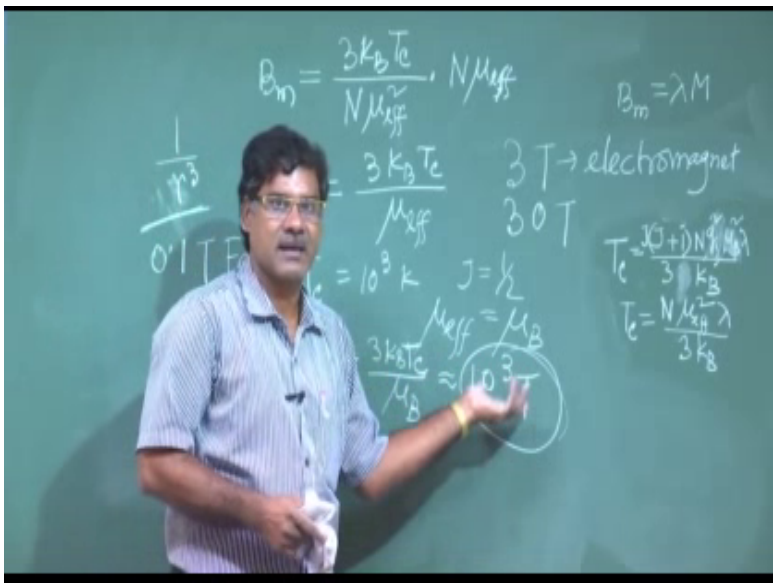
Solid State Physics
Prof. Amal Kumar Das
Department of Physics
Indian Institute of Technology, Kharagpur

Lecture - 68
Magnetic Property of Solids (Contd.)

(Refer Slide Time: 00:14)



So, we have calculated the T c we have calculate the T c it is related with the exchange this interaction dipole interaction energy constant lambda. So, now B m is equal to lambda M. So, if I calculate B m molecular field and that I will get lambda, what will be the lambda then 3 K B T c divided by N mu effective square n into m. So, this form spontaneous magnetization, so its maximum value can be N spontaneous magnetization, saturation magnetization. If it is maximum value can take saturation magnetization because as I told that when alpha tends to infinity alpha tends to infinity then b j equal to 1, so M equal to m s.

So, m is value N mu effective right N mu effective. So, from here what we are getting B m equal to 3 K B T c divided by mu effective. So, if you calculate this molecular field for say iron for iron ferromagnetic material, so its T c around to T c around I think 1043 Kelvin. So, if we take just approximate into the 3 Kelvin and if you take J equal to half means only we considering spin not this orbital contribution. So then it will be mu effective will basically mu effective will be equal to mu B. So, you know the mu B

value, K B value T c if we take this and then B m b m equal to then effectively for this condition this T c by mu B if we put on this value this T c Kelvin 10 to the power 3. So, it gives approximately I think it is filled in the order of 10 to the power 3 Tesla in the order of magnitude this 10 to the power 3, this is comes around 1500 Tesla there anyway. So, we are taking this if it will be 10 to the power 3 Tesla.

Now 10 to the power 3 Tesla is a huge very high value, because in lab we use electromagnet and now maximum magnetic field from electromagnet, we get it is 3 Tesla only 3 Tesla from electromagnet in our lab electro magnet. And using the superconducting magnet, nowadays one can get say maximum around this 30 Tesla field. So, in lab it is difficult to produce even 30, 40, 50 Tesla say it is not possible to get more than 50 Tesla or say 100 Tesla. So, we cannot generate magnetic field in lab more than say it is 100 Tesla right, but here internal field it is 1000 Tesla more than 1000 Tesla, it is a huge field.

