Atomic and Molecular Physics
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Lecture - 19
Structure of an atom (Contd.)

So, in last class we have discussed about the LS coupling, LS coupling.

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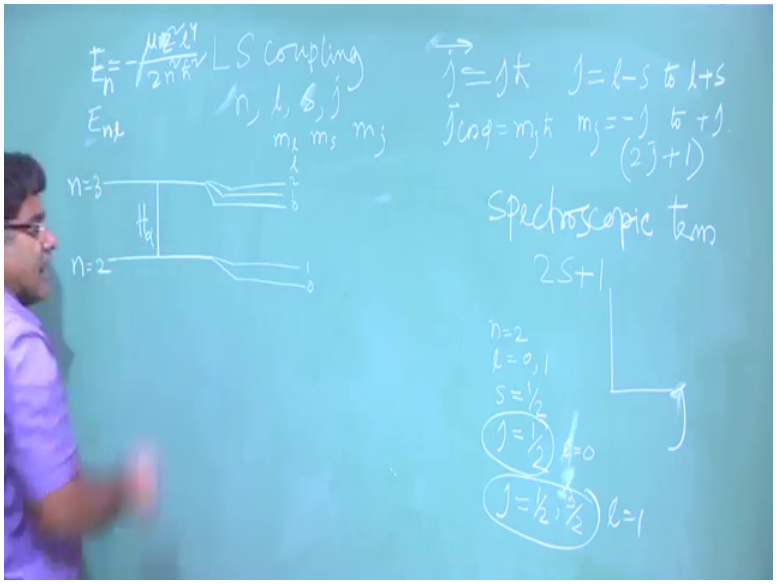

And we got the resultant angular momentum j and the j equal to angular momentum j, j equal to j h cross, so j value will take l minus s 2 l plus s right. And we have seen that that corresponding n j value is basically j cos phi it is a called space quantization m j h cross and the j value m j value, it can take minus j to plus j is total number of value 2 j plus 1 as 2 l plus 1 for m l, 2 s plus 1 for m s.

Now, I will define a spectroscopic term generally we present the energy level with spectroscopic term. So, that is spectroscopic term. So, its notation is 2 s plus 1 l j that is the notation. What is the meaning of this? s is spin angular momentum, j is total angular momentum and l is the angular orbital angular momentum.

So, this notation will be clear let us consider now this we have n f, n f principle quantum number, n f quantum number that is the principle quantum number, angular quantum number, orbital angular momentum and then spin angular momentum and then total

angular momentum. And then corresponding m l, m s, m j right. So, let us now try whether we can explain the fine structure of hydrogen atom. So, what was the fine structure? What was the, so from Balmer series this transition between n equal to 2 this is n equal to 3. So, these are the H alpha line, H alpha line.

Now, for n equal to for n equal to 2, only considering up to s considering up to s yes. So, n and l considering this we are not able to explain this, this it was splitted into 3 lines, but it consist of 5 lines. So, now, considering s and then j now we are trying to. So, here this energy level if I try to draw that using this spectroscopic term. So, for this n equal to 2 what we are getting n equal to 2 I think here.

Let me write n equal to 2 l will be 0 1, then s will be half s will be half it is because it is 1 electron, 1 electron and j will be for n equal to l equal to 0 and s equal to half because j value it can take l minus s to l plus s right. So, l minus s say l if you take 0 and this s of l minus s it will give minus half. So, its value is really we take all, we take positive value negative value we do not take. So, here j value it is for l equal to 0 j value will be minus half and l plus s will be plus half. So, minus half minus is not allowed. So, we will get j value half.

And if you consider l equal to 1 then we will get s equal to half. So, again l minus s to l plus s, I will get half and 3 by 2. So, j value I will get for l equal to 1. So, this, let me write this is for l equal to 0. For each l again I will get 2 s plus 1 number of splitting. So, so 2 s plus 1 if l equal to 0, it is a 1 and if I get this. So, here next j value will be half and 3 by 2, 3 by 2, this, for this value we will get for l equal to 1. So, now, here n equal to n equal to 2; So, now, it is splitted according to l it was splitted into n equal to 2 means l, equal to 0 1 because n minus 1, it will be splitted into 0 1. So, I think this way it splitted and n equal to 3 it will be splitted into 3 lines. So, this was 0 1, this was 0 1 2, things for l value ok.

