

**Atomic and Molecular Physics**  
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**Lecture -40**  
**Hydrogen like atom in electric field**

So far, what we have discussed that hydrogen like atom, now if it is in external magnetic field. So, what will happen? So, when it is in external magnetic field. So, we have seen that that spectral line which is which is 1 line without external magnetic field. So now, under the magnetic field these lines, spectral lines are split it into different number of lines ok. So, for we have seen that in case of normal Zeeman Effect, where magnetic field is very high. So, magnetic interaction term is very strong compared to  $l s$  coupling term.

So that is the basically condition. So, far and there we have we have we have seen that each spectral lines means before applying magnetic field it splitted into 3 lines, it is splitted into 3 lines. So, this it does not matter which spectral line what is the  $l$  value, what is the  $n$  value. So, every spectral line will be splitted into 3 lines and that is called Lorene stifled. So far, so, in this case  $l s$  coupling should be very week compared to the magnetic interaction term and. so, we basically neglect this  $l s$  coupling interaction and as I mentioned that magnetic field should be very high and how, how much it should be high. So, that is condition is  $b$  will be greater than  $z$  to the power 4. So, even  $z$  to the power 4 tesla. So, even for hydrogen atom  $z$  equal to 1. So, you need greater than 1 tesla field. So, for second atom helium  $z$  equal to 2 for that  $z$  to the power 4  $z$  is 2.

So, you need basically, no its  $z$  to the power 4 means 16 tesla magnetic field. So, 16 tesla magnetic field in laboratory it is very very difficult to produce because gel electromagnet in laboratory it can produce maximum 2 to 3 tesla magnetic field and had then the 2 to 3 tesla gel use superconducting magnet and using superconducting magnet doing the experiment is also difficult, but anyway.

So, even using the superconducting magnet, field moral is generally less than 50 tesla one can produce so, generally it is even whatever I know, I have seen in our normal labs. So, magnetic field around 10 to 20 tesla we are using. So, even 30 tesla also one can get. So, that is enough I live I just consider that 50 tesla.

So; that means, that for lithium  $z$  equal to 3. So, 3 to the power 4 means 81 tesla magnetic field. So, hardly we can see using the using the electromagnet hardly we can see the see the normal Zeeman effect in case of hydrogen atom. Using the using the super conducting magnet we can maximum we can see the normal Zeeman effect for helium , but other than helium.

So, for higher  $z$  this lithium, beryllium etcetra etcetra we cannot see the normal Zeeman effect, but in our laboratory to demonstrate normal Zeeman effect, generally people use neon and mercury source neon and mercury atom. So, neon this I think  $z$  value is 10 and mercury  $z$  value 80. So now imagine how how big how how much filed you need if this is the condition for normal Zeeman effect that  $b$  has to be greater than  $z$  to the power 4.

. So,. So, then we cannot see this normal Zeeman effect for for neon and mercury. T hen how you are getting this normal Zeeman effect in our laboratory. So, reason is that main reason is that that  $l s$  coupling has to be very very small compared to the magnetic interaction term. So, if either  $l$  or  $s$  or both or 0 for some atom, then  $l s$  coupling term varies is 0 ok. So, whatever the magnetic field that is really higher than the 0 ok. So, we will see the normal Zeeman effect.

So, that is the, that is why for normal Zeeman effect criteria is, we always tell in terms of magnetic field higher magnetic field it is not the main criteria. Criteria is always one has to compare with the  $l s$  coupling term. So, this magnetic field should be high enough. So, that this magnetic interaction term has to be will has to be very very greater than the  $l s$  coupling term, ok. So, that is the, that is the main condition for normal Zeeman Effect. So, for next intermediate magnetic field again we are telling in terms of magnetic field. So, again here main condition is that this  $l s$  coupling term and magnetic interaction term, it has to be comparable.

