Atomic and Molecular Physics
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Lecture -08
Structure of an atom (Contd.)

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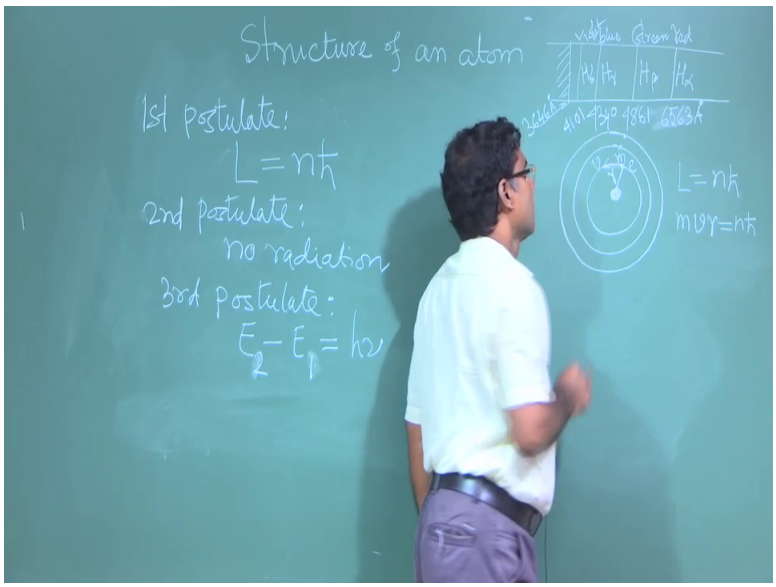

So, we will continue our discussion on Bohr model, so that was the three postulates. So electron, so; what was the model? So, nucleus Rutherford model it will rotate in a orbit

Now, first postulates of Bohr it tells that it can rotate in a definite orbit, this orbit or this orbit or this orbit. The orbit it is defined by this first postulates, the angular momentum has to be quantized and it will be n h cross. So, it has to be h cross; 2 h cross, 3 h cross, 4 h cross like this. So, that way one can find out the orbit and they are stationary orbit, fixed orbit. And then second postulate is telling; there will not be any radiation when they are rotating in this stationary orbit.

So, from first postulates we get L equal to n h cross angular momentum. So, that is basically m v r; if radius is r its mass of electron is m and its velocity is v. So, this m v r equal to n h cross, so from first postulates we can get and then here you see from Rutherford postulates or model. So, electrons are rotating revolving in a orbit; so, its charge is E and these nucleus; the charge is Z e. So, Bohr actually; he considered one electron system basically hydrogen atom or hydrogen like atom.

So, 1 election system if system is having 1 electron then he was trying to explain this hydrogen spectra. So, hydrogen like atom is 1 electron it is like helium. So, it will be 1 electronic if we take out, so it will be hydrogen like atom, but it is nucleus; so lithium. So, it has 3 electrons; so if you take out 2 electrons. So, it will be this nucleus; its charge will be plus 3 e. So, in generally we have taken Z e; Z e is atomic number; so, they are revolving. So, as we know this coulombs force Z e into e, so Z e square by r square.

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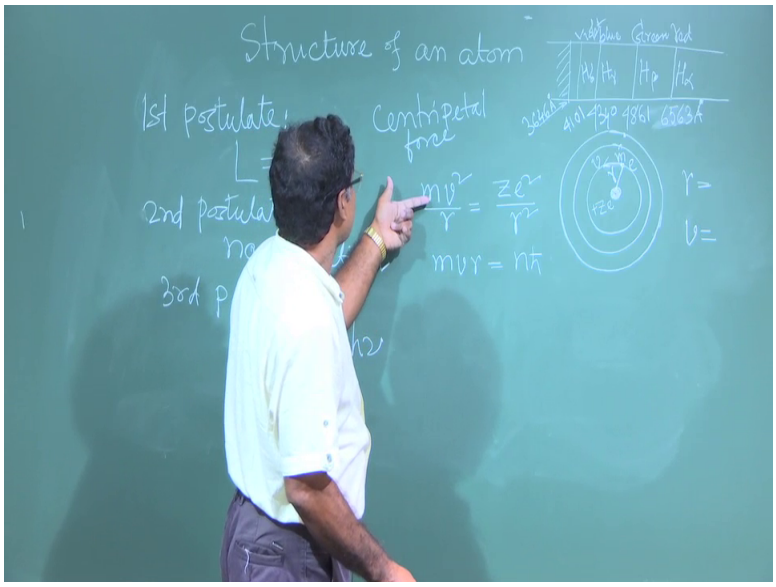

So, that is the force attracted force; so electron will try to come fill force towards the nucleus and when it is revolving, so there will be centripetal force; so, centripetal force will be equal to the Coulomb force.

