Modern Optics
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Lecture – 01
Maxwell's equations and electromagnetic waves

Welcome to the course on Modern Optics. The course modern optics has the prime focus for light matter interaction. Light as you know is an electromagnetic wave. When an electromagnetic wave travels through a medium, several exciting phenomena happen; this is basically due to the interaction of the material medium with the electromagnetic wave. So, in order to understand this phenomena, we will briefly touch upon and visit the basics of the propagation characteristics of electromagnetic waves in free space material media, the medium could be lossy, it could be isotropic, anisotropic and so on and so forth; we will discuss all those things.

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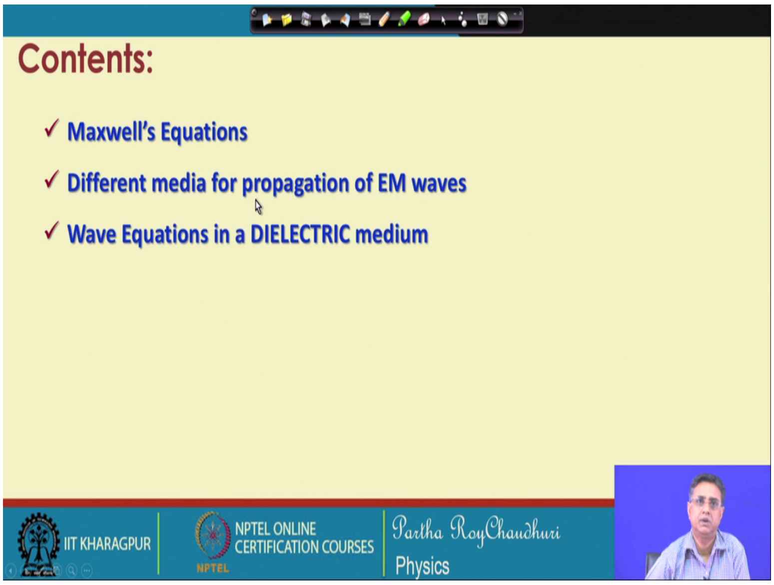

So, the content of the course is the Maxwell's equation and Electromagnetic Waves this under this we will discuss Maxwell's equation, the origin, how Maxwell's equations came up, these are basically experimental laws. And then we will talk about the different medium; media where the electromagnetic waves propagate; followed by that the wave equation in a dielectric medium.

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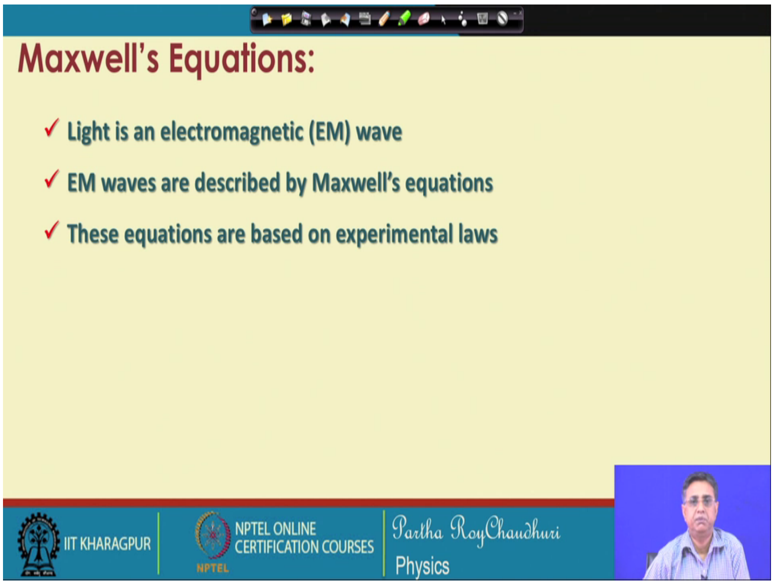

So, as you know that light is an electromagnetic wave and this electromagnetic waves are best described by Maxwell's equation. And these equations are based on the experimental laws.

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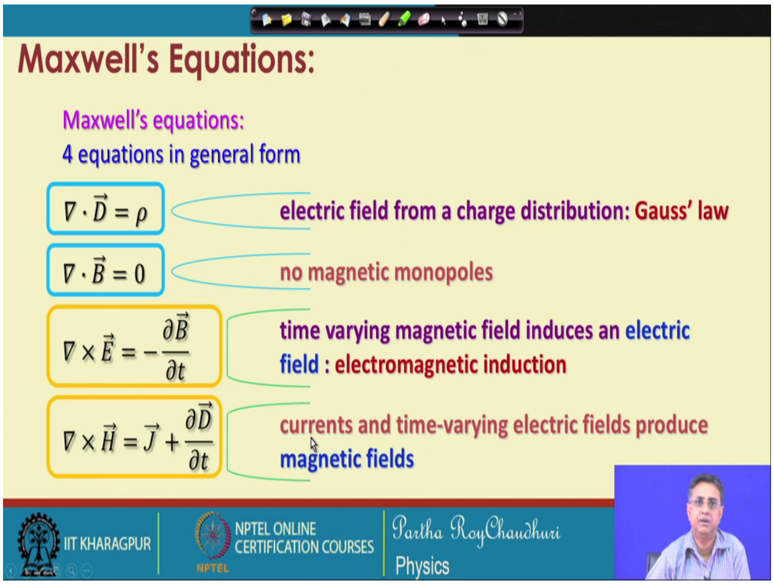