They are closely all the magnetic interaction term has to be will be a higher than the  $l s$  coupling term. They are comparable their magnitude is comparable ok. It is close to each other. There is no very high difference. So, under this condition, what about the effect of magnetic field on spectral lines, we see. So, that effect his call Paschen back effect and there you have seen that whatever in normal Zeeman effect we see 3 lines and in Paschen back effect. There we see each line is splitted into 2 is formed the doublet basically; that means, they are closely spaced ok, but they are separable ok. So, so they form the

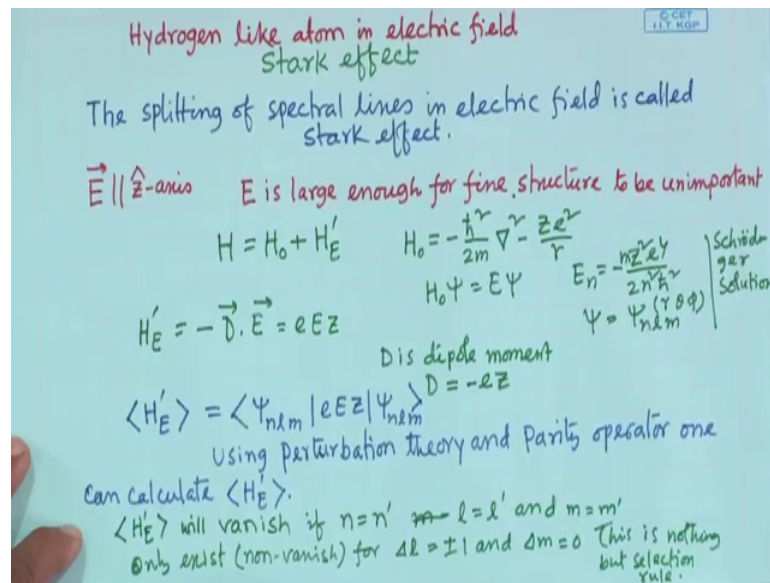
doublet. So, will get basically is 3 normal Zeeman splitting or spectral lines will be 3 doublets, 3 doublet means will get the 6 lines ok. So, that is the Paschen back effect so.

Now, if magnetic field is very very weak. So,  $l s$  coupling term dominate, then the magnetic interaction term. Then we have to we have to consider consider this relativistic correction, then Darwin term. What basically we have to consider the Dirac Hamiltonian ok. We cannot neglect this relativistic term and the Darwin term because they are comparable with the  $l s$  term although they are smaller the  $l s$  coupling term ok. So, when here  $l s$  coupling term will be stronger then the magnetic interaction term. So, in that case we have seen the magnetic effect on the spectral lines and that is anealogic Zeeman effect and we have seen that that transition between the  $p$  half to  $s$  half.

So, it is without magnetic field is the 1 line ok, but it is after applying weak magnetic field it is splitted into 4 lines and so, this in case of hydrogen this fine. In case of sodium this this line we tell that sodium  $d_1$  line. So, sodium  $d_1$  line, one can say that it is splitted into 4 lines and other  $P_3$  by  $2s$  half that transition it is that line, it is in case of sodium it is called  $d_2$  line and this  $d_2$  line is splitted into 6 lines. So, in Anomala Zeeman effect the splitting of the line are more ok.

So, this is because of strong magnetic, for strong  $l s$  coupling interaction now, other now if I apply electric field in place of magnetic field, if I apply electric field. So, what happens? So, for that I want to discuss, I will not do much mathematics because this it is not as simple as this Zeeman Effect. So, calculation is slightly longer and one need to really need to use more quantum mechanics mechanical calculation. So, I will avoid calculation, but I will logically I will try to explain, what is what is the effect of electric field on hydrogen.

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Like atom. So, So, hydrogen like atom in electric field. So, this effect is called basically Stark effect ok. So, what is Stark effect? That is basically splitting of spectral lines in electric field is just like similar to the Zeeman effect or Paschen back effect.

