Modern Optics
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Lecture – 13
Wave Propagation in Layered Structures

So, today we will be discussing electromagnetic waves in interfaces and layered structures.

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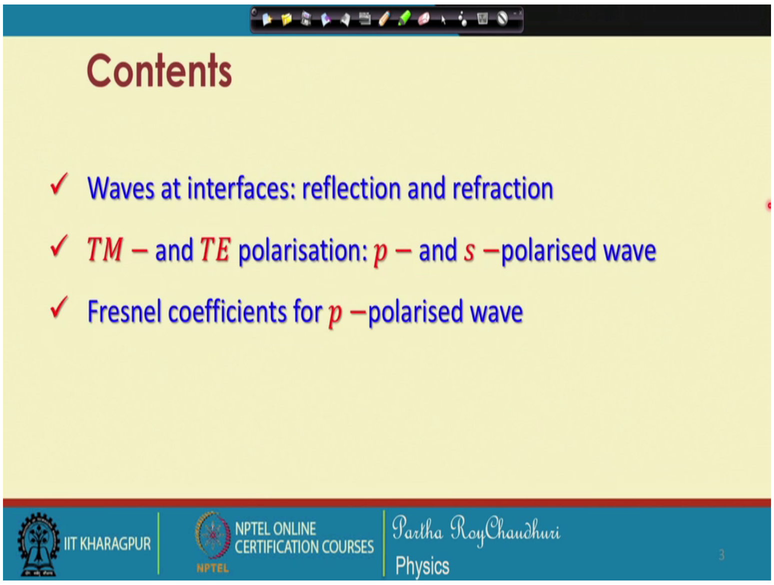

We have seen the wave propagation in isotropic medium, anisotropic medium and also at metallic conducting medium. With that background and knowledge about the propagation characteristics of electromagnetic waves, we will now study the waves at interfaces their reflection and transmission properties and we will see that the waves can have two different configurations or add a mixture of both the configurations and we will call them the TE and TM polarisation or the s and p-polarised wave.

Then, we will look at the reflection and transmission properties of the p-polarised wave in terms of the amplitude reflection amplitude transmission. And we will try to study this particular p-polarised wave in little more detail. so that the knowledge can be can be carried forward to the discussion of s-polarised wave and a comparison of both the polarisation properties of the electromagnetic waves at the interfaces.

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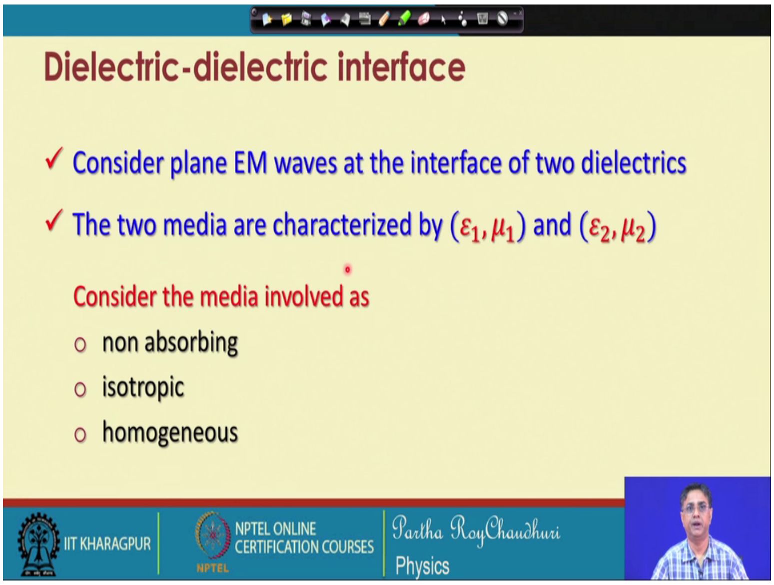

So, in order to do that we will assume the properties of the media who are involved in forming the interfaces, it should be non absorbing. The individual media will be isotropic and homogenous. Now, let us consider the plane electromagnetic waves at the interface of two dielectrics and these two dielectrics are characterized by the properties the permittivity and the permeability as epsilon 1 and mu 1 for the first medium that is one of the media and the respective quantity is epsilon 2 and mu 2 for the other media

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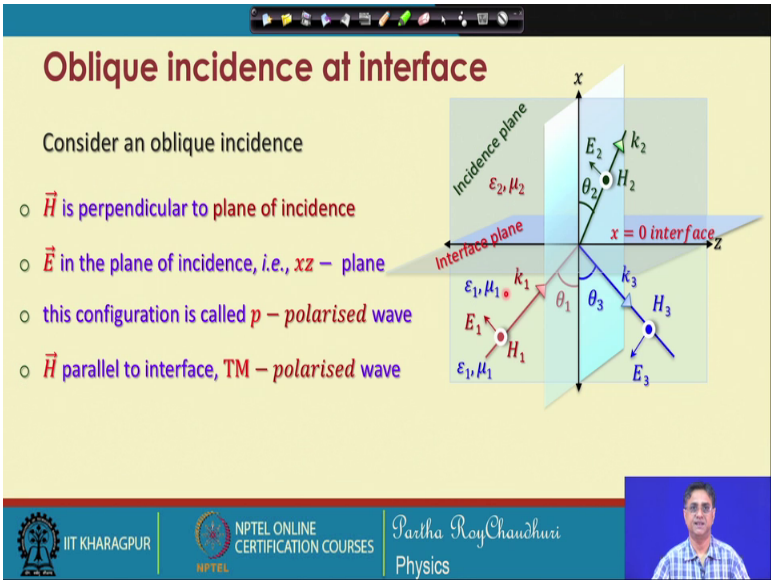

So, let us first consider oblique incidences at the interface. Look at this figure you have a wave which is incident at the interface and it gets reflected part of the wave is transmitted. Let us consider that the magnetic fields are perpendicular to the plane of incidence by plane of incidence we mean, this plane this plane. So, the magnetic field lines are out of the paper that is perpendicular to this board and electric fields are lying in the xz plane such a configuration is called a p-polarised waves configuration.

This is all with respect to the structure the name p-polarised because each field that is the magnetic field is parallel to the interface field, this is the interface plane. It is vertical plane is the interface plane since the magnetic fields are verticals are parallel to the interface that is why it is also called TM-polarised that is transverse magnetic polarised wave. So, with this configuration we will now study the reflection and transmission properties.

