Modern Optics
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Lecture – 14
Wave propagation in layered structures (Contd.)

So, we have seen waves the basic configuration of electromagnetic waves at the interface of two dielectric media.

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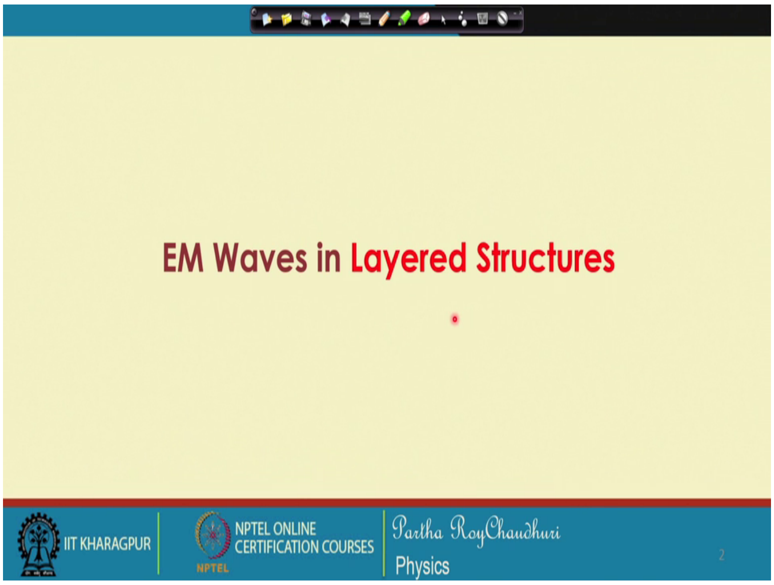

EM Waves in Layered Structures

And, now we will consider you have seen that the reflection amplitude reflection and amplitude transmission coefficients for the p-polarised wave.

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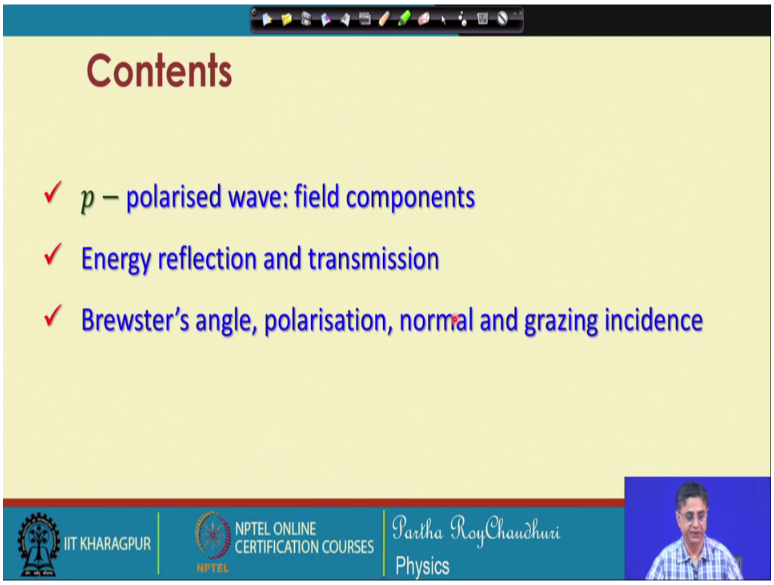

Contents

- ✓ p – polarised wave: field components
- ✓ Energy reflection and transmission
- ✓ Brewster's angle, polarisation, normal and grazing incidence

We will continue this p-polarised wave in terms of how to evaluate the field expressions, field components at the different regions. And, from the from the known expression of the energy amplitude reflection coefficient and amplitude transmission coefficient how to calculate using the pointing vector along the direction of propagation perpendicular to the direction of propagation and so on.

The energy reflection and energy so, the amount of energy that is associated in the process of reflection and transmission, that also we will try to understand. Then we will consider few representative cases in the case of p-polarized wave. For example, normal incidence, grazing incidence then if there is no reflection, then we will consider this individual cases to know more about this p-polarized like when they undergo a reflection transmission at the interface of two dielectric media.

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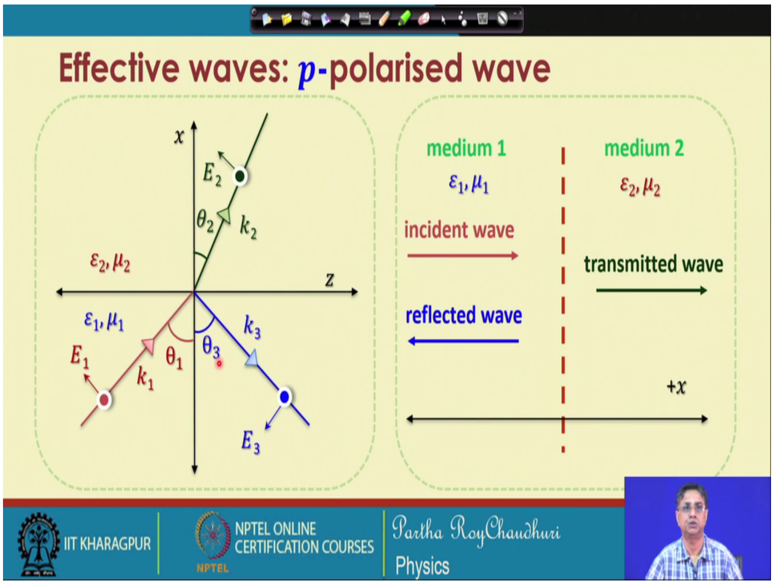

Effective waves: p-polarised wave

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So, so the effective arrangement for this p-polarized wave I try to show here that you have a medium and this is the interface. So, the interface you have an incident media incident electromagnetic wave and this electromagnetic wave is reflected back and the transmitted wave. So, effective arrangement of this configuration you have the electric field which is lying in the in the x z plane. This is your x axis, this is your z axis and perpendicular the plane of the paper is y axis. So, the magic fields are along the y direction k 1 theta 1 f E 2 theta 2 k 3 theta 3; all this things are by now it is known and we have defined those parameters.

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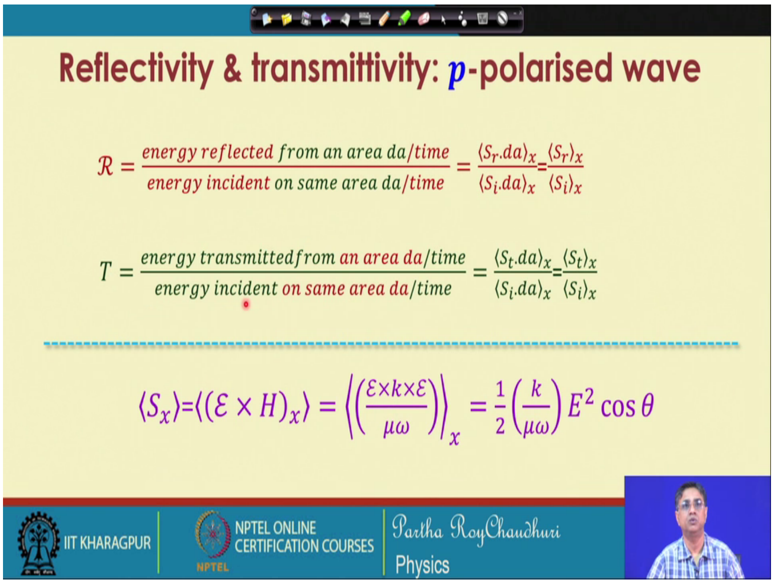

Reflectivity & transmittivity: p-polarised wave

$$\mathcal{R} = \frac{energy\ reflected\ from\ an\ area\ da/time}{energy\ incident\ on\ same\ area\ da/time} = \frac{\langle S_r.da\rangle_x}{\langle S_i.da\rangle_x} = \frac{\langle S_r\rangle_x}{\langle S_i\rangle_x}$$

$$T = \frac{energy\ transmitted from\ an\ area\ da/time}{energy\ incident\ on\ same\ area\ da/time} = \frac{\langle S_t.da\rangle_x}{\langle S_i.da\rangle_x} = \frac{\langle S_t\rangle_x}{\langle S_i\rangle_x}$$

$$\langle S_x\rangle = \langle(\mathcal{E} \times H)_x\rangle = \left\langle\left(\frac{\mathcal{E}\times k\times\mathcal{E}}{\mu\omega}\right)\right\rangle_x = \frac{1}{2}\left(\frac{k}{\mu\omega}\right)E^2\cos\theta$$

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Now, the reflection and transmission in terms of the energy we want to evaluate. So, the energy reflection coefficient is defined as the energy reflected from an arbitrary area da small infinitational area da per unit time divide by the energy incident on the same area da per unit time. So, we have a infinite similar area da on which the energy is incident that amount to be we want to calculate. And how much energy is reflected from that area, this ratio will give me the energy reflection coefficient; in terms of the pointing vectors.