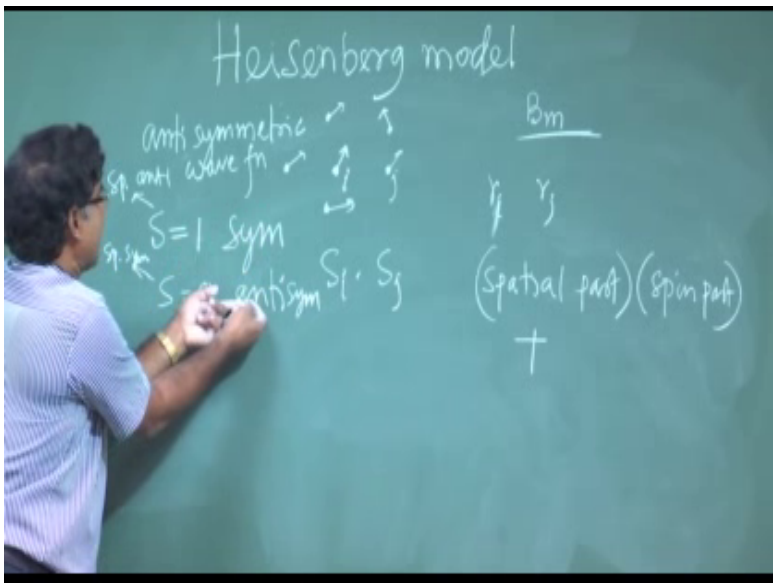
So, now, question is this from where this field is coming this internal field from where it is coming, so that is the question. And answer is the way the Weiss considered as if this molecular field this originates from the dipole-dipole interaction magnetic dipole-dipole interaction and magnetic dipole-dipole interaction it is that if you one can calculate if taking the distance it depends on distance. So, the dipole interaction that energy it is I think it its inversely proportional to that energy is inversely proportional to r cube dipole that energy is dipole-dipole interaction that energy is proportional to 1 by r cube.

And one can show that dipole-dipole interaction, it is hardly it can give hardly can give around 0.1 to Tesla or within 0.34 Tesla this dipole-dipole interaction, it can give equivalent magnetic field, equivalent magnetic field. So, the way the Weiss field we considered molecular field considered then we applied the is called this mean field theory mean field the approximation mean field the approximation this B m we have taken as a mean field the approximation means this all the dipole this they experience the same field. So, it does not depend on the location of these dipole. So, on the average all that dipole are having feeling this same will that is called the mean field approximation. So, whatever calculation we have done that is based on the mean field approximation.

So, point is that it cannot be it cannot be this Weiss molecular field it cannot be origin cannot be dipole-dipole interaction because dipole-dipole interaction that is magnetic

interaction that interaction is generally very weak compared to electrostatic interaction. Electrostatic interaction coulomb energy which gives coulomb energy, so coulomb interaction, so that comparatively this very high So, there must be it is not the dipole-dipole interaction origin of this Weiss field, molecular field it must be the electrostatic-electrostatic interaction, coulomb interaction, so that is the point. Now, to explain that one what is to find out the origin of this molecular field origin of this molecular field and its strength is very high it is 10 to the power 3 Tesla in that range.

(Refer Slide Time: 09:53)



So, so to find out the origin then Heisenberg, so Heisenberg proposed that propose one model proposed it is called Heisenberg model, Heisenberg model of ferromagnetism. So, it tells that this origin of this molecular field it cannot be magnetic interaction, magnetic dipole interaction, there must be electrostatic interaction coulomb interaction and that that could be the origin of this field or that could give this high internal field. So, I think is in Heisenberg model there, so he considered that in crystal this whatever we are permanent dipole moment we are telling, so he considered that they have moment, so that we are considering this spin moments spin they have spin.

Now, in surrounding others spin is there, others spin is there, others spin is there. So, if it is the ith spin, so if it is this is jth spin, so if you just if you take simple model two electron spin two electron spin. So, this ith spin S i, and another one is S j. So, they will interact they will interact they will interact. How they will interact, so basically if the

position. So, ith and j th electron if they have in they are in lattice, so they have position spatial coordinate say r i and r j they are spatial coordinate r i and r j and they have spin part S i and S j.