So, centripetal force what is that? So, that is basically m v square by r; these are standard you know. So, this is one equation and second one is we are getting from here; what we are getting? So, let me write this way just in opposite way. So, basically m v square by r equal to Z e square by r square and other one; this m v r equal to n h cross. So, from here we can get expression of radius r and velocity of electron in the orbit.

So, just is simple you can find out; so, m v square here r square. So, by m; I will take this and then this r square, if I take this side; so r square will not be there. So, m v r equal to n h cross, so this are n; this I can write as a this.

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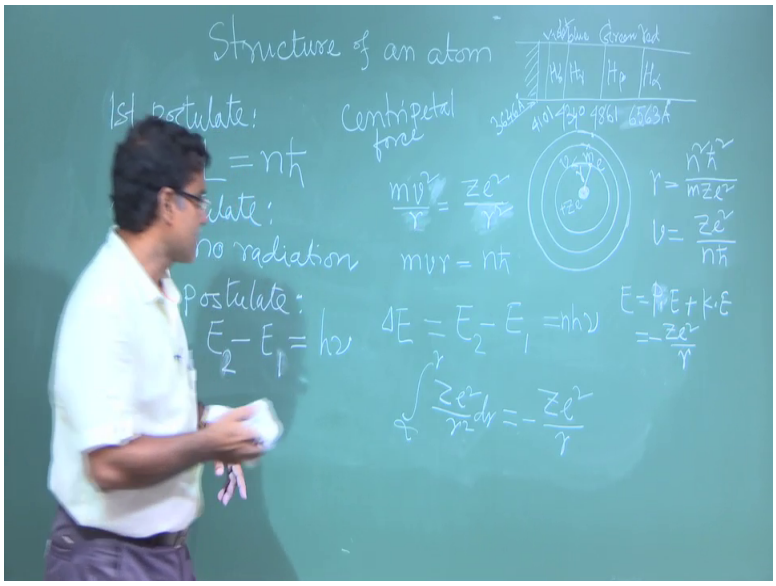

I can write n square h cross square by m r equal to Z e square. So, r equal to n square h cross square by; m Z e square, so r equal to n square h cross square by m; Z e square.

So, let me go back to the m v square r equal to r square; so that was our original. So now, if I want to find out m v square by m v square by r equal to Z e square r square, so if this r; this will go. So, this I will take here; so m v r equal to n h cross, I can write n h cross into r equal to Z e square. So, r equal to Z e square by n h cross; so, sorry not r; m v r, this is v; it will be v. So, v equal to; I will get Z e square by n h cross.

So, one can find this from this two basic equation, let us go back. So, m v square by r that are; so, that is the original relation, these two relation. Now, radiation what is the energy of the; when it will jump from one level to the another level? So from third postulates; so energy E equal to, so say E 1, E 2 minus E 1. So, this will give us energy difference; so I can write del nu.

So, that will come out as a radiation and it is it h nu of; one can write n h nu. So, what is the energy expression? I can write E equal to kinetic energy plus potential energy of the electron plus potential kinetic energy plus potential energy of electron; what is the potential energy? So these the Z e square; r square this force. So, potential energy different from infinity to taking an distance r; what is the work to be done? So, Z e square by r square d r; see if you integrate, you will see that is comes minus Z e square by r. So, that will be the potential energy.

So, E equal to I will write minus potential Z e square by r and then kinetic energy half m v square kinetic energy is half m v square.

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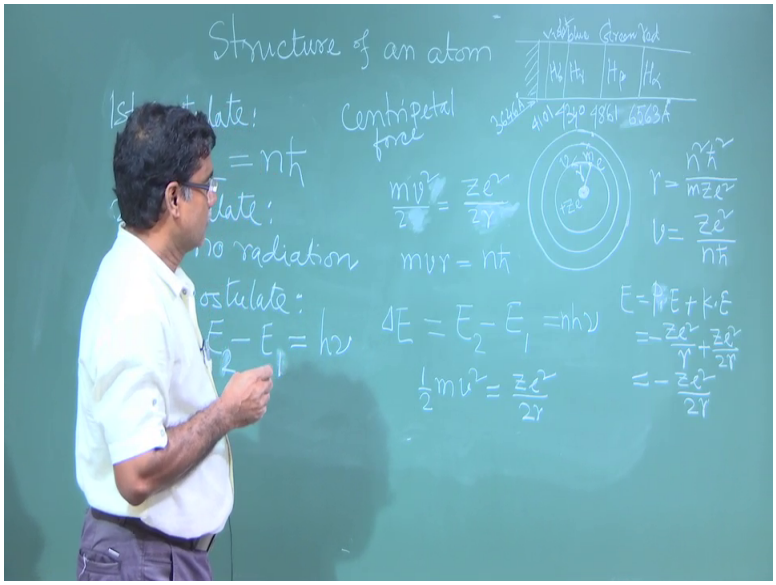

