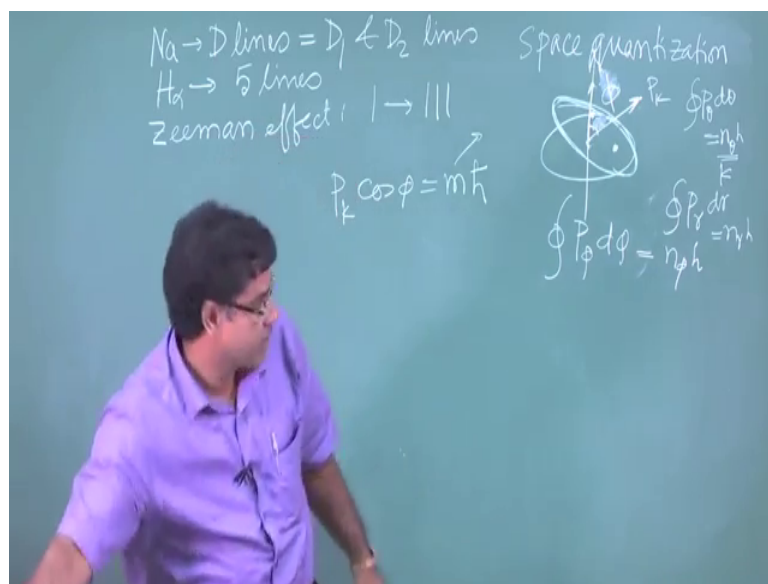


**Atomic and Molecular Physics**  
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**Lecture - 15**  
**Structure of an atom (Contd)**

So, I will discuss about the Moore quantization, why we require Moore quantization that sort I explained in last class.

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So, if we have seen that sodium D1 lines sodium D lines, it is basically composed of 2 lines D 1 plus D 2 lines right and then that was not explained using Bohr Somerfield theory and then hydrogen H alpha line is composed of 5 lines that also not explained, and then also Zeeman Effect. So, any spectral line is splitted under magnetic field splitted into 3 lines. So, why it is happening so that is also not explained in case of normal anomalous Zeeman Effect will come later ok.

So, far we have considered that electrons in a main orbit, so it is a basically planar motion. So, this plane can be take in any direction in space. So, there is no restriction of the orientation of the planes of orbits in space, it can orient in any direction. So, now this Somerfield again put this condition that, no it cannot orient in any direction, here it is again quantized, so that is called space quantization. So, electrons are rotating in a orbit elliptical orbit. So, it has angular momentum. So, a direction of angular momentum is

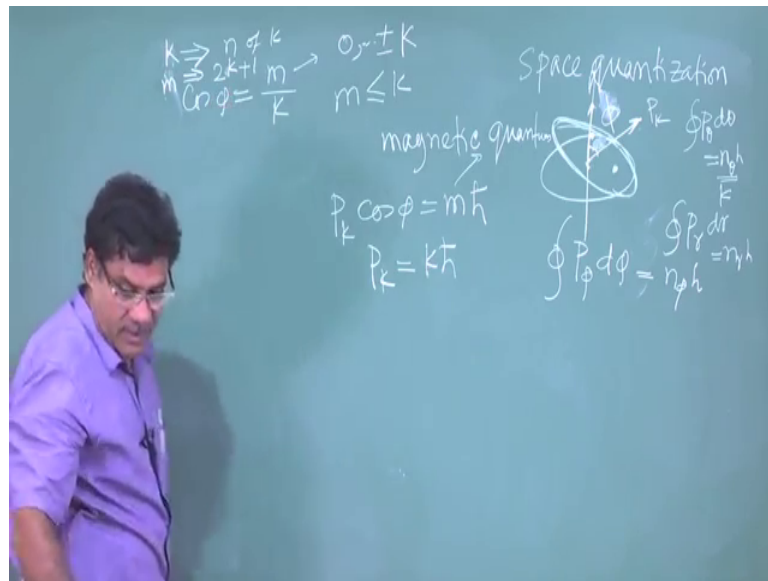
basically perpendicular or to the plane of this orbits it. So this is  $P_k$  say  $P_k$  right. So, earlier I was writing  $p$  and corresponding this quantum condition was basically  $P_\theta D_\theta$  equal to  $n_\theta h$  right.