Because, the pointing vector which is associated with the reflected wave is S r which is E cross H and then for the area da we write this S r dot da and take the time average value of the x component of this. Because, we are interested if you go back we are interested in the energy reflection and transmission only along the x direction. So, this is the effective transfer of energy reflected energy from this wall and transmitted energy through this wall. So, it is only the x component that we are interested.

So, this gives you this S r of x S i of x and similarly for the energy transmission coefficient, we went to calculate we define in the same way that is the ratio of energy incident on an infinitesimal small area da per unit time. And, the ratio of this to this energy transmitted from an area da the same area per unit time. Therefore, in the same way we can we can express the energy transmission coefficient as the pointing vector associate with the transmitted wave and the average value of time average value of this.

So, you can represent in this form S t x, x component of the pointing vector averaged the time average value. So, S x can be written as if we write in terms of E and H fields this should E. So, E cross H x component this will be E cross k cross E, this is all known divided by nu into omega which is equal to because k cross E by omega we know that is equal to k by omega. So, you can write this expression in terms of this E square cosine square theta.

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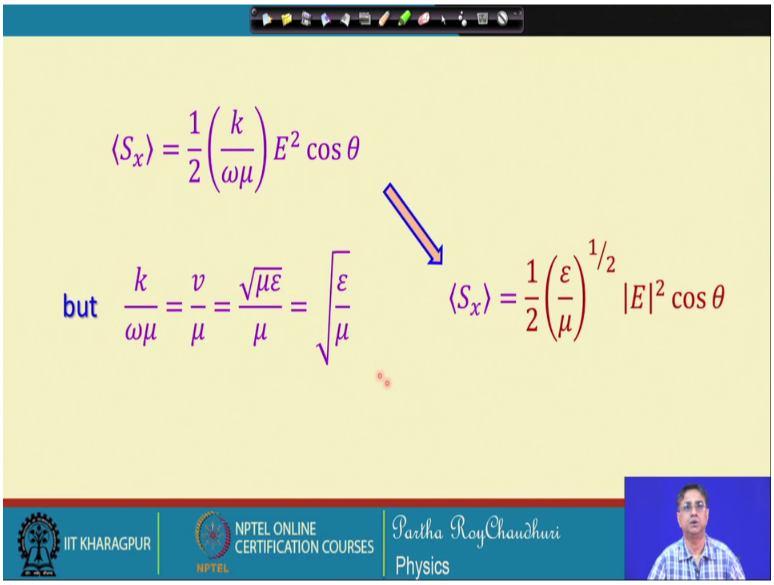

$$\langle S_x\rangle = \frac{1}{2}\left(\frac{k}{\omega\mu}\right)E^2\cos\theta$$

but $\dfrac{k}{\omega\mu} = \dfrac{v}{\mu} = \dfrac{\sqrt{\mu\varepsilon}}{\mu} = \sqrt{\dfrac{\varepsilon}{\mu}}$

$$\langle S_x\rangle = \frac{1}{2}\left(\frac{\varepsilon}{\mu}\right)^{1/2}|E|^2\cos\theta$$

So, this S x half equal to half k mu omega E cross cosine square theta, we will translate this S x as k by mu omega you know k by mu omega may be written as v by mu is equal to mu epsilon under root by mu. So, effectively this gives you the square root of the permittivity of the ratio of the permittivity to the permeability.

So, this is your k by omega mu and this is also the also called the reciprocal of the mu by epsilon not under root is the characteristic impedance of the medium, we will see this later. So, now S x can be written in this form half epsilon by mu square root of that E square cosine square theta. So, we will use this expression for S x for the reflected and transmitted wave to evaluate how much part of the energy is reflected or transmitted in the process of undergoing reflection and transmission from the from the interface.

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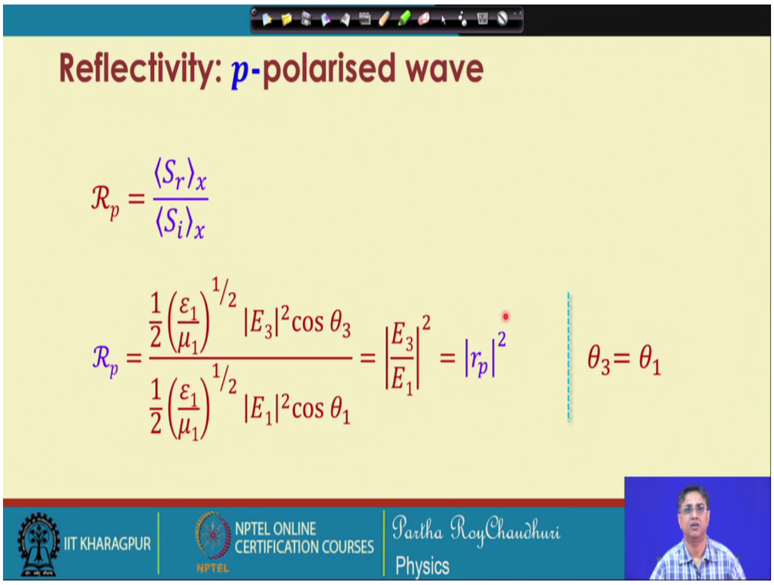

Reflectivity: p-polarised wave

$$\mathcal{R}_p = \frac{\langle S_r \rangle_x}{\langle S_i \rangle_x}$$

$$\mathcal{R}_p = \frac{\frac{1}{2}\left(\frac{\varepsilon_1}{\mu_1}\right)^{1/2} |E_3|^2 \cos\theta_3}{\frac{1}{2}\left(\frac{\varepsilon_1}{\mu_1}\right)^{1/2} |E_1|^2 \cos\theta_1} = \left|\frac{E_3}{E_1}\right|^2 = |r_p|^2$$

$$\theta_3 = \theta_1$$

So, reflection energy reflection coefficient given by this can now be written in terms of this epsilon 1 by mu 1 square root of that in the numerator, which is to represent the time averaged value of the pointing vector along the x direction, in this form for the reflected wave. And, this is the same quantity for the incident wave, but interestingly because the incident waves medium and the reflected wave medium there the same. So, we have seen that epsilon 1 mu 1 and epsilon 3 mu 3, they are the same. So, they will cancel and as a result you just get end up with this expression that E 3 square by E 1 square because, theta 1 is also equal to theta 3.

So, this quantity also goes out, this cosine theta 1 cosine theta 1 in the denominator. They will be cancelled, this part will be cancelled. So, we are just left with E 3 by E 1 mod square which is equal to the nothing, but the r p square this also we have seen the amplitude reflection coefficient. So, energy reflection coefficient and the amplitude reflection coefficient they are related directly by just square of the amplitude reflection coefficient. So, this is an interesting finding, but this is not the same case when we will consider the amplitude transmission coefficient a connection for the amplitude energy transmission coefficient.