You know these four Maxwell's equation del dot D equal to rho; del dot B equal to 0; del cross E equal to minus del B del t; and the last curl equation del cross H equal to J plus del D del t. So, this first equation so called electric field from a charge distribution has actually come from the Gauss' law of electrostatics. And the second law which comes from the; from that no magnet monopole exist this fact, which is also a consequence of Gauss' law in magneto statics.

The third law that is del cross E is an outcome of the Faraday's law of electromagnetic induction which tells that the time varying magnetic field induces an electric field. The last one that is del cross H equal to J plus del D del t is the one which where Maxwell actually contributed by putting incorporating that this displacement current. So, this is current and time-varying electric field produce magnetic field. This fact is in incorporated in this law.

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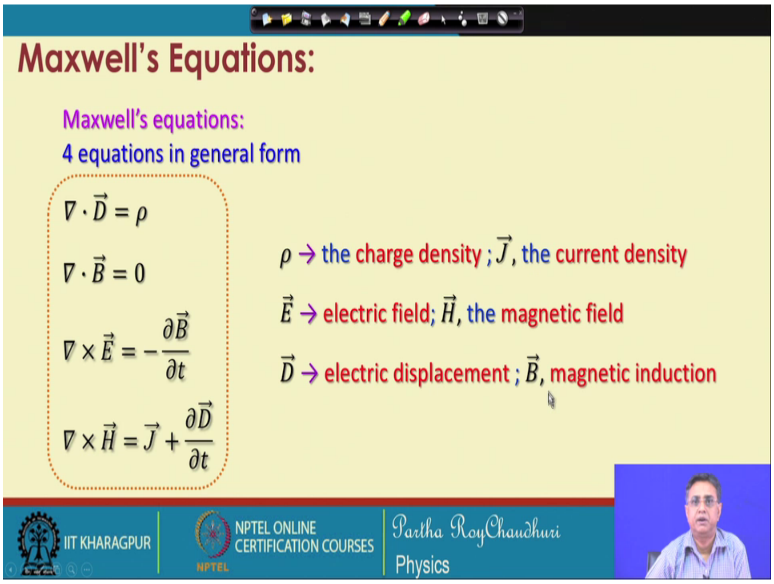

Now, this Maxwell's four equations looking at the symbols and the notations rho is the charge density; J is the current density; E the electric field and H, the magnetic field. D represents the electric displacement and B the magnetic induction vector.

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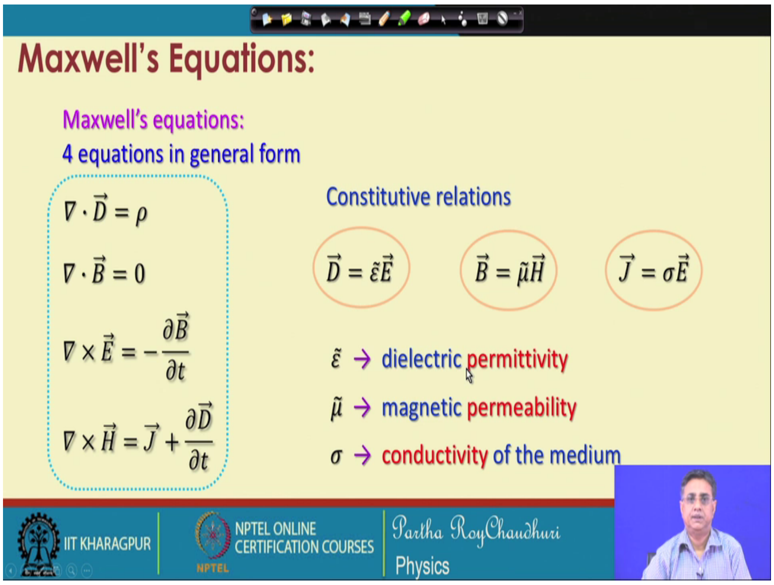

There are constitutive relations which are associated with these equations particularly with D, B and J. This D is equal to this epsilon into E, this epsilon in general is a tensor 3 by 3 9 component tensor; but in a simple medium, it reduces to a scalar quantity. Likewise B is also connected to the magnetic field through this tensor mu, which is in general a 3 by 3 9 component tensor. And for a simple medium and for most dialectics this mu is a scalar and has a constant value. J is the current density which is again connected to the electric field by the equation sigma. This is actually the outcome of the ohm's law. Now, this e dielectric permittivity as I have told is the 3 by 3 tensor. Magnetic permeability is also a nine component tensor; sigma the conductivity of the medium. These are the basic definitions and the terms which are associated with the Maxwell's equation.