Now, if I consider j value splitting due to j value. So, how it will be splitted? So, that is the question how energy depends on the j value. So, I have to I have to write that one, I have to write that one. So, from Bragg, from Bohr not Bragg, E n equal to minus n z square E to the power 4 by 2 n square h square right then later on we found that E n k now I will write E n l, E n k now I will write E n. So, that is basically minus E n. So, again, let me write.

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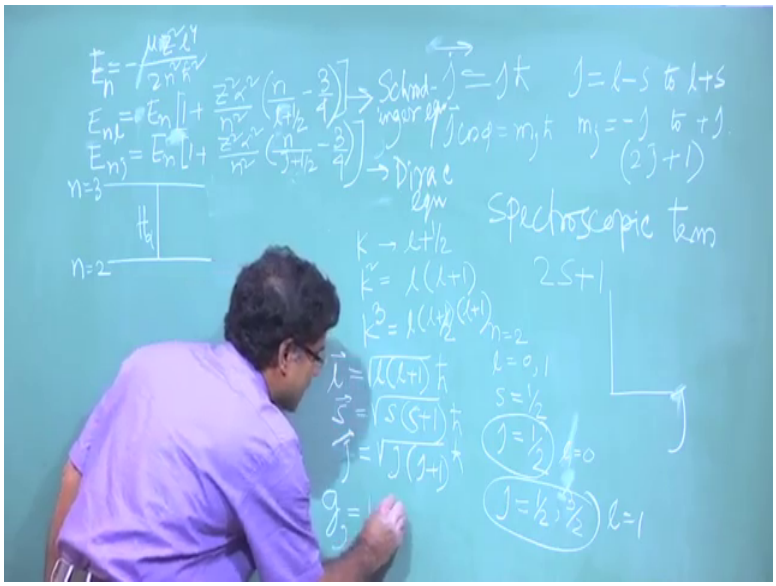

So, if I E n here taken positive. So, if I define E n like this. So, minus I should not write. So, this I will get 1 plus 1 plus z square alpha square by n square into n by k. So, k now I am writing l. Now, l value when l value is 0 k value was k, I was writing k value 0 was not allowed k 1 2 3. So, now, if I write just l l value is 0 then it will be infinity right. So, it is not feasible.

So, l here it is written l plus half it is not arbitrary it basically comes from quantum mechanics so that we will see later on. So, instead of just k replacing by l, it is coming l plus half minus 3 by 4. So, this k as I was replacing k by l and suddenly I took k equal to 1 2 3, but I took l equal to 0 1 2 3. So, that I have taken because just to match with the modern notation and ultimate result. So, this result comes from the quantum mechanics when you will solve the Schrodinger equation.

So, this comes from Schrodinger equation, Schrodinger equation, if you solve Schrodinger equation. So, this result come interestingly see using just this from Bohr-Sommerfeld model we got this expression where n the here it was n by k. Now, k is replaced by l plus 1 with the difference that l plus half with the difference that k value was 1 2 3 4 and l value is 0 1 2 3 4 a slight change and k here we have replaced by instead of l, l plus half. So, why we are writing this 1 because this is the correct result we got from the quantum mechanics is basically Schrodinger equation.

So, Schrodinger equation is basically the non-relativistic equation and non spin in Schrodinger equation spin of electron was not considered. So, solving that equation without considering the spin of the electron and without considering the relativistic particle is non-relativistic. So, that is the Schrodinger equation. And solving the Schrodinger equation for this problem will solve we will see later on.

So, exactly same similar not same I will tell similar almost same expression as expression we got whichever came from the Bohr-Sommerfeld model just with slight change. And we have seen this only we are playing whatever earlier we earlier we earlier of expression from Bohr-Sommerfeld model whatever result we got same result similar result we are getting from quantum mechanics only with slight change just it is affecting l affecting k, k is replaced by l.