So,  $n_\theta$  that I mentioned that  $n_\theta$  we have taken as a  $K$ . So, now this index also I am using  $k P_k$ . So, that the directions now this if this planes rotate in another direction say this way another orientation. So, the direction is this it is  $P_k$  now this direction is this. So, similarly another direction so this  $P_k$  direction it can be any direction in the space, now quantum condition is applied again no in space it cannot orient in any direction; this also it will be quantized like this and another quantization was that  $P_r D_r$  equal to  $n_r h$  each. So, third quantization space quantization it was considered that  $P_\phi$ , what is  $\phi$  is basically if you consider forget this one, if you consider a reference direction in space if it is this. So, then the direction of the angular momentum  $P_k$  this is the direction.

So, what about the angle between this reference direction and this angular momentum direction, so, this angle is  $\phi$  this angle is basically  $\phi$ . So, another third coordinate we are using  $r_\theta \phi$  in 3 dimension basically  $r_\theta \phi$  that so another number parameter or coordinate that is  $\phi$ . So, this  $\phi$  coordinate also will be quantized, so angular momentum for this  $\phi$  corresponding so  $d_\phi$  right. So, it will be say  $n_\phi h$  so this is another quantization. So, here just simple way we can tell that find out that, this  $P_k$  or this orientation plane of the orbit, we search that it is direction angular momentum direction if you take the projection along this fixed direction, this projection will be  $P_k \cos \phi$ . So, that will be quantized if I just write forget this one just using the similarity I have written.

So,  $P_k \cos \phi$  will be  $m h$  cross; so  $m$  can take different value, but it is a integer value. So,  $P_k \cos \phi$  will be  $m_\phi h$  so this is the  $m$  is quantum number and this quantum number is called magnetic quantum number.

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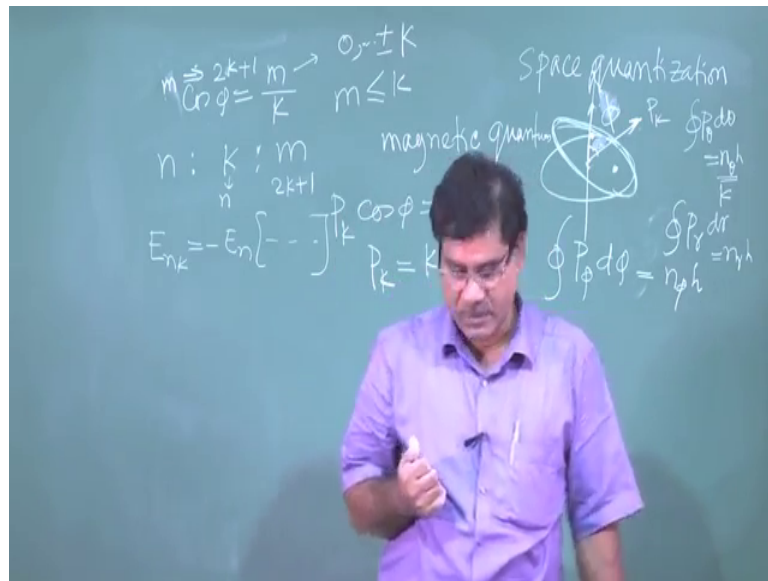
So, this is fine  $P_k \cos \phi$  equal to  $m \hbar$  and we know that  $P_k$  equal to  $K \hbar$  cross right  $P_k$ . So, from here what we are getting if I take ratio of these 2, so I will get  $\cos \phi$  equal to  $m$  by  $K$ . So, for a particular value of  $n$  we know this  $n$  number of  $k$  value will be there.

Now, for a particular  $k$  for a particular angular momentum again it is quantized in space and this quantum number has come. Now  $\cos \phi$  values it can be maximum value is plus minus 1 limit value and 0 also possible. So, when  $\cos \phi$  is 0 say  $n$  will be 0, when  $\cos \phi$  equal to plus minus 1, so this 2 can be equal  $n$  can be plus minus  $k$  then.

So,  $n$  value is allowed 0 value and plus minus up to plus minus  $K$  because,  $n$  can be  $n$  cannot be greater than  $K$  it can only be less than equal to  $K$ , because of this  $\cos \phi$ . So, what we are getting? So for a particular  $k$  value for a particular  $n$  value we will get a  $n$  number of  $k$  value and for a particular  $k$  value we will get  $2m + 2k + 1$  number of  $m$  value,  $2k + 1$  number of  $m$   $n$  value  $m$  value we will get right say  $k$  equal to 2.