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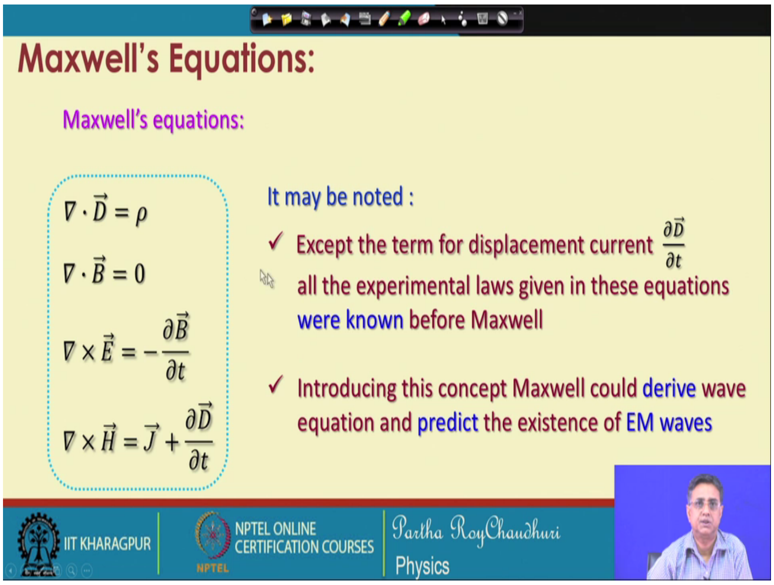

Now, going back with the history of Maxwell's equation except the term, which represents the displacement current that is this del D del t all other laws were already given in these equations are were known before Maxwell. Now, Maxwell introduced this concept of this displacement current. And by introducing this displacement current, he could derive the wave equation and he could predict that electromagnetic waves exist. This we will see in the later stage that how the experimental laws put together you see the Maxwell's equation; and from there you arrive that the wave equation. From there you calculate a very important quantity that is the velocity of the wave and how you verify the experimental law.

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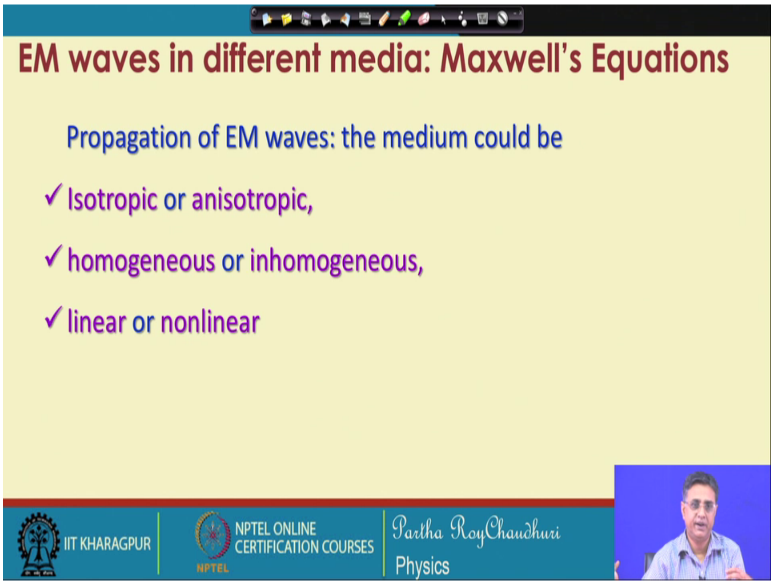

Well propagation of electromagnetic waves can happen in different medium natural medium; it could be isotropic or anisotropic. So, let us first of all try to look at what is the isotropic. If the material, if the medium through which the electromagnetic wave propagates is independent of the direction of the of the propagation of the wave that is it behaves identically whichever direction the electromagnetic wave propagates, then it is called isotropic. If the behavior of the material in terms of the permittivity or more particularly the refractive index if it depends on the direction of propagation of the electromagnetic waves, then the material is called the anisotropic material.

So, these are there are two broad varieties of materials, homogeneous and inhomogeneous. If the material part composition everywhere will within the medium is the same, then we call it is the homogeneous medium. And if the composition varies from different from place to place, for example, I start with a pure quartz at one end and in the medium the other end it becomes ordinary glass, then the material composition has varied and it has become an inhomogeneous material.

Linear and non-linear, this is another important aspect of the medium. If the properties of the medium particularly the permittivity or the refractive index if it depends if it is a function of the of the electric of the intensity of the electric field, intensity of the incident electromagnetic wave, then it becomes non-linear. If it does not behave with the intensity, does not behave differently with the intensity of the incident reflect incident electromagnetic wave then it is a linear medium. So, these are the three broad varieties of material medium through, which electromagnetic waves are propagate and we will study each of them in this discussion.

