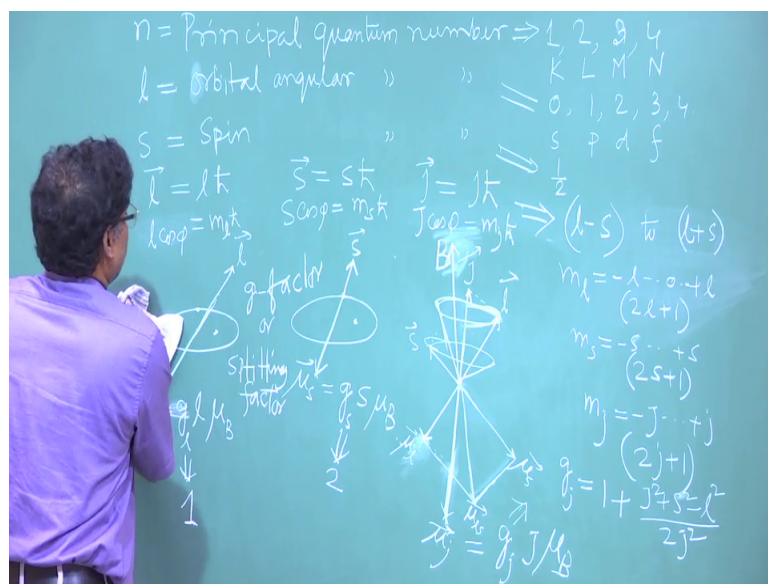


**Atomic and Molecular Physics**  
**Prof. Amal Kumar Das**  
**Department of Physics**  
**Indian Institute of Technology, Kharagpur**

**Lecture - 18**  
**Structure of an atom (Contd.)**

So, we will start considering the spin angular momentum. So, far, what are the quantum number, we have seen that is  $n$  that is the principal quantum number ok.

(Refer Slide Time: 00:34)



So, you can take value 1, 2, 3, 4, 5 and corresponding is 1, 2, 3, 4, 5, etcetera and this we tell K shell, L shell, M shell, N shell, right and then angular quantum number and imphal quantum number. So, the better, we should write because we have another quantum number that is a angular spin quantum number.

So, this  $l$  we tell is orbital angular quantum number. So, this  $l$  value it takes for a particular  $l$  value, it can take 0 1 2, 3, 4, etcetera. So, its name is given is p d f g etcetera. So, then we got the other quantum number, it is the spin quantum number, this is also angular momentum spin angular momentum orbital angular momentum. So, spin quantum number spin quantum number and this it can take value of it can tell take value of, and then 2 more quantum number, we have seen that that is basically active that is basically active for when we apply magnetic field or electric field some external agency we apply.

So, that was I will write that later on one is  $m_l$  and another one is  $m_s$ ; so, corresponding here  $m_l$ ,  $m_s$  are there so that is a above to fixed direction in space that direction is realistic in the direction of the external magnetic field or electric field ok so that I will come later, so, without magnetic field or electric field or external agencies. So, what are the quantum numbers?

So, these 3 are the quantum number now we have seen the quantization relation we have seen the quantization relation this for  $l$ , we have seen for  $l$ , we have seen  $l$  angular momentum, it is quantized this is  $l \hbar$  cross  $l$  is magnitude, and  $l$  can take value as I told and then similar  $S$   $s$ , we can write swing arrow  $S$   $s$  cross  $S$  can take value of.

So, here this  $l$  angular momentum; so, this basically electron; so, it is a direction of the angular momentum is you can take this  $l$  and this  $l$ . Now, it is for the particular orbit its fixed its normal to the plane now then you are saying this now orbit this in space is quantized. So, it is giving  $m_l$  so that I will come later now in this of  $S$  also electron; so, it has spin motion and for that will spin angular momentum so, it also direction  $s$ , right. So, now, for same electron there are 2 angular momentum.

Now, question is that whether they are independent or they are coupled actually they are coupled is called  $l_s$  coupling is called  $l_s$  coupling  $l_s$  coupling. So,  $l$  and  $S$  there are they are not independent they are coupled together and keep the resultant angular momentum. So, when they coupled they keep they give the resultant angular momentum. So, that is the that called total angular momentum it is represented by  $J$  it is represented by  $J$  ok. So,  $l$  is  $n$   $J$   $j$  is the total angular momentum and representation of this  $J$  is basically it is a. So, I I will not draw this orbit.