So, means minus 2 minus 1 0 plus 1 plus 2, so that will be the value of  $m$  so we will get  $\phi$  value. So,  $k$  equal to 2 so it is we will get, so this is  $n$  basically  $n$  value will be and this for how many number of  $k$  for a particular  $n$  value. So,  $n$  number of  $k$  value we will get for a particular  $n$  value, we will get  $n$  number of  $k$  value. So, I think I am not writing properly, so I think just so for a particular  $n$  value.

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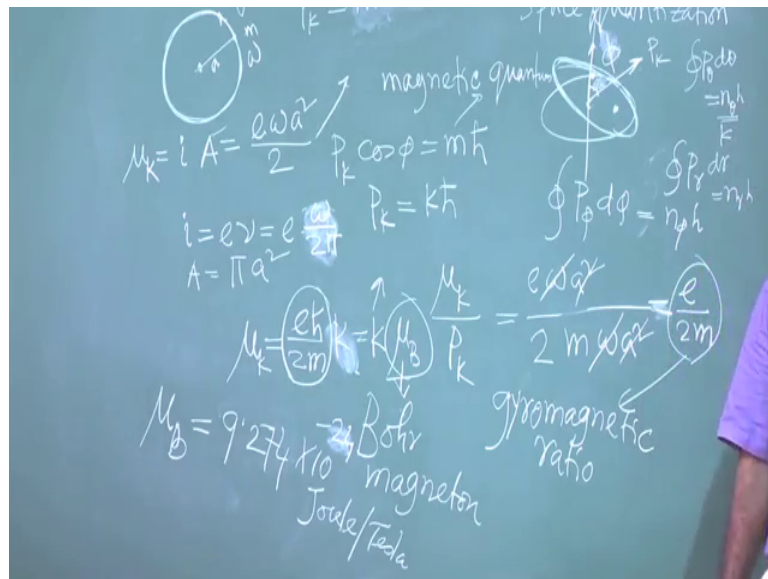


So, I will get how many k values, and I will get for a particular k value I m value. So, it is a here I will get for a particular n value, whatever this if it is n value is n. So, here I will get this n number of k value and for a particular k value if it is k then, I will get 2 k plus 1 number of n value. So, this is the space quantization and considering the space quantization the energy express is not change energy expression is not change  $E_{nk}$ , whatever we got minus n etc whatever I got. So, this it is will not change. So, whatever the sodium D 1, D 2 lines whatever the this hydrogen h alpha line it is fine structure of hydrogen h alpha line.

So, that nothing to do with that 1 after introducing the quantum quantization, but this quantum quantization was able to explain the normal Zeeman effect; it was not able to explain the anomalous Zeeman effect bit we will see later on, but it was able to explain the normal Zeeman effect. How? So that is what I want to discuss, so when electron you see when we are applying magnetic field, if magnetic field interact with the electron or atom. So, then there will be change of energy due to interaction change there will be change of energy potential energy mainly. So, now that energy we have to find out and we have to see whether it depends then on this value depends on m or not, if it depends on n then in energy level another quantization quantum number will be introduced and we may see the more splitting of the energy levels. So, how it interact? So basically it is a magnetic interaction magnetic field.

Now, if atoms have magnetic sum magnetic property, sum it shows the magnetic property. So, then this interaction is possible, electric field will interact because electron have charge electronic charge. So, electric field will interact but magnetic field why it will it should interact. So, if electrons have or this it is electronic orbit moving electron have the magnetic property then only there is a possibility of interaction. So, it is true that this electrons in orbit it has magnetic moment, so that we will find out. So, say for simple phase let us take a circular orbit.

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So, electrons are moving radius is say  $a$ , it is mass is  $m$ , electron mass is  $m$  and it is velocity is  $v$  it is moving. So, it will have this angular momentum  $P_k$  so that we have seen.

Now, when electron is moving in a orbit with velocity  $v$  and then from electro dynamics we know this orbit will act as a magnetic shell, if current circulate in a closed loop then that loop act as a behave like a magnetic moment or magnetic cell and this magnetic moment of this orbit is cell will be nu equal to nu equal to it is  $i$  into  $A$ , area of the loop and  $i$  is the current passing through the loop.

So, now  $i$  what is  $i$ ? When charge electron  $e$  is moving. So, electron charge  $e$  it is moving, now current is per unit time how many times you see it is passing through a point so that  $s$  the current. So, if I take frequency  $\nu$  means per unit time it is  $\nu$  times it is circulating; that means, that charge  $e$  it is circulating  $\nu$  times. So, total charge

circulating in per unit time that will be  $e \nu$ , so that will be the current basically  $i$  equal to  $e \nu$  I can write.