Kinetic energy half m v square; so from here itself you can see m v square. So, half m v square; so this r, so I have to put 2 here. So, this r will go; so, half m v square equal to Z e square by 2 r. So, this half m v square equal to Z e square by 2 r.

So, plus Z e square by 2 r; so this total energy kinetic energy plus potential energy, you will get minus Z e square by 2 r; Z e square by 2 r. Now, if you put the value of r you; so what we will get energy expression.

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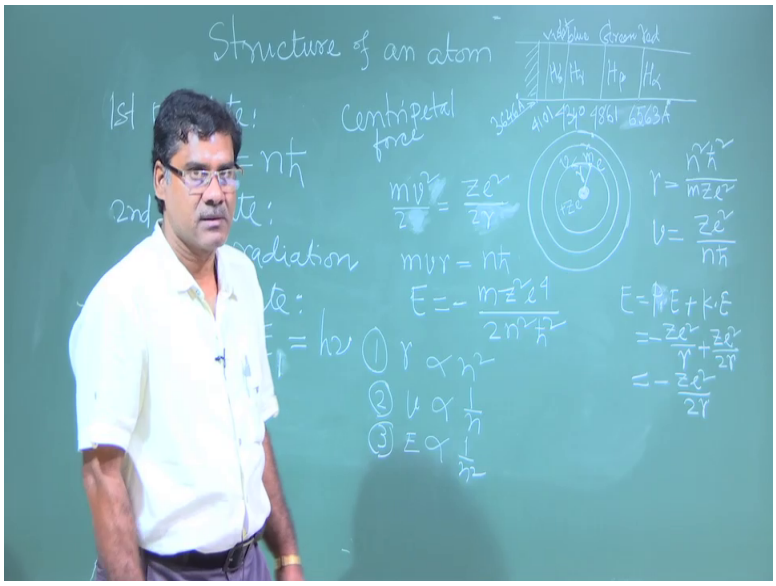

So, I will remove it; so I will get energy expression E equal to, so n square an m Z e square. So, I will get minus m z square E to the power 4 divide by 2 m square; z square. So, that is the energy of electron in a particular orbit basically n th orbit; so from here you can see, you can conclude that this radius; one can conclude that radius of the orbit is proportional to the n square; that means, when we will go this first orbit whatever the radius second orbit radius will be four times; third it will be 9 times.

So for first orbit; if is the r 0; so, second one it will four times of that one third; one it will be 9 times of that one. So, one can draw the orbit and one can just speculate this; how the radiuses of the orbits are varying. Second is velocity; velocity is basically inversely proportional to orbit number first orbit; what was the velocity? Second orbit velocity will decrease, third orbit velocity will decrease.

So, it is inversely proportional and energy also it is minus. So, inversely proportional and energy also it is minus sign; is there third energy also; it is proportional to the 1 by n square. So in principles if you know the radius velocity energy of the first orbit, in terms of that you can express the energy velocity radius of the other orbit.

So now this energy difference; when it is going from one orbit to the another orbit, so these the energies.

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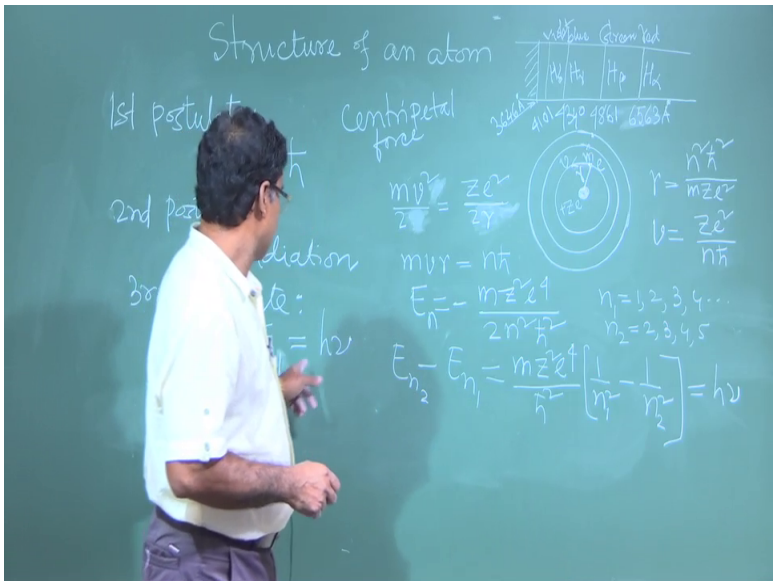