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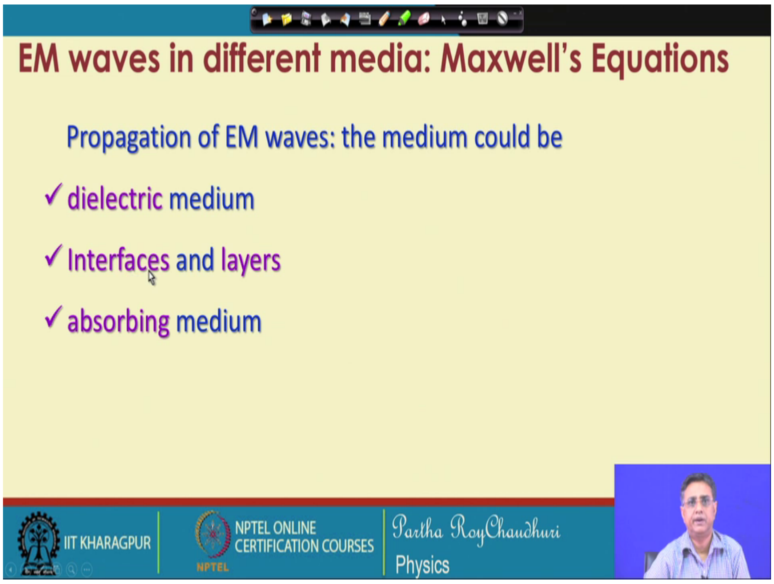

The medium could be dielectric medium also; it could be interfaces and layers. It could be an absorbing medium lossy medium as well. For example, when you take the interfaces the of two dielectric media and if an electromagnetic wave has to propagate to this medium, then how this structure interacts with the electromagnetic wave we will study that. And if the medium is lossy for example, any conducting medium, then how it works that also we will study in this discussion.

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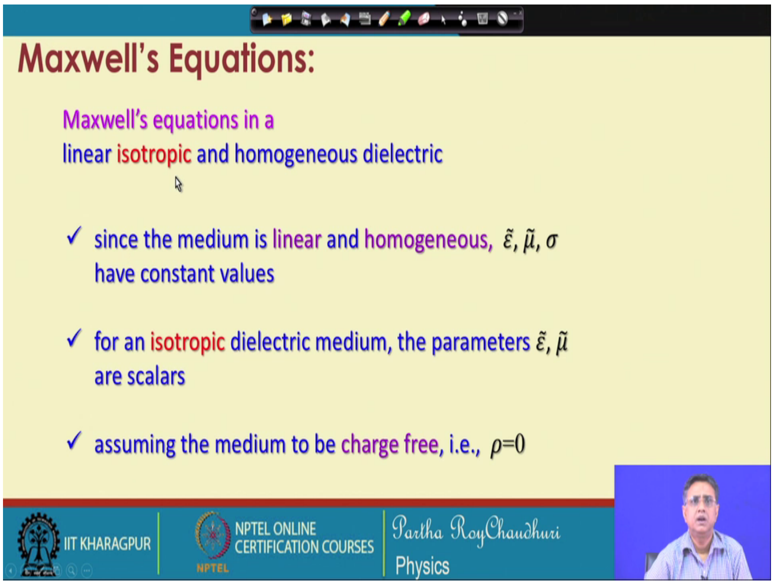

Maxwell's equations in linear isotropic and homogeneous dielectric this particularly the first thing that we would like to emphasize is that this is the case of a free space or vacuum that we will study and we will arrive that the different equations. Since, this medium is linear and homogeneous then the permittivity, permeability and the conductivity all these parameters assume a constant value. They are no more tensors, if the medium is very simple. And because it is an isotropic medium dielectric medium, so these quantities are also scalars. Assuming the medium to be charge free that is if you assume that there is no free charge in the medium, which is the most common case then we put rho equal to 0.

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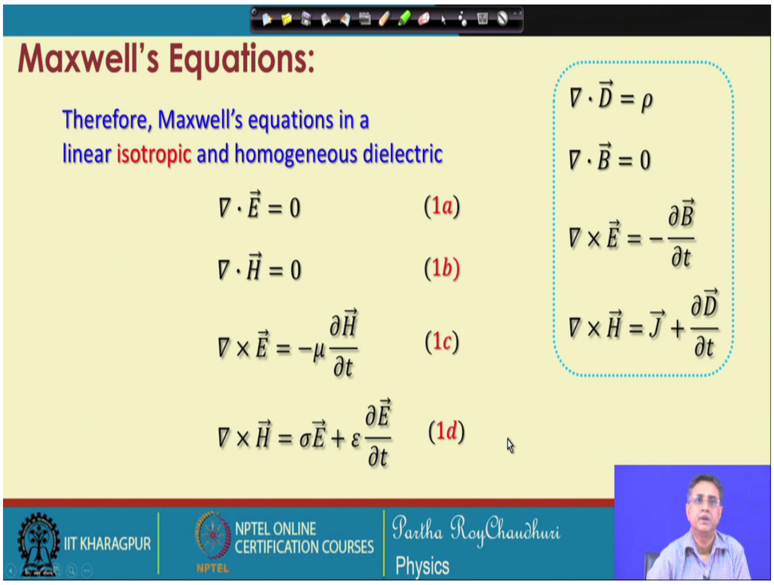

Putting that rho equal to 0 that is charge free reason, this equation becomes this, because D you have simplified by epsilon into E, where E is a con epsilon is a constant. And similarly, del dot B has also become simplified, because mu is again constant. Del cross E this equation will take the form of this mu times del H del t. I have use that constitutive relations. And for this equation, del cross H also, the J is now again replaced by the constitutive relations sigma into E. So, these are the equations set of four equations, which now represent the isotropic homogeneous and linear dielectric medium. And let us see how we organize this equation to arrive at the wave equation. So, let us take the curl of this equation.

