 2 square. Then you can rewrite this equation for the expression for the transmission amplitude for the s polarised wave, in this form that is twice n 1 cosine theta 1 which is a very well known equation for the parallel polarization of optical waves at the interface

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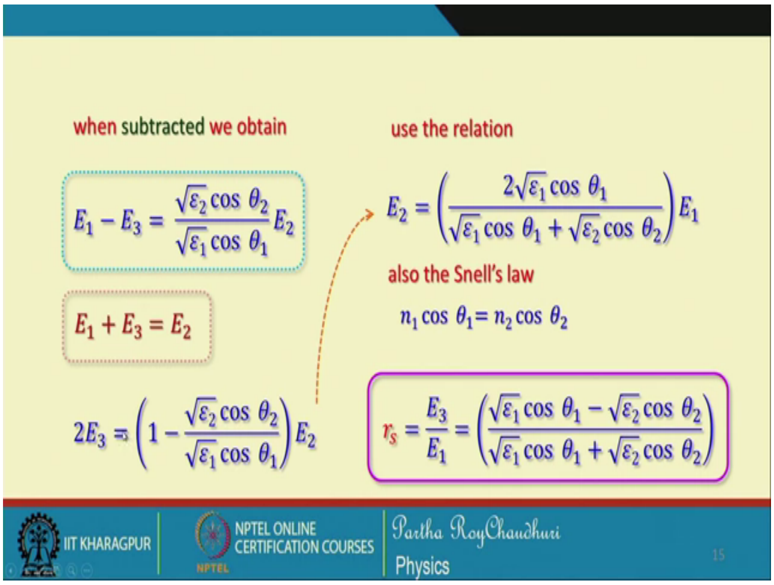

And, the next step is to do the other part of the manipulation that is when you subtract these two equations. This equation is for the tangential component of the magnetic field this is for the electric field continuity. So, if I subtract these two equation then you will get twice E 3 equal to 1 minus this quantity multiplied by E 2 which gives you that, but I need a relation which will involve E 3 and E 1, but this is in terms of E 2.

So, I will have to replace this E 2 in terms of E 1, but knowing the E 2, E 1 connection from the from the previous relation that is the transmission amplitude E 2 and E 1 they are related in this way. Therefore, we can use that relation for E 2 equal to this quantity into E 1 and also from the Snell's law, we can write n 1 cosine theta 1 equal to n 2 cosine theta 2 this using this we can arrive at this equation.

So, r s that is E 3 by E 1 that is the amplitude reflection coefficient E 3 is the reflection amplitude E 1 is the incident amplitude of the electric fields. So, the ratio of the incident amplitude to the reflected amplitude of the electric fields can be represented in this way just by substituting the values of E 2 in terms of e one in these equation we can arrive at this relation.

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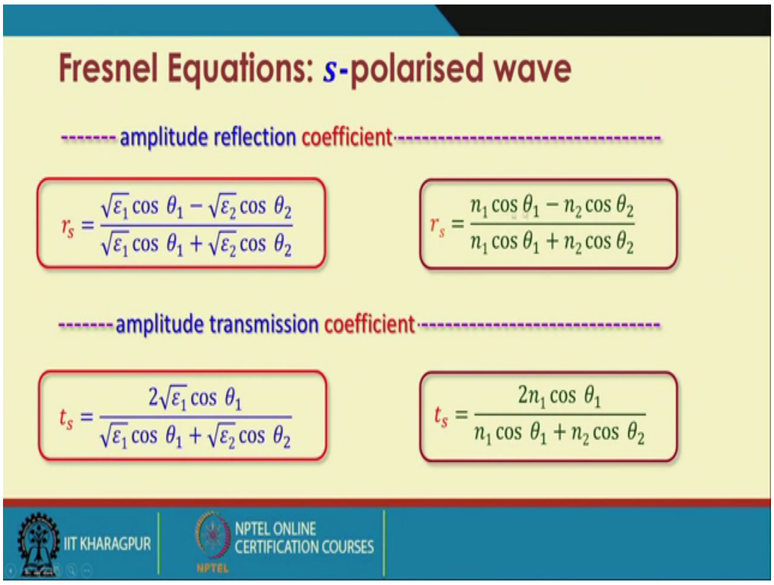