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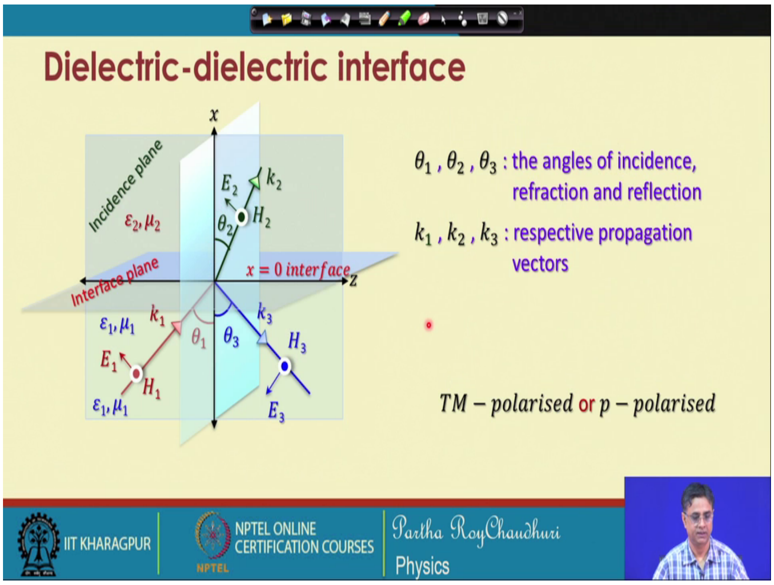

But, we also have to define the angles of incidence and angle of reflection and angle of transmission that is refraction as theta 1, theta 2 and theta 3. k 1, k 2 and k 3 these are the propagation vectors along the incidence direction, refracted direction and along the reflected direction. So, with this configuration the wave that we will discuss is the TM-polarised or p-polarised waves.

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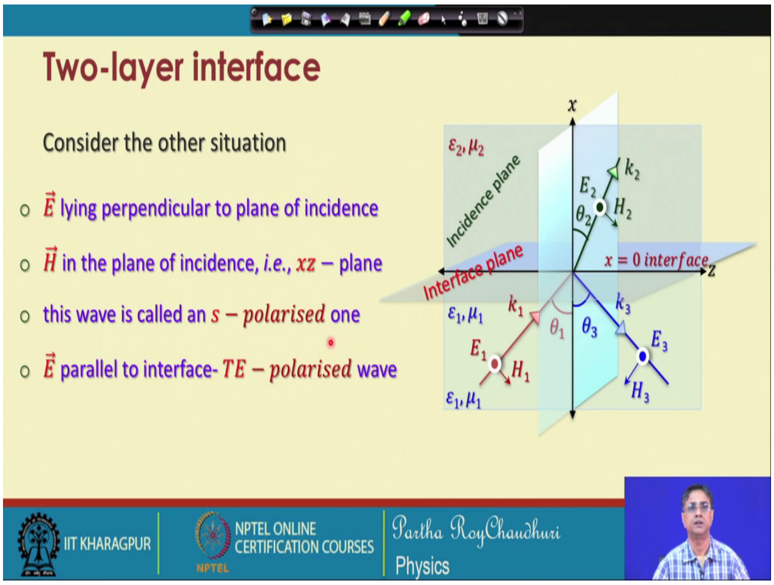

There could be another situation, just complimentary situation that now that the electric field is perpendicular to the incidence plane. That is, the plane of incidence that is the vertical plane that is out of the out of the paper and magnetic fields are laying in the xz-plane such a configuration is known as s-polarised one. This is also called TE-polarised transverse electric polarised wave because the eclectic fields are parallel to the interface.

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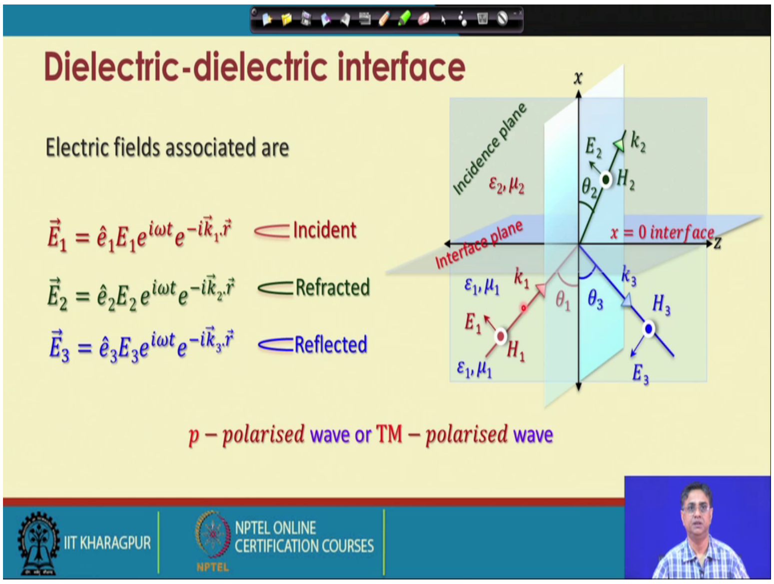

So, initially we will look at the properties of the medium in terms of the reflected and transmitted rays. Let us consider the electric fields plane wave fields associated with the incident wave as E 1 e to the power of i omega t, e to the power of minus i k 1 r k 1 is the propagation vector along the direction of incidence. E 2 similarly we have an expression that is e to the power of i omega t e to the power of minus i k 2 dot r and likewise for the third part that is a reflected part we have this expression. So, these are the plane wave expressions for the electric fields in three different places that is the incidence wave, the refracted wave and the reflected wave for the case of p, TM-polarised wave.

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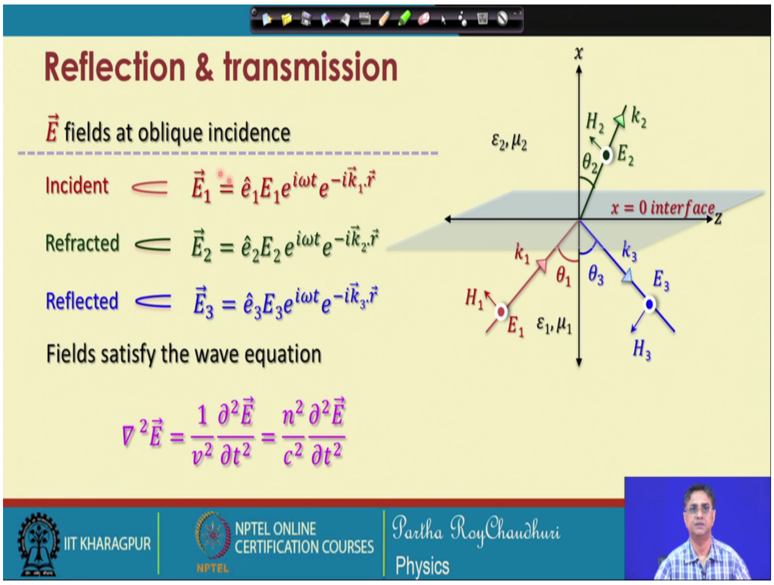

Now, E fields at oblique incidence we have this electric fields and each of them and also the magnetic field components will satisfy the wave equation. This we have seen earlier. Now, V square we can replace by n square by c square, where n square is the refractive index at the medium involved.