So, electron one electron it is an orbit it has angular momentum orbital angular momentum spin angular momentum and then due to this  $l_s$  coupling we will get the total angular momentum. So, how they are coupled question is how they are coupled so that we can consider that. So, this electron is the say orbital angular momentum and say this is the spin spin angular momentum ok. So, now, they are coupled and the resultant  $J$ . So, these 2 are vector basically right, their direction and their magnitude is half and this  $l$  have different magnitude depends on.

So, now say it is  $l$  magnitude; so, now, resultant one; so, vectorial addition, we can get this resultant angular momentum so that will be basically this so that is  $J$ ; that is  $J$  we

write here that is  $J$ ; that is  $J$ . So, from vectorial addition, we can get the resultant angular momentum. So, it is not applied only for this case ok so, for as a whole this vectorial addition it is applicable in many cases of this for this atom. So, actually this model was developed is called vector model of an atom. So, this is one part of that. So, vector model of at very famous model successful model vector model of atom because considering this model actually when.

So, so far we have we are discussing only one electron system hydrogen atom or hydrogen like atom or alkali atom because it has similarity with the hydrogen atom or electron system. So, these are the one electron system, but for many electron system when atom is having many electrons. So, each electron will have angular momentum spin angular momentum orbital angular momentum. So then also for individual electrons they have this momentum whether they are coupled or individual. So, that question also will arise and we will see that yes they are coupled and following the vector model one can explain the structure of the atom of many electron system.

So atom having many electrons for that this vector model is very successful model, so, this under this model one can consider that we found this total angular momentum from lns of a one electron system ok. So far for  $l$  this is the  $l$  equal to  $l h$  cross for  $S$   $S h$  cross. So, this  $J$  is called total angular momentum and this similarly this  $J$  will follow the same way into see it. So, now, another angular momentum is total angular momentum. So, this also one can write this same way  $J S$  cross now what will be the value of  $j$ . So, this  $J$  is total angular momentum this  $J$  value it will take  $l$  minus  $S$  to  $l$  plus  $S$  it will take  $l$  minus  $S$  to  $l$  plus  $S$ .

Now, you; so, here let me discuss more this how they are coupled how this  $l$  is coupled. So, it is telling that this  $l$  will precise will precise  $l$  will precise taking  $J$  around the  $J$  taking  $J$  axis it will precise ok, it will precise this  $J$  will precise I think I should I should grow symmetrically. So, it is form a cone basically. So, above this axis  $J$  will precise like this. So,  $J$  will precise  $l$  will precise this taking the  $J$  axis. So, around the  $J$ ,  $l$  will precise around the  $J$   $S$  will precise  $S$  will precise around the  $J$   $S$  will precise.

So, thus they are coupled with  $J$ . So, into this precision they are coupled with  $j$ . So,  $J$  value I have as I told this it can take  $l$  minus  $S$  to  $l$  plus  $S$ . So, then if I come to the similarity, if I come to the similarity, next one is basically  $ml$ . So,  $ml$  value  $ml$ , it can

give it is nothing, but a space quantization and I think it is a how it is it can it is  $l \cos \phi$   $l \cos \phi$ , right that was quantized. So, that was  $m_l S$  cross similarly  $S \cos \phi$  it is  $m_s s$  cross here also that  $J \cos \phi \cos \phi$ .

So, this is about a about a constant direction right in space see you can take as a field direction say magic field direction. So, they are making angle this  $\phi$  this also making angle  $\phi$ , but its  $\phi$  is variables. So, initial angle may not be same, but they are in spaces. So, space quantization. So,  $J \cos \phi$  it has to  $m J S$  cross. So, is a see it is a angular momentum or the angular momentum. So, it will for the; we can follow the same way for all cases. So, what are the  $m_l$  value  $m_l$  value  $m_l$  value you can take minus 1 to 0 plus 1 plus 1.

So,  $2l + 1$  it is the number of  $m_l$  values; similarly,  $S$  value that is  $m_s$ . So, it can take minus  $S/2$  plus  $S/2$  since it is half. So, it will also take  $2S + 1$   $2S + 1$  since  $S$  is half only this value is allowed for one electron. So, it will be  $m_s$  will be  $2m_s$  will be  $2$  plus or other minus 1 and then similarly 1 can write  $m_j$   $m_j$ , it will take value minus  $J$  to plus  $J$  it will also  $2J + 1$  value it will have also  $2J + 1$  value.