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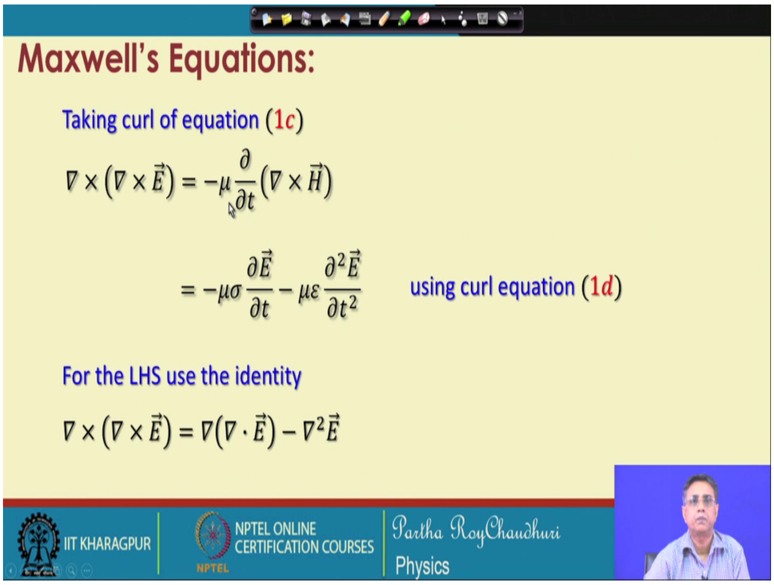

Let us take the curl of this equation. So, this equation will give you del cross H del del t of that. Now, because time and space this derivative they are independent, so we can put in this form. And because del cross H is also known to us from here, so we just substitute the value of del cross H. So, we end up with an equation which is the curl equation from 1 d. For the LHS, we use a very well known identity that del cross del cross E is equal to this using this identity.

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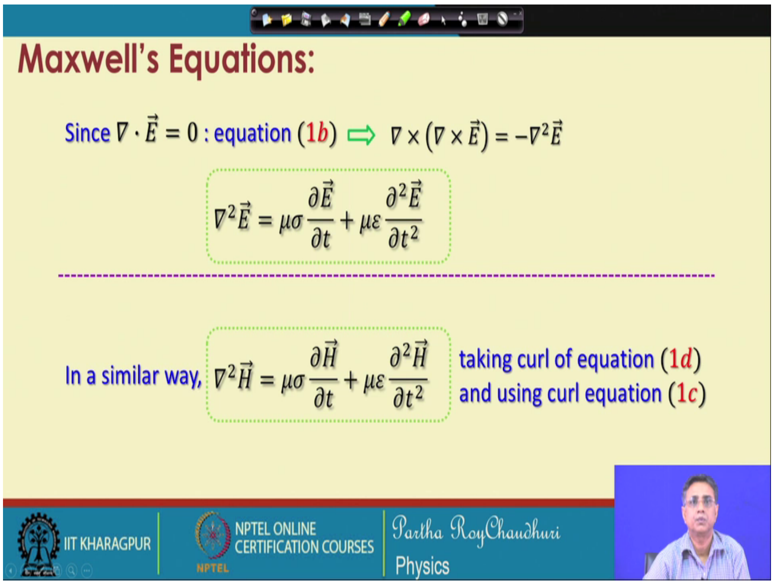

Now, we have this equation del dot E equal to 0. If I use this in this equation, then this quantity becomes 0. So, this identity becomes del cross del cross E is equal to minus del square E the Laplacian of the electric field vector. So, arranging both the sides, we can write this equation del square E equal to this quantity, and the second order time derivative of the electric field. If we proceed in the similar way starting with the third the fourth equation, and then if we substitute the third equation into that, then we will end up with a similar equation for the electric field, del square H equal to del H del t times this mu into sigma, and the second order derivative time derivative of the magnetic field. So, we have a pair of two equations representing electric field and the magnetic field.