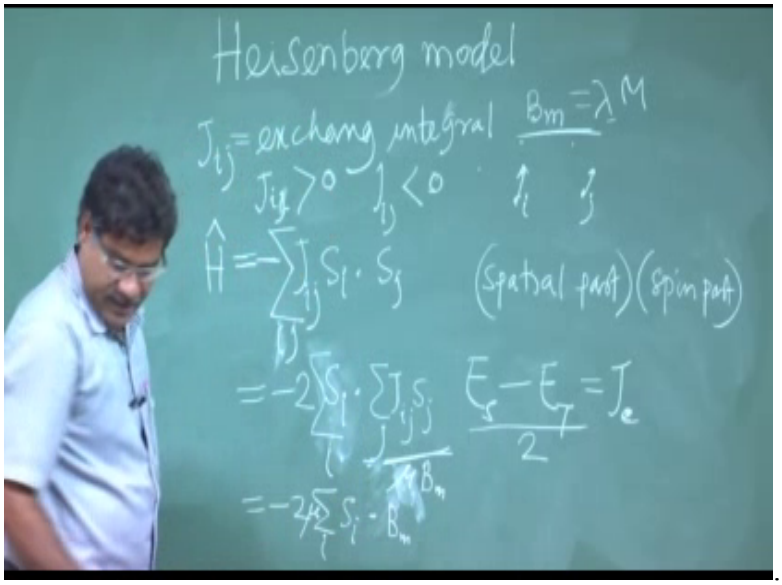
Now, if you find out, this is now two electron system, and if you find out the wave function for this two electron system, wave function has to be this since it is electron has to be anti-symmetric wave function. And to overall this wave function of this system will be anti-symmetric. So, now this wave function have two part; one is spatial part, another is spin part. So, one is spatial part, another is spin part. So, two spin is there if they are parallel. So, you will get S equal to 1, you will get S equal to 1. So, S equal to 1, this is basically give triplet state, triplet state and if one can show I do not know you are quite familiar with the quantum mechanics are not, but just believe me.

So, this triplet state, so its spin part its wave function is symmetric function its wave function is symmetric wave function; and if they are anti parallel then S equal to 0, you will get singlet state. So, singlet state and that is anti-symmetric. Now to become this wave function in general, this is anti-symmetric. So, it has spatial part and this spin part, if spin part is symmetric, so these has to be anti-symmetric. So, overall it is anti-symmetric or spin part is anti-symmetric then these part has to be symmetric then overall it will be anti-symmetric.

So, for this spin part, spatial part is anti-symmetric; and for this one, it is anti-symmetric, so this special part will be symmetric. What does it mean, so how this wave function is say this function, so give this waves this wave function is found. So, it has basically it will have two part this and this and this two parts come from the exchange in spatially as well as from spin part. So, if I just talk about this spin part. So, if this state, it goes from symmetric it exchange if it exchange, so spin both are up if just exchange. So, one is up and one is down if it then what will happen when it will exchange, so it will go from anti-symmetric to symmetric. So, simultaneously spatial part will go from anti-symmetric to spatial part will go from symmetric anti-symmetric. So, here information is that spatial part it is related with the distribution of the charge, it is related with the distribution of charge and I have spin part, so nothing to do in that magnetic moment of the spin, spin direction is important.

So, whenever this change in spin there is exchange between the spin there is exchange between the spin then simultaneously there will be exchange in the exchange in the spatial part means they are will be change in the distribution of the charge. So, whenever this spin part is effected much changing then simultaneously spatial part charge part also its changes. So, thus this wave when this spin or whatever dipole whatever we are telling this they interact with each other, because of that it is simultaneously it is getting this charge that is they are spatial charge they are interacting. So, these are called exchange interaction.

(Refer Slide Time: 18:58)



So, the energy because of that this energy or in terms of Hamiltonian on one can write. So, it was written for this that J i j, one constant term it is called integral exchange integral J is called exchange integral or exchange energy constant. So, this energy that is exchange between S because of S i and S j. Now, this is the two electrons system now I have in my crystal I have many I have many, so and this exchange energy this is taken as minus this minus one can derive this basically. And then for considering all spin, we can take i summation over i and j . So, these the Heisenberg model for this for this exchange energy, which we will show later on that this exchange energy constant this related with the it is related with the lambda.

So, that means, this difference between these two here just we thought it is magnetic interaction, but here it is showing no, no, it is not magnetic interaction it is basically

electrostatic interaction because charge is involved whenever this something happening with the spin. So, simultaneously charge is also changing means interacting, so that exchanging that, so that is why it is called exchange interaction. If they exchange spaces spin part is changes, so simultaneously spatial part also will change. And yes and this J i j, it is J i j, it is called exchange interaction constant or exchange integral, because here summation we are taking, so integral or exchange energy constant exchange energy constant.