Therefore, to summarize this that we have been able to estimate the reflection amplitude for the s polarised wave as this or it may be expressed in these way when we mention the permittivity, but if we if we express in terms of the refractive indices, then we can write in this equation for the amplitude transmission coefficient we can write this equation in this form. So, t s will be equal to this so we have these two equations for the reflection amplitude coefficient and this is for the transmission amplitude coefficient.

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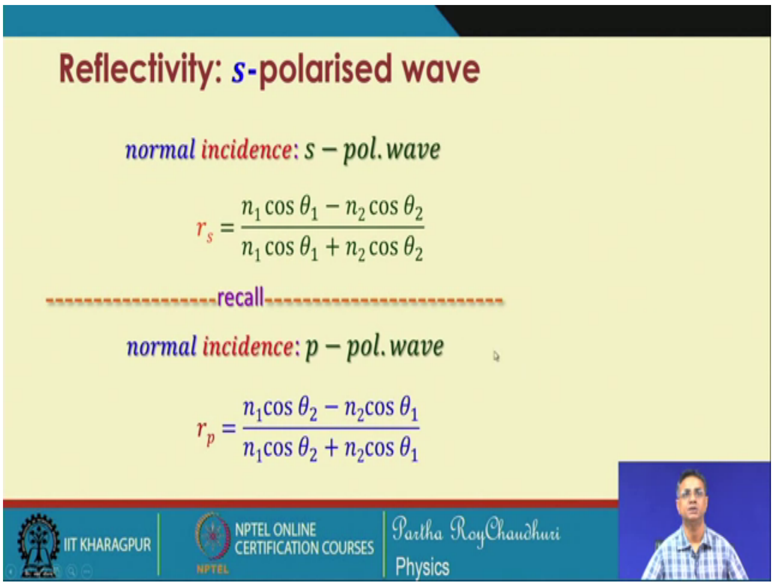

So, we will use these two equations to look at the other possibilities for normal incidence of the s polarised wave this is one very interesting case, that we can use this cosine theta 1 to be equal to 1 for normal incidence. And, then we will we can arrive at a relation that for normal incidence the reflection amplitude coefficient will be n 1 minus n 2 by n 1 plus n 2. And, this in this connection we will also recall the for the normal incidence of the p polarised wave which is given by this we can substitute the value of theta 1 and theta 2 as 0. And, you can see that both of these equations will give you the same amplitude reflection coefficient whether it is a it is an s polarised wave or a p polarised wave.

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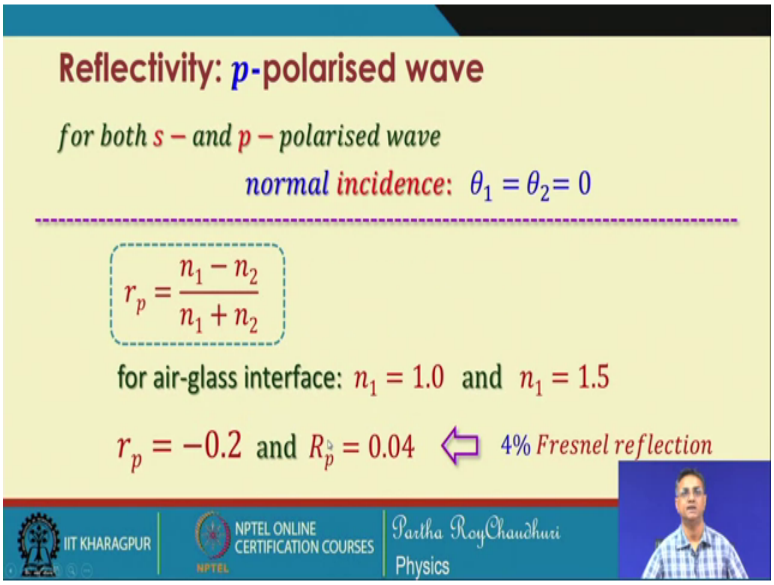