So,  $\nu$  equal to  $e$  I can write  $2\pi$  by  $\omega$  angular frequency,  $2\pi$  by  $\omega$ . So, that is the  $i$  and what will be the area? Area is the circular orbit it is  $\pi a^2$ . So, your  $\nu$  will be  $\nu$  here this magnetic moment of this electron, it will be  $\pi a^2$  sorry it is just like did mistake it is not  $2\pi$  by  $\omega$  yes  $\omega$  by  $2\pi$  because  $\omega$  angular velocity is equal to  $2\pi$  into frequency. So, then you are getting if I multiply this  $2$ . So,  $\pi$  will go. So, I will get  $e \omega a^2$  by  $2$ . So,  $e \omega a^2$  by  $2$   $e \omega a^2$  by  $2$ . So, that is the magnetic moment of the electron motion is rotating in a orbit. So, it depends on the electronic charge it is angular momentum and this radius ok.

So, you remember that this is  $e \omega a^2$  and this from here that Bohr radius was yes. So, that it turns up Bohr radius also we can define this. So, before that what we can do. So, what we are getting? Here this electron is rotating in a orbit. So, it has angular momentum  $P_k$  and since it has and since it has angular momentum means it is in motion in orbit. So, corresponding magnetic moment we are getting. So, that magnetic moment corresponding if it is for say  $k$  s orbital's if you consider. So, then we can tell this is the  $\mu_k$ . So, this electron is in  $k$  th orbital. So, this will get cor  $P_k$  and corresponding magnetic moment we will get  $\mu_k$ .

So, now this  $P_k$  is your  $P_k$  equal to  $K h$  cross  $P_k$  equal to  $K h$  cross, and then we have relation that  $P_k$  equal to  $h$  cross and from here if you put  $P_k$  it is  $K h$  cross  $\cos \phi$   $P_k$  I can write also  $K h$  cross  $\cos \phi$  equal to  $m h$  cross from there we took the ratio  $n$  by  $k$  that is that has to be equal to  $\cos \phi$ . So, here now  $\mu_k$  equal to here if I take the ratio of  $P_k$  and  $\mu_k$ , these  $2$ , if I ratio of these  $2$ . So, if you write  $\mu_k$  by  $P_k$  if I write  $\mu_k$  by  $P_k$ . So, what I will get? I will get here  $e \omega h^2$  by  $2$  and here I will get  $K h$  cross. Now from this from here we would like to find out  $\cos \phi$  equal to  $m k^2 P_k$  I can actually  $P_k$  I can also write in terms of this angular momentum,  $P_k$  I can write. So, that is there  $k K h$  cross is there, but here since  $\nu_k$ , I have written in terms of  $\omega$  and a square.

So,  $P_k$  one can write angular momentum what is angular momentum  $m b r$  right  $m b r$  mass into  $v$  into  $r$ . So, here  $r$  we have taken this  $a$  and  $b$  one can write since we have considered this angular velocity is  $\omega$ . So,  $v$  you can here, since we have used

omega. So, here I can use omega. So,  $v$  equal to  $\omega r$  or  $\omega a$ , basically  $m \omega a$  square  $P k$  equal to  $n \omega a$  square, here instead of that one let us because I have to use same way with same parameter. So, now, here  $P k$  equal to I can write  $m \omega a$  square.

So, this is giving me this equal to a square will go and I will get  $e \omega$  by  $\omega a$  also will go. So, I will get  $e$  by  $2 m$ , it is very interesting to see that  $\mu k$  by  $P k$  equal to constant  $e$  by  $2 m$ ,  $m e$  are constants. So, these are constants. So, this constant is called or this ratio is constant. So, it is called  $e$  by  $2 m$ , is called gyro magnetic gyro or people play gyro whatever gyro or gyro magnetic ratio. So, so ultimately what I am getting. So,  $\mu k$  equal to I am getting  $e$  by  $2 m$  into  $P k$ , now  $P k$  1 can replace  $K h$  cross. So, I am writing  $h$  cross here.

Now, here this again you can see this is constant and this constant if I write that  $k$  into this constant is  $\mu v$ . So,  $\mu v$  is called Bohr magneton, it is called Bohr magneton and it is very interesting to see that this magnetic moment is quantized and that it is quantized in terms of this it is smallest unit  $\mu v$ , it just  $\mu v$  is like the charge of the smallest unit of charge you know this electron charge whatever  $e$ . So, in  $e$  we take a now we taken as a universal constant of charge.