Now, here, so this exchange either they are now how they are, so if it is ferromagnetic ordering, if ordering is ferromagnetic, so then they I think J i j, if it is ordering ferromagnetic means ferro, so this J i j is 0 sorry greater than 0. And if it is anti parallel anti-ferromagnetic coupling i j, so then it is less than zero. So, what does it mean? So, this exchange integral or exchange energy constant, so here as I told that that either spin part is triplets states triplet symmetric state or it is singlet anti-symmetric state. So, this system two electron system are whatever this or this many electron system also. So, it will have two energy state it will have two energy state one is for triplet another for another is for singlet state. So, with this energy is for singlet state, it is E s and for triplet state it is triplet state it is E T

Now, if it is exchange singlet and triplet exchange. So, one has to overcome this energy E s minus E T. So, during the exchange from triplet to singlet or singlet to triplet, so this much energy is required explains of this of the energy due to this exchange is this. So, these basically is taken the exchange energy difference between the E s and E T it is the exchange energy it is taken as exchange energy constant. Depending on the system, this energy state for singlet triplet fixed. Now, if any exchange from triplet to singlet or singlet to triplet any exchange occur, so then energy either increase or decrease by this one. So, exactly it is comes by this two half of that J e half of that.

So, this one can find out. So, one has to write the wave function spatial part, spin part form the anti-symmetric wave function for the system electron system from then it comes. So, that I have not done it may be difficult for you. So, these exchange energy constant. So, difference between the singlet and triplet state. So, this exchange from one state to another state singlet to triplet, triplet to singlet, so this much energy this required for that system.

So, now, what is the meaning of that that J e is greater than 0, J i j, so these the average exchange energy if I take and it is constant. So, in that case, so it is greater than 0 means E s is greater than E T, it is positive means E s is greater than E T. I think I have written correctly this is E s that is the problem if we do not derive. So, we have to yes E s, so it will be E s minus E T. So, E s will be when it is positive E s is greater than E T, E s is greater than E T. So, if energy is Hamiltonian means is a energy, its energy will be negative when this is positive energy will be negative. And when this one is less than zero means E s is less than E T, so E s singlet state will be preferable.

When J is negative singlet state will be preferable that means, and anti parallel configuration preferable. And when j is positive, so then E s is greater than E T means E T will be lower than E s, so triplet state that energy is lower, so it will be preferable, so ferromagnetic configuration parallel configuration will be preferable. So, that is the there is the significant why we take j is greater than 0 for ferromagnetic ordering and j is less than 0, for anti ferromagnetic ordering, so that comes from here.

So, this Hamiltonian one can write minus now imagine that in. So, now, in this case we are dealing with the spin part, but dealing with the spin part anything exchange in spin part automatically this spatial part will be involves; that means, electrostatic-electrostatic interaction will be active. So, here you know that if I have ith one here, this ith spin ith electron, so S i S j, so here basically jth one is here. So, this if I take other one, so S i S j, now S i if I fixed one that is here. So, now, S j, j equal to 1, 2, 3, 4, 5, they are many is. So, they are interacting with this they are interacting with this right like for this configuration.

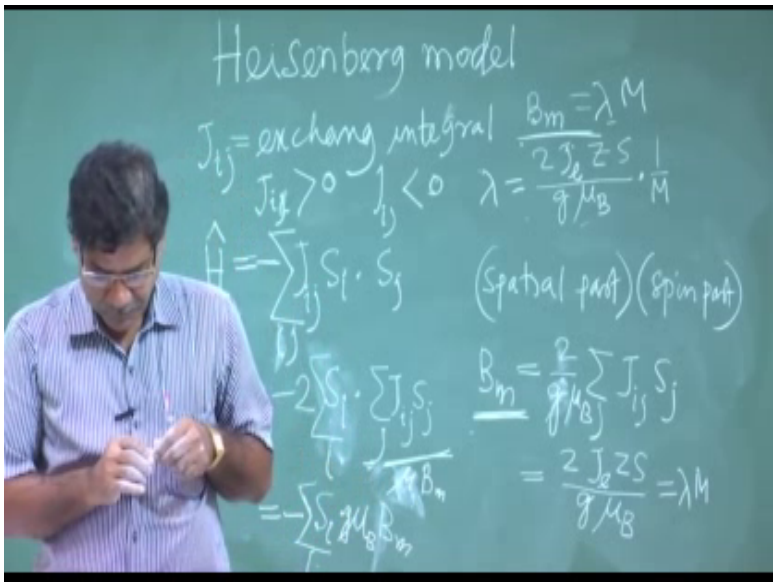
So, what is happening. So, this S i one this is one. So, it is getting it is interacting with this all others right that is indicated by the j, j equal to 1, 2, 3, 4, 5. So, in case of B m, here also we have considered this similar concept this one. So, whatever the rest of the things they had influence on it, so that average influence we took as a molecular field right. So, if I write this one J i j as is taken as a constant J i j. So, here one can write I can write 2 sum over i is greater than j. So, just to avoid double counting to avoid double counting because i j, so i j now if you go for same one and two, so two and one, so that is why, so to just the double counting you are considering taking here considering taking here 2.