Now, magnetic moment magnetic moment for  $m_l$ ; so, the magnetic moment as I mentioned this  $S$  cross is opposite direction we write for  $l$  corresponding this will be  $m_l$  it will be in opposite direction. Similarly, it will be opposite direction, this  $S$ , this also it will be in opposite direction  $k$ ; so, it will be  $J$  in opposite direction. So, later on I will see magnitude. So, this is  $\mu_J$  this is  $\mu_J$  right. So, here is what relation we got  $m_l$  equal to  $l \mu_B$   $l \mu_B$ . So, let me write here  $g_l$  and  $\mu_S$  was. So, it was  $g_s \mu_B$ ;  $S \mu_B$  ok. Similarly,  $\mu_J$   $\mu_J$ , it will be  $\mu_J$  it will be  $g_J$   $J$  in same way I have written in same way I.

So,  $g_l$  value is basically we have seen this is 1  $g_l$  value is one  $g_s$  value is taken this.. So, this  $g$  is called is called  $g$  factor is called  $g$  factor or splitting factor is called  $g$  factor or splitting factor, why it is called that I will we will see later on. So,  $g$  factor  $g$  is called  $g$  factor. So,  $g$  is called basically  $g$  factor or splitting factor splitting factor ok, it is called  $g$  factor or splitting factor right now if I considers; so, this I have seen  $S$ .

Now, here this one is  $g_s$  we have taken 2 for experimental because here is we have seen this electron this  $dp$  spin, it is a magnetic moment in physics magnetic moment is basically  $\mu_B$ ; so, it is half  $g_s/2$ . So, it may be and here it is  $l \mu_B$  for  $l$  equal to 0 it is 0

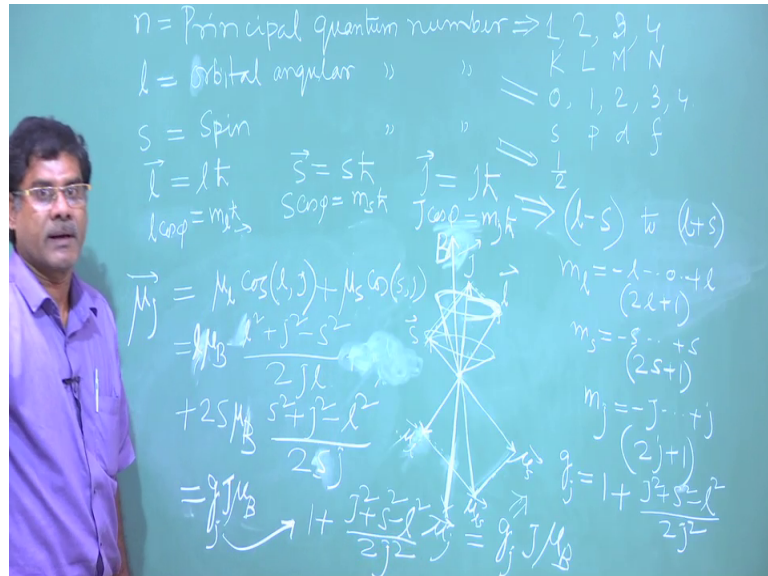
l equal to one it is  $\mu_B$  l to 2 it is  $2 \mu_B$  etcetera, but here we will discuss later on and let me first find out what will be the value of  $g_j$  what will be the value of  $g_j$   $g_j$  value is not 1, 2 or 3 it is different value. So,  $g_j$  value I can write now, but I will derive it.

$g_j$  is basically one plus one plus  $J^2$  plus  $S^2$  minus  $l^2$  divided by  $2 J^2$  square  $2 J^2$  square  $g_j$  value is one plus  $J^2$  plus  $S^2$  minus  $l^2$  by  $2 J^2$  square. So, that that how it is coming that I want to show now; so, for that; so, this is  $J$  corresponding magnetic moment  $\mu_J$ , now this is  $l$  corresponding magnetic moment  $\mu_l$   $\mu_l$ ; this magnitude will be same in terms of  $\mu_B$ . So, this is  $\mu_l$  in opposite direction and this is  $S$ ; so, corresponding sorry; so, what about the magnitude here is equal to half basically. So, but here it is not half maybe it is  $\mu_B$ . So, this magnitude will be doubled in terms of  $\mu_B$  magnitude will be doubled.