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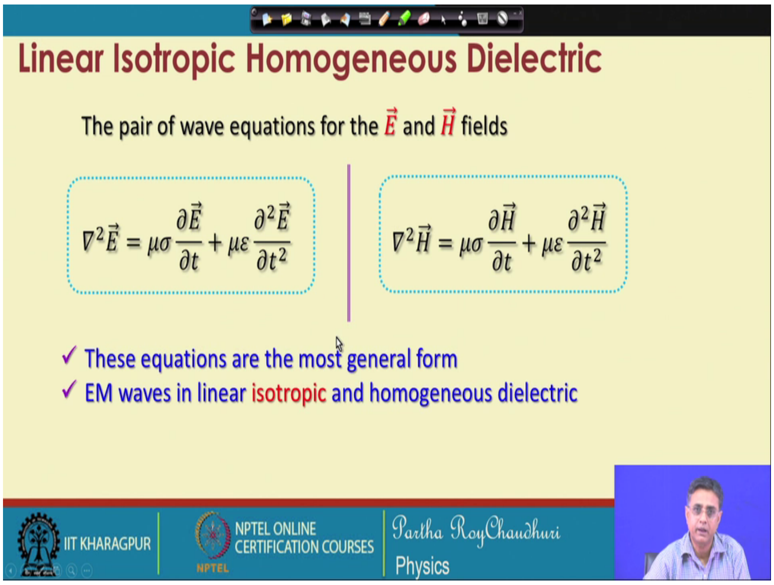

Now, this pair of wave equations for the electric and magnetic field, we can see that these equations are the most general form. Because if I use the specific properties of mu and sigma, epsilon etcetera, then they will reduce and take the appropriate form of the med of the medium, through which will study the properties of the electromagnetic waves. Electromagnetic waves in a linear isotropic and homogeneous dielectric medium, these are the most general form.

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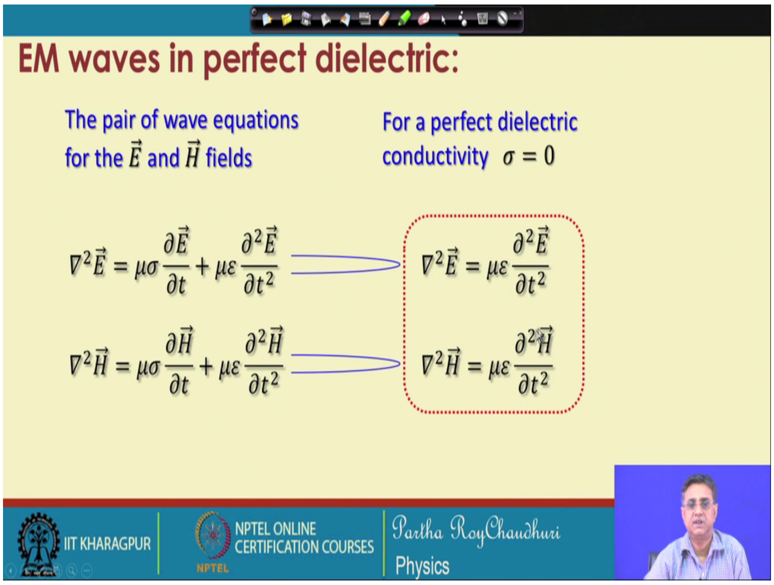

Now, the pair of wave equation this E field and H field for we will study this equation for a perfect dielectric conductivity, where sigma equal to 0. A dielectric perfect dielectric is represented by this property that sigma equal to 0, then this equation will reduce to this equation because you have put sigma equal to 0 here. And similarly for this equation, we will end up with this equation. So, the reduced form of the pair of wave equations; for this case of perfect dielectric are represented by these two equations.

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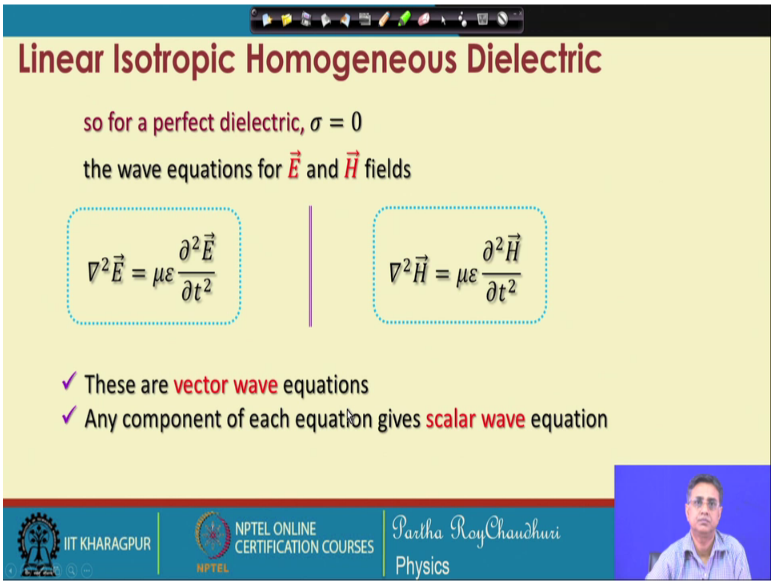

So, for a perfect dielectric sigma equal to 0, the wave equations for E and H fields are this as you have seen, these are vector equations. But if you see that if I take the component of E, for example, Cartesian components we will see later the E x, E y and E z that each of the components will also satisfy this equation. Similarly, for the magnetic field, in general this is the vector wave equation, but each of the components will satisfy this wave equation, and then that equation which will be satisfied by the components will be called the scalar wave equation.