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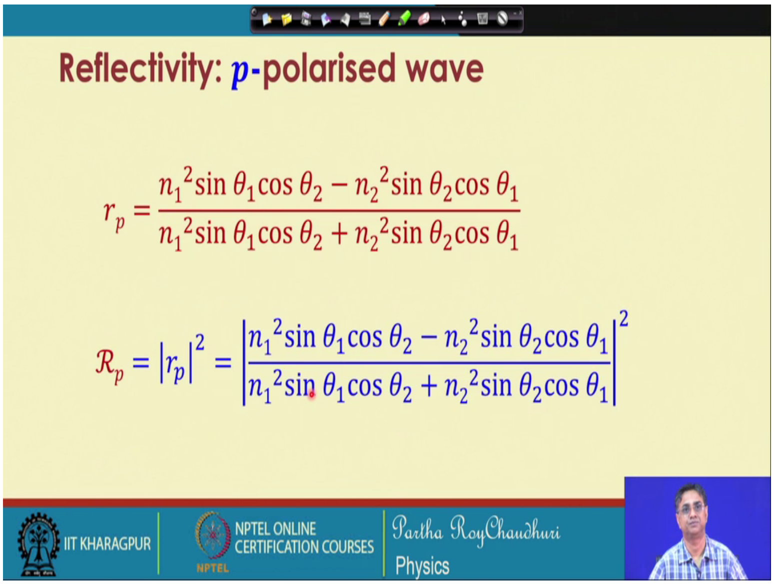

Reflectivity: *p*-polarised wave

$$r_p = \frac{n_1{}^2\sin\theta_1\cos\theta_2 - n_2{}^2\sin\theta_2\cos\theta_1}{n_1{}^2\sin\theta_1\cos\theta_2 + n_2{}^2\sin\theta_2\cos\theta_1}$$

$$\mathcal{R}_p = |r_p|^2 = \left|\frac{n_1{}^2\sin\theta_1\cos\theta_2 - n_2{}^2\sin\theta_2\cos\theta_1}{n_1{}^2\sin\theta_1\cos\theta_2 + n_2{}^2\sin\theta_2\cos\theta_1}\right|^2$$

But, now r p we have seen is given by this. So, capital R p that is the energy transmission coefficient will be equal to the square of this quantity, square of r p square.

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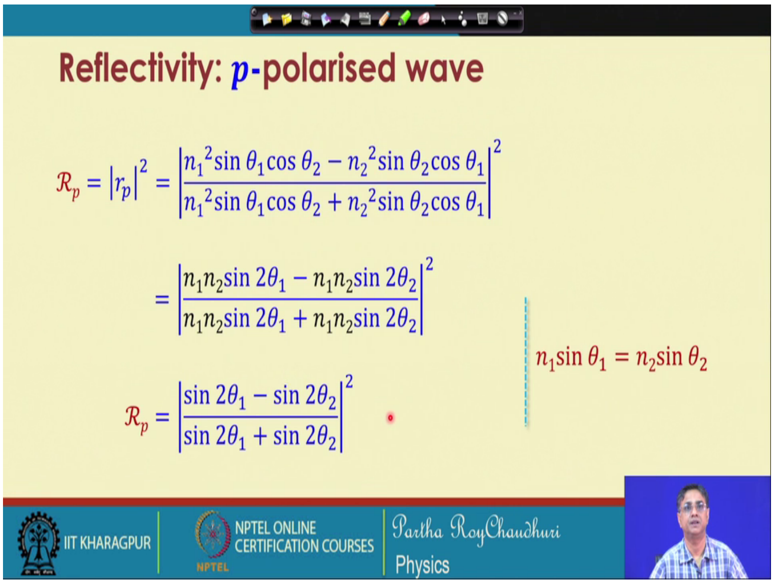

Reflectivity: *p*-polarised wave

$$\mathcal{R}_p = |r_p|^2 = \left|\frac{n_1{}^2\sin\theta_1\cos\theta_2 - n_2{}^2\sin\theta_2\cos\theta_1}{n_1{}^2\sin\theta_1\cos\theta_2 + n_2{}^2\sin\theta_2\cos\theta_1}\right|^2$$

$$= \left|\frac{n_1n_2\sin 2\theta_1 - n_1n_2\sin 2\theta_2}{n_1n_2\sin 2\theta_1 + n_1n_2\sin 2\theta_2}\right|^2$$

$$n_1\sin\theta_1 = n_2\sin\theta_2$$

$$\mathcal{R}_p = \left|\frac{\sin 2\theta_1 - \sin 2\theta_2}{\sin 2\theta_1 + \sin 2\theta_2}\right|^2$$

And therefore, r p square we can simply we can just simplify this expression in terms of because n 1 sin theta 1 can be can be taken as n 2 sin theta 2. So, you have cosine theta 2 and sin theta 2 n 1 and n 2. So, we write in this from n 1 n 2 sin 2 theta 1 sin 2 theta 1 and here n 2 n 2 sin n 2 sin theta 2 is equal to n 1 sin theta 1. So, this will give you n 1 cosine theta 1. So, it should be 1, this should be 2 and similarly for the denominator also, but if you reverse it remains the same.

So, this is how we can we can write this expression, that energy reflection coefficient is equal to sin 2 theta 1 different sin 2 theta 2 divide by some of this sin 2 theta 1. So that means, this is the reduced form that means, that it does not involve any n 1 or n 2. It is only the theta 1 and theta 2, the angle of incidence and angle of refraction. They are going to decide the amount of energy that will be reflected; the amount of energy that will be reflected from the interface.

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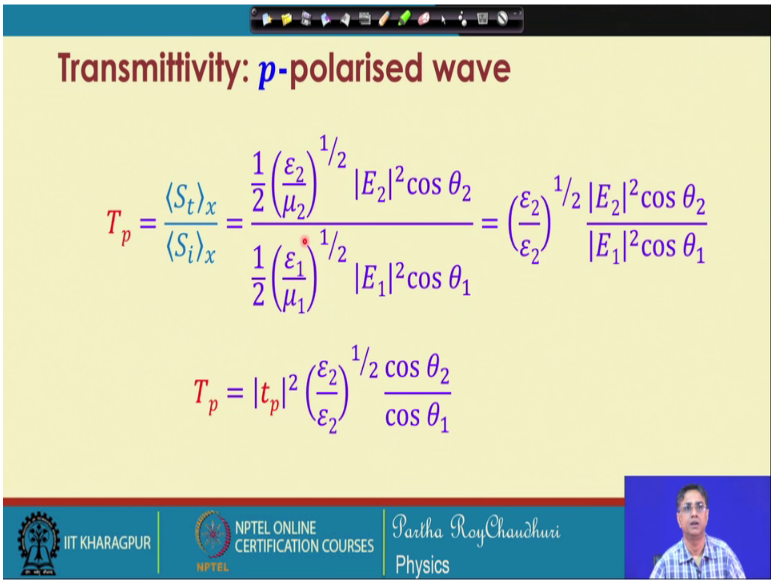

Transmittivity: *p*-polarised wave

$$T_p = \frac{\langle S_t\rangle_x}{\langle S_i\rangle_x} = \frac{\frac{1}{2}\left(\frac{\varepsilon_2}{\mu_2}\right)^{1/2}|E_2|^2\cos\theta_2}{\frac{1}{2}\left(\frac{\varepsilon_1}{\mu_1}\right)^{1/2}|E_1|^2\cos\theta_1} = \left(\frac{\varepsilon_2}{\varepsilon_2}\right)^{1/2}\frac{|E_2|^2\cos\theta_2}{|E_1|^2\cos\theta_1}$$

$$T_p = |t_p|^2\left(\frac{\varepsilon_2}{\varepsilon_2}\right)^{1/2}\frac{\cos\theta_2}{\cos\theta_1}$$

Now, the remaining part that is the energy transmission coefficient we can evaluate in the same way. The pointing vector associated with the transmitted wave and the pointing vector associated with the reflected wave, with the incident wave and you can write this expression because, we have written the general expression for the pointing vector. So, in case of the transmitted wave we will have to write in this form that is cosine theta 2 and E 2 square and for the first medium that is for associated with the incident wave we write this in this form.

But, this time you can see is since mu 1 and mu 2 we have assume that that is there non-magnetic and is equal to mu 0 so, they will cancel. And, it effectively gives you epsilon 2 by epsilon 1 sorry, there is a mistake it should be epsilon 1 epsilon 2 by epsilon 1 under root of that multiplied by this two. And, this time cosine theta 2 and cosine theta 1 theta 1 theta 2 being different they will be retained in the expression for energy transmission coefficient. So, we can write this equation in this form, that energy transmission coefficient is equal to t p square amplitude transmission coefficient square multiplied by the ratio under root of the ratio of the dielectric permitivities. This should be epsilon 1 and the ratio of the angles cosine of the angles.

