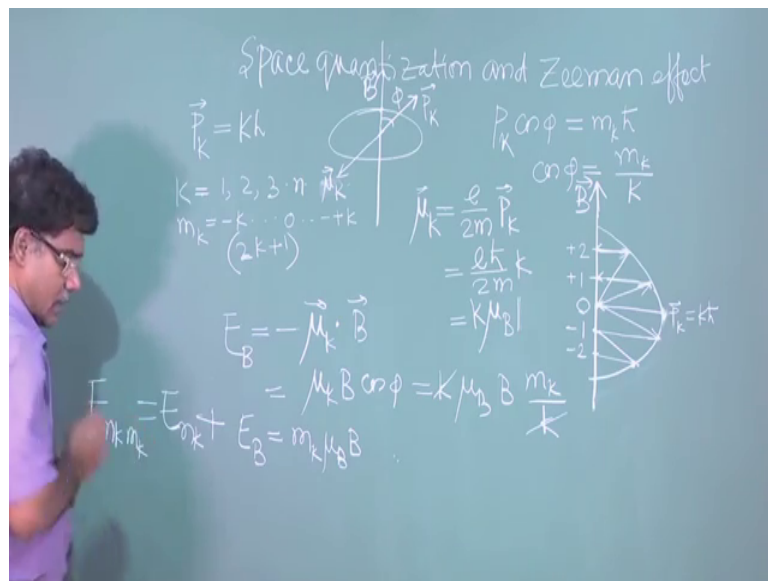


**Atomic and Molecular Physics**  
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**Lecture - 16**  
**Structure of an atom (Contd.)**

So, we are discussing about the space quantization and Zeeman Effect.

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So, we have seen that fine structure of hydrogen h alpha line and then for sodium d lines, d1, d2. So, that was not able to explained using the Bohr Sommerfeld model as well as when magnetic field is applied strong magnetic field is applied then its spectral line is splitted into 3 lines. So, that is normal Zeeman effect and that also not explain by Bohr Sommerfeld model. So, it seems there are more lines means more splitting of energy levels. So, we need more quantum quantization or quantum number.

So, far what we have seen that P k that is Kh cross angular momentum and what is that this, there is a direction of this one there is a direction of this one and that direction is electron is rotating in a in a elliptical orbit and its angular momentum k is basically is a is perpendicular to the to the plane of orbit or orbitals. So, this basically P k direction; So, now, this plane itself can orient in any direction in space. So, now, that is restricted here also it is quantized that direction of P k is quantized with respect to a fixed direction in space.

So, a fixed direction in space is this, (Refer Time: 3:14) the direction in which we apply magnetic field of electric field. So, in this from this, so it is quantized. So, condition for this quantization if it is  $\phi$  if the angle is  $\phi$ . So,  $P_k \cos \phi$  is projection of this direction along this reference direction in space;

So,  $P_k \cos \phi$ , it can be it is quantized. So, it will be such that the orientation of  $P_k$  is such that  $P_k \cos \phi$  it has to be quantized in  $Kh$  cross. So, we have seen this  $\cos \phi$   $\cos \phi$  equal to from this  $2 M_k$  by  $k$ . So, we have seen that  $k$  can take value,  $k$  can take value 1, 2, 3 up to  $n$  principal quantum number and for each  $k$  again  $M_k$  can take value,  $M_k$  can take value because  $\cos \phi$  can be 0 it can be maximum plus minus. So, on

So, from here as I described earlier; So,  $M_k$  can take value, minus  $k$  to 0 plus  $k$  ok. So,  $2k + 1$  number of  $M_k$  value for a particular  $k$  value ok. So, this quantization is basically for a particular value  $k$  value, if  $k$  value is a say 2 for  $k$  value 2. So, it can have different orientation, now which orientations are allowed. So, if this is the  $P_k$  value, if this is 1 orientation,  $P_k$  equal to  $K h$  cross ok. So, its projection along this along this direction, along this referent direction in space projection will be 0. So,  $M_k$  value basically is 0,  $M_k$  value is basically 0 and then other direction. So, it can it is are allowed it should be plus 1, it should be plus 2 and minus 1, minus 2 ok.

So, these are the for  $k$  equal to 2,  $m_l$  is 5 values are allowed, quantized value along this fixed direction. So, that direction has to be such that the projection, projection of this  $P_k$  direction it has to be plus 1 for  $M_k$ , this value has to be plus 1. Similarly this will be this will be plus 2, this is another direction ok. I think direction I should write projection in this direction, projection in this direction.