In case of magnetic field, in case, electric field similar things will may happen and that is called basically Stark effect ok. Now, take electric field and apply apply it. So, if you take this direction of the electric field line like magnetic field in space is along the z direction ok. So, electric field is applied along the z direction and atoms are there. So, now, this electric field also is taken z electric field large is in general electric field is large and this electric field is large enough for fine structure to be unimportant. So, what is the meaning of this statement? Meaning of the statement is that in case of Zeeman effect we have seen. So, in atoms this l s coupling term is basically it is responsible for the fine structure right. So, this has strong influence on the atomic spectral lines.

so fine structure here, fine structure to be unimportant means l s coupling term is weak compare to the term of electric field term related to the electric field. So, electric interaction, magnetic interaction term and this case electric interaction term that is very very strong than the fine structure means l s interaction term. So, this is the case is similar to the to the normal Zeeman Effect you know, there will we have considered that this magnetic interaction or magnetic field is large enough.

So, that this magnetic interaction term is very very high than the spin orbit interaction term ok. So, here also this we have considered this case only, for other case will not discuss or other case have no much importance. So, here fine structure is negligible compared to the electric interaction term. So, now, Hamiltonian, then naturally  $h$  equal to  $h_0$  unperturbed Hamiltonian plus this perturbed term. In this Schrodinger Hamiltonian right because  $l s$  coupling term is negligible means this relativistic correction as well as Darwin term, they are even smaller. So, we they are also negligible. So, here  $h_0$  is nothing, but the Schrodinger Hamiltonian.

Now, this has been taken as a unperturbed Hamiltonian plus this perturbed Hamiltonian ok. So, now, perturbed Hamiltonian  $H' = -\vec{\mu} \cdot \vec{B}$  due to electric field so, that is like there we have considered this magnetic moment dot  $\vec{b}$  in case of magnetic interaction term or energy. So, that was minus  $\mu$  dot minus  $\mu$  magnetic movement dot  $\vec{b}$ . So, similarly here in case of electric field electric dipole moment dot that is  $\vec{D} \cdot \vec{E}$  minus  $\vec{D} \cdot \vec{E}$ . So, this is similar to this magnetic interaction terms. So,  $\vec{D}$  is nothing, but the dipole moment, dipole moment is charged that to charge separated by distance.

Now, along the  $z$  directions we are considering. So, this we have taken that distance  $z$ . So, electronic charge  $E$  that is minus charge. So, minus we have written. So, minus  $E z$  ok. So, that is the dipole moment of electrons in hydrogen like atoms. So, this basically also we see this  $ye$ . So, which electron this dipole moment will defence it is that that electron is higher staying, either it is staying at a  $S$  orbit or  $P$  orbit or  $D$  orbit so, that also factor that will see in calculation. So, in our case this that magnetic interaction term perturbed term, perturbed Hamiltonian that is equal to ultimately  $E$ , capital  $E$  electric field into  $z$ . So, this the our Hamiltonian perturbed Hamiltonian..

Now, consider. So, here that energy unperturbed Hamiltonian, we know the wave function as well as we know the energy term. So, energy term is this and this  $\psi$  this suitable Eigen function is  $\psi_{nlm}$ , in case of whatever the for Schrodinger equation for Schrodinger solution, whatever we got the wave function. So, that is the wave function we can use and now using this wave function we have to find out this perturbed term ok. So, expectation value of this Hamiltonian. So, that will be that will give the energy change due to this perturbed term. So, that is we have written here in Dirac notation. So, that 1 has to calculate this now calculation of this again, one has to use quantum

mechanics and its. So, basically one has to use this perturbation theory and parity operator.

So, now using this perturbation theory and parity operator one can calculate this term. Perturbation Hamilton correction term one can calculate. So, I will not calculate this one. So, I will avoid this calculation, but I can logically I can tell you this, the effect of this electric field. So, now, we see this term. So, here basically this, what we have written this one ok. So, it will vary it will it will depends on this value of this term it will depend on NLM right ok. So, this term vanish if  $n$  is equal to  $n$  dash,  $l$  is equal to  $l$  dash and  $m$  is equal to  $l$  dash.