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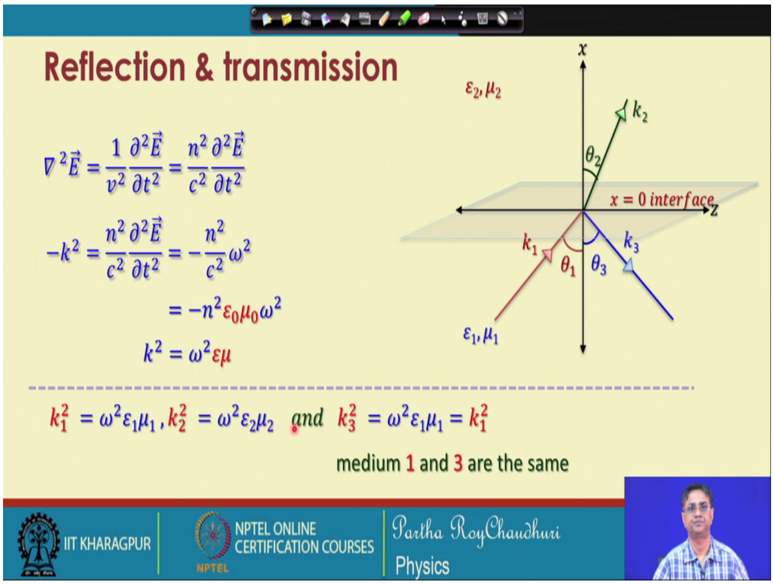

From this we can we can write down this equation in terms of k square that is second order derivative of this electric field in the plane wave form will give you minus k square into E. And this del 2 E del t square will give you minus n square omega by c square into omega square.

Therefore, you can equate these two that k square equal to omega square mu into epsilon. This means that for k 1 this will be omega square epsilon not mu 1 omega is the frequency which does not change whether it is reflected refracted and k 2 square from here will come out like this omega square epsilon 2 mu 2 and for k 3 square. We have this expression, but because the mediums one and three in our assumption. They are the same medium that is the first medium that is why k 3 square is equal to k 1 square and we will use this property when we will be discussing further.

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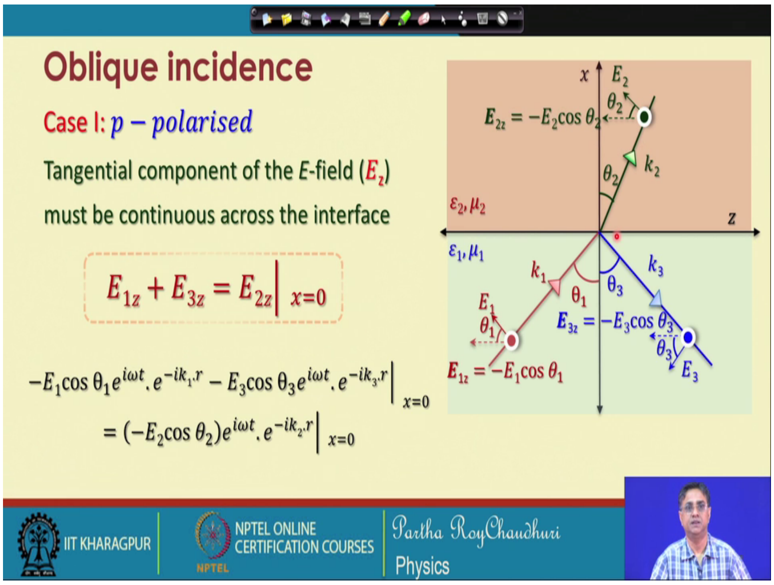

Now, let us consider few example cases. First is the in the case of p-polarised wave that is the present configuration that we have undertaken, the boundary conditions are the tangential component of the electric field must be continuous across the interface. The tangential component that is that is this component, this component and this component the tangential components tangential with respect to the interface they must be continuous.

So, we resolve this electric into two components parts, that is one is one is the one is the longitudinal component, the other one is the transverse components. And, so, for the for the incident wave we write E 1 z which is because this vector is directed in the negative direction of z, this is your z-axis, this is your x-axis and perpendicular to this plane is the y-axis. Therefore, E 1 z equal to minus E 1 cosine theta, this is very clear. And, similarly for the for the reflected wave you have minus E 3 cosine theta 3 which is the component tangential component of the reflected wave electric field and also for the transmitted wave the refracted wave the tangential component will be equal to all of them are incidentally in the negative direction of z.

So, these all tangential components with minus signs will be put together. So, we can write this tangential components continue continues continuity condition that this electric incident electric fields, tangential components, reflected electric field tangential components will be equal to the transmitted electric field tangential components that is E

1 z plus E 3 z equal to E 2 z. So, they must be continuous across the interface that is at x equals to 0 at this points. Let us see.

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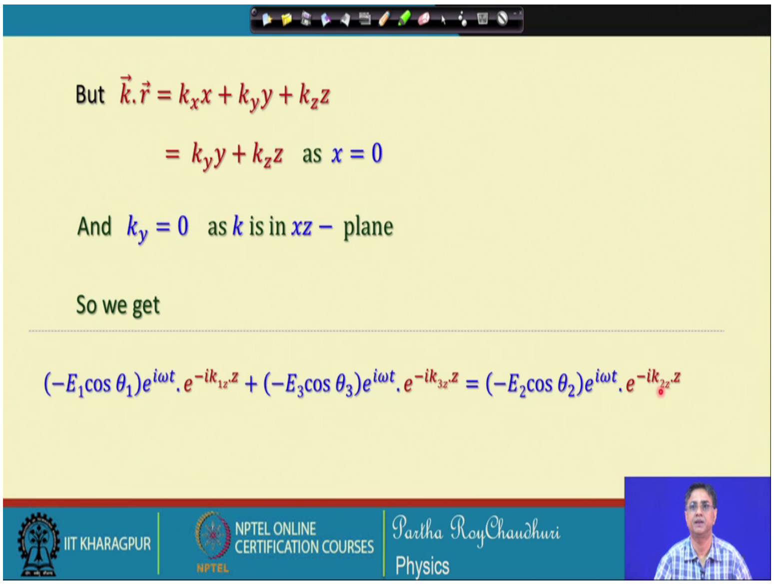

Now, k dot r in general is equal to k x x, k y y is the is the plane. So, this because we are considering the reflection refraction at x equals to 0. So, x equal to 0 gives you k dot r equal to k y y plus k z z. Further, because k y because if we look at the configuration there is no there is no y component of the electric field. The magnetic field is purely along the y direction. So, there is no y component of the electric field; that means, k y equals to 0.

As k is as k is in the xz plane so, we get this equation that is we we connect the all the three components through this equation that is E 1 cosine theta 1 e e to the power of i omega t, e to the power of minus i k 1 z z plus similar expression for the reflected wave k 3 z, z equal to this the expression for the transmitted wave e to the power of i k 2 z z.