So if I take write basically n energy of the n th orbit, so from third postulates; I can write E n. So, n it goes from n 2; n 2 E n 1; so, this you can get. So, this n 1, n 1, n 2 is higher than n 1; so basically when from a higher energy level to the lower energy level jumps then its emit and other it actually absorb. So, here we are considering the emission of radiation from atom due to which jump of electron from one orbit to the another; from higher orbit to the lower orbit; lower energy orbit.

So, this E for this I can write m Z square e to the power 4 by h cross square; then since minus sign is there. So, you will get 1 by n 1 square n 1 square 1 by n 1 square minus 1 by n 2 square; where n 1, n 2; n 1 can take 1, 2, 3, 4 etcetera, n 2 can take 1, 2, 3, 4 , 5 etcetera.

So, this will be kept; so it will this energy difference will give us radiation and that radiation is energies; h nu according to third postulates. So, this energies h nu; so from here I can write.

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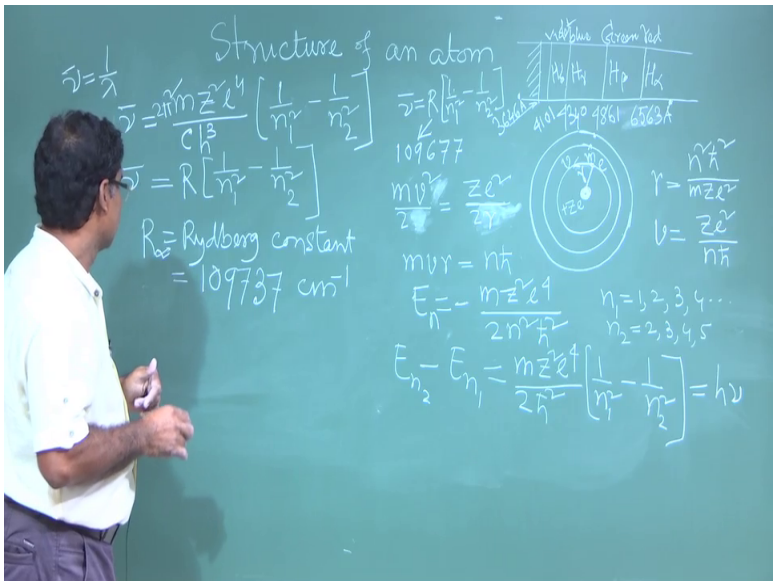

I can write this nu equal to m Z square e to the power 4 divide; by now here I am missing 1, 2; I have to put 2; basically here is 2 and h square; h square by 4 pi square; so 4 pi square; so 2 basically; 2 pi square.

So, that I can take off, so here I can write 2 pi square m z square; e to the power 4. Now, h square h cross; it will be h that is square and now the other side h nu. So, it is h cube and then 1 by n 1 square minus 1 by n 2 square and generally as I mentioned that Balmer empirical formula that was written that nu bar equal to r 1 by n 1 square minus 1 by n 2 square. So, that is the empirical formula in 1885; it was given to explain this atomic spectra.

From Bohr model whether we can get this or not; so that is the first if you get. So, it will be able to explain that one and we are going to achieve that one. So, nu equal to it is basically c by lambda; so this is written 1 by lambda is 1 by lambda is clearly its written nu bar wave number is nu bar; so, nu bar. So, this we can write c nu bar.

So, actually you are getting then nu bar. So, if I write c here; so then I can write nu bar here. So, nu bar equal to this; so now to compare with this Balmer's empirical formula, that is why we have take. So, nu bar equal to here r 1 by n 1 square minus 1 by n 2 square. So, this is exactly; this theory gives exactly this empirical formula, which was able to explain this spectra hydrogen spectra.

So, but unfortunately so this now r is known and it is called Rydberg constant; and its value from here all are constant. So, if you put the value and that value is 10937 centimeter inverse; if put the value and this now this was slightly different, then this value means this constant was choosen in such a way; so that is it fit the experimental data; it explain experimental data.