So, here 2 states, this basically transition between 2 states that gives the spectral lines no. So, here this, what are the, what are those states ok. So, they are NLM can be same or can be different ok. So, from this property of this wave function as well as I told this wave function is basically the wave function, simultaneous wave function of this parity operator. I have not discussed this parity operator. So, parity operator, in terms of parity operators, this operator has the same wave function simultaneous wave function of  $\psi$  NLM.

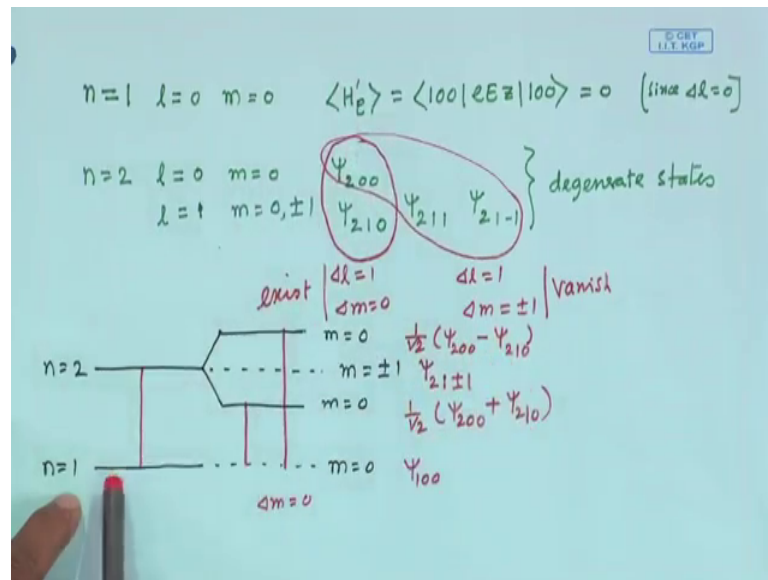
So, because this  $H_{mH0}$  and this operator  $P$  they common ok. So, that is that is from hydrogen like atom it has one can prove it. So, that is why this parity operator also very important for calculating this one. So, here just I am just telling the result if you calculate. So, one can show that this  $H E$  dash, this term will vanish if  $n$  equal to  $n$  dash  $l$  equal to  $l$  dash and  $m$  equal to  $m$  dash.

So, only exist means this will not vanish; only it will exists ok. If or for  $\Delta l$  equal to plus minus 1, means  $l, l$  dash should not be same and simultaneously  $\Delta m$  equal to 0 ok. So, when this condition will be satisfied, then this term we exist and this is nothing, but the selection rule ok. So, here basically I am telling you the result in terms of selection rule, but if you calculate then basically you can find out the selection rule ok. So, after calculation you will show, you will be able to show that when  $\Delta l$  equal to plus minus 1 is nothing, but selection rule.

So, avoiding the calculation, I am using the reverse logic, as if this I accepted the selection rule and applying the selection rule, I am telling that this term will exist only when  $l$  equal to  $l$  dash  $l$  plus or  $l$  minus 1 and  $l$  equal to  $l$  dash. So, simultaneously they

have to satisfy this condition. Then this term will have some value otherwise it is 0, if it is 0 means there is the effect of magnetic field on the lines, on the energy levels right ok. If it exists only then we can tell that there is effect of electric field on the energy levels.

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So, now, just let us consider a simple example that in case of hydrogen atom; so, for  $l$  equal to 1 so,  $l$  equal to 0,  $m$  equal to 0. So, sometimes I am writing  $m=1$ . sometimes I am writing  $m=0$ , but both are the same ok. So, for this energy level this presents 1 energy level, this presents one energy level  $n$  equal to 1 and  $l$  equal to 0  $m$  equal to 0. So, for  $l$   $n$  equal to 0, this is the energy level. So, now, for this, before applying magnetic field this is the energy level  $n$  we know this  $E_1$ , what is the value.