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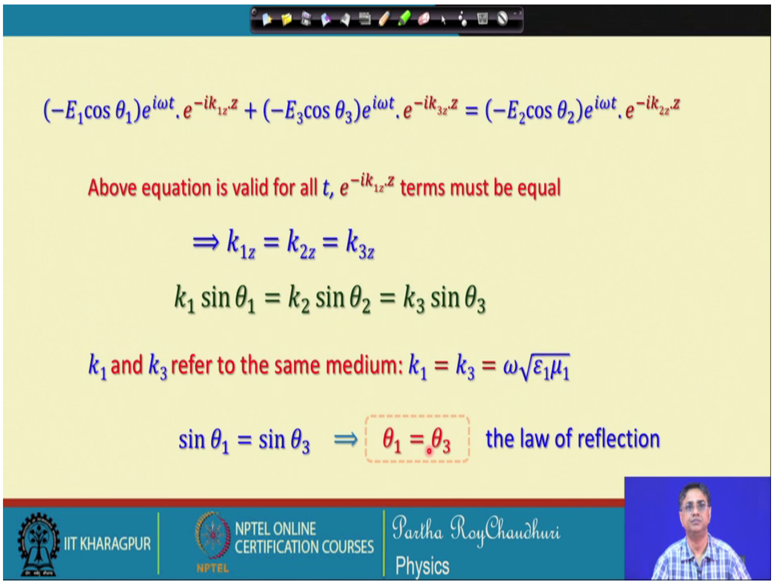

Now, this equation will be valid for all times and for this validity it is required that e to the power of i k 1 z z; these terms must be equal. Therefore, we directly conclude that k 1 z equal to k 2 z equal to k 3 z, but k 1 z is k 1 sin theta 1 k 2 z is equal to k 2 sin theta 2 and k 3 z will be equal to k 3 sin theta 3. Therefore, each of them are equal to the other. So, k 1 and k 2 because they refer to the same medium we have seen that k 1 equal to k 3. Therefore and this is equal to omega under root of epsilon 1 mu 1.

So, from here we immediately conclude k one sin theta one is equal to k 1 sin theta 3, that that tells you that sin theta 1 must be equal to sin theta 3. So, that is theta 1 must be equal to theta 3. So, this is nothing, but our old law of reflection that the angle of incidence must be equal to the angle of reflection.

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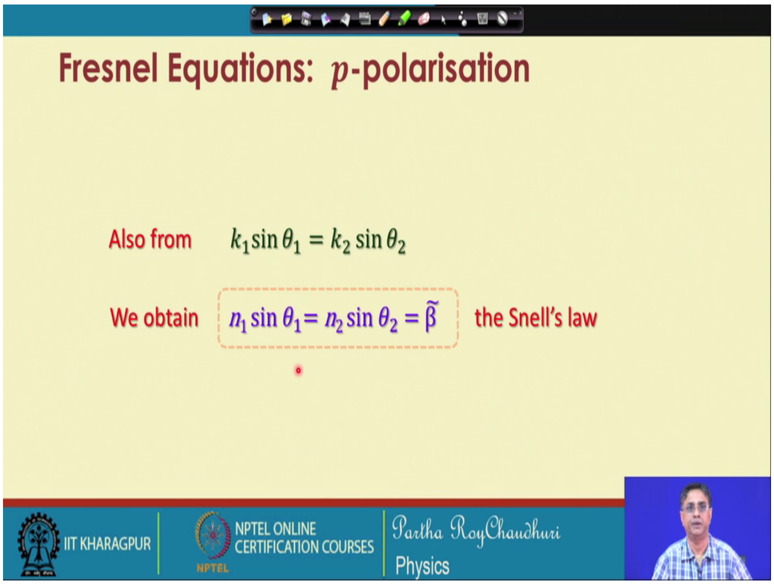

And, also from the above condition k 1 sin theta 1 is equal to k 2 sin theta 2 theta 2. So, when we connect the two medium that is mu 1 epsilon 1 mu 2 epsilon 2, that is, k 1 and k 2. So, therefore, from here because k 1 is k 0 n 1 sin theta 1 and k 2 equal to k 0 n 2. So, n 2 sin theta 2 they must be equal to some constant beta delta which is the invariant of the combination of the medium. So, therefore, this shows the Snell's law which is again our old law that is the law of reflection and refraction ok.

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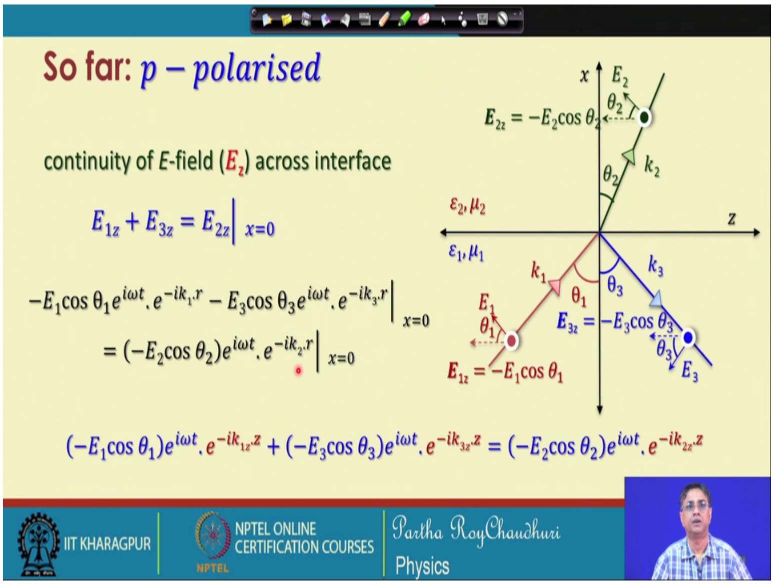

Now, so far all that we have discussed is that the continuity of the electric field, the tangential component of the electric fields across the interface which has given us this equation E 1 z plus E 3 z is equal to E 2 z at x equals to 0. If we optimize use this condition that e to the power of i omega t e to the power of i omega t and e to the power of i omega t, here all are same. Then we can write down the same old expression which you have seen earlier in this form we will remove this e to the power of i omega t.

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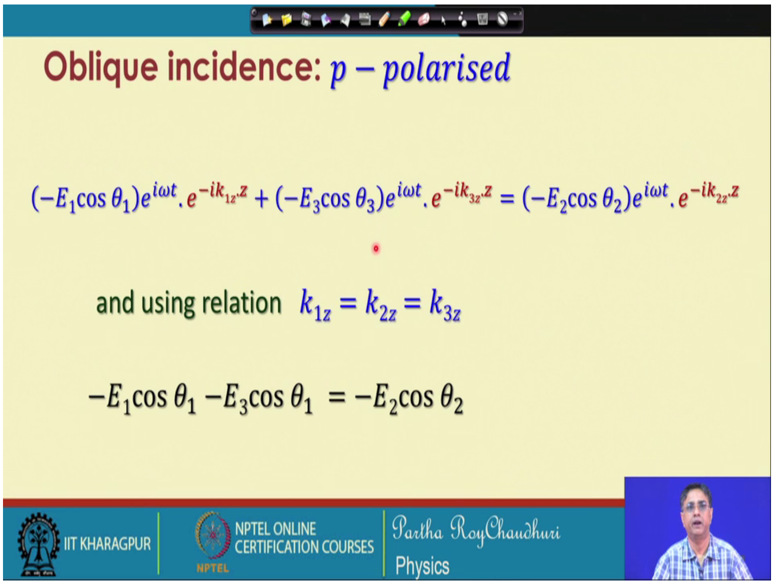

Therefore, and also we will use the relation k 1 z, k 2 z and k 3 z all of them are equal they are the z component of the tangential component of the propagation vector. So, k 1 z, k 3 z, k 2 z all of them are equal.