So, let us check that relationship for both s and p polarised life the normal incidence theta 1 will be equal to theta2 equal to 0 and if I substitute in these equation I get n 1 minus n 2 by n 1 plus n 2 to represent the amplitude reflection coefficient. But, this is true the same thing will come out if I substitute in these equation that is cosine theta 1 and cosine theta 2 are put to be equal to 1 then r p will be again n 1 minus n 2 by n 1 plus n 2.

So, this is r p and r s both are given by this n 1 minus n 2 by this reminds you about an interesting consequence that for air glass interface we usually quote a Fresnel reflection of 4 percent has taken place. So, this can be quickly made out from here if you put n 1 equal to 1.0 for air refractive index and for glass that is n 1 equal to 1.5 then r p will be equal to minus 0.2 just substitute here and then the amplitude reflection will be 0.04 that is 4 percent Fresnel reflection right.

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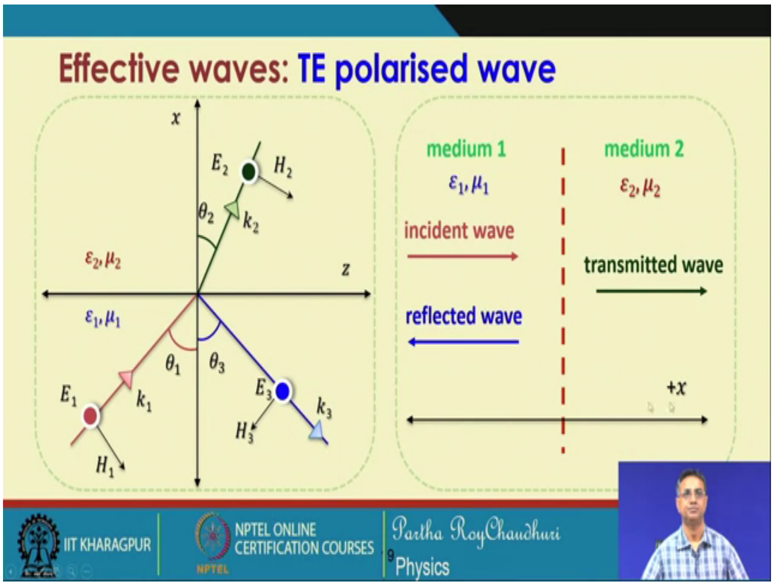

Now, the effective arrangement of this is like this you have an incident wave which is effectively traveling along the x direction reflected wave is in the backward direction because this is the interface which is perpendicular to this and the transmitted wave is from the interface. So, this with this arrangement we need to calculate how much is the x component of the energy that is going along this or z component of the energy that is going along the z direction along the direction of the interface in order to find out how much is the energy reflection.

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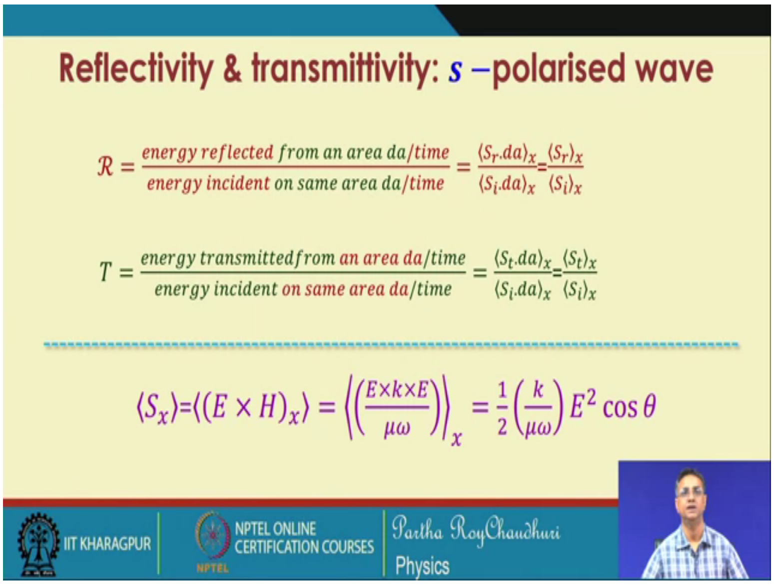