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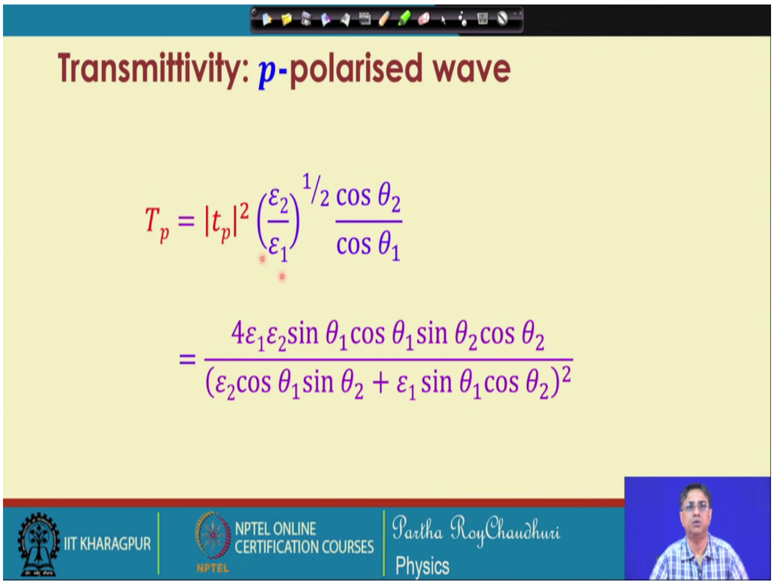

Transmittivity: p-polarised wave

$$T_p = |t_p|^2 \left(\frac{\varepsilon_2}{\varepsilon_1}\right)^{1/2} \frac{\cos\theta_2}{\cos\theta_1}$$

$$= \frac{4\varepsilon_1\varepsilon_2 \sin\theta_1 \cos\theta_1 \sin\theta_2 \cos\theta_2}{(\varepsilon_2 \cos\theta_1 \sin\theta_2 + \varepsilon_1 \sin\theta_1 \cos\theta_2)^2}$$

So, this is how we can write this expression T p equal to this, which can be can be written in this form. Because, if we use this expression for t p that the transmission amplitude transmission coefficient, if we use that expression we can write T p equal to this equation.

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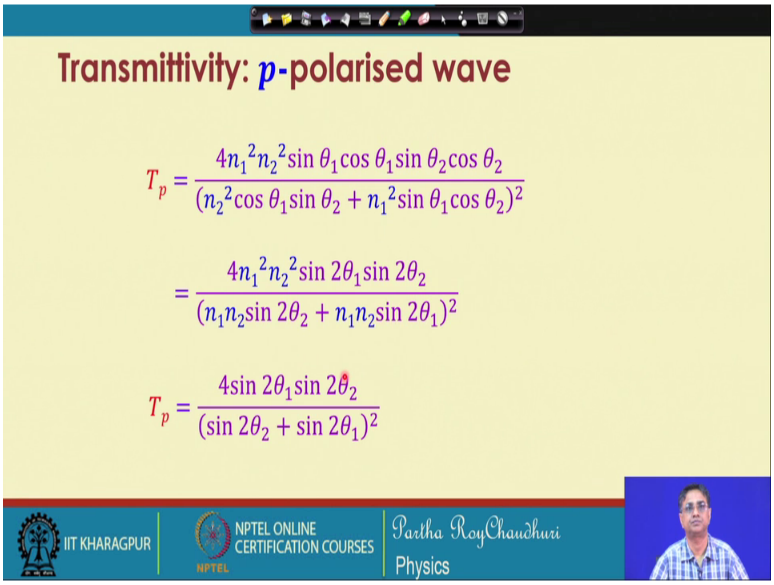

Transmittivity: *p*-polarised wave

$$T_p = \frac{4n_1^2 n_2^2 \sin\theta_1 \cos\theta_1 \sin\theta_2 \cos\theta_2}{(n_2^2 \cos\theta_1 \sin\theta_2 + n_1^2 \sin\theta_1 \cos\theta_2)^2}$$

$$= \frac{4n_1^2 n_2^2 \sin 2\theta_1 \sin 2\theta_2}{(n_1 n_2 \sin 2\theta_2 + n_1 n_2 \sin 2\theta_1)^2}$$

$$T_p = \frac{4\sin 2\theta_1 \sin 2\theta_2}{(\sin 2\theta_2 + \sin 2\theta_1)^2}$$

So, the energy transmission reflection of energy transmission coefficient can be again expressed in this form. And, if I use the same principle that is n 1 sin theta 1 equal to n 2 sin theta 2 which comes directly from the Snell’s law. We can write down the expression for the numerator n 1 sin theta 1 cos theta 1 will give you sin 2 theta 1 n 2 sin theta 2 cos theta 2 will give you sin 2 theta 2 multiplied by 4 and multiplied by 4 in the denominator.

So, you can do some algebra, some simple manipulation of the above expression using n 1 sin theta 1 equal to n 2 sin theta 2, we will end up with this expression. And so, you can write this compact you know expression for the energy transmission coefficient for the p-polarized wave using this. So, we have both the energy reflection coefficient as R p and the energy transmission coefficient as capital T p for the p-polarized wave.

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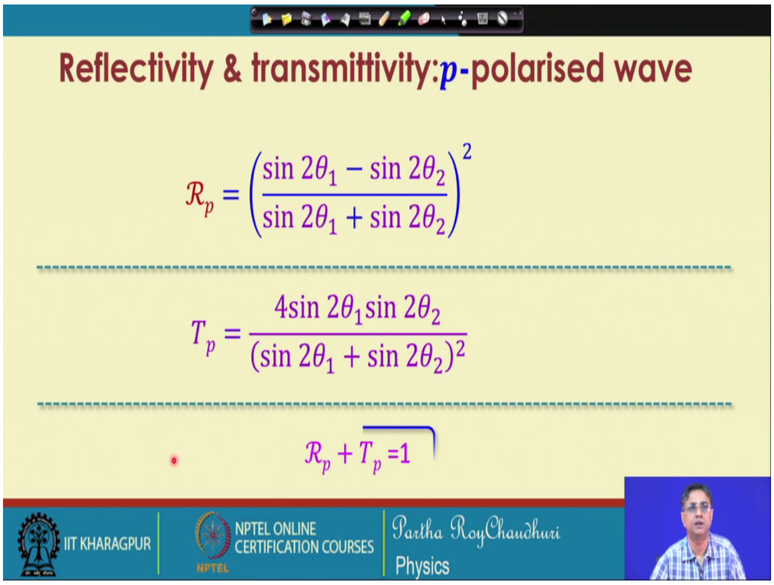

Reflectivity & transmittivity: *p*-polarised wave

$$\mathcal{R}_p = \left(\frac{\sin 2\theta_1 - \sin 2\theta_2}{\sin 2\theta_1 + \sin 2\theta_2}\right)^2$$

$$T_p = \frac{4\sin 2\theta_1 \sin 2\theta_2}{(\sin 2\theta_1 + \sin 2\theta_2)^2}$$

$$\mathcal{R}_p + T_p = 1$$

So, these are the reflectivity and transmissivity. So, this two if we put together you get R p plus T p equal to 1. So, very evident this square of this difference then, if you add 4 sin 2 theta 1 sin 2 theta 2 denominator it remains the same so, it become say equal to 1. So, that tells you that the total energy which is incident is equal to, if you assume it is equal to 1 unity so, that remains the same. So, that is through this process of reflection and transmission the conservation of energy remains valid.

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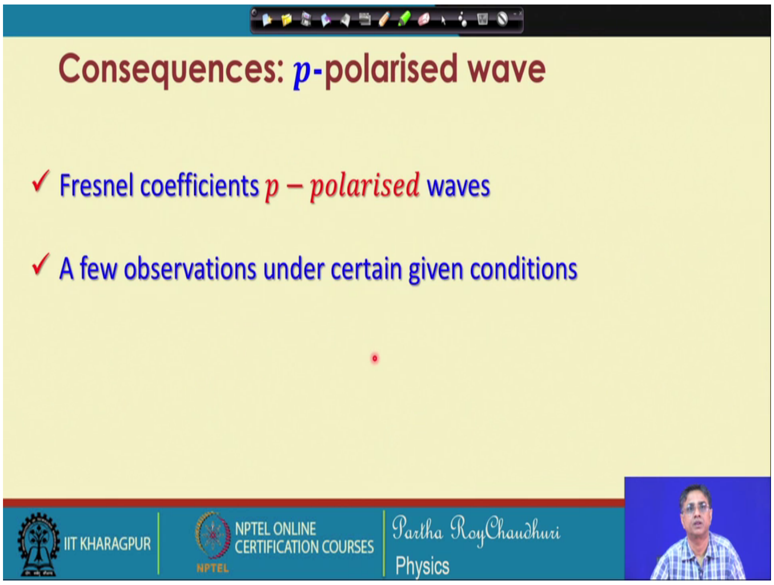

Consequences: *p*-polarised wave

- ✓ Fresnel coefficients *p – polarised* waves
- ✓ A few observations under certain given conditions

Now, with this understanding that we have been able to evaluate the reflection amplitude reflection, amplitude transmission coefficients as well as the energy reflection and energy transmission; that is the reflectivity and transmissivity. So, knowing all these parameters we will now, study few representative cases, few special conditions which are under certain conditions; what happens with the reflection or transmission.