So, you can also eliminate this term e to the power of i omega t, e to the power of i omega t, e to the power of minus this term this term and this term. So, then we end up with a reduced form of this continuity equation that minus E 1 cosine theta 1 which gives the expression for the electric field for incidence wave reflected wave which will be equal to the electric field component of the transmitted wave.

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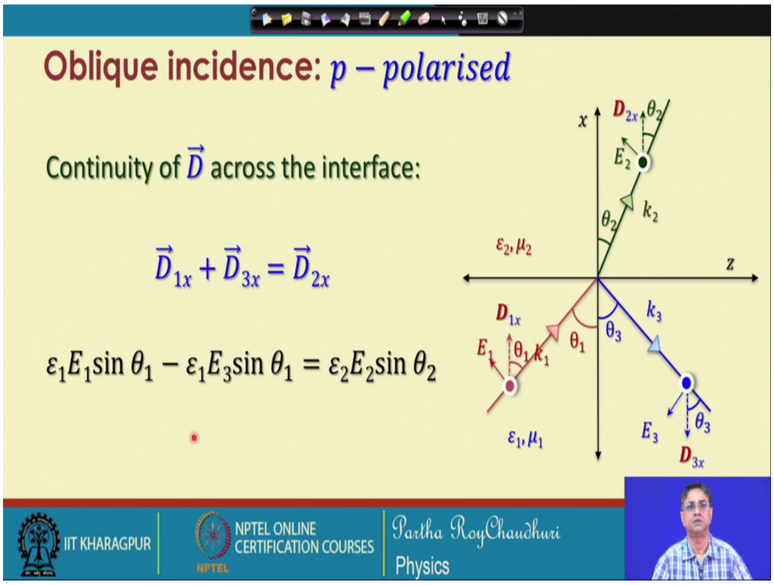

Now, we will consider the other condition continuity condition that is the displacement vector across the interface, the x component. x component of the normal component of the displacement vector will be continues that is D 1 x plus D 3 x equals to D 2 x. Now, D 1 x will give you epsilon 1 E 1 sin theta 1 and similarly for D 3 x we get because epsilon 1 and epsilon 3 they are the same. So, we will write epsilon 1 E 3 and sin theta 1. We have seen that theta 1 and theta 3 they are also equal. So, we can write this left hand side in this form and for the right hand side D 2 x we will write epsilon 2 the permittivity of the second medium, the electric field component and sin theta 2 in terms of that.

So, this is the second continuity condition that is the normal component of the displacement vector will be continues across the interface.

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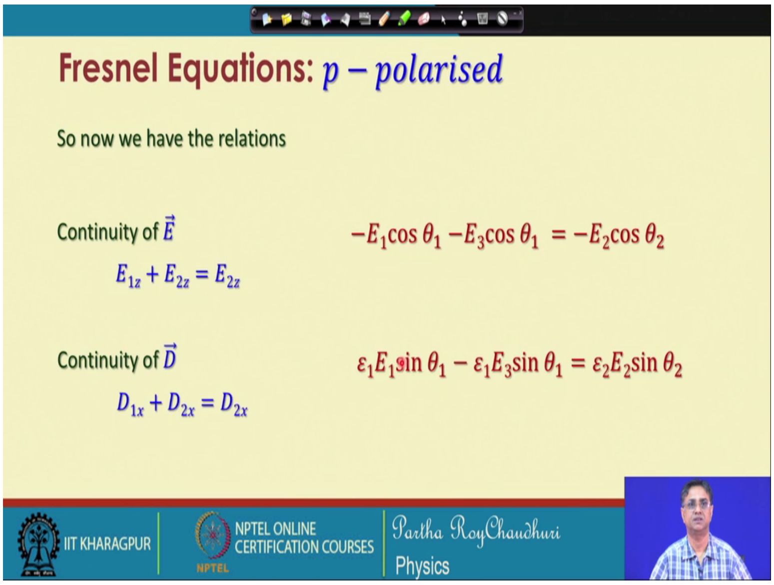

So, we have two continuity conditions and we will put together these two conditions to see how we can extract some relations with for E 1 and E 3, E 1 and E 2. So, in summary we have this electric field continuity condition and for the displacement vector we have this continuity condition this one has given in this equation, whereas the equation for the continuity of the normal component of displacement has give you this equation.

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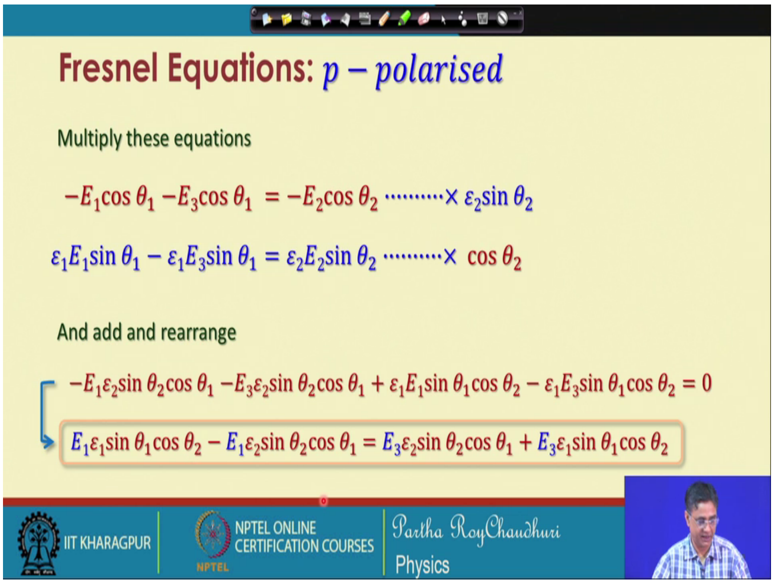

Now, we will do some algebraic manipulation with these two equations and we will see how the electric field components are connected with the incident field to the refracted field, incident field to reflected field. Now, I have these two equations to do this algebra, let us first multiply this first equation with epsilon 2 sin theta 2 and the second equation we will multiply with cosine theta 2. As a result you see that this first equation will become this first part will be minus epsilon to E 1 sin theta to cosine theta 1.