So, the energy reflection coefficient that is the reflectivity as we have defined earlier is the energy reflected from an area da per unit time an elementary area da per unit time and the energy incident on the same area da per unit time if you take the ratio that gives us the reflectivity. So, in terms of the pointing vectors we can write this the s component of the pointing vector because the reflection is along the x direction.

This reflection or transmission for both the things whether it is positive or negative x axis it is only along the x direction. And, similarly the transmission energy transmission coefficient that is the transmittivity is defined in the same way that is energy transmitted from an elementary area infinitesimal elementary area per unit time by the energy that is reflected on the same elementary area da per unit time. So, that again gives you this pointing vector ratio relationship for the x component for the transmitted wave and we know that this S x writing exclusively in terms of the electric and magnetic field vectors we the who are already known to us by now for this s and p polarised cases individually all the components. So, we can easily evaluate this expression.

Now, writing for H equal to k cross E by mu omega we can reduce this expression to this form and then on evaluation we can get that the s component of the pointing vector when time averaged I will get this equation. So, we will now evaluate this expression in order to find the energy reflected or transmitted that is energy flowing along the x direction if it is in the negative x direction it is the reflection if it is in the positive x direction it will be the transmission.

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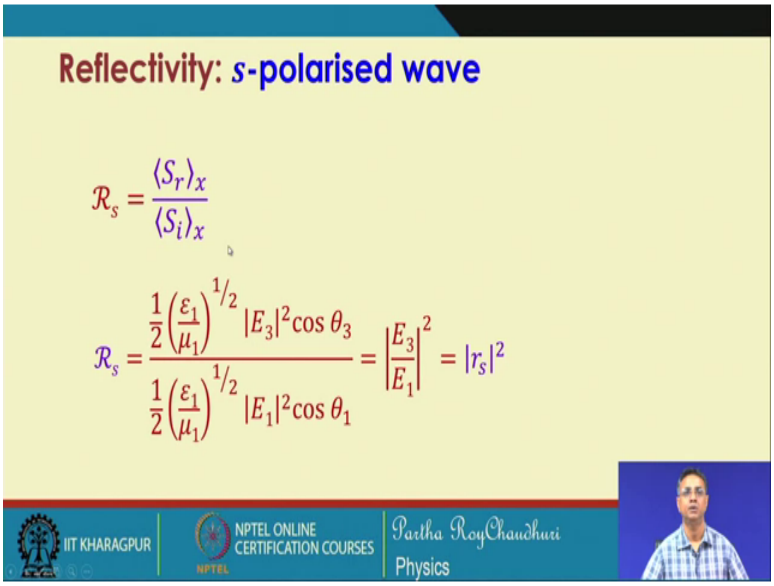

So, let us calculate that for the reflectivity for the s wave we can write in this equation. So, on evaluation you can see that because it involves only the first medium that is the lower medium. So, epsilon 1 by mu 1 epsilon 1 by mu 2 these things will cancel out and giving rise to E 3 by E 1 square mod square which is equal to r s square.