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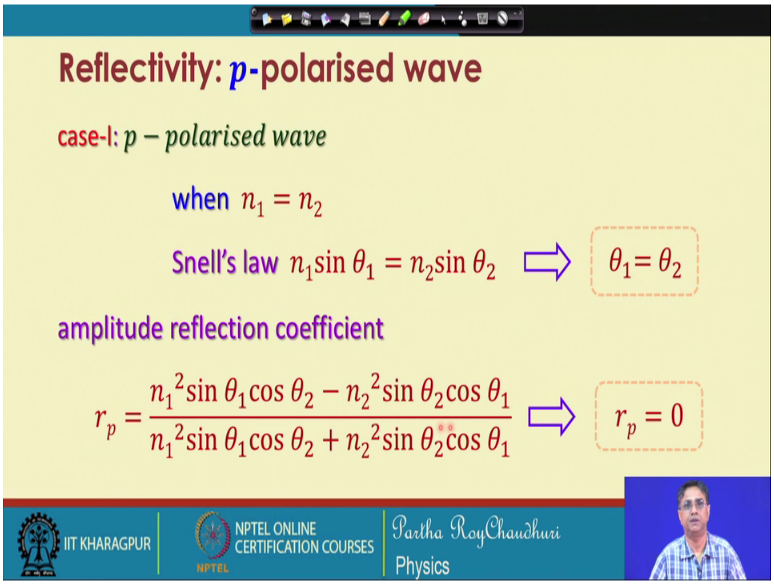

Reflectivity: p-polarised wave

case-I: $p-polarised\ wave$

when $n_1 = n_2$

Snell's law $n_1 \sin\theta_1 = n_2 \sin\theta_2 \Rightarrow \boxed{\theta_1 = \theta_2}$

amplitude reflection coefficient

$$r_p = \frac{n_1^2 \sin\theta_1 \cos\theta_2 - n_2^2 \sin\theta_2 \cos\theta_1}{n_1^2 \sin\theta_1 \cos\theta_2 + n_2^2 \sin\theta_2 \cos\theta_1} \Rightarrow \boxed{r_p = 0}$$

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So, let us take this condition that in the equation for reflectivity or amplitude reflection coefficient and transmission coefficient, if I use this condition the first case that is n 1 equal to n 2. If you use this condition that both the media are having the same refractive indices; in that case, what will happen n 1 sin theta 1 equal to n 2 sin theta 2, this tells you that theta 1 must be equal to theta 2. If theta 1 is equal to theta 2, there are two possibilities: one is that these two media are the same exactly the same; that means there is no interface.

So that means, the ray will not undergo any reflection or transmission, it is simply penetrate through the medium we under deviated and undistorted theta 1 remains exactly equal to theta 2. And, there is a second possibility if I use this n 1 equal to n 2, you find that r p equal to 0; because, n 1 sin theta 1 n 2 sin theta 2. So, all of them will make this right hand side equal to exactly 0. Now, for this case n 1 equal to n 2 we have seen theta 1 equal to theta 2, amplitude reflection coefficient r p equal to this.

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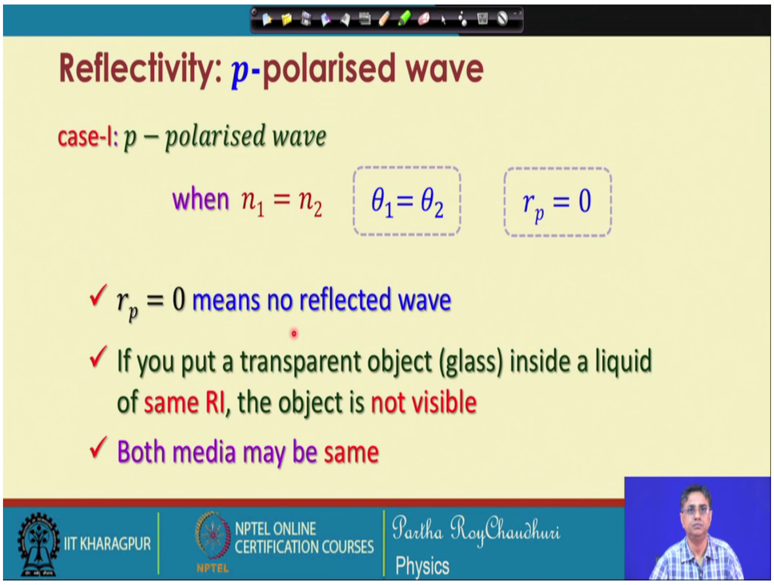

Reflectivity: p-polarised wave

case-I: $p-polarised\ wave$

when $n_1 = n_2$ $\boxed{\theta_1 = \theta_2}$ $\boxed{r_p = 0}$

- ✓ $r_p = 0$ means no reflected wave
- ✓ If you put a transparent object (glass) inside a liquid of same RI, the object is not visible
- ✓ Both media may be same

This means that there is no reflected wave, the second possibility that now we are interested. First one is that the two media at the same media, that is it is one medium through which the wave is traveling and therefore, because there is no interface theta 1 remains theta 2 and the wave goes undeviated, undistorted. But it could so, happen that there to different media, but their effective indices are the same. In that case we put r p equal to 0; that means, there is no reflected wave. It means that, if you put a transparent objective glass inside a liquid of same refractive index this situation, the object will not be visible because there is no reflection.

If there is no reflection from any object we cannot see that object and the first possibility that we have already discussed that is both the media may be the same media. So, this is one special situation when n 1 equal to n 2 and we plug in into the Fresnel's condition, Fresnel equation and we have seen that it could be two way possible.

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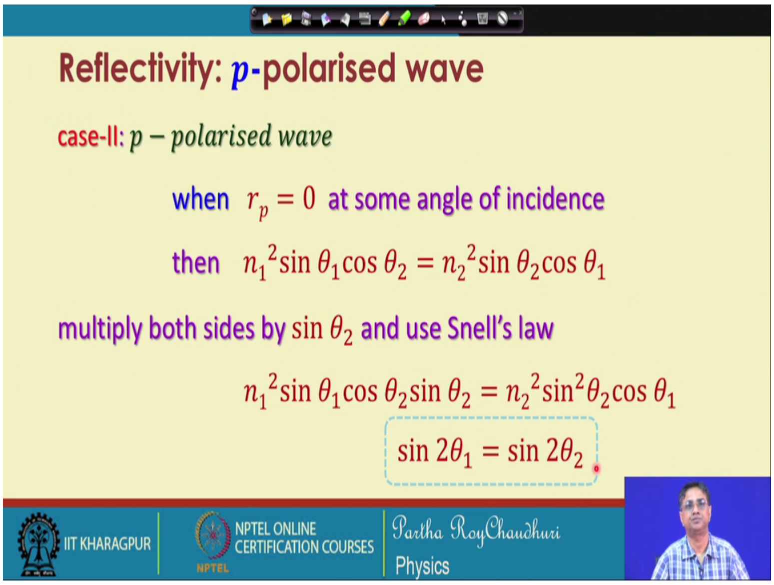

Reflectivity: p-polarised wave

case-II: $p-polarised\ wave$

when $r_p = 0$ at some angle of incidence

then $n_1^2 \sin\theta_1 \cos\theta_2 = n_2^2 \sin\theta_2 \cos\theta_1$

multiply both sides by $\sin\theta_2$ and use Snell's law

$$n_1^2 \sin\theta_1 \cos\theta_2 \sin\theta_2 = n_2^2 \sin^2\theta_2 \cos\theta_1$$

$$\boxed{\sin 2\theta_1 = \sin 2\theta_2}$$

Now, second thing is that if you assume that r p itself equal to 0, the reflection amplitude reflection coefficient is 0 that is that may happen for some angle of incidence. For some angle of incidence the reflection amplitude is 0 that is there is no reflection. When does it happen? If n 1 square sin theta 1 cosine theta 2 is equal to because, I put r p equal to 0 from where I can write this equation. Now, if you multiply both sides by sin theta 2 and use Snell's law, we can simply show that sin theta 2 1 sin 2 theta 1 equal to sin 2 theta 2.