Here also you will get epsilon 1 E 2 cosine theta 2 cosine theta 1. So, you can add both of them. If you add both of them then you can you can see this equation, just by adding these two you can see this because they will cancel. So, from here if we arrange in a way that all the electric field E 1 are on the left hand side and E 3 are on the right hand side then you can write in this form.

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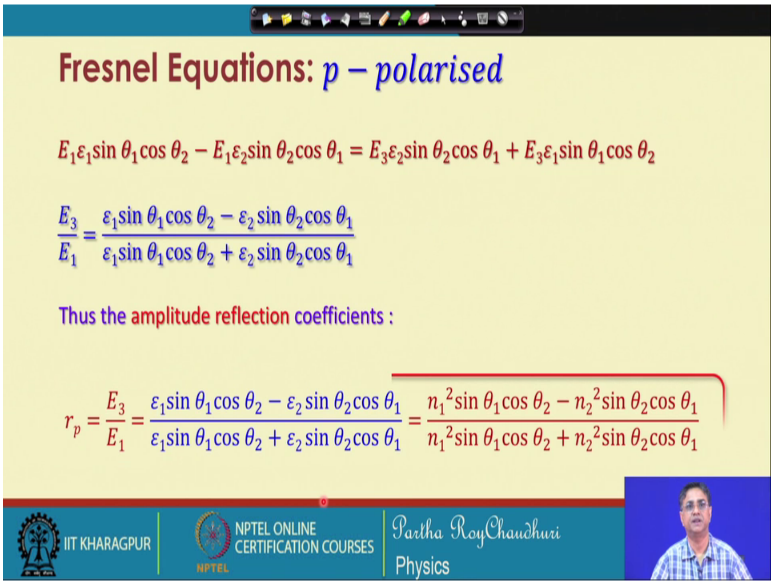

So, that we can take E 1 bracketed out from this expression E 1 will take it out. So, we can write this from this equation E 3 by E 1 is simply the ratio of the sin and cosine components in this form. Therefore, E 3 by E 1 we can readily write down in this form, but what is E 3 by E 1? E 1 is the incidence amplitude of the electric field whereas; E 3 is the reflected amplitude of the electric field.

Therefore, the amplitude reflection coefficient which is given by E 3 by E 1 we will call this r p because we are working with the p-polarised line that is the TM polarization therefore, r p is equal to E 3 by E 1. This we can write directly in this form in terms of the permittivity constants of the permittivity values of the two medium involved like sin theta 1 cosine theta 2, sin theta 2 cosine theta 1 and the same expression at the bottom, but with a in the denominator, but with a plus sign between them.

And because E 1 the permittivity for this dielectric is nothing but the square of the refractive indices; so, we can write E 1 equals to n 1 square and epsilon 2 is equal to n 2 square. So, this the other form of this equation for the amplitude reflection coefficient for the p-polarised light r p.

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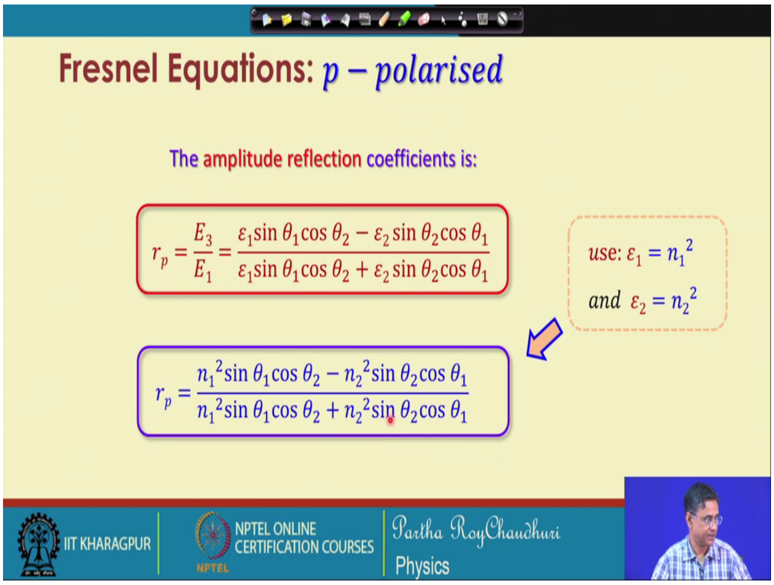

So this amplitude reflection coefficient now we have we have seen that this equation and we have also seen that we have used this epsilon 1 equal to n 1 square epsilon 2 equal to epsilon 2 square and we have got this form which is now the amplitude reflection coefficient is in terms of the concerned refractive indices and the angles of incidence and angles of refraction. So, these are the things which are going to decide the amplitude reflection coefficient. So, it primarily depends on the refractive indices and also the angle of incidence which generates which heals the angle of refraction also because they are connected by the refractive indices indices properties.

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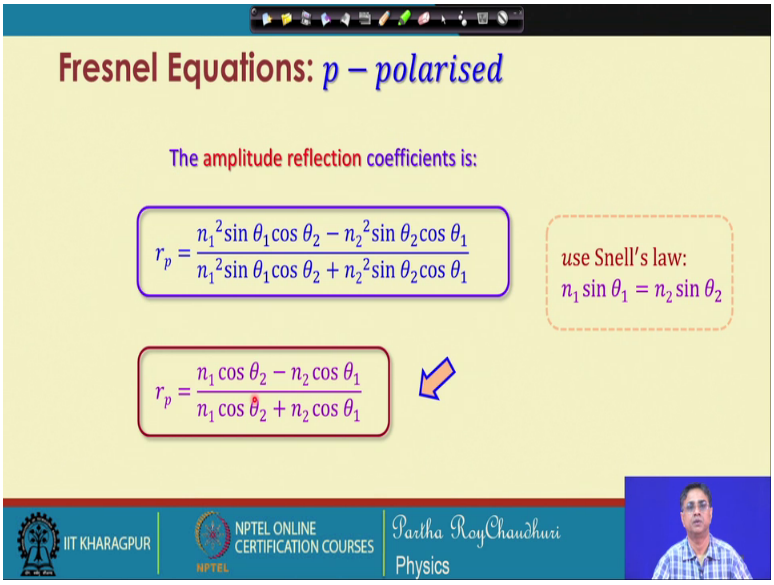

So, amplitude reflection coefficient we can even write in some modified form or reduced form rather we can say that r p is equal to this in terms of the refractive indices. But, now n 1 sin theta 1 is by the Snell's law we have just now seen that n 1 sin theta 1 for the first medium this is the angle sin of the angle with the refractive index is equal to n 2 sin theta 2 for the sin second medium.

Therefore, using this connection we can again rewrite this equation that n 1 sin theta 1 if we remove one that will give me n 2 sin theta 2 which will also appear here n 2 sin theta 2. So, we can take out outside from this numerator one n 1 sin theta 1 or n 2 sin theta 2 and likewise, from the denominator we can also take out one n 1 sin 1 or n 2 sin theta 2 and they will cancel outside.