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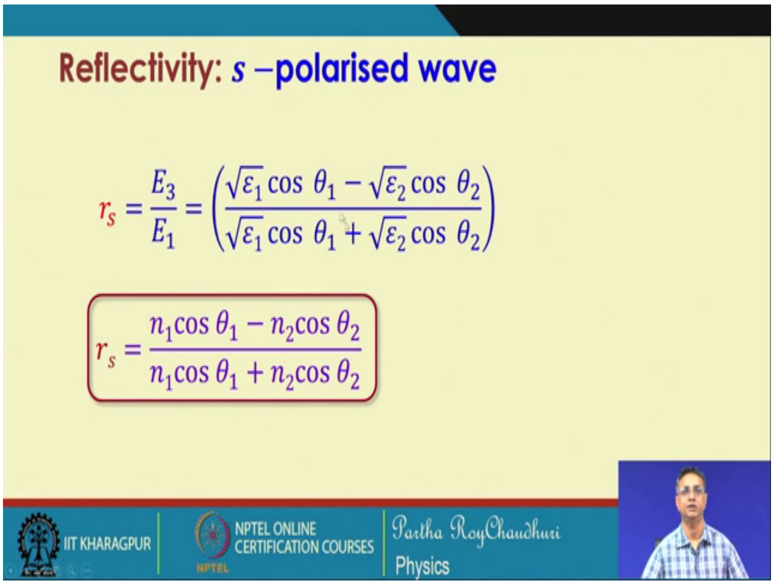

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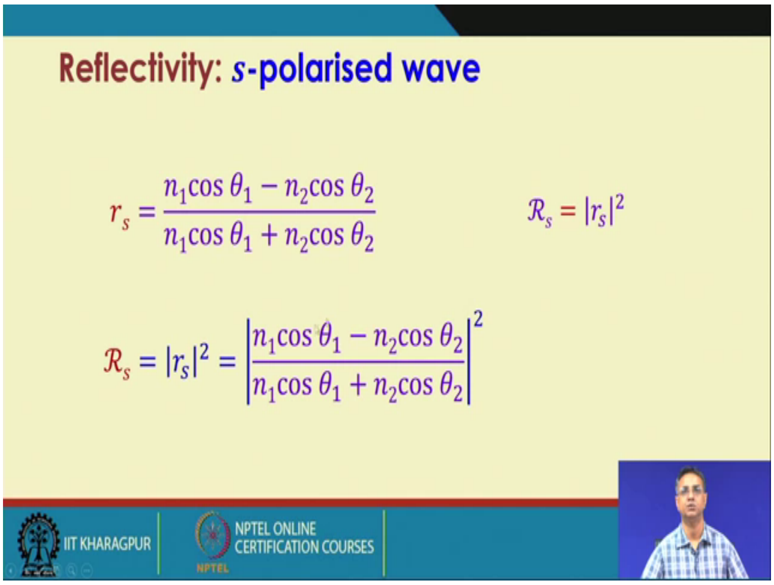

And, then the reflection amplitude reflection coefficient if I use for r s square then we can write this equation as just the mod square of this r s square.

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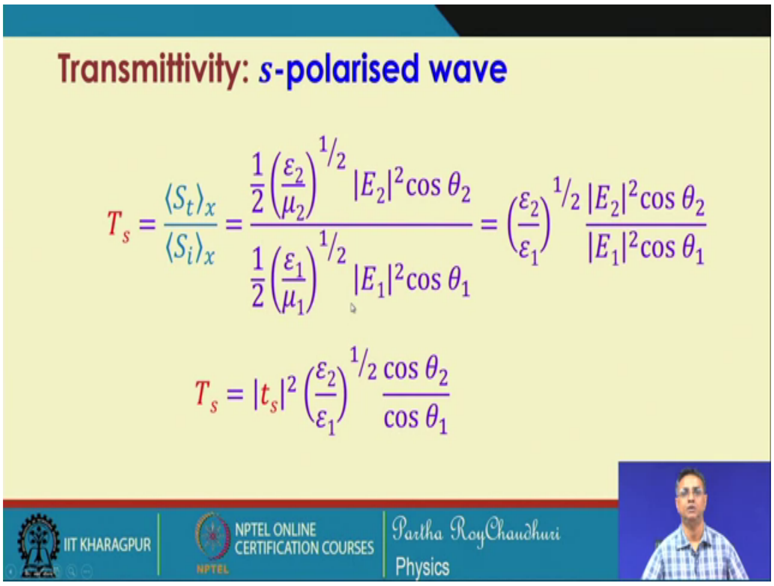

