Atomic and Molecular Physics
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Lecture – 39
Hydrogen like atom in magnetic field (Contd.)

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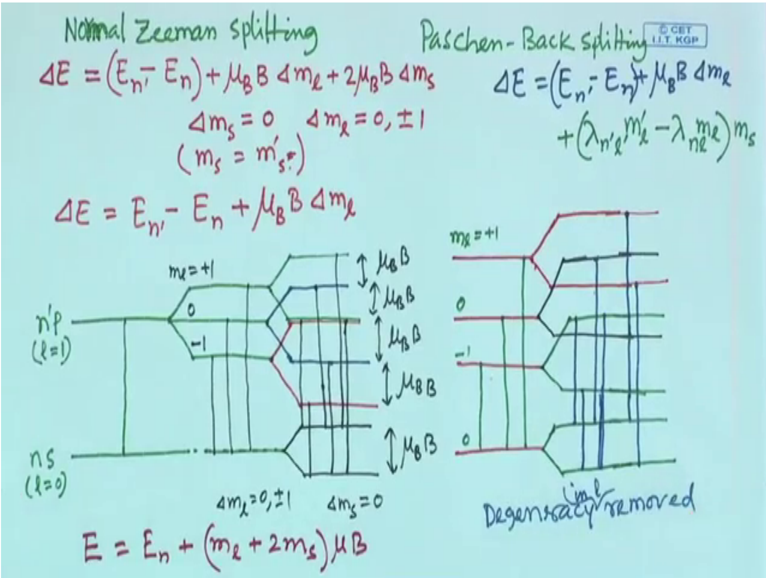

So, we continue our discussion on the Paschen back splitting Paschen back splitting, there is a for intermediate magnetic field. So, if I compare this normal Zeeman effect and Paschen back Zeeman effect, not effect Zeeman effect, so, Paschen back effects. So, here, just I have I have compared through the splitting of the spectral lines in case of normal Zeeman effect and other one is Paschen back effect.

So, normal Zeeman may be here it was very high magnetic field or spin orbit interaction coupling was negligibly small or vanish.

So, there, whatever what was the energy expression for spectral lines? So, del E equal to E n dash minus n plus mu B B del m l plus 2 mu B B del m s. So, here, I have written, so, but basically, I do not need this term. Because, only transition is possible when del m s is 0 ok. And in case of Paschen back splitting what happens? So, this del E equal to the same one ok. So, same one I have written del del m s, I have not written here because, I do not need here itself. So, plus and additional term you have got for intermediate magnetic field ok because, this l S coupling was not neglected in this case.

So, now, if you see splitting, so, let us take 2 energy level. N S means l equal to 0 and n dash P where l equal to P l equal to 1 ok. So, 2 energy level. Now, these energy level before apply magnetic field, what will happened? So, if there is a transition, transition is permissible allowed because, del l equal to plus minus 1. In this case, either plus 1 or minus 1 whatever does not matter. So, without applying magnetic field, we will get one line one transition.

Now, what happens after applying magnetic field? After applying magnetic field, what will be the energy? Is here I have shown ok. So,. So, E n means this is the E n energy before applying mil magnetic field. Now, after applying magnetic field, this is the things we will see. So, this E n plus that that term ok, that term mu B B del m l. So, del m l means, here, you can tell this energy term, actual energy terms I have to show you, actual energy terms was E n plus m l mu B plus 2 m s mu B ok.

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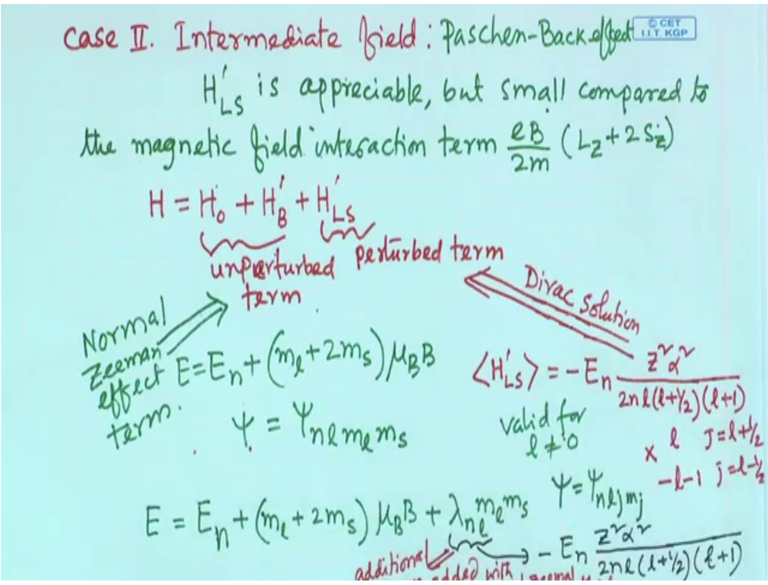