As a result, you can write down this equation in this compact and very small reduced form that the amplitude reflection coefficient for the p-polarised light can be written as n 1 cosine theta 2 minus n 2 cosine theta 1 divided by some of these two factors, n 1 cosine theta 2 plus n 2 cosine theta 1 and this equation is very often used for calculating the Fresnel reflection coefficient energy reflection coefficient etcetera, ok.

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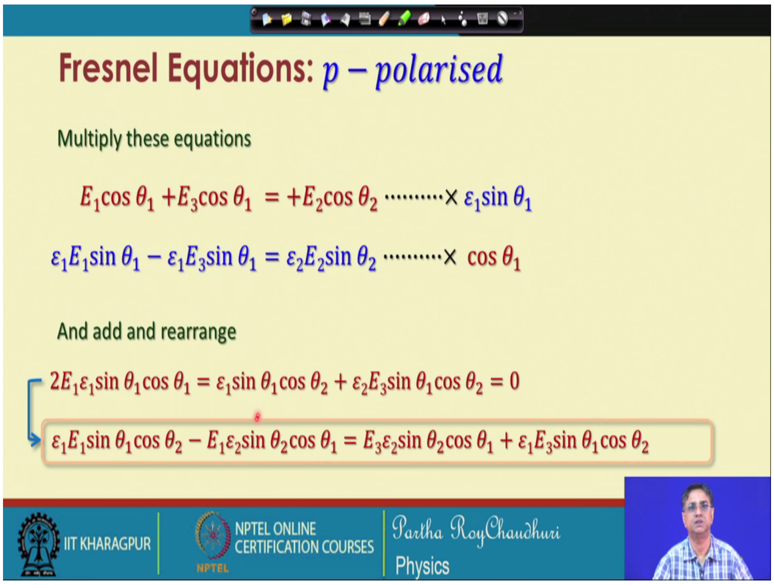

Now, for calculation of the energy transmission amplitude transmission coefficient we again work with the with those two equations that is this continuity condition for the electric field and for the normal component of the displacement vector and this time we multiply the first equation with E 1 sin theta 1 and the second equation with the cosine theta 1. Then, if we add we can see from here we can rewrite this equation as this twice E 1 epsilon 1 sin theta 1 cosine theta 1 and so on which is equal to 0.

Because, here we try to eliminate E 2 from both the equations and then if you bring all the E 1 to left hand side and E 3's to the right hand side, then again you can write in this form. The equation can be written in this form.

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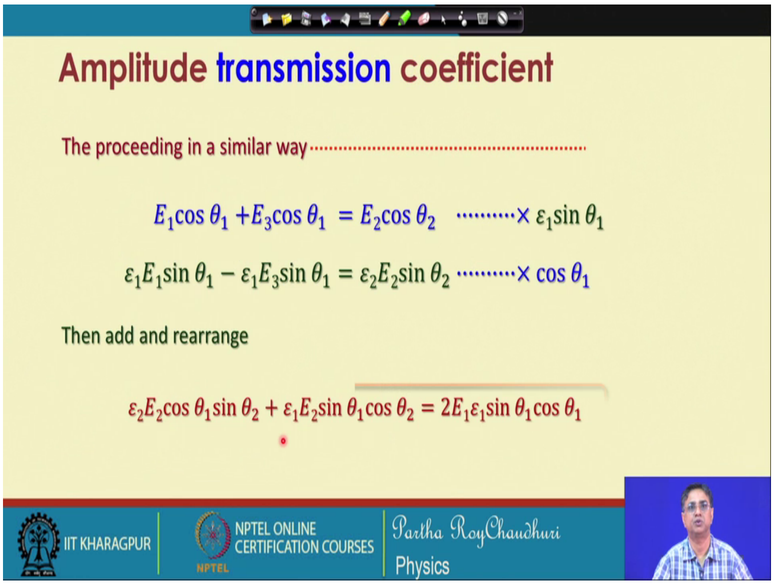

Then proceeding in a similar way by multiplying this we add and rearrange we can we can write down this equation. This step we have shown earlier. So, E 2 epsilon 2 E 2 cosine theta 1 and E 2 E 1 cosine theta 2.

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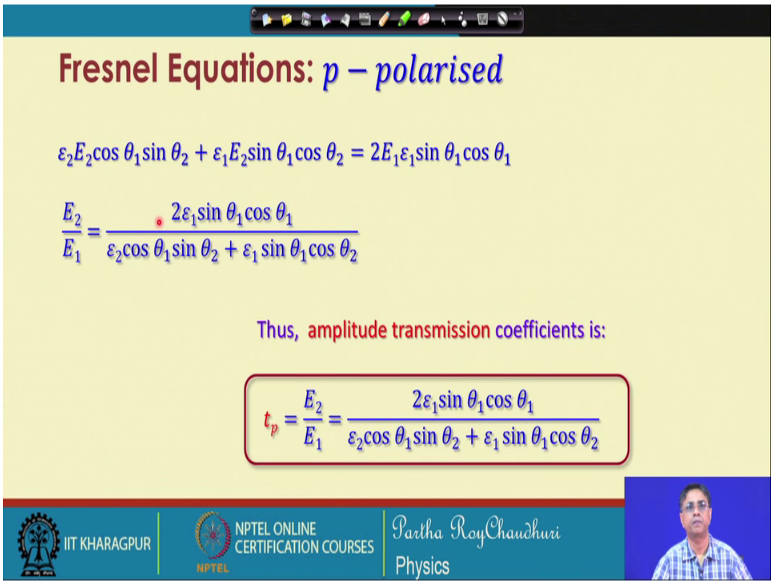

Then, we will get the amplitude transmission coefficient, that is, E 2 epsilon 3 E 2 cosine theta 1 sin theta 2 this. So, that means, this time E 2 and E 1 they are now connected E 2 and E 1. They are now connected by these equations. Thus the amplitude transmission coefficient which we will write t p for this p-polarised light and we can write E 2 by E 1 because it is the ratio of the incident electric field amplitude to the transmitted electric field amplitude. So, we will end up with this equation which is this expression which is in terms of the permittivity constants.