But, for the transmittivity because it involves both the lower and upper media both the media across the interface; so, this epsilon1and epsilon 2 will now be retained in the expression. And, this transmission energy transmission coefficient will be in terms of the amplitude transmission coefficient as well as a multiplier of the angle of incidence and angle of refraction with the respective permittivity constants.

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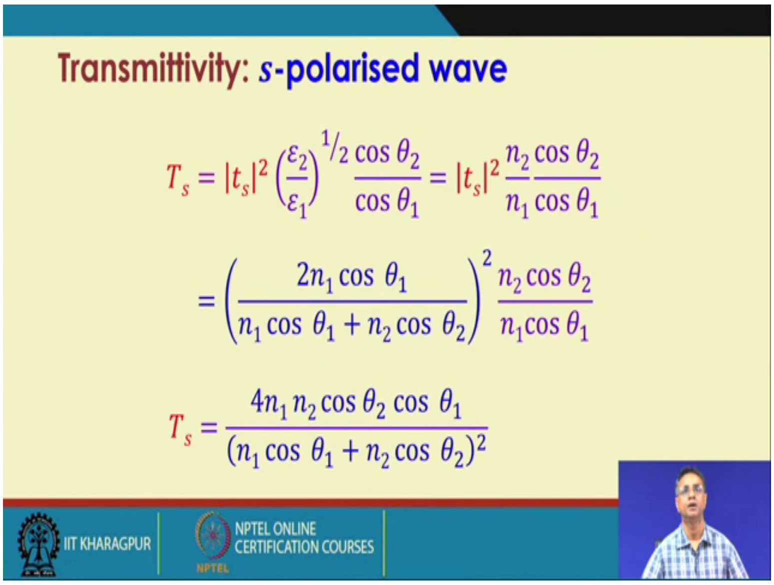

So, transmission with some algebra you can show that this transmission amplitude energy transmission coefficient will be equal to this.

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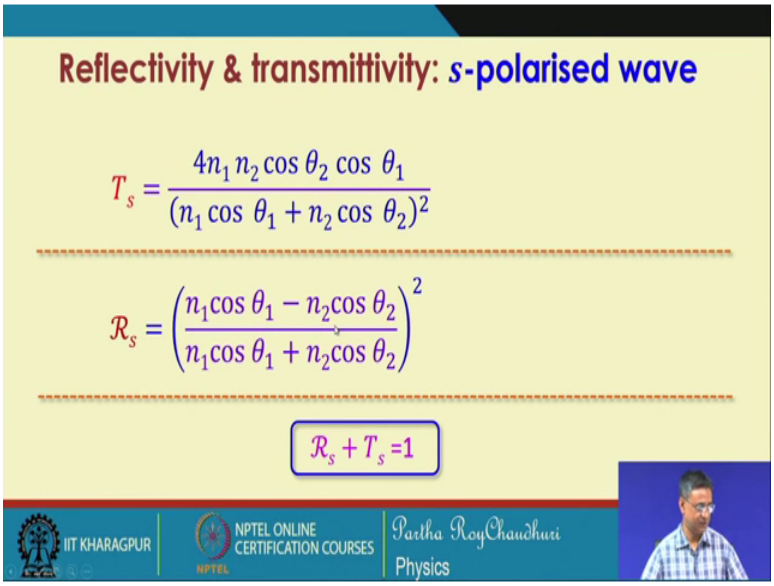

Now, T s and R s we have been able to exclusively determine from the knowledge of the refractive indices and the angle of incidence, angle of refraction transmission.