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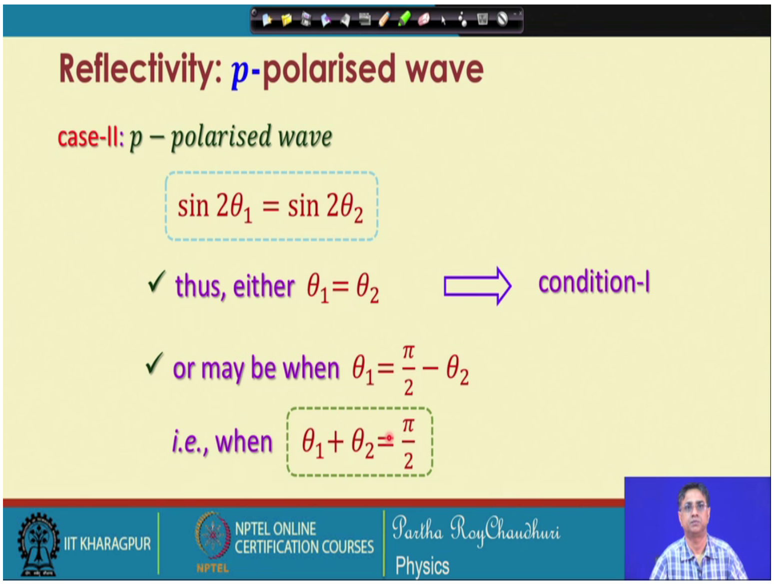

Reflectivity: p-polarised wave

case-II: $p-polarised\ wave$

$$\boxed{\sin 2\theta_1 = \sin 2\theta_2}$$

✓ thus, either $\theta_1 = \theta_2$ $\Longrightarrow$ condition-I

✓ or may be when $\theta_1 = \frac{\pi}{2} - \theta_2$

i.e., when $\boxed{\theta_1 + \theta_2 = \frac{\pi}{2}}$

This could be true, if theta 1 is equal to theta 2 is a direct condition theta 1 equal to theta 2 or it may so, happen that theta 1 equal to pi by 2 minus theta. That is pi theta 1 plus theta 2 is equal to pi, this is also possible.

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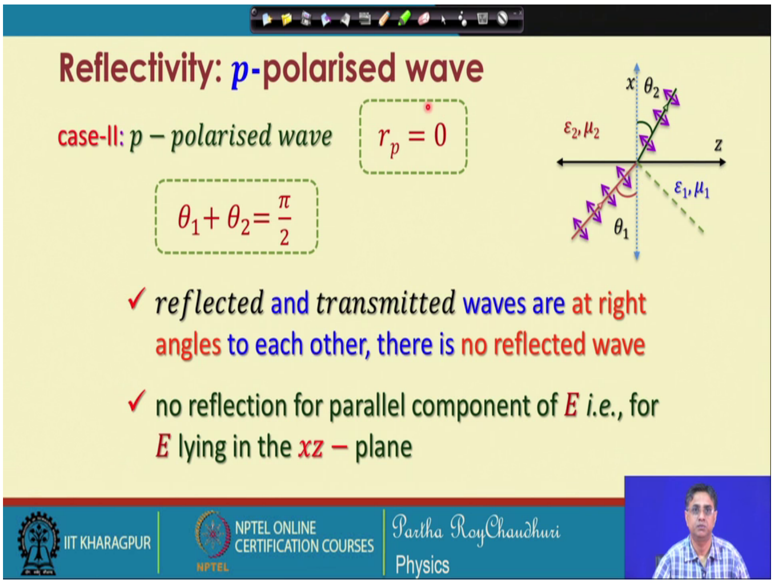

Reflectivity: p-polarised wave

case-II: $p-$ *polarised wave*

$\boxed{r_p = 0}$

$\boxed{\theta_1+\theta_2=\frac{\pi}{2}}$

✓ *reflected* and *transmitted* waves are at right angles to each other, there is no reflected wave

✓ no reflection for parallel component of E *i.e.*, for E lying in the $xz-$ plane

Now, for this case when there is no reflection and theta 1 plus theta 2 equal to pi, this situation is already known to us this involves the Brewster’s condition. In this case the reflected and transmitted waves are at 90 degree. These two waves reflected and transmitted waves are at 90 degree to each other and there is no reflection when there is only that p-polarised to f, this p-polarised to f.

And, there is no s polarised f, there is no parallel component of the electric field only the only this component the electric fields are lie in the exact plane. So now, reflection for parallel component of E and for the situation that is E lying in the exact plane, so, this gives you a situation that the waves are incident and they will be transmitted out. There will be no reflection because for which you get this condition r p equal to 0.

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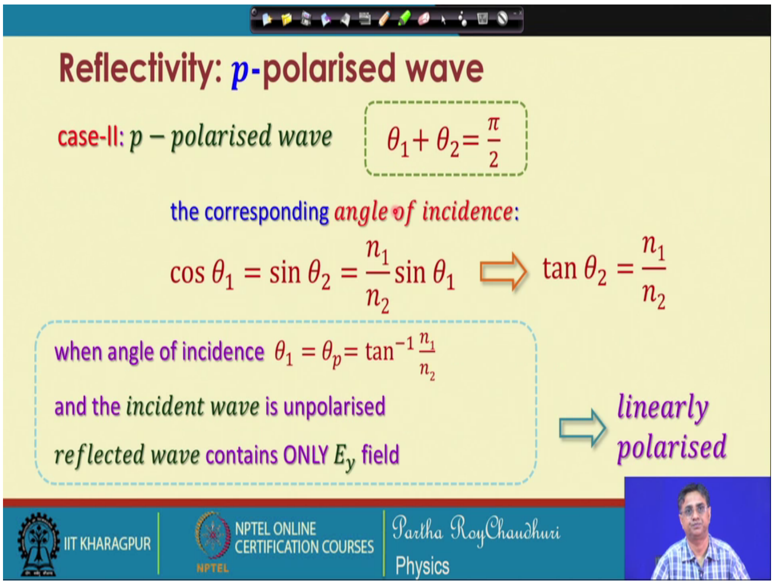

Reflectivity: p-polarised wave

case-II: $p-polarised\ wave$ $\boxed{\theta_1+\theta_2=\frac{\pi}{2}}$

the corresponding *angle of incidence*:

$$\cos\theta_1=\sin\theta_2=\frac{n_1}{n_2}\sin\theta_1 \Rightarrow \tan\theta_2=\frac{n_1}{n_2}$$

when angle of incidence $\theta_1=\theta_p=\tan^{-1}\frac{n_1}{n_2}$

and the *incident wave* is unpolarised

reflected wave contains ONLY E_y field

⇨ *linearly polarised*

Now, this theta 1 plus theta 2 equal to pi by 2, for this the corresponding angle of incidence we are interested because, this will happen only for a very special, only for a fixed for a specific angle of incidence which will which will depend on the two media involved. So, the condition is cosine theta 1 equal to cosine theta 1 equal to sin theta 2 because, by using this theta 1 plus theta 2 be equal to pi by 2 will give you sin theta equal to cosine theta 1 sin theta 2 equal to cosine theta 2. And, from here by using Snell’s law we can write this equation. The consequences very straight forward, tangent of theta 2 is equal to n 1 by n 2 which is again a very known condition, which is again a very this should be 1 tangent of theta 1, sorry there is a mistake this would be theta 1.

So, when angle of incidence theta 1 is equal to theta p, that is this p-polarized angle of incidence which is equal to tan inverse of n 1 by n 2, then the incident wave and if you assume that the incident wave is unpolarized and the angle of incidence is this. Then the reflected wave contains only E y because, the component of the electric field which is lying in the exit plane, for the incident wave will not be reflected. Only the component which is perpendicular to the exit plan, that is along E y that will be transmitted through. Such a situation is the linearly polarized wave. So, we have an incident wave which is which has this component E y and also exit plane component and it gives you a linearly polarized light output.