So, whatever delta m l we have written, that basically m l dash minus m l. So, both m l dash and m l is there. So, so this m l equal to 0 here, l equal to 0 means, m l equal to 0. So, m l equal to 0. So, there is there will not be any energy change. So, this is there for m l equal to 0 this is there and then due to m s term, due to m s term 2 m s right 2 m s then into mu B 2 m s then into mu B. So, that will give you the for m s equal to plus half and m s equal to minus half 2 m s means, plus 1 and minus 1 ok. So, there will be symmetrical splitting of this one. So, one is this. This whatever the original energy plus mu B and what about the original energy minus mu B. So, that is coming 2 m s mu S ok. So, m s is plus half and minus half. So, this splitting we will get and then for next one l equal to 1. So, m l will be plus 1 0 minus 1, right.

So, now, when m l is plus 1, so, energy I think, we should I should write what is the energy E equal to energy was E n plus m l plus 2 m s and then mu B ok. So, then it will be easier for me to explain. So, now, you see this now, for m l equal to plus 1 0 minus 1 ok. So, m l equal to 0. So, there is this. So, energy will remain n or E n dash whatever. So, this for m l equal to 0. So, m l equal to plus 1 and minus 1. So, it is mu B. So, that is mu B into B that is magnetic field and this Bohr magnetron. So, here, you will get this whatever the original energy. Now plus mu B and other one for minus 1, it will be minus mu B.

So, now this line is splitted into 3 lines ok. One is original energy, other is mu B higher and other one is mu B lower, ok. Now, for each case, each case, so, when m l equal to plus 1, this is the energy. Now, for m s, for m s again, 2 m s is there. So, m s equal to plus half means, this energy will increase by mu B plus mu B. So, energy increase by plus mu B. And other one minus m s, energy will decrease by minus mu B. So, this minus mu B ok. Similarly, for each case, plus mu B minus mu B from original energy. For these also plus mu B and minus mu B. So, this energy separation. Here, energy separation are basically, mu B B ok. Between this successive energy levels, this all are this energy separation (Refer Time: 07:14) that is why, I told that this energy splitting. It has symmetry.

Now, here, if you see, if you see this, considering the transition rule, del m l equal to 0 plus minus 1 and del m s equal to 0 ok. So, now, if you consider that transition, ok, if I consider this one, here, so, from here, if I can start from here ok; that means, if I start from here, I think I will take another pen. If I start from, let me check. So, in this case, only if I consider this energy splitting, so, here, I can see this transition from this line to this line. It is allowed because, delta m l. In this case, this is 0 and this is minus 1. So, it will be minus 1 or plus 1 whatever. So, this is allowed. From here to here also allowed, del m will be 0 and from here to here also allowed because, this is plus 1 and this is 0 this mean; that means, that del m l will be plus or minus 1. So, it depends you are taking difference from final to final one minus initial one. So, generally, final one minus initial one, one has to take. So, if you set that rules. So, one case, it will be plus 1 and another case it will be minus 1.

So, this 3-term show as if I will get these lines. in this case, now, if I consider this m s, del m s is equal to 0, then for each line, if we consider this line, so, this line now it is it has 2 energy level ok. And here also, it has 2 energy level. Now, we consider for these lines. So, if I consider this. So, from here to here, it is allowed del m s has to be 0, it is allowed.

But here to here, it is not allowed ok. So, in this case, if it is plus m s equal to plus half, so, this case also m s equal to plus half. So, del m s will be 0 for this case if it is m s equal to plus half. So, other one will be minus half, then m s plus half to m s minus half. So, del m s change will be plus 1 or minus 1 ok. So, that is not allowed. So, from here to here, it is not possible.

So, similarly, for m s equal to minus half this one is m s equal to minus half. So, m s equal to minus half to m s equal to minus half is possible m s equal to minus half to m s equal to plus half is not possible ok. Because, del m s so, wherever this so, m s dash equal to m s. So, there only transition possible. Other case transition is not possible. So, here, just one example I give you. So, in this case, this one line, now considering the spin magnetic moment, considering the that m s. So, considering that one, so, we are getting basically 2 lines ok. But, these 2 lines the energy of these 2 lines are same if you see this symmetrical way, because, from here, you see this whatever the difference of energy.