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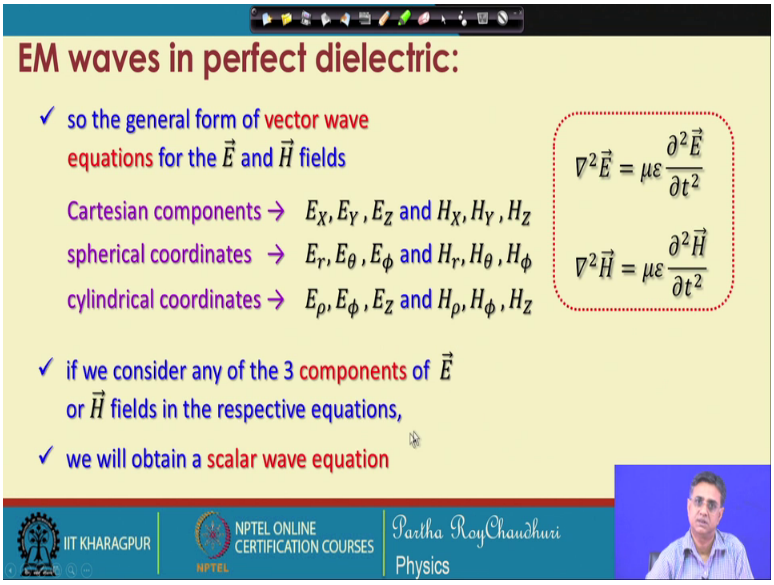

So, the general form of the vector wave equations for E and H fields or now this but the Cartesian components of this equation E x, E y, E z they will be again satisfied as I have mentioned before. If you consider 3 components of E or H field in the respective equations, then the same equation will represent the scalar wave equation.

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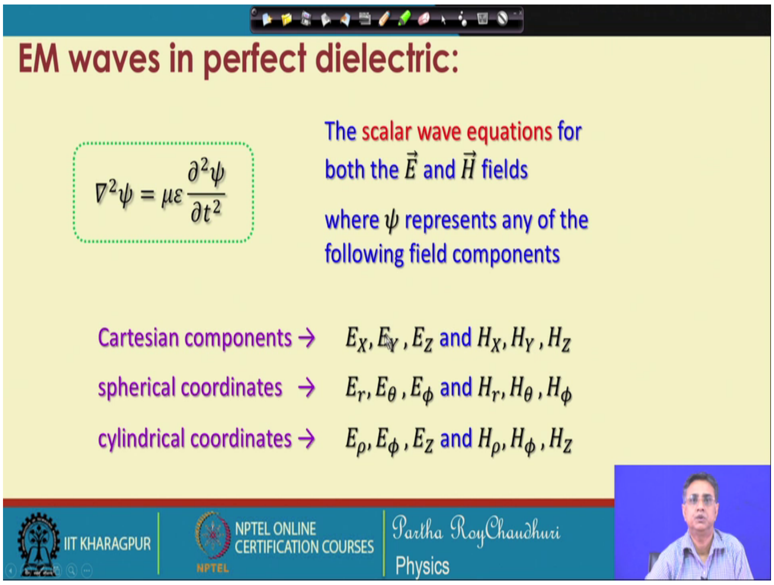

Now, because each of the components, all the 6 components of the electric and magnetic fields can be represented by these two equations; we can generalize and we can write one compact single equation for that in terms of psi. So, there are psi the equation for psi will represent the scalar wave equation, where the where the psi can assume can take up any value of this E x, E y, E z or H x, H y, H z. The Cartesian components are given by this E x, E y, E z and so on spherical coordinate in spherical coordinates these components will be E r, E theta, E phi; and cylindrical coordinates it will be like this.

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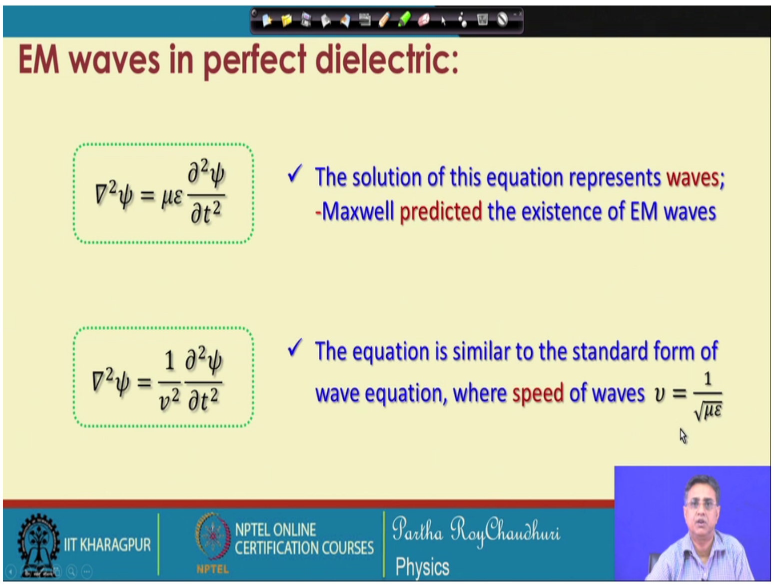