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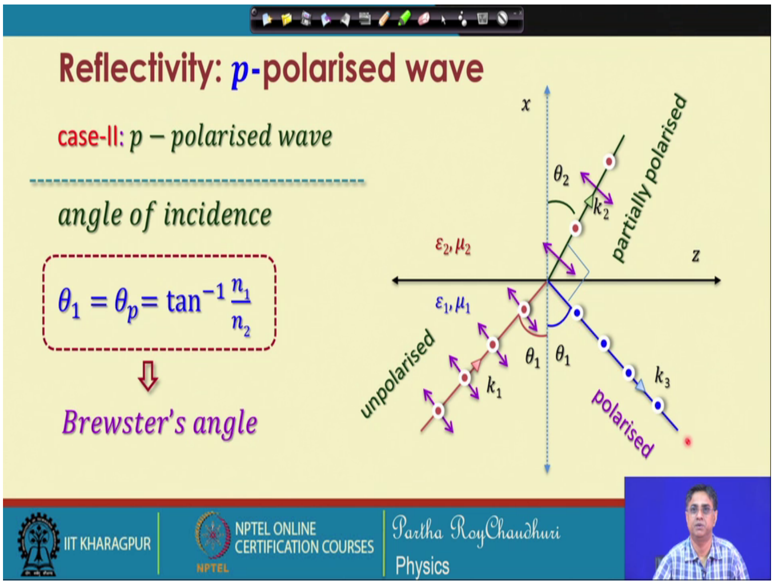

Reflectivity: p-polarised wave

case-II: $p-$ polarised wave

angle of incidence

$$\theta_1=\theta_p=\tan^{-1}\frac{n_1}{n_2}$$

⇩

Brewster's angle

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So, you can see the situation the incident wave has both the polarizations the s polarized wave and p-polarized wave, but the transmitted wave will be partially polarized and the reflected wave we will contain only the perpendicular component, that is the E y component of the electric field. So, this light is purely polarized, but this happens only at a certain angle theta 1 equal to theta p equal to tan inverse of n 2 by n 1 by n 2. So, this specific angle of incidence for which the reflected wave is purely polarised is called the Brewster's angle.

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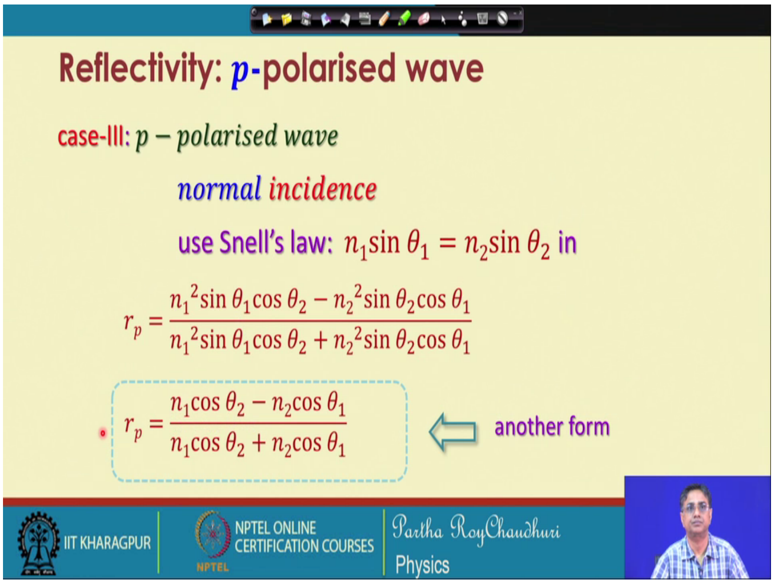

Reflectivity: p-polarised wave

case-III: $p-$ polarised wave

normal incidence

use Snell's law: $n_1\sin\theta_1=n_2\sin\theta_2$ in

$$r_p=\frac{n_1{}^2\sin\theta_1\cos\theta_2-n_2{}^2\sin\theta_2\cos\theta_1}{n_1{}^2\sin\theta_1\cos\theta_2+n_2{}^2\sin\theta_2\cos\theta_1}$$

$$r_p=\frac{n_1\cos\theta_2-n_2\cos\theta_1}{n_1\cos\theta_2+n_2\cos\theta_1}$$

⇦ another form

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Now, we will consider two more cases: that is the normal incidence and the grazing incidence. For the normal incidence we use Snell's law n 1 sin theta 1 equal to n 2 sin theta 2 in the expression for this r p. This we have seen that another form of this amplitude reflection coefficient can be written in this way r p equal to n 1 cosine theta 1 minus n 2 cosine theta 1 by sum of these two components. So, we will use this expression for normal incidence.

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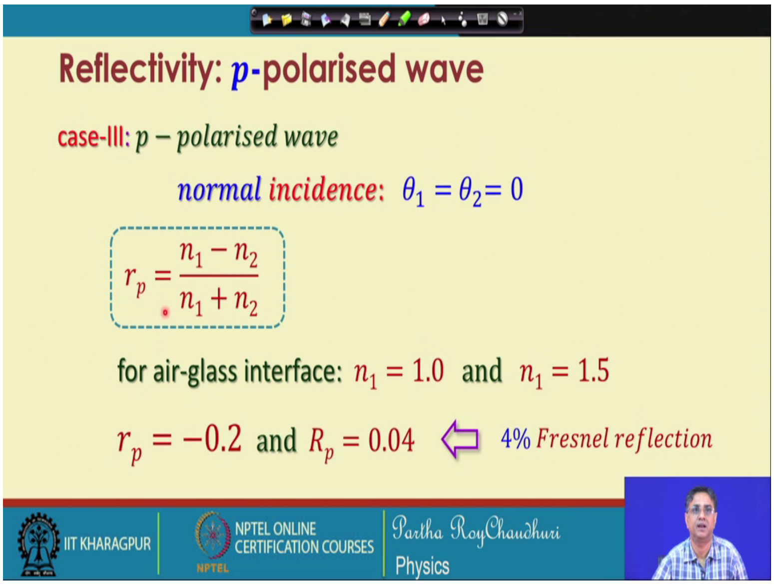

Reflectivity: *p*-polarised wave

case-III: $p - polarised\ wave$

normal incidence: $\theta_1 = \theta_2 = 0$

$$\boxed{r_p = \frac{n_1 - n_2}{n_1 + n_2}}$$

for air-glass interface: $n_1 = 1.0$ and $n_1 = 1.5$

$r_p = -0.2$ and $R_p = 0.04$ ⇦ 4% *Fresnel reflection*

Normal by normal incidence what do you mean that, the angle of incidence is 0, angle of reflection, angle of transmission all of them will be eventually 0. So, r p we can write as n 1 minus n 2 from this expression, from this expression cosine theta 2 equal to 1 cosine theta 1 equal to 1 and so on. So, you get r p equal to n 1 minus n 2 divide by n 1 plus n 2 and this is a very well known expression for Fresnel reflection, for the air glass interface; where n 1 equal to 1 and you assume for glass n 1 equal to 1.5.

And, this gives you r p equal to 0.2 and this energy reflection is 0.04 that is 4 percent. So, this is usually termed as the 4 percent Fresnel reflection from air glass interface. And, this is the reason that from ordinary glass in the air we can see some reflected light, we can see partially our face if you look at the glass surface.