So, now additionally in one case whatever, so, here, I can see that, for these lines what happens? So, this mu B. So, it has gone up by mu B and for this one also, it has gone up by mu B. So, these lines energy of this line is basically same energy of this line. In other case also, one can tell that ok, whatever the energy it has come down by mu B and this line also whatever energy it has come down by mu B. So, both has come down by mu B. So, energy difference is same as this energy difference ok. So, basically, these 2 line is nothing, but this same this line ok. So, but transition wise, it has 2 lines, but energy are same. Similarly, for other 2 lines, one can show that I will get 2 lines, but their energy are same. And this case also, these 2 lines (Refer Time: 11:57) energy are same. So, basically, we will get in we will get in strong magnetic field, we will get basically only 3 lines. Whatever this 3 lines we will get 3 lines ok, so, that is normal Zeeman effect.

Now, in Paschen back splitting, there, so, this term is there so; that means, this type of splitting will be there. Now, in energy level, additional term is added and that term depends on l value and m s value ok. So, whatever the symmetric was there, now, this term is contributing some additional energy and thus yes.

So, and this term is not basically is not symmetric term ok. Because, this lambda. This lambda n dash l dash is not same as lambda n l because, in this expression, n and l are there.

So, here, this here, this lambda n l l lambda n dash l dash these have different value and then it depends on l and n also ok. So, this change of energy here, this asymmetric ok. So, that that is why, so, that, but these are small term is not very is compared to this term. So, this change of energy will be small ok. But, so, this will get the symmetry of this energy splitting. And so, there will be asymmetrics. break of this energy level asymmetric break of this energy level.

So, then, because of this asymmetric break of the energy levels ah, but transition rules are same. So, earlier, whatever these 2 level was the having the same energy, now their energy will not be same. So, but they are they are closed this 2-value energy value will be closed. So, that is why, we tell this will see this doublets. So, each line whatever 3 lines was observed in normal Zeeman effect in Paschen back effect. So, each line will be will form doublet closely spaced 2 lines we will get. So, here basically, we will see 3 doublet instead of 3 lines, 3 doublet means, it is a we will get six lines ok. So, ah. So, this is called Paschen back splitting.

So, normal Zeeman effect, it is basically for high magnetic field. L S coupling is is vanished or negligibly small compared to the magnetic interaction term ok. And in Paschen back effect, this magnetic interaction term is higher. But ah, but it is it is comparable, but this L S coupling term also comparable is is small. But it is it is close to the L S couple this magnetic interaction term and because of that, so, this asymmetry splitting of the energy levels. So, that is why, the de degeneracy basically removed and we get each line whatever normal Zeeman effect each line. So, it will show as a doublet in in intermediate field and that is that is that is called the Paschen back effect ok.

So, next I will consider the if magnetic field is very is weak is very low; that means, third case 3 is weak magnetic field ok.

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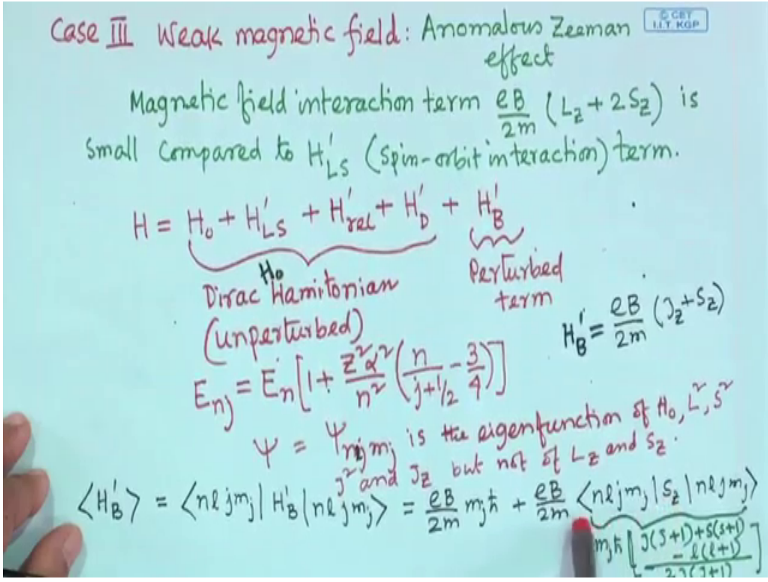

In this case, whatever Zeeman effect we will see, that is called the anomalous Zeeman effect. So, now, magnetic field interaction term, as you know this is the magnetic field interaction term and this term is now very small. It is it is small compared to the compared to the L S coupling term. So, so how small how weak that we have to compare with some other reference. So, here, reference is the spin orbit interaction term. So, spin orbit interaction term.