Now, if you calculate this one electric field effect ok. So, one has to calculate, this here for this energy level ok. So, this  $n$  equal to 1,  $l$  equal to 0  $m$  equal to 0; that means, I can write  $\psi_{100} E \times z \psi_{100}$  ok. So, obviously here,  $l$  equal to 1 dash,  $m$  equal to  $m$  dash and  $n$  equal to  $n$  dash. So, it will be as I discussed. So, this term will be zero this term will be 0 ok. So, here I have. So, in this case  $\Delta l$  equal to 0. So, that is not allowed ok. So, now these lines, there will be after applying electric field these lines energy will not change it will remain the same. Now, for  $n$  equal to 2,  $l$  equal to 0  $l$  equal to 1.

So,  $m$  equal to 1 equal to 0,  $n$  equal to 0. and  $l$  equal to 1,  $m$  equal to 0 plus minus 1. So, in this case basically. So, this wave function is for this  $l$  equal to 2,  $l$  equal to 0 and  $m$  equal

to 0,  $\psi_{200}$  and for other case  $\psi_{2n}$  equal to  $l$  equal to 1,  $n$  is 0,  $n$  is plus 1,  $m$  is minus 1 ok. So, basically for  $n$  equal to 2,  $E_2$  energy, under this energy there are 4 Eigen state, 4 Eigen state, 4 states having the same energy. So, basically these are degeneracy state. Now, for this degeneracy state calculating this  $H$ , expectation value is more complicated any way.

So, again I will tell following some logic what we what is happening. So, if you calculate this term. So, if you take same wave function in both places. So,  $\psi_{200}$  this  $\psi_{200}$  so, obviously it will be 0. So, if you take individual this 1, then for all cases it is 0 ok. So, there will not be effect of electric field on the energy level. Now, if you take these 2, if you take these 2. So, calculate this expectation value in terms of  $\psi_{200}$  this and this one,  $\psi_{210}$  ok. So, in this case  $n$  is not same no,  $n$  are same  $n$  are same fine, but  $l$  here 0 and here one. So,  $\Delta l$  equal to plus 1 or minus 1 and  $m$  is 0,  $m$  is 0 and  $m$  is in this other case  $m$  is 0. So,  $\Delta m$  is equal to 0.

So, then for this 2 wave function, then this will not be equal to 0, it give some value ok, one can calculate that value and the for taking this one with other this 2, we can see that in this case  $l$  equal to one  $n$  are same,  $n$  equal to  $n - 1$ , this is 0 and this is 1 fine.  $\Delta l$  equal to 1 or plus minus it satisfy, but  $m$  equal to 0 and  $m$  equal to 1. So,  $\Delta l$  will be plus or minus 1 ok.

So, that is not allowed [vocalized-noise ] because only allowed only this term will exist when  $\Delta l$  equal to plus minus 1 and  $\Delta m$  equal to sorry  $\Delta l$  equal to plus minus 1 and  $\Delta m$  equal to 0 ok. So, but for this for these 2 term for this and these it is  $\Delta l$  equal to plus 1 or minus 1 whatever that is fine, but  $\Delta m$  equal to it is plus minus and it is not 0. So, then this term will vanish. So, only you will have here this combination, this combination is allowed and so, here for  $l$  equal to 0  $m$  equal to 0. So, this term vanish. So, so this originally here you see for  $l$  equal to 0  $m$  equal to 0. So, there will be no change of the energy level, no change of the energy level, but for it is for other. So, degeneracy will be removed for this 2 step.