So, projection; So, it is a this type of diagram generally you can see in book to represent this space quantization, to represent this space quantization. So, these are the possible orientation of  $P_k$  angular momentum in space. So, this is the space quantization. Now, now we have seen that we apply magnetic field, we want to explain the Zeeman effect. So, if we apply magnetic field in this direction in this direction. So, magnetic field will interact with the atom, if atom have some magnetic moment and it is we have seen that it has magnetic moment, we have find out this  $\mu_l$ ,  $\mu_k$  equal to  $e$  by  $2m P_k$ .

So, that is the relation we have seen and now  $P_k$  is  $Kh$  cross. So, if I write this way  $K h$  cross. So, this is defined by  $\mu_B$ ,  $K \mu_B$  ok. So,  $\mu_B$  is the unit of the of the magnetic

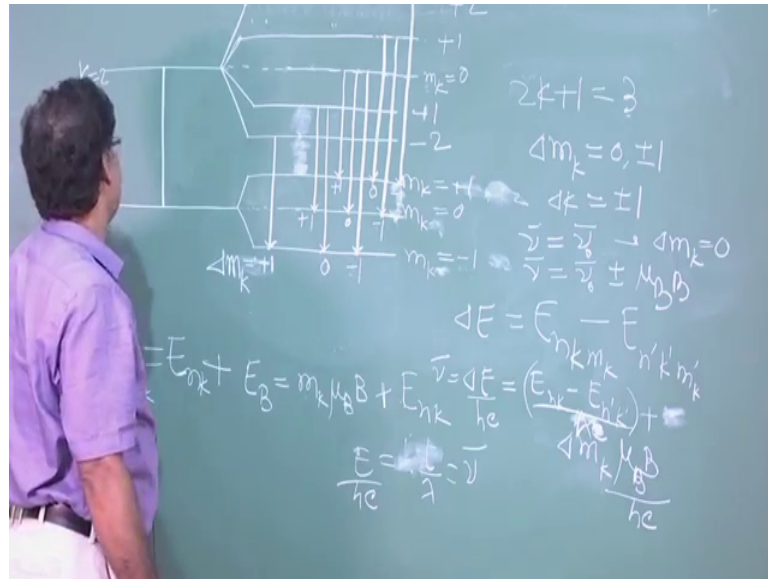
moment, it is called Bohr magnet term as I described earlier. So, So, now, energy, when I apply magnetic field this energy I have the energy due to magnetic field, it is defined that  $\mu_K$ , if magnetic moment of atom or electron is  $\mu_K$  and if we apply magnetic field. So, it is a potential energy we will change this ok.

So,  $\mu \cdot B$  from electro dynamics. So, once can one get this definition. So,  $\mu_K \cdot B$ , now this I can write, you see angle between. So, this is the B direction, this is the b direction, this is the b direction. Now, this high angle is between P k and this B, but  $\mu_K$ ,  $\mu_k$  generally it is just opposite direction of P k because this electronic charge. Here e whatever this relation between these 2, is basically opposite direction because this e is negative, charge is negative. So, that is why it is that direction of magnetic moment and this angular momentum angular momentum that direction is in the opposite direction.

So, this will be, this is the direction of corresponding magnetic moment for this angular momentum P k, corresponding angular momentum P k, they are in opposite direction ok. So, now angle between these 2 is  $\phi$ . So, it is in opposite direction. So, you will get basically  $\mu_k$ , you will get basically  $\mu_K B \cos \phi$  because this negative will go because of this opposite direction 180 degree difference. So, now, your this cycle right  $\mu_K$  is basically, I can write  $k \mu_B$ ,  $k \mu_B$ , B is there and this  $\cos \phi$ , I can write  $M_k$  by K. So, ultimately I am getting  $e B$ , ultimately I am getting  $e B$  equal to  $e B$  equal to  $M_k$ ,  $\mu_B B$ ,  $M_k \mu_B b$ .

So, that is the energy of the system due to magnetic field; So, now, in presence of magnetic field, what will be the total energy of energy level. Now, energy level of energy, now this energy will be added. So, energy will be our it was  $E_{nk}$  plus  $\mu b$ . So, that will be the energy and that energy now it depends of if n k and M k, in presence of magnetic field. So, here I should use  $E_{nk}$ . So, that will be the energy of energy levels in presence of magnetic field. Now, you see if I, now I have energy level.