So, if you replace k by l plus half k square if you replace by l l plus 1 k cube if you replace by l l plus 1, l l plus half and l plus 1. So, whatever from Bohr-Sommerfeld model whatever energy expression was whatever we will get or we have got. So, there in terms of k we got now if you replace k k square k cube by this l plus half l l plus 1 k square l plus this. So, you will get basically the same energy expression or other things from quantum mechanics.

So, next energy, next energy is in terms of j in terms of j, E n j, E n j it will be n same we are getting just in case for the alpha square by n square then you will get n, so here j plus half j plus half minus 3 by 4. So, you see there is a similarity and just k replaced by l k replaced by j. Now, as I told this following this not l or j l plus half or j plus half and we are getting and these energy expression in terms of j total angular momentum, this result we obtained from the Dirac equation quantum mechanics Dirac equation. So, this result comes from Dirac equation.

So, probably I will not show you because this is the, for higher level, but at least 1 can write from the symmetry. So, this quantum mechanics Schrodinger equation Dirac equation this is slightly complicated calculation. So, in Dirac equation is basically relativistic equation and this, and the spin of the electron was (Refer Time: 17:02). So, everything was considered for realistic case everything was considered and this is the final correct result which explains atomic spectra correctly. So, this is the energy expression.

Let us consider this one or just here we are extending this for Bohr-Sommerfeld model whatever the energy we expression we got just we are using the same expression, but just replacing now l by j. So, j contain this both l and s. So, the j may the right maybe the right angular momentum which can explain the fine structure of the atom that we would like to see. So, j is varying like this.

So, in this expression you can see there is no l, there is no l. So, this I do not need in terms of l I do not need, so in terms of j I can, I can. See the splitting of energy level. So, for and since I have written this 1. So, let me correct this other things in quantum mechanics. So, whatever we are writing l equal to l h cross. So, from quantum mechanics actually this l is replaced by is l is replaced by square root of l l plus 1.

So, this is the ultimate result. So, just slight modification comes h cross then s right square root of s, s plus 1. So, this I am writing from similarity, but this comes from the quantum mechanics. So, then j this right square root of j j plus 1 h cross then g j. g j I wrote 1 plus j square a square minus l square. So, j square it will be right j j plus 1 plus s is plus 1 minus l l plus 1 divided by 2 j j plus 1. So, 2 j square was there, 2 j j plus 1.

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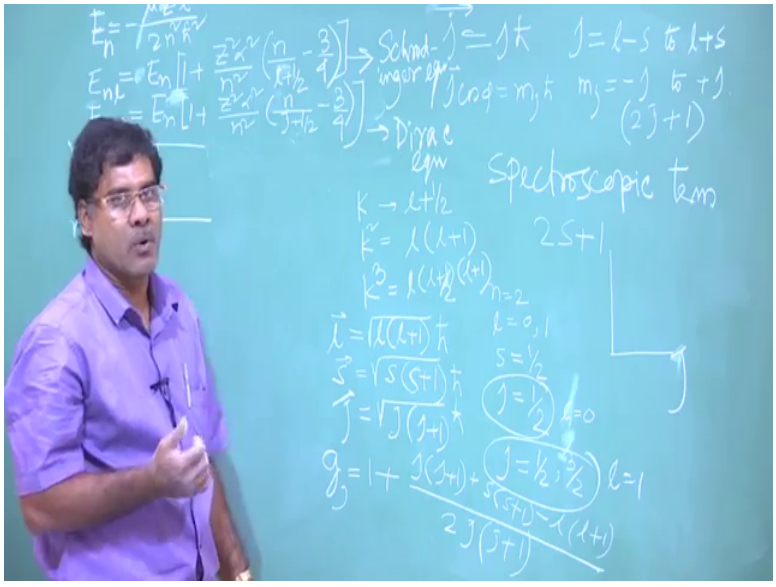

So, these are the just since we are at the end of this end of this success we will show I will show you that this we have the end of the success for explaining all atomic spectra for single electron system all atomic spectra hydrogen or this other also alkali atoms is similar to of hydrogen atom single electron system. So, now, onwards just I will write

this whatever the result we got from quantum mechanics, but this results are similar the slight with slight modification we got.

