Solid State Physics
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Lecture - 67
Magnetic Property of Solids (Contd.)

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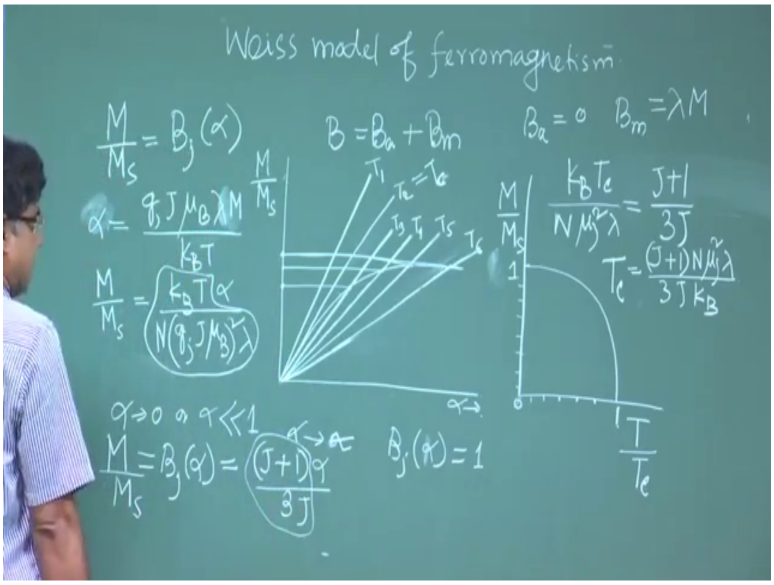

So, you were discussing about the Weiss model of ferromagnetism, in Weiss model of; so, in this model basically this internal field was taken is called molecular field. So, this field and if we apply field B a; applied field, external field. So, that is the total field that permanent dipole moment will experience.

So, whatever the paramagnetic formulas we have derived. So, in that formula if you replace this that B with this B a plus B m, and then that will be valid for ferromagnetism as for this Weiss model. So, then we have put the B a equal to 0, means external field equal to 0. And then whatever we will get that is the because of internal field; molecular field, and this molecular field was taken by Weiss as a as a B m is B m is proportional to magnetization, and this proportionality constant is lambda.

So, lambda is basically is shows the strength of interaction. So, interaction of what? So, this as I mentioned this is dipole moment; permanent dipole moment, they interact with the neighbour’s others dipole moment. So, as if this other neighbour’s dipole moments have influence on these dipole moment. And that is the that influence is expressed that dipole-dipole interaction is expressed by this molecular field. So, and what we have seen that that 2 expression we have derived; this one is M S; M by M S that is Brillouin function alpha. And so, if you plot this Brillouin function; so, let me plot just one half. So, this it is basically like this, and this is a function it is the M by M S, and this is the alpha. As a function of alpha this the B j curve. And the; and that another expression we cut that is for alpha, alpha is; alpha was what is that alpha? Alpha was g j J mu B, right? And then magnetic field B. So, B a equal to 0. So, B m and then B m is replace; was replaced by lambda by lambda M divided by K B T, right.

Now, from here we got this; M by M S equal to what we got? This K B T alpha divided by g j J mu B square, and then it was lambda. Yes, because for M S one this; or this one N will be there, M S is N g j J mu B, I think that you have; last time I have derived is so that; so, these 2 equation; so, M by M S is function of alpha B j and this M by M S this function of alpha. So, that solution has to satisfy this both equation. So, solution will be the intersecting point of these 2-curve. So, if I; so, I have drawn this one, if I draw this one. So, this if I take it is not straight line if it will be straight line. So, that is for some temperature, and then other temperature. So, this curve for no, I think I have to draw only straight line.

So, all this curves at alpha equal to 0. So, is intersecting with this with this curve. So, in this case M m is 0 alpha is 0. So, we do not have interest. In other case so for these say this is for temperature, see T 1, this is T 2, this is T 3, T 4, T 5, T 6 etcetera. So, here you can see that here T 2 is basically is the gradient, is a gradient of this of this curve, B j curve, right. So, it intersect at alpha equal to 0. So, about at higher temperature than T 2. So, here basically we have plotted this one for different temperatures T 1 T 2 T 3 etcetera. So, so this above this T 2 higher temperature T 1 is higher greater than T 2. So, it is not intersecting this curve, but below T 2, T 3 is less than T 2, T 4 is less then T 2, etcetera.

So, we are getting intersecting point with this curve right, with this curve. So, other side also will get negative value, because these are symmetric curve; so, I have not drawn that part. So, here so these T 2 is basically; it will this is the critical temperature, or curie temperature, right. So, above curie temperature there is no magnetisation, but below curie temperature, there is a magnetisation we can see, there is a magnetisation, right.

So, these are the magnetisation believing by M S value. So, this magnetisation is when without external magnetic field we are getting some; we are getting magnetization, and these are basically call the spontaneous magnetization without external field the material which is showing the spontaneous magnetization. So, that material is basically this ferromagnetism, ferromagnetic material. So, so from here one can basically draw. So, I can get M by M S value for different T value.