In case of normal Zeeman effect, we have seen that the that is spin orbit interaction term is very, very small compared to the magnetic interaction term. So, this can be neglected for intermediate magnetic field. We have compared this magnetic interaction term with this spin orbit splitting interaction term and we have considered that still magnetic interaction term is higher than this spin orbit interaction term, but this time this term is smaller, but it is close to the magnetic interaction term ok. So, we cannot neglect we have considered.

Now, third case, in case of weak magnetic field, we are telling that this term is is higher than the magnetic interaction term. Now, we see, this term is higher and other term in Hamiltonian this relativistic correction term Darwin term. Earlier, we have neglected that one because, these are the smaller than the L S interaction term. Now, earlier we have neglected L a L S term. It was very small. So, other term we neglected. But now, what about the magnetic interaction term?

So, that term is really very, very weak. So, compared to that this L S coupling term is strong we are considering. So, now, we cannot neglect this relativistic term and Darwin term ok. So, unperturbed Hamiltonian. So, that is basically Dirac Hamiltonian we have to consider. So, this H 0 plus this H dash L S plus H dash relativistic correction term plus Darwin term ok, H D dash.

So, this let us consider this is a this H 0 this H 0 is Dirac Hamiltonian H 0 we are writing as a for representing the unperturbed Hamiltonian. And this other perturbed Hamiltonian is this interaction term. Because, perturbed term always has to be small term correction term. So, now, it is our perturb term. Now, this now nice because, we have to know the solution of the unperturbed term ok. So, this is the Dirac Hamiltonian. So, we know the solution what are the solution E n j energy equal to E n 1 plus z square alpha square by n square n j plus half minus 3 by 4. So, this solution energy is known to us and corresponding wave function psi n l j m j was the Eigen function ok. And this Eigen function was for we know we have checked earlier. See earlier, that is a H 0 l square S square j square j square and j z ok.

So, this wave function was the simultaneous wave function of this operator, but it was not the wave function of L z and S z ok. So, this is important if it if L z and S z would be the this psi n l j m j wave function of this l z and s z. Then, just from Eigen equation one get could get the Eigen value. So, for, but unfortunately this not the wave function for L z and S z. So, in our interaction term L z and S z is there. So, for these we cannot use this psi n l j m j as a Eigen wave function. So, we have to basically follows the perturbation method to find out this value ok. So, our, so, how to that correction term? How to find out that you know this in Dirac rotation I have written? So, this basically psi n l j m j then, this H B term H B dash Hamiltonian correction term ok, non-perturbation term and then, this psi n l j m j. So, this in Dirac rotation. we have written and one has to find how to find out that that one has to know. So, now, H B dash, if we put H B dash, if we put this is the term ok.

So, I will get E B by 2 m l z. yes. So, so here, I have written, so, H B term. Since this psi n l j m j is not the wave function for l z and s z, so, we have tried to convert it to j z because, it is the wave function for j z. So, L z plus 2 S z. So, L z plus S z can be written as j z plus S z. So, that is why, we have just modified this one in terms of J z plus S z.

So, now for j z it is. So, this Eigen value will be if we apply on this wave function it will give m h cross. So, that is why, e b by 2 m m h cross m j h cross j m j h cross plus mu b b by 2 m. Now we have to find out this expectation value basically for s z expectation value of s z and one has to do that I have not done because I am trying to avoid this Dirac calculation because for that really student should know quantum mechanics. So, ah, but I am just showing the result. So, the student later on when they will complete the quantum mechanics course. So, they will be able to fill up the gap means they can just calculate how this is giving this value ok. So, if you calculate this expectation value. So, it will come m j h cross J J plus one plus S S plus one minus l l plus one divided by 2 J J plus one ok.

Now, here m j h m j h. So, e b by 2 m this common term was there if we take common. So, it we will get basically 1; 1 plus this term ok. So, that is why we have written here .