So, that is why I am following some logic and writing this. So, here j here j is energy in terms of j it does not depend on l you can see. So, here this energy it will be splitted. So, this n equal to 2 n equal to 3, for j value it is similar for j for here, what I got? I have erased j I got n equal to 2 n equal to 2 l equal to 0 1 l equal to 0 and 1 then j equal to s equal to always half, s equal to always half s equal to always half for 1 electron system. So, j I will get here half for this l and for 1 I will get half and 3 by 2.

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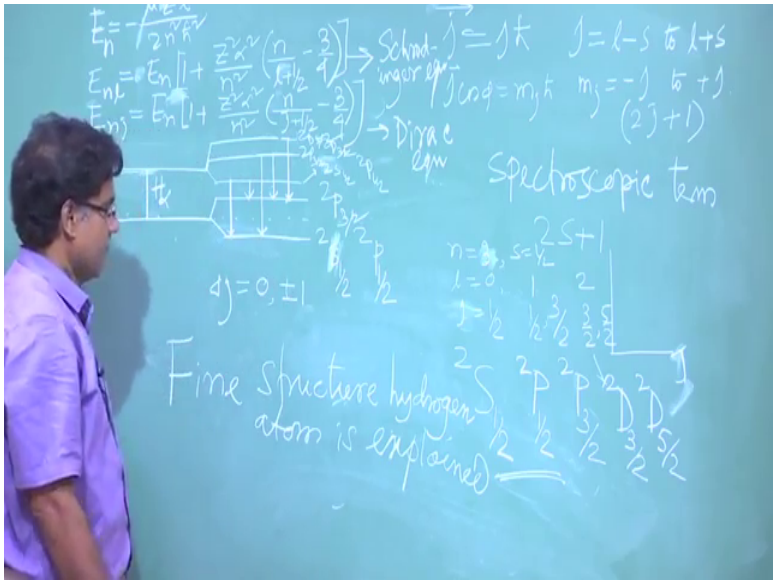

So, for l equal to 0, j half. So, actually l equal to 0 and l equal to 1, l equal to 0 and l equal to 1. So, as I told this 2 energy level. Now, for l equal to 0 j equal to half j equal to half l equal to 0 j equal to half. So, I will get basically for j value I will get this way and here j equal to s equal to half sorry l equal to 1, I will get half and 3 by 2 j equal to, so it does not depend on l. So, j equal to half and this j equal to half it will be same line.

So, it is it has basically 2 2 2 line and for j equal to 3 by 2 it will be just above. So, it is the normal this one and that is splitted into 2 this. Now, generally now we write this energy level in terms of spectroscopic term. So, here you can see for this what is the spectroscopic term l. So, this l, it will be s it will be s right s and s is half it is 2.

So, all the time it is for single electron system it is called multiplicity. We will see later on what is the meaning of this multiplicity and j is half j is half. So, this will be 1 spectroscopic term and this 1. So, this is for this and this 1 l equal to 1 means p, l equal to 1 is p and then s half means 2, 2 s plus 1 and this j is half.

So, j is half I will write and this one here l is l, is 1 s is half and j is 3 by 2. So, these are the spectroscopic term here 3 lines, but for these two spectroscopic term energy are same. So, I can write here 2 energy state 2 states, 2 s half and 2 p half s l are different, but energy does not depend on l. So, it depends on j. So, j are same. So, this is same 2, but it contains 2 lines. Having same energy they are degenerate states and this 1 is 2 p 3 by 2.