So, here basically the here this original one, it is for it is for basically ok. So, here one this term is  $l$  equal to 0  $m$  equal to 0 this and this term is this wave function is basically  $l$  equal to 1 and  $m$  equal to 0. So,  $m$  equal to 0 this term is there. So, for other term where  $m$  equal to plus minus 1, in that case we have seen that this wave function will vanish



this, this 1 not wave function this term will vanish. So, this term originally this because  $n$  equal to 2, here 4 states ok.

So, this for  $m$  equal to plus minus this 2 states, this they remain the degeneracy state its energy will not change. So, basically there wave function is  $\psi_{2,1} + \psi_{2,-1}$ , as if and this 2 and now here will get only splitted into 2 lines this and this other one This one basically  $m$  equal to 0 and other one also  $m$  equal to 0. Here this 2 states, this will get basically due to this for this 2 states it is splitted into 2 states, but it the mixture states.

So, that is why wave function for this state is basically comes form this 2 combination of this 2. So, this one can write this wave function for energy for this energy one by route 2 normalised  $\frac{1}{\sqrt{2}}$   $\psi_{200} - \psi_{210}$  and other is  $\frac{1}{\sqrt{2}}$   $\psi_{200} + \psi_{210}$ . So, so from the combination of these 2, will get these 2 states and their. So, this 2 now non degenerate and that wave function will get.

So, will get 2 energy levels energy are different from the original energy and their wave function generally in this form 1 is 1 can show, 1 is symmetric another is anti symmetric kind of wave function for this 2 energy level. So, so  $l$  equal to 1 and  $n$  equal to 2 effect of electric field. So, one has to find out  $l$  equal to 3, one has to find out calculate. So, just for this 2 logically I could tell you the splitting of the energy levels due to magnetic, due to electric field and if you consider the transition. So, 1 line in this case as if this it is splitted into 2 lines. So, for other  $n$  equal to 3 we can show that is splitting into more line

So, there is effect of electric field due to the electric field that is the splitting of unperturbed energy level for degenerate energy level. So,  $z$  is degeneracy is lifted, not all but, in some extent, if it lifted or also we could get splitting off into 4 energy level, but it is splitting into only 2 ok. So, partially degeneracy is removed. So, you will get obviously more lines and that is the, that is called the stark effect ok. So, I could tell you following the logic without calculating in details because calculation will be it will be difficult form under graduate student without knowing advance quantum mechanics ok. So, but hopefully I could give you just some favour of this effect of electric field. So, I think that is what I taught you about the 1 electric system.

So, I have discussed all things and also for 1 electron system I have shown you the quantum mechanical, how quantum mechanics is used and get the all information although all information we have got from the old quantum using the old quantum

theory, but same things we same energy expression etcetra etcetra, we got from quantum mechanical calculation and further more information also of course, we have got, but for spectroscopy spectral analysis.

So, we need basically this energy expression and some ye. So, because transition among this energy levels. So, that gives the spectral lines. So,. So, this I have discussed. So, for multi electric system I have I think using the paper model 1 has to find out the total l, total s then will get total j and then ye.

So, basically whatever for 1 electron system whatever the selection rule etcetra, whatever the applicable for multi electron also all are applicable, using the Pauli exclusion principal, using the home store. So, I have shown you how to get the down state of a of the atoms right. So, and from there basically one can find out the spectroscopic term and if you find out the spectroscopic term you can basically tell the transition tell about the spectral line ok.

So, I think and then I have shown you of course, this quantum mechanically what is the what is the, I think what is called this stark effect what is the Zeeman effect and what is the Paschen back effect and what is the Liang sheet ok. So, without details calculation I tried to explain and you will understand more when you will you will you will know more about quantum mechanics. So, I think for undergraduates student if you. So, so far whatever I thought about the atomic physics. So, that should be enough. So, next from next on word next class onwards will teach basically molecular physics, physics of molecules. So, physics of atoms, I think we have we have most of the things whatever required for the under graduate student. So I will stop here.

Thank you for your attention.