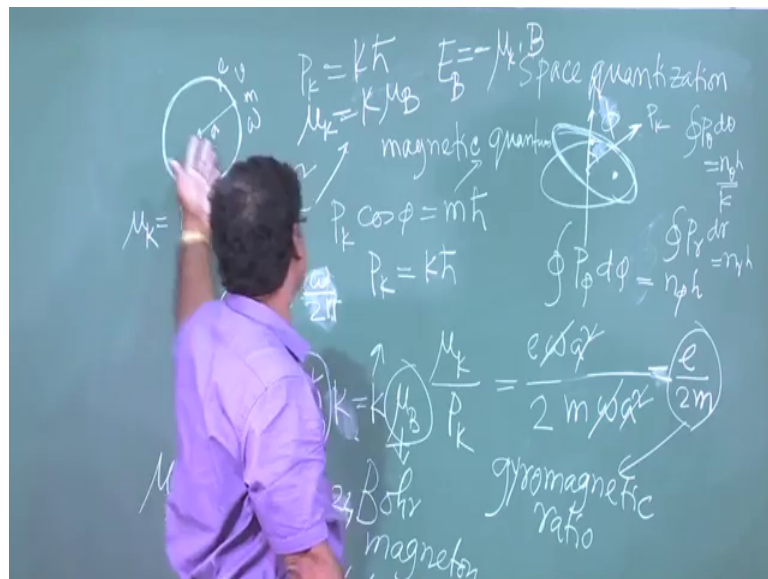
So, and we express the other (Refer Time: 26:54) in terms of this  $e$ . So,  $2 e$ ,  $3 e$ ,  $4 e$ ,  $5 e$  etcetera. Similarly here this magnetic moment that is we express in terms of this  $\mu v$ ,  $2 \mu v$ ,  $3 \mu v$ ,  $4 \mu v$ , because this  $k$  value you know this it can take 1 2 3 4 5 value. So,  $\mu 1$  for  $k$  equal to 1 for  $s$  cell it will be  $\mu v$ , for  $p$  cell sub cell basically it will be  $2 \mu v$  then  $3 \mu v$ ,  $4 \mu v$ . So, these are one basic unit of magnetic moment it is called Bohr magneton it is called Bohr magneton why it is called Bohr magneton? Now these  $e h$  cross by  $2 m$  this is.

Now, this will come this comes basically when electron this moment this is the moment of electron in a hydrogen this Bohr orbit in a Bohr orbit  $n$  equal to 1. So, Bohr (Refer Time: 28:18) orbit is always. So, Bohr (Refer Time: 28:19) orbit is always  $n$  equal to 1 and this radius is  $0.51$  angstrom. So, that is the fixed an electron in that orbit it will show the magnetic moment, because it shows when it is moving in a orbit. So, it shows the magnetic moment.

So, that magnetic moment whatever the magnitude of that magnetic moment that is  $\mu$ , and it is value most probably  $\mu$  equal to  $\mu$  value 8 I have to see.  $8.78$  into  $10$  to the power  $10$ ,  $8.78$  to into  $10$  to the power  $10$  joule let is write joule or tesla. yeah I think you need a joule per tesla no no no this  $9.274$  into  $10$  to the power joule per second it is wrong value I think I have to it is wrong value, it is a  $9.279$  point what is that value  $9.274$  into  $10$  to the power minus  $24$  joule per tesla yes.

So, that s the this is the universal constant for magnetic moment and now we got the relation  $\mu$  k like P k like P k we got  $K$  h cross and  $\mu$  k is basically what we got k mu v k.

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So, this is in terms of  $\hbar$  cross it is the  $\hbar$  cross is the unit of angular momentum and  $\mu$  v that is the unit of unit of magnetic moment. So, this is the relation. So, what happened? So, atom have electron and electron are in motion in different orbits. So, it has magnetic moment, now, if I apply magnetic field.

So, we will get the there will be interaction between the magnetic moment and magnetic field and due to this interaction there will be change of energy and that energy potential energy that is from electrodynamics one can get this energy due to magnetic field say  $b$  (Refer Time: 31:26). So, that is minus  $\mu$  k dot  $b$ . So, that will be the energy. So, this. So, now, there will be change of energy of the electrons spins energy levels of the atoms. So, that we find out in next class. So, I will stop here.



Thank you.