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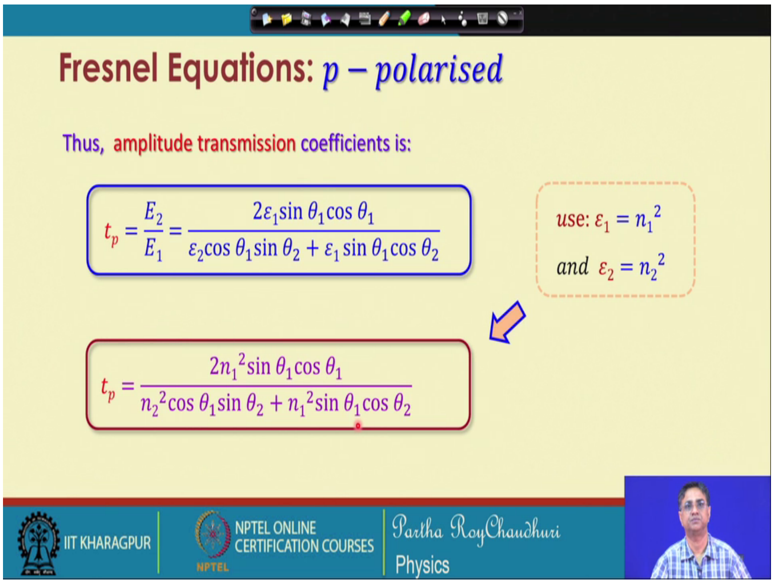

And, this t p in terms of permittivity constant this expression can be written in terms of the refractive indices in the same way as we have done in the case of reflection amplitude coefficient. So, twice n 1 square sin theta 1 sin cosine theta 1 divided by this denominator.

And, we will again use Snell's law to this expression to have a reduced form of this equation .

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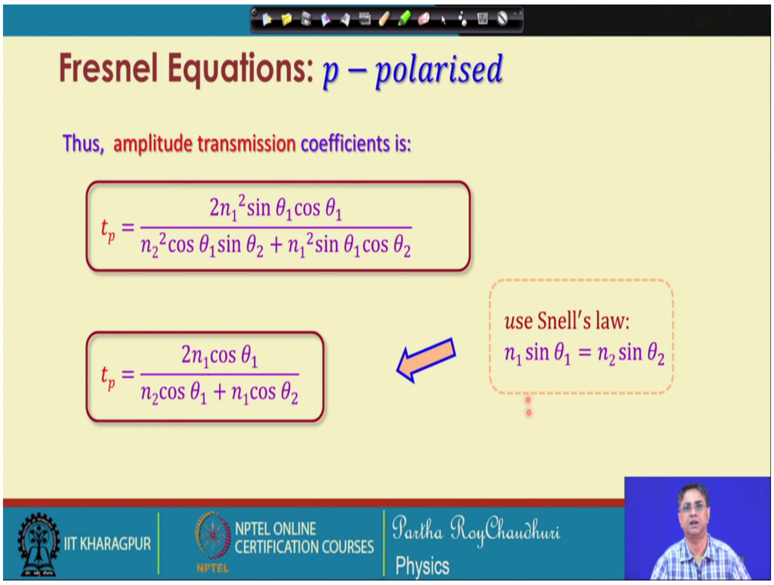

So, if I use Snell's law for this equation you can see that n 1 sin theta 1 will be equal to n 2 sin theta 2. So, from both the places we can take out n 1 sin theta 1 n 1 sin theta 1 here n 2 sin theta 2 they are appearing. So, you can take out an n 1 sin theta 1 from the numerator you can take out. As a result we will get t p this amplitude transmission coefficient in this reduced form, that is, twice n 1 cosine theta 1. This expression is also very often used in Fresnel equation transmission amplitude coefficient calculation.

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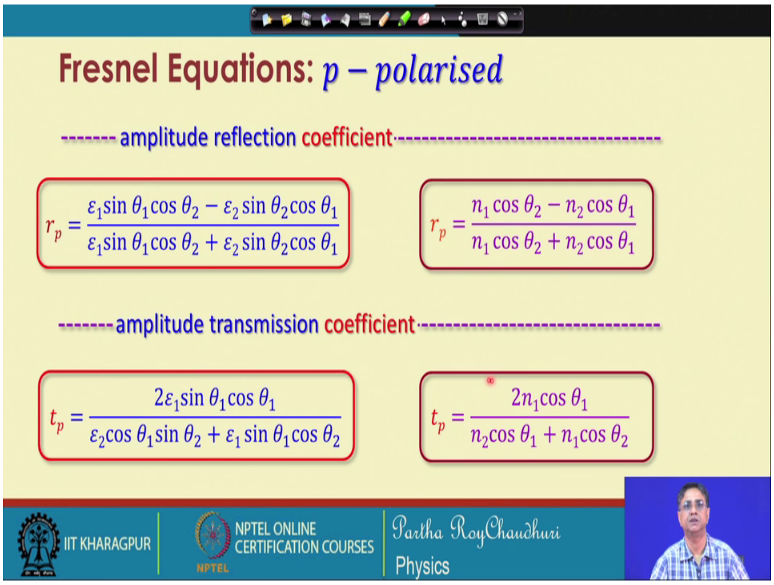

Therefore, to summarize that we have amplitude reflection coefficient in the form of this or in terms of only the refractive indices and the angle of incidence and angles of refractions. We can write this equation and for this we can write the transmission amplitude in terms of this equation.

So, till now we have been able to see how this electric field amplitude are related to the from the incident wave to the reflected wave and to the transmitted wave in terms of the amplitude reflection coefficient and amplitude transmission coefficients.

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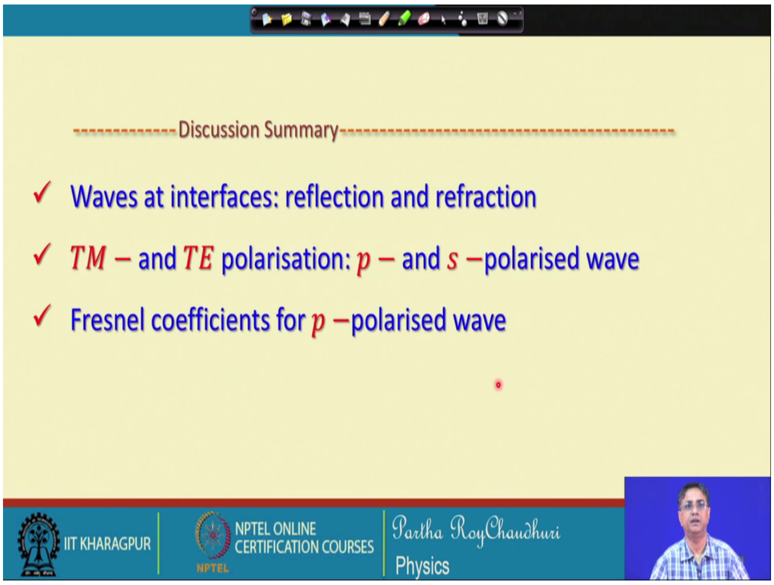

And, then we will discuss more about the details about the energy transmissions, pointing vector and other things in this discussion in the next section. So, to summarize the discussion we have seen the waves at interfaces, particularly the reflection and refraction. We have also see the both the configuration that is TM and TE polarization, how the electric field magnetic field will be oriented with respect to the the structural geometry that is the interface plane, the incidence plane and we call this TM wave, TM-polarised wave as p-polarised wave and this TE-polarised wave as s-polarised wave and we were discussing the Fresnel reflection coefficient, amplitude reflection coefficient for p-polarised wave and we will continue this discussion in the next section.

Thank you.