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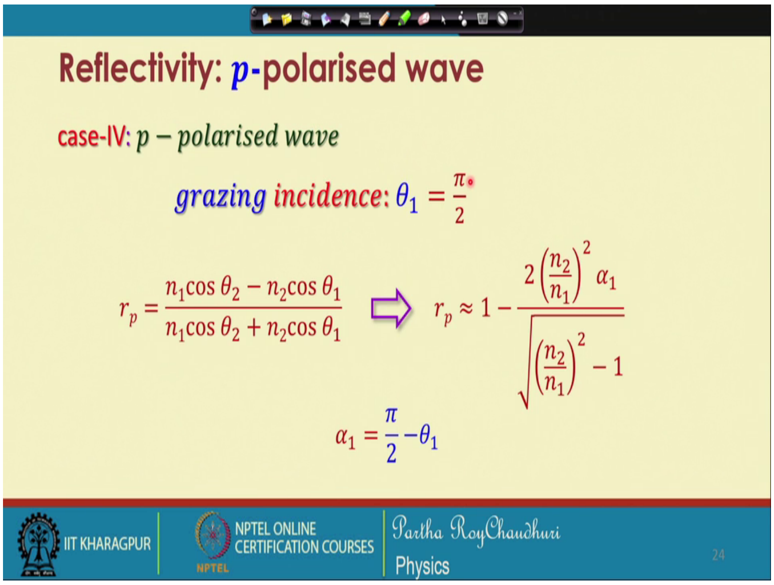

Reflectivity: p-polarised wave

case-IV: $p-polarised\ wave$

$grazing\ incidence: \theta_1 = \frac{\pi}{2}$

$$r_p = \frac{n_1\cos\theta_2 - n_2\cos\theta_1}{n_1\cos\theta_2 + n_2\cos\theta_1} \Rightarrow r_p \approx 1 - \frac{2\left(\frac{n_2}{n_1}\right)^2 \alpha_1}{\sqrt{\left(\frac{n_2}{n_1}\right)^2 - 1}}$$

$$\alpha_1 = \frac{\pi}{2} - \theta_1$$

Then the possibility that the incidence the angle of incidence is very large that is at grazing angle that is theta 1 we will use theta 1 equal to pi by 2. So, r p will be equal to n 1 cosine theta 1, where we will use theta 1 equal to pi by 2. And, if you do some calculation some algebraic manipulation, you can show that r p will be approximately equal to 1 minus n 2 by n 1 square twice of that and, with this denominator with a factor alpha 1 where alpha 1 is nothing, but pi by 2 minus theta. We have substituted for pi by 2 minus theta 1 is equal to alpha that is the small angle which is the wave is making with the interface, this angle is alpha.

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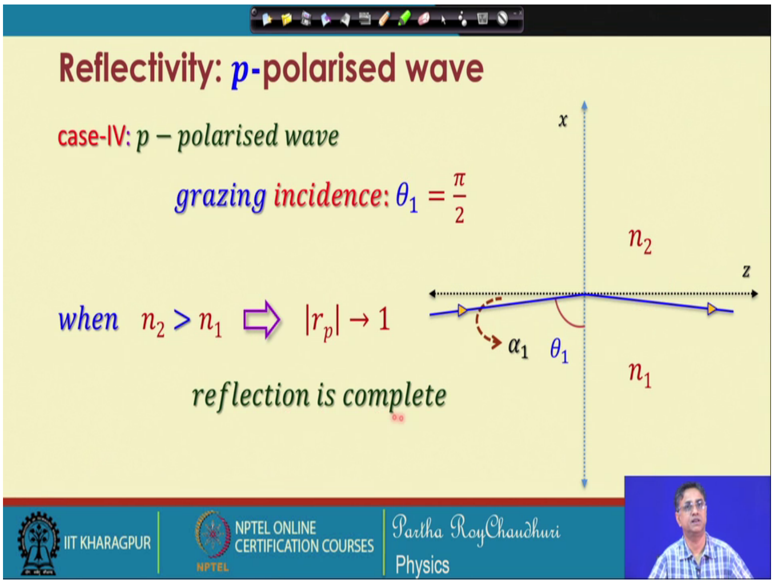

Reflectivity: p-polarised wave

case-IV: $p-polarised\ wave$

$grazing\ incidence: \theta_1 = \frac{\pi}{2}$

$when\ \ n_2 > n_1 \Rightarrow |r_p| \rightarrow 1$

$reflection\ is\ complete$

So, you can see this figure a small angle alpha 1 which they wave mix with the interface and at grazing angle it reflected, it is reflected out. And, from this expression you can see that you can see that, for this situation when n 2 is greater than n 1 this r p tends to unity. That means, the reflection is almost complete that is entire part of the way will be reflected and there is no transmitted wave.

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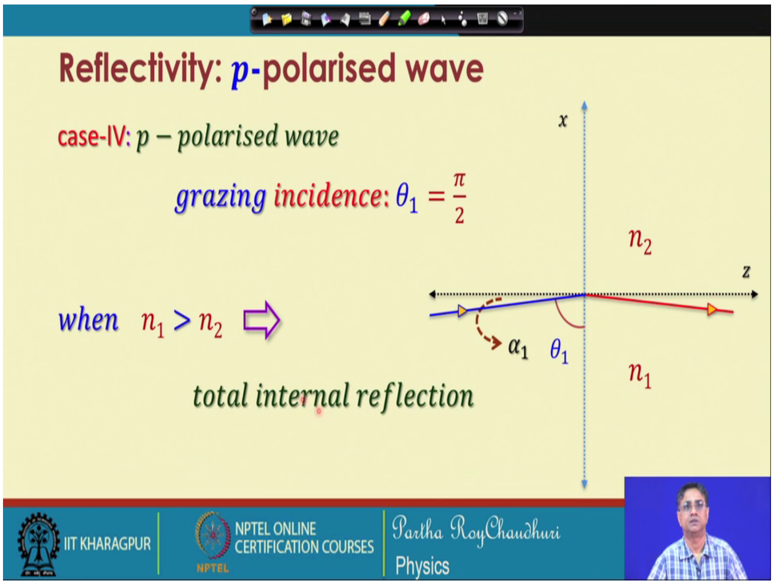

Reflectivity: p-polarised wave

case-IV: $p-polarised\ wave$

$grazing\ incidence$: $\theta_1 = \frac{\pi}{2}$

$when \quad n_1 > n_2 \Rightarrow$

$total\ internal\ reflection$

For the other situation that is, if n 1 is greater than n 2 n 1, this medium having refractive index higher than this one; the rarely fad me rear medium. In that case this angle is more than the critical angle of incidence as a result the wave we will undergo total internal reflection. So, in the case both the case that is when n 2 is greater than n 1 then also the reflection is complete and the other case when n 1 is greater than n 2, then the wave undergoes a total internal reflection.

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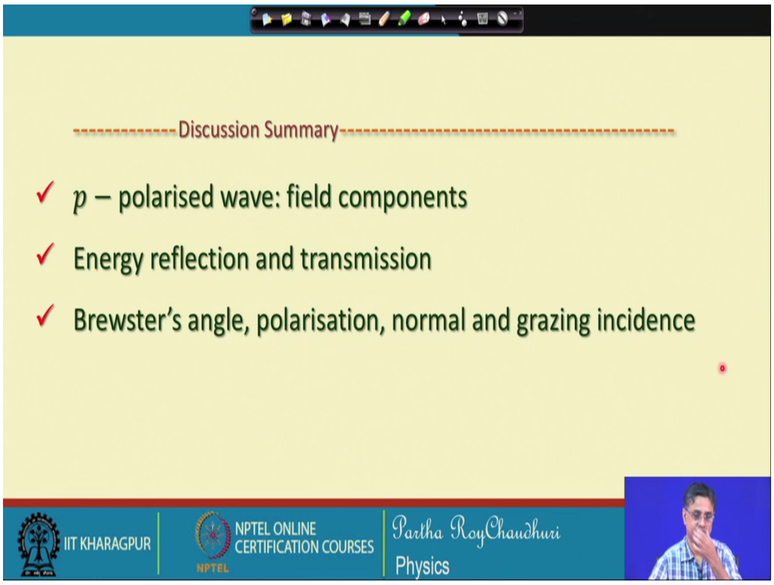

-------------Discussion Summary--

- ✓ p – polarised wave: field components
- ✓ Energy reflection and transmission
- ✓ Brewster's angle, polarisation, normal and grazing incidence

Thus, we have we have discussed all the possibilities with a p-polarized wave starting from the amplitude reflection, amplitude transmission coefficient, their energy reflection and energy transmission, that is the reflectivity and transmissivity. Then, we have also discussed some representative situations when r p equal to 0 or n 1, the media are having the same refractive indices and what is the consequence of all these things.

With this knowledge and background, in the next section we will consider the s polarized wave that is t polarized wave and we will look at the energy reflection and transmission coefficients and other properties. We will also look at the total internal reflection, we will also compare the situation, certain things will be same for both the polarization waves polarized waves that also we will consider.

Thank you.