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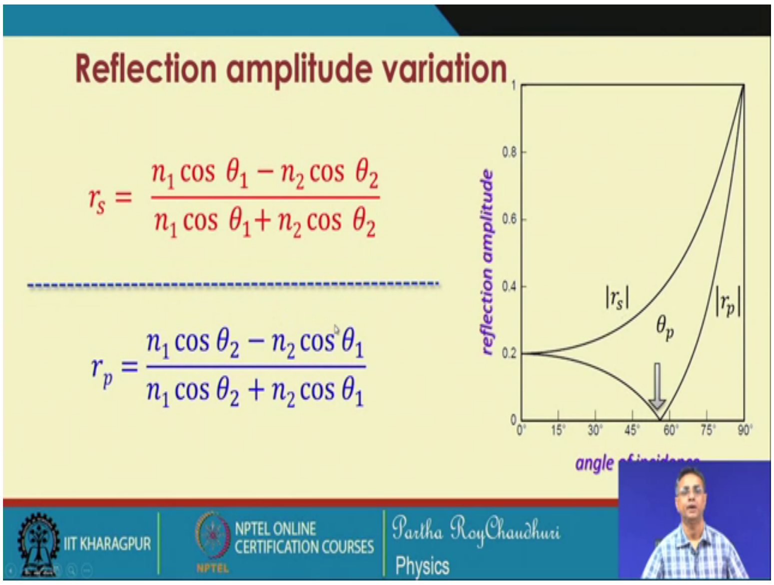

And, you can show that this if you add these two it is very easy to show that r s plus r t is equal to 1 square of this plus will which will have some n 1 n 2 cosine theta 1 cosine theta 2. So, if you do this it will become plus. So, both of them will cancel and will give you unity. So, that means, this shows the energy conservation, the energy in the incident wave, and in the reflected wave, put together the reflected, and transmitted wave put together will be equal to 1. If you assume the incident wave amplitude is energy is unity

Now, there is a comparison want to show that the reflection amplitude for s wave can be written in this form and you can see that the r s varies like this say as a function of the angle of incidence theta 1 whereas, r p varies in this way and at this point which we have discussed and we will see it again that this is the Brewster's angle now which depends on the refractive index of the of the media involved. The minimum value of the reflection amplitude for the p polarised light, but for s polarised light there is nothing as such because they are continuous the electric fields are parallel to the plane of the interface.

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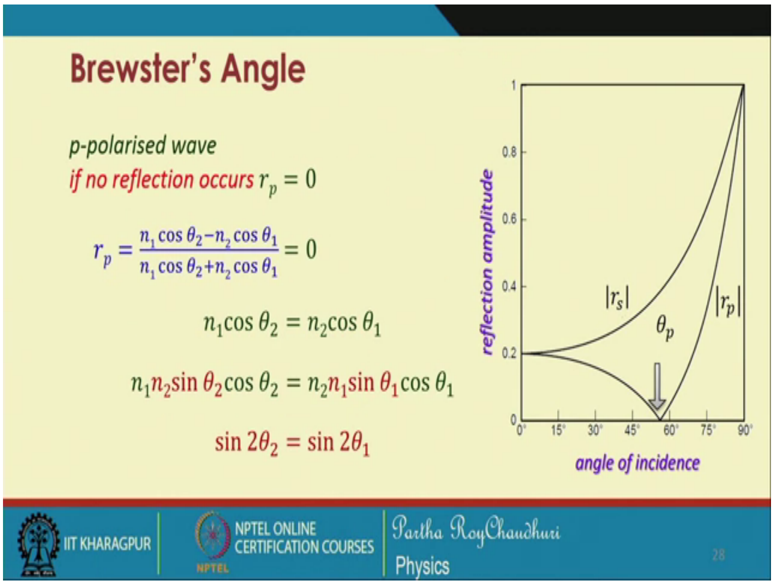

So, for p polarised life if there is no reflection r p equal to 0 and that gives you the condition that n 1, n 2 sin theta, if you connect these two you can write that sin 2 theta 2 equal to sin 2 theta 1.