So, here now this i and j cannot be exchanged. So, whenever this i one, so j has to be 2, 3, 4, 5. So, otherwise whenever i equal to 1, so j can be also 1. So, this double counting whatever here was coming, so that taking two we have considered that that exchange between i and j. So, this I can write like this. Now, you see I have spin S i, this S i. So, interacting with this one with the rest of the parts, so I can write sum over J i j interacting i with j and this S j. So, this is telling this all j which interacting with this i. Now, for all i, now this is one and rest of the others are interacting, so I can express this way. Now, I can vary this i.

Now, this part this part what is this what this part this part some effect of the others on this. So, there the B m is exactly same effect of the others on this that even is field. So, this I can replace with B m this I can replace with B m this I can replace with B m. Now, this is energy B m is field. So, energies mu B g, I think this magnetic moment dot this B m magnetic moment dot this B m will give the energy. So, I can write g j I can write, so this I can replace with B m molecular whatever this one, and then magnetic moment g j g mu B, you write g mu B. Now, g can be if it considering only spin g will be 2. I think I have to put g s mu B, g s mu B, yes have to put g s mu B because here I am considering only spin, so g s mu B B m.

Now, that now that this spin I am not writing because already this spin is there right already this spin is they are spin is there. For that spin energy will be I think I will write if it is spin energy is this moment is mu B is moment is mu B. So, mu B B m, mu B B m I can write or just simply I can write you forget this one just I can write mu whether it is mu j of or mu s or mu m whatever, so that is the magnetic moment dot B that will give you the energy. So, this I can write equal to minus 2 sum over i S i dot B m, and that mu I can write here outside is constant mu I can write here, mu I can write here. So, ultimately here what we are getting here if you so this is my original things. Now, for considering this equivalent this molecular field, I can here write this one.

(Refer Slide Time: 36:51)



So, from here this B m is basically one can write from here I have to get, but two is there. So, here I think I am missing two 1 by mu B 1 by mu B or mu 1 by mu, this would be equal to this is m. So, mu are there yes. So, sum over j sum over j J i j S j. So, this the B m molecular field. And yes so just I am missing this some, I am missing one, I am missing two. So, this is the original concepts. So, this I can get like this, this I can get like this.

So, what I should do, I think what mistake I have done, this will be equal to know if I considered this one, so here is B m field is that one. So, this field is now applied on all the spins, so that has come from this others region j 1. Now, I am considering now well from j 1 whatever coming that we have taken has a B m, now B m interacting on this is S i only S i only. So, now, what about the spins are there all are s 1, 2, 3, 4, 5 then B m applied on that. So, here what I did mistake is that, so I have to consider this B m field and that if equivalent energy B m field and equivalent energy is mu B m, equal energy is mu B m, so that one applied on S i.

Now, S i is varies as a i equal to 1, 2, 3 etcetera whatever the number is there. And this mu dot b the energy is minus mu dot B, so that is why minus. So, this I am writing from the concept of this B m I have spin 1, 2, 3, 4, 5 that is given by i; and on this spin this B m field is acting, so energy will be mu B energy will be mu B or just I am writing mu into that field. Now that energy will come from all S i. Now, I am taking summation on

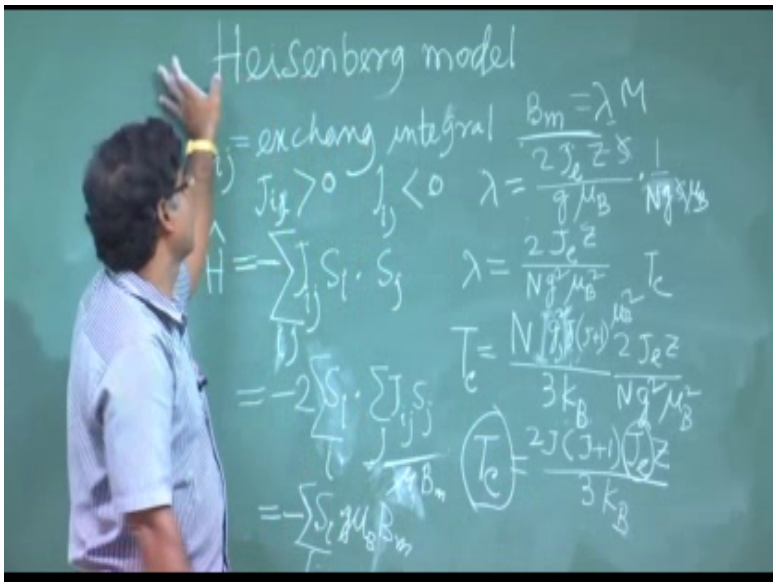
this S i. So, this mu this S i is there, so that is why S already is there. So, that is why this if I write this mu this value g j j mu B right since it is only spin, so that is s is always g g s mu B. So, this sometimes if you write here g mu B, so g s mu B kind of things g s mu B that is the moment into this B m.