Similarly, here you can see here you can see I think I have to take slow slightly below let me take slightly below I have to draw. So, this is n equal to 3 n equal to 3 and this is the spectra and now here for n equal to 2 we have seen now you see for n equal to 3 for n equal to 3. So, 1 more we will get l equal to 0 1 and 2. So, for 2 what, j value, j value will you will get l minus s l plus s, this s is half. So, you will get 3 by 2 and 5 by 2 j value you will get this. So, spectroscopic term for this will be this is l equal to 2 means S P D. So, D is half and 3 by 2 and D you will get, you will get this is 2 and this 5 by 2.

So, here basically again this we will get, you will get, this 2 you will get. Now, it will be these 2 lines you will get an additional line you will get for additional line you will get for. So, this will be one line, this will be another line because 3 by 2, 3 by 2 and this is 5 by 2. So, I will get basically 3 lines one is this, one is corresponding with 3 by 2 and another is corresponding to 5 by 2.

So, this is basically this 2 S half, 2 S half, 2 P half 2 P half and this will be 2 P 3 by 2, 2 P 3 by 2 and 2 D 3 by 2, 2 D 3 by 2 and this one will be 2 D 5 by 2, 2 D 5 by 2. So, here now we see now let us consider transition, transition here selection rules is considered that del j equal to 0 plus minus 1. These are selection rule. It was it comes from quantum mechanics, but to match with the, to match with the to match with the experimental result. So, that time earlier time the selection rule was taken on ad hoc basis.

So, here now if I this is the selection rule. So, let us see the transition. So, transition from this level to this level from n 3 to n 2. So, if I consider this one, here j half here j half. So, this transition is possible because del j equal to 0 is allowed now here j half and here j 3

by 2. So, del j will be plus minus 1 whatever or either plus 1 or minus 1 this is also allowed. So, this transition is allowed. Now, if I consider this one, here 3 j is 3 by 2 and here j is half. So, half minus 3 by 2 will give 1. So, this transition is allowed this transition is allowed and this transition 3 by 2 and this j here j here 3 by 2, so del j 0. So, this transition also allowed, this transition also allowed. Now let us start from here j is j is 3 j is 5 by 2, so j is 5 by 2. So, 5 by 2 and here j is half.

So, half minus 5 by 2 it is 2. So, del j value 2 is not allowed. So, this transition is not allowed and here this energy level j is 3 by 2. So, 5 by 2 to 3 by 2 del j is plus 1 or minus 1. So, this is allowed this is allowed. Now, you see how may lines 1 2 3 4 5 that is what experiment will observe.

So, now, considering the total angular momentum which contains, which is the resultant of 2 angular momentum, one is orbital angular momentum another is spin angular momentum. So, due to the consideration of the spin angular momentum additional angular momentum now we are able to successfully explain the fine structure of hydrogen atom.

So, this I have shown for 1, 1 for all, for all spectra in different series Balmer series or Limen series everything now one can explain that it consist of not single line it consist of many lines. In this case it is 5, but for other case it maybe 3 it maybe 8 etcetera. So, fine structure of hydrogen atom, fine structure of hydrogen atom is explained. So, here without considering quantum mechanics I have replaced j k by l and then similar way l by j these are because k is angular momentum and then k we have considered l.

So, with some slight modification of the value, now l is the angular momentum now similar the j is similar. So, just l we have replaced by j. So, and that result here just I mentioned that this is the result it comes actually from the quantum mechanics so but without quantum mechanics also basis we are able to derive the expression of energy that energy comes also from the quantum mechanics.

But here advantages is that, it is neither quantum mechanics neither classical mechanics we tell all quantum theory is the mixture of this both concept, but it is easy to visualize when we will start quantum mechanics and derive the change. So, its abstractive, it is difficult to understand difficult to visualize, but ultimately that is the mechanics which

can give the correct result, but this results are it cannot discard the as a whole it cannot discard, but partially just it has modified.

So, spectroscopic term now how to write you know, how to level this energy level using the spectroscopic term and then considering the selection rule one can see the transition. Then now, I will stop here.

Next class, I will explain this other aspect of fine structure and Zeeman effect. So, let me stop here.

Thank you.