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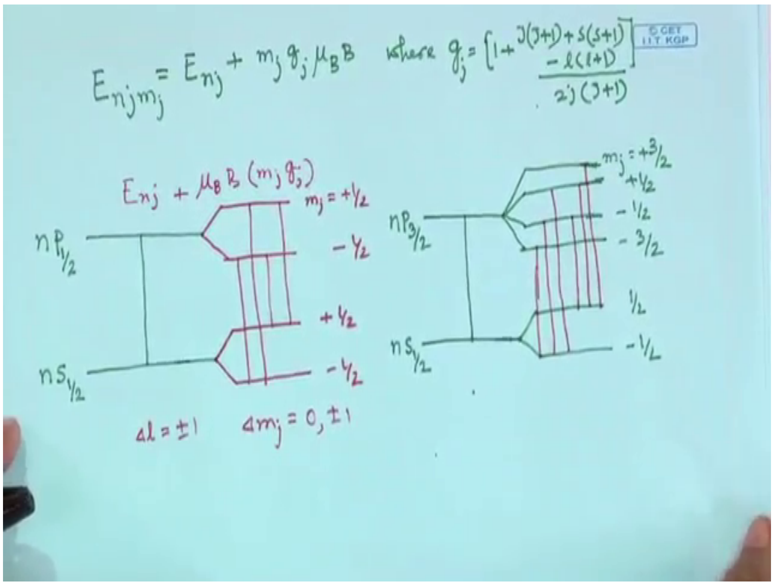

So, that E n j m j is equal to E n j plus m j g j mu B B. So, for for yes. So, g j g j is basically one plus J J plus 1 plus S S plus 1 minus l l plus 1 by 2 J J plus 1 as I mentioned earlier. So, So, basically, as I as I as I told you, from here if you E B by 2 m m j h cross if you take common, so, then you will get 1 plus this one ah. So, that is what we have got and this one we have written as g j. So, g j m j mu B B. So, our now in weak magnetic field the energy what will the energy of the of the energy levels. So, that is this. So, it it is it is this equal to E n j without magnetic field plus m j g j mu B B ok.

So, now, you consider this 2 energy level says half and p half S half and P half because, now this we have to we have to consider this because, it depends on j, not l right. So, because, energy expression it is a is does not depend on l. So, it depends on j and n. So, we have to we have to use spectroscopic term or energy level for for for j value. So, j equal to half. So, S half L equal to 0 j equal to half and L equal to 0 p and j equal to half. So, now, between these 2 line. So, L is 0 and L equal to 1 ok. So, transition is possible before applying magnetic field is transition is possible and it gives one spectral lines ok. Because, it fulfil this selection rule del l equal to plus minus 1 right.

Now, see that now, this for additional term additional term. So, it will be splitted into 2.

Because, m j value will be if j value is if j value is half m j value will be plus half minus half. So, plus half and minus half, you will get you will get you will get the energy and plus half and minus half here you have to see j g term is there. So, that term depends on j s l ok. So, for a particular J and for a particular S this level l l also same. So, for this level the g j term will be same ok.

Now, depending on the m j value depending on the m j value. So, it is splitted into 2 it is splitted because here l equal to 0 now for l equal to one this one L equal to one. So, here also we will get for m j j is half. So, m j will plus half and minus half again will get 2 splitted line now here imagine that because of this g j. So, this splitting will not be symmetric it will be asymmetric symmetric means with respect to this original one. So, it is by same amount it it is going up and by same amount it is going down. So, that is that is we called symmetric breaking symmetric splitting, but in this case it will not be symmetric splitting it will not be symmetric splitting ok. So, that is why, whatever considering this selection on del j equal to 0 plus minus one ok.

Considering this one, this 4 transition will be possible 4 transition will be possible considering the j value ok. So, and due to asymmetric splitting all 4 lines we will see. Otherwise, this is if it is symmetric. So, energy of these could be the same energy of the this one ok. If it is symmetrically splitted ok, but due to g j term it is asymmetrically splitted. So, the energy of this one will be different than this one ok. So, we will get this four-splitting ok. From one line splitting into 4 similarly.

So, this is the it may be hydrogen line or it can be sodium d 1 you know, sodium d 1-line we get from p half to s half and d 2 line we get 3 by p 3 by 2 to s half ok. So, this sodium d 2 line. So, sodium d 1 line will be splitted into 4 lines and sodium d 2 line will be splitted into 3 lines you know not a six lines again. Here this s half splitted into 2 s this is the p 3 by 2 j is 3 by 2. So, m j will be 4 value plus 3 by 2 plus half minus half minus 3 by 2 ok.

So, it will give 3 4 value for 4 m j value and g j will be also different because, there is one has to calculate this one ah. So, that is why, this it will not splitted symmetrically it will splitted asymmetrically.

And that is why, here whatever the transition is possible, so, here one can show that these six transitions possible ok. Considering del m j equal to 0 plus minus 1 six transitions is possible. So, this one line before without magnetic field will be splitted into six lines ok. So, ah. So, in anomalous Zeeman effect in weak field, this splitting is one energy level splitting into more lines. So, this is the basically anomalous Zeeman effect already I have explained earlier. So, now, sequentially within the quantum mechanics how things are coming that I have discussed. Now, I think I will stop here.

Thank you.