So, this value was basically taken that is the experimental value; these value was taken 1, ,0, 9, 6, 7, 7 , 1, 0, 9, 1, 0, 9, 6, 7, 7, 1, 0, 9, 7, 3, 7. So, it was slightly higher this value was slightly higher than the experimental value; otherwise everything is fine. So, these value can explain can get reproduce this wavelength or in terms of wave number.

So, that is why we write this r infinity; why we are writing infinity? So, you know that again it was started to re think why this discrepancy? Why this difference is coming? Then it is realized that this actually here, we have considered the nucleus is having infinite mass.

So, it is at rest; it was considered the nuclear nucleus is at rest and only electron is rotating moving. So, this is only possible when this nucleus mass will be infinity, but it is not the case; its mass is finite, mass is finite; mass m capital M and so one has to consider that finite mass of the electron and if it is mass is finite then it is not only the electron is moving. So, electron and this nucleus both are moving with respect to the center of mass.

So, if we considered the center of mass.

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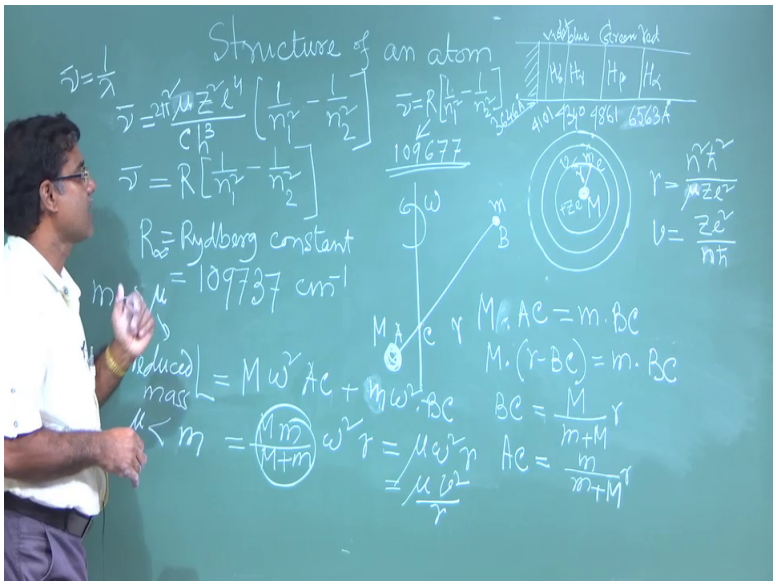

So, it is the electron; it is the nucleus. So, both are rotating with respect to a axis passing through the center of mass. So, this center of mass c; this is capital M, this small m it is; say this is B, it is rotating with omega angular velocity omega. So, from here one can find out; so, whether I should complete it. So, from center of mass m into AC; that distance it has to be equal to the m into BC; that distance if this distance between this distance is nucleus and this is r.

So, basically AC I can replace AC I can replace m into r minus BC; r minus BC equal to m; BC. So, you will get BC equal to BC equal we said m plus m and other side equal to m by m plus m into r; similarly we can find out AC; AC equal to m by m plus m r.

So, total angular momentum will be L total angular momentum l. So, this angular momentum of this one m omega square r m omega square not r with respect to this; m omega square AC plus m omega square sorry small m omega square BC. So, put value of AC and BC and you can show that it will come like this; this m omega square r. So, this if you write mu omega square; r mu omega square r or it is equal to mu omega into r is v basically; so we can write.

So, earlier we took this angular momentum m b square by r; now it is writing mu v square by r. So, m if you replace m by mu then all expressions are valid; so in that case we are considering that nucleus is not at a rest, it is also moving. We nucleus as a whole with respect to centre of mass; so this mu is called reduced mass; why? Because mu is

less than m from here; you can show mu is less than m. So, now the same just we have to replace this find energy, where is energy? So, this m we have to replace by; so mu is less than m.

So, then now if you consider so this Rydberg constant; so mu is less than m; so, it will be reduced and it will basically reduce and it will get this value. So, now fact is the; so this Rydberg constant, these values for when you are considering the nucleus is of infinite mass; it is at rest, but in reality it is not the case and one has to consider this that; both electron and this nucleous, both are rotating with respect to the center of mass and we will get reduced mass and then it will give a checked that Rydberg constant, which is experimentally found. And now this from both; you need first time this spectra, hydrogen spectra was explained. So, I will stop here and I will continue in next class.

Thank you.