So, this and this; so, this is more or less this or this will be the  $\mu_S$ , now vectorial addition of these 2 vectorial addition of these 2 will give you vectorial addition of these 2 if  $l$  and then this  $S$   $S$  more or less more or less it is. So, vectorial addition will be this and then if  $l$  just. So, here we can see the direction of  $\mu_J$  and direction of the vectorial addition of these  $\mu_S$  and  $\mu_l$  are not same. So, this we tell nearly  $\mu_l$   $S$   $\mu_l$   $S$ .

So,  $\mu_l$   $S$  is different from the  $\mu_J$  here this vectorial addition will give an  $J$  a corresponding  $\mu_J$  and now if we consider the individual  $\mu_l$  just opposite to  $l$   $\mu_S$  opposite to  $S$ . Now, resultant of this  $\mu_l$  and  $\mu_S$  is  $\mu_l$   $S$  then that magnitude and direction is not same as  $\mu_J$  it is different. So, I think I will I will erase it; so, now, you see if I want to find out  $\mu_J$ .

(Refer Slide Time: 25:21)



So,  $\mu_J$ ; so,  $\mu_J$ ; now you can take the component of  $\mu_S$  along this along this  $J$  direction along the  $J$  direction all right; so,  $\mu_J$  we have to we can to we write  $\mu_S$ .

Let me first write  $\mu_L$  I can write  $\mu_L$  component of this around the  $\mu_J$  direction or  $J$  direction. So, this will be  $\mu_L$  angle between  $l$  and angle between  $l$  and  $J$  angle between  $l$  and  $J$  right plus  $\mu_S$  angle between  $\mu_S$ ;  $S$  angle between  $S$  and  $J$   $\mu_S$  angle between  $S$  and  $J$  or  $\mu_S$  and  $\mu_J$  so, that will give  $S J$  ok. So, from here  $\mu_L$  we can to  $\mu_L$  we can to  $l \mu_B$  sorry  $l \mu_B$  right. So,  $J l J l$  is  $1 + \cos l j$ ; so,  $\cos l j$ ; so, it will take it is tangent plus ok. So, this  $\cos$  value you know this. So, it will be  $l^2$ . So, what are the; what are the.

So, here  $l S$  and  $J l S$  and  $J$  if angle if we considers  $l S$  and  $J$ . So,  $S$  what will be the angle  $\cos l J$  and  $S$ . So, that angle you can you can write it is angle opposite to this which one is the angle  $l j$   $l j$ , this is the angle right, this is the angle and this will be  $S$ , this will be  $S l$  is  $l$  is  $J$  and this will be the  $S$  right, this will be the  $S$ ; so, this will be the  $S$ . So,  $\cos J l l$  squares this  $\cos J l$ , this will be  $l^2 + J^2 - S^2$  divided by  $2 J l$  right plus  $\mu_S$  plus  $\mu_S$ .

So,  $\mu_S$  is I can write  $S \mu_B$   $g_s \mu_B$  right that we have written  $g_s \mu_B$ . So,  $g_s \mu_B$ . So,  $g_s$  is  $2$   $h$  is half that is why we write this is only  $\mu_B$  or no. So, we will write to  $S \mu_B$   $g_s \mu_B$ . So,  $S$  is half  $S S$  is  $S$  is there  $g_s$  is  $2$ . So, I will write in  $2 S$  because  $g_s$  is always two. So, then angle  $\cos S J \cos 2 n J \cos n j$ ; so, this, so, you can consider this; so, one. So,  $J S$  at opposite one is  $l$ . So, this angle is just opposite to  $l$ . So, it will be

we can write here so, plus  $2 S \mu b$   $2 S \mu B S J$ . So, I will get  $S^2$  plus  $J^2$  minus  $1$  square divided by  $2 S^2$  to  $S J S^2 S J$ .

Now, if you just simplify it is done just you simplify it one step you will get. So, you have basically a  $\mu b$  you have  $\mu b$  you have  $\mu b$  and then you will see from here itself I think  $J$  will come because  $J$  is everywhere is theirs  $J$  is everywhere in theirs and from here you will get  $g_j$  some constant  $g_j$  and  $g_j$  will be this. So, I am not doing the next step just please do it and then you will see  $g_j jmb$  where  $g_j$  will be one plus  $J^2$  plus  $S^2$  minus  $1$  square divided by  $2 J^2$ . So, if you just do the next step.

So, I have derived it ok. So, I got the relation for  $g_j$  and now everything in our hand, I think; these are the sufficient these are the sufficient quantum number in our hand and next we will try to see whether this number of quantum quantization can explain the file structure of atom. So, I will stop here in the next class I will explain it.