Now, the solution of this equation represents waves. Maxwell predicted the existence of electromagnetic waves, how he predicted after organizing this wave equation, he could arrive at the at the quantity v which represents the velocity in terms of this. So, this is the standard wave equation and this is the wave equation for electromagnetic waves, where psi represents the component of the electric fields in any of the coordinate systems. So, if you compare these two values, you get v equal to 1 upon under root of mu and epsilon all right.

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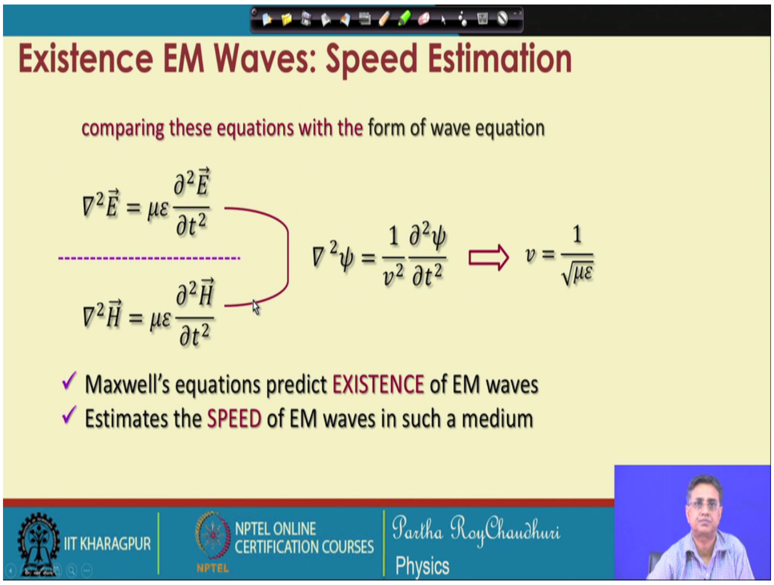

So, comparing these equations with the form of wave equation, we get that v equal to 1 upon mu naught epsilon 1. By this comparison, Maxwell could predict that the existence of the electromagnetic waves; and from this equation, he also predicted the speed of the electromagnetic waves in free space by knowing the values available to him in that time.

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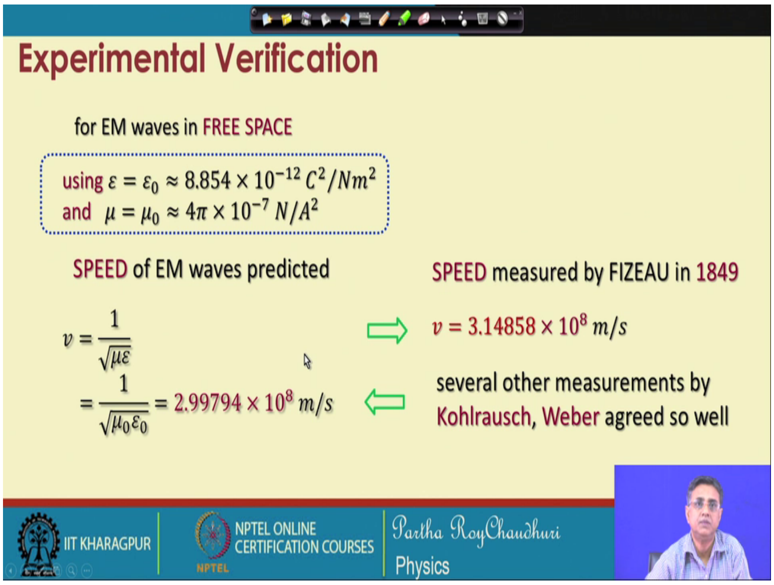

For electromagnetic waves in free space, the best value which was known to Maxwell was for epsilon naught, which is a scalar; and for free space the value is this. And for most dielectrics for free space the mu naught the permeability is given by this equation; these are very usual. And using these two numbers, the speed of the electromagnetic waves could be predicted by plugging into this equation. If I use these numbers, then the value of the velocity of the speed of the electromagnetic waves will come out to be this.

Now, in those days in around 1849, the speed of electromagnetic wave, speed of light waves were measured by several people FIZEAU's experiment is very well known and he measured the speed of light as this value. You can see that these two numbers are very close at least by the order and by the magnitude also. Maxwell commented that these two numbers are not accidental and during those time several other measurements by these people, Weber agreed also this number so well. So, there were several experimental measurements for measurement of the speed of light and Maxwell simultaneously predicted the estimated the value of the speed of light and these numbers where matching. So, by doing this, he could established that electromagnetic waves that is the light is the form of electromagnetic waves.

Thank you.