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So, It is a for a particular n value, for a particular n value ok. There are different numbers of; there are n number of k value ok

So, now if I take this 2 K say K equal to 1 and K equal to 2 ok. K equal to 1, k equal to 2 before applying magnetic field, that was the energy according to this. Now, energy of this will be it depends on  $M_k$ , for each K, we have how many number of l value, 2 a, 2 k plus 1. So,  $M_l$  value will be 1,  $m_l$  value will be 1. So, this will be 1 ok. So, now, its energy,  $M_l$  value will be 1. So,  $M_k$  it is a 0 value. So, this energy of these will after applying magnetic field, it will not change it will not change. It will remain the same, same energy, you have  $M_k$  is 0. So, this is  $M_k$  k is here 1, k equal to 1. So, this is 0 value.

Now, for K equal to 2 no, K equal to 1, 2 k plus 1 sorry. So, K equal to 1 it has to be 3. So, it has to be 3. So, this 1, 1 is this  $M_k$  that us for 1, it is for 0, it is energy will plan change, then when  $M_k$  is plus 1. So,  $M_k$  value for K equal to 1  $M_k$  value is plus 1 and minus 1 as well as 0. So, plus 1. So, energy will be up and minus 1 so, more negative. So, it will go down, but it will go down by equal amounts. So, this is  $M_k$  equal to this is it is for it,  $M_k$  equal to 0  $m_k$  equal to minus 1. Then, another splitting of this energy lines, energy lines. So, that will be plus 1 ok.

So, now this K equal to 1, this 1 energy level in presence of magnetic field it is splitted into 3 lines. So, one is original and 2 more lines and what is the separation of these 2 is basically  $\mu_B B$ , this separation ok. So, similarly from here we will get 5 line, it would

be splitted into 5 original and then 2 more. So, here now  $M_k$  value for  $k$  equal to 0, this is plus 1, this is plus 2, this is 1, minus 1, this is minus 2 ok. So, now, energy levels are splitted depending on the  $k$  value  $2k + 1$  in number and one can find out what will be the energy

So, now this there will be transition there will be transition following the here also selection rule is considered  $M_k$  equal to 0 plus minus 1. So, that is the selection rule and  $\Delta k$  equal to plus minus 1 ok. So, here  $\Delta K$  is 1. So, whatever the one energy is 1 spectral line ok.  $\Delta K$  is plus minus 1. So, now, this energy level here after applying magnetic field following this summation rule, if I draw then you can see that if I start from here. So, minus 2 to plus 1 is not allowed because final to initially if I take difference plus 1 minus of minus 2. So, it will be plus 3.

So, this is not allowed, this also not allowed difference will be plus 2, this is allowed minus 1 plus 2. So, it is plus 1. So, you will get this transition ok, you will get this transition, if I start from here. So, here to here it is possible because it is 0, this is 0 and then, from this to this also allowed, this to this also allowed and this minus 1 to minus 1 also allowed. So, you will get this and then if I start from her I will I will get this is allowed, this is allowed, this is plus 1 difference and then this to this also allowed, sorry, this to this also allowed, if 0 difference and we will get from here to here this is also allowed (Refer Time: 20:09). So, this is difference  $\Delta m$  minus one ok.

Now, if I see from here. So, I will see this, plus 1 to plus 1 difference is 0  $\Delta M_k$  is 0, this is allowed, this also allowed minus 1 and this to this, minus 1, minus of plus 1, this minus 3 is not allowed and 2, 2 to plus 1 means plus 1 minus plus 2 means minus 2. So, it is minus 1. So, this will be allowed, this will be allowed transition and then this to this is not allowed, this to this is not allowed ok. So, how many lines? I am getting I am getting 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. So, something mistake it should be 9, just let us check it. So, if I start from here. So, this is allowed it is a delta. So,  $\Delta M_k$  equal to, in this case minus 2 to minus 1.

So, basically initial to final is plus 2. So, difference is plus 1. So, it is plus 1,  $\Delta M_k$  is plus 1. So, this is allowed and then if I consider from here to here, it is allowed from here to here, from here to here it is not allowed. Now, go to the next energy level, from here minus 1 to 0, sorry minus 1 to plus 1. So, this is not allowed, minus plus 1 to final minus

of minus 1. So, plus 1 is 2. So, that is not allowed. So, that was the mistake. So, this is not allowed, this is not allowed, this is not allowed because this is coming plus delta m equal to plus 2. So, this check this one, this one is difference is plus 1. So, this is plus 1, delta M k, this one is minus 1, 0, delta M k 0.

This one between these 2, it is plus 1, plus 1, this one is 0, delta M k 0, this one delta M k is minus 1, minus 1, this 1 is 0, this 1 is minus 1 and this one is, why this one I have done, this is not allowed, it is not this one, it is still there here, just to here. So, plus 2 to plus 1. So, minus 2. So, it is a minus 1. So, minus 1 ok. So, these are the transition are possible and here you can look that, you can look that for delta M k plus 1, how many are there 1, 2, 3 lines, delta M k 0 how many are there 1, 2, 3, delta M k minus 1, how many are there 1, 2, 3 ok.