So, for all S i for all S i i equal to this summation are s t, so that will be equal to this one. So, now, if you equate this one then here, so that is why I was missing this two. So, let me write 2 by g mu B, so this one. So, this is relation between this molecular field and the integral exchange integral. So, just from here one can explicitly find out this, this relation with the T c. So, here if I take now see if I take this interaction if I consider only that exchange interaction is effective among the nearest neighbour.

So, j average and that interaction is a if I J i j whatever written if I write that j e, so that is a nearest neighbour. So, this distance are same, so J i j that will be same for all, so J e for nearest neighbour. If I take J e then B m that is the only for nearest neighbour, so I can write this one. So, taking nearest neighbour that is why J e same for all. So, I can write two J e for each pairs. Now, if I have z number of nearest neighbour, so have to multiply by Z for each one this for each this Z that S divided by g mu B, so that will be that will be the B m and now this B m we have taken lambda M we have taken lambda M. So, is lambda M.

So, then lambda equal to one can write lambda equal to I can write lambda equal to by M, so 2 J e that is e exchange one not that is, so Z S by S by g mu B and this M s, so this is I think lambda equal to this by this 1 by M. So, M we have to take M value, what is M value. So, M mu B, n mu effective in this case I should now this problem is taking whether I will take this yes, yes. So, I have to take this M. So, M I can write N g S mu B right. N is the number, so for each one that is the moment. So, I can write this.

(Refer Slide Time: 46:14)



So, from here I can get lambda equal to 2 will be there J e exchange integral then Z it is there divide by what you are getting N you see here you are getting, so S, S will go N I am getting g square mu B square. So, here this is the relation between this lambda and J e. Now, T c, what is T c, T c was the mu effective T c. What was the T c, whether it is there, T c was this have to T c is that is yes T c was yes. So, basically T c, what is T c, T c was M I think 3 K B I stopped that I just stopped you remember N mu effective square then lambda divided by 3 K B. So, c it was the mu 0, and this for T c this is lambda.

So, now if you put this lambda value from here that is 2 J e Z by N g square mu B square. And then mu effective you remember mu effective was what was that mu effective was g j square J j plus 1 mu B square. So, basically here this mu effective was g j J j plus 1 mu B sorry I think this was square, this was square right yes. So, then T c you are getting N, N will go, g square will go, mu B square will go, I will get J j plus 1, J j plus 1, 2 is there, J e this exchange is equal to do not mix with this Z by 3 K B. So, that is the exact relation for between T c and that between this and this.

So, I think these the Heisenberg model. From this Heisenberg model, so this we can find out the origin of this B m molecular field that is basically coming from the electrostatic force, electrostatic energy, electrostatic interaction, spin part and spatial part. If any exchange in spin part there will be exchange in spatial part. Spatial part is nothing but the distribution of the charge right it charge distribution, say affect the charge so that

interaction among the charge it is electrostatic in nature. And that whatever the molecular field theory concept that is correct, but they are that is magnetic dipole interaction kind of concept was taken, but this B m is basically that we are getting from this from this Heisenberg model also, and there is a connection.

So, this express in terms of exchange interaction wherever is here dipole interaction, so that exchange energy constant J and here this lambda they are connected and that connection we have shown. And T c also in terms of exchange integral exchange energy constant one can write, so that is all about the Heisenberg model, which is very successful to explain the high molecular field. But result are same whatever the from Weiss model we have got, here also will get the same result, but approach is different approach are more realistic and this Hamiltonian in this form is very useful for explaining the ferromagnetic property of material.

So, I will stop here, I will continue next class.

Thank you.