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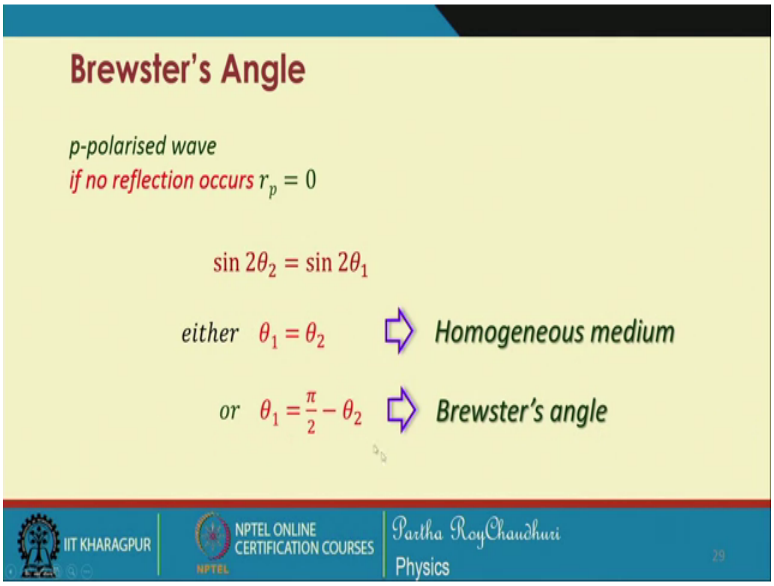

This will give you two possibilities that either theta 1 equal to theta 2; that means, there is no interface this is all one medium continuous and the wave does not get bent or distorted because there is no interface. So, theta 1 and theta 2 it emerges out straight along the direction and if theta 1 is equal to pi by 2 minus theta, which will also satisfy this equation that tells you theta 1 plus theta 2 equal to the equal to pi by 2 which is the condition for the Brewster's angle.

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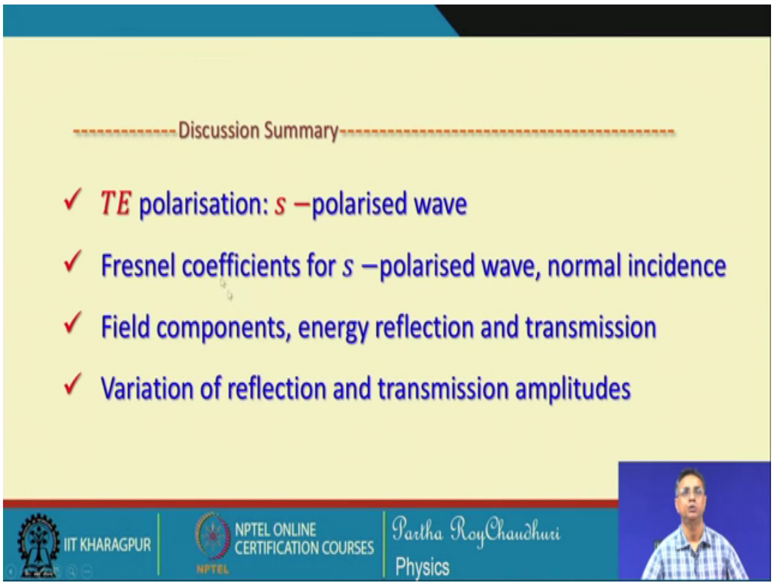

So, in this discussion we have derived the reflection and transmission amplitude coefficients for the s polarised life, then we have considered the case of normal incidence. We also worked out the individual field components for the electric fields and the magnetic fields all in terms of the reflected wave transmitted wave and in terms of the incident wave amplitude also, utilizing the reflection and transmission amplitude coefficients.

Then we have also compared the variation of the reflection and transmission amplitudes as a function of the incident angle and we could locate the Brewster's angle for the p polarised wave from this discussion. We will continue this discussion for the total internal reflection.

Thank you.