So, and you can see that for it is, depends on delta M k when delta M k. So, transition between 2, if I in general if I, if I consider that delta energy difference because of transition, that will be  $E_{n k m k}$ , that energy level minus E, say n dash k dash in general M k dash. So, that is the energy difference and corresponding transition this spectral lines you will get. So, if you take difference like this. So, what you will get the delta E, you will get, if we write this minus of this dash 1. So, I will get  $E_{n k} - E_{n dash k dash}$  ok. This one and then I will get M k let me write here  $M k - M k dash$  and then mu BB, mu BB ok.

So, this  $M k - M k dash$ ; It is basically  $\Delta M k$   $\Delta M k$ . So, this I can write  $\Delta M k$ ,  $\Delta M k$ ,  $\Delta M k$  ok. So, so this is the energy difference. Now if I divide by c, I will get lambda because energy, what is the relation? Energy h, E equal to h nu, h nu and nu equal to nu equal to c by lambda ok; So, if I divide it by h c. So, I will get one by lambda. So, this is basically nu bar ok. So, I will get nu bar, I will get nu bar 1 by lambda, if I divide it by hc. So, this will be divide by h c h c it will be divide by h c. So, does not matter ok.

Now, this spectral line is how many I will get because this is the selection rule  $\Delta M k$  h 0 and then del, then nu bar I can write, nu bar I will get original one original one. So, these transition this transition original one between these two. So, between these two, I will get the original one. So, this what about the original wave number of this one so, that, that I will get here. So, this for delta M k, it can be 0 and then I will get delta M k

plus 1,  $\Delta m = k \pm 1$ . So, then I will get  $\bar{\nu}$  equal to specifically I will get  $\bar{\nu}$ ,  $\bar{\nu}$  then this plus 1 and minus 1. So, I will get plus minus  $\mu_B$   $\bar{\nu}$   $\mu_B$  ok.

So, here you can see this for  $\Delta M = k$  this original line and 2 more line will get the (Refer Time: 28:10) number is just is the difference by this  $\mu_B$  ok, plus  $\mu_B$  or minus  $\mu_B$ . So, that is why whatever line lines we are getting their energy are not different, only the 3 lines are having the same energy, only 3 different energy of spectral lines will get. So, for  $\Delta M = k$  equal to 0. So, this all 3 will have the same energy because these are symmetry. You know this it is shifted by this by same amount, you know this  $\mu_B$  is the splitting is in terms of  $\mu_B$  ok, this 2  $\mu_B$ , 1  $\mu_B$ , here also 1  $\mu_B$  ok,

So, that is why energy of all 9 lines are not same, it is only we will get 3 group of lines. So, 3 lines of are having same energy. So, that corresponds to this for this for these 3  $\Delta M = k$  equal to 0 plus 1 and minus 1. So, you see here, it will all we have taken arbitrary  $k$  value,  $k$  equal to 1,  $k$  equal to 2. These are 1 spectral line for any  $n$  value ok. So, with these 3 lines, it does not depend on, see here also from expression you can see ok. It does not depend on  $n$  and  $k$  and even yeah. So, only this restriction transition rule is followed ok. It does not depend on  $n$  and  $k$ . [noise]

So, we are getting any spectral lines having any  $n$  and  $k$  value, but under magnetic field it will be splitted into 3 lines, it will be splitted into 3 lines. So, as I told this called the normal Zeeman effect. Any spectral line under high magnetic field is splitted into 3. So, now, apply introducing the space quantization, space quantization ok. We are able to explain the normal Zeeman effect. So, that is very this is, in that sense it is very successful so, but still introducing this  $M = k$  space quantization we are not still able to explain the fine structure of hydrogen atom and oh yeah and sodium or alkali atom also like sodium  $d_1$ ,  $d_2$  lines. What is the origin of that, what is the origin of 5 spectral lines of  $h\alpha$ .

So, these are the experimental fact and that cannot be explained, as well as anomalous Zeeman effect in that case it depends on a spectral lines it depends on the  $n, k$  value, depending on that one spectral line another weak magnetic field it is splitted into different number, say it can be 4, it can be 6 etcetera, but it is higher than the 3, energy levels energy spectral lines. So, that was not also one cannot explain. So, again we are getting. So, there are something left, which will give more quantization and we will be

able to explain this observation of more spectral lines as we have seen from sodium d lines, we have seen from h alpha line and for also anomalous Zeeman effect ok. So, we need more quantization.

So, I will discuss in next class, I will stop here.

Thank you.