So, what we are seeing that this that you know that when alpha also when alpha tends to infinity. So, what will get? This B j, B j alpha that function is basically 1. So, in that case we will get this magnetisation M equal to M s, that is spontaneous magnetisation you will get that is the equal to M S; this saturation magnetisation, it is saturation magnetisation. So, you will get higher value; you will get higher magnetisation. And that happens? Alpha tends to infinity means, this is the alpha tends to infinity means T tends to 0, T tends to 0 right, T tends to 0. So, here T tends to 0 said this we will get maximum; see will get maximum, spontaneous magnetization.

Now, if you put; if it put the you see alpha is a having temperature it is lower. So, we are going towards the lower temperature towards the lower temperature. So, this value are very they are they are separation very small. So, for lower temperature you will get. So, this basically you see this slope, slope change is very small here, and this slope change is here in this area it is higher. So, that is slope basically depending on the slope these value is decided, because slope is this by this, right.

So, what about the way? Slope is varying so that one can. So, it is varying like this. So, if I plot here in this direction T. So, with respect to this T c, with respect to that T c. So, these value is basically 1, and these value is basically 1 right, and then you just. So, between 0 to 0 to 1; 0 to 1. So, this the variation of these; the variation of spontaneous magnetization without applying any magnetic field, external field.

So, this is very sensitive to the temperature. Spontaneous magnetization is very sensitive to the temperature, and it varies like this, and one can show basically after proper calculation proper form that probably I will show you just. So, here from here what we are seeing from Weiss model we are seeing that, this magnetization spontaneous magnetization exist in the ferromagnetic material below a certain temperature; that is curie temperature, that is T c curie temperature. Above curie temperature; so, magnetization is 0 above curie temperature magnetization is 0, right.

So, above this curie temperature if you apply magnetic field then what will happen? If we apply magnetic field, then what will happen? So, that if you want to find out then you will see that it behaves like paramagnetic material, it behaves like paramagnetic material. So, basically in case of ferromagnetic material below T c behave as a ferromagnetic material. And above T c behave like a paramagnetic material. So, these the so these the T c is the basically phase transition temperature T c is the basically phase transition temperature between the ferromagnetic phase and paramagnetic phase. So, ferromagnetic material have 2 phases, one is ferromagnetic and another is paramagnetic. And this is the phase transition temperature.

So, this as I mentioned that T c T 2 equal T c so how? So, this T c is related basically it is related with this with this lambda T c is related with this lambda in the depends on the interaction constant; interactional energy constant. So, how they are related that if you want to find out. So, what we have to find out? That at this temperature T 2 equal to T c at this temperature the gradient of these 2, gradient of these gradient of these will be equal to the gradient of this at alpha equal to 0.

So, gradient of this is so obviously, is this a K B This part, gradient of this is will be this part right. This the gradient now these will be equal to the gradient of this one, and these we are considering at alpha considering at alpha is very, very less than 1 or alpha tends to 0, alpha tends to 0 at this point right alpha tends to 0 at this point. So, when alpha tends to 0 or alpha is very, very less than 1, in this condition this B j alpha, that B j alpha one can show, it will be I think j plus 1 alpha divided by 3 J. One can show this if alpha is very, very less than 1, this that will be yeah, yes that will be this ok.

So, I think I can show you. So, basically yes how I have got it. So, let me show you I think this you have to use Taylor series. You have to use Taylor series, but somehow is not displayed I need to display, but somehow is not displayed. Anyway, you know this cos hyperbole we have cos hyperbole cot hyperbolic x right. So, it is Taylor series if x is very, very less than 1. So, this the 1 by x plus x by 3 plus other term is there x is very, very less than 1. So, what can keep up to this. So, using this approximation. So, B j that you know 2 j plus 1 by 2 j, cot hyperbolic it cot hyperbolic 2 j plus 1 by 2 j alpha. So, so

2 j plus 1 by 2 j alpha that if you take x. So, you can cot hyperbolic x it can be replaced by this.

So, another term was this minus what was the another term minus 1 by j cot hyperbolic, alpha by 2 j 1 by 2 j, cot hyperbolic alpha by 2 J. So, here alpha by 2 j can be taken as x, and you can get this one, that you can replace with this one. And then if you proceed then you will get this. So, if possible I will show you sometime. So, just if you use this one if you use this. So, that is basically now in this condition is M by M S is this is M by M S is this right. So, this gradient will be j plus 1 by 3 J right gradient for this for this at alpha is very, very small alpha tends to 0 at this point at this point. So, it will be j plus 1 by 3 J. So, this this will be the gradient right.

So, this gradient at alpha equal to 0 or alpha tends to 0 at this point. So, this will be equal. So, if you equate them and why it will be equal? So, this gradient we have taken at T e equal T c. So, this here you can you can write T c. So, I think have to write; here I can write. So, what I am getting? K B gradient K B T c divided by N. Let me write this one mu square. Just mu square let me right mu square or mu j square whatever; if I write mu square or mu j square I can write. So, mu j is g j mu B, and then lambda that will be equal to j plus 1 divided by 3 J right.

So, from here you can get T c, you will get T c equal to j plus 1 N mu j square right, N mu j square lambda by 3 J K B by 3 J K B right. So, that will be the T c, and now here actually j is there j square is there. So, one can remove the j squares in that case I have to write this one mu j explicitly. So, what I can do?

So, here I can write g j square, here I can write j and then I can write mu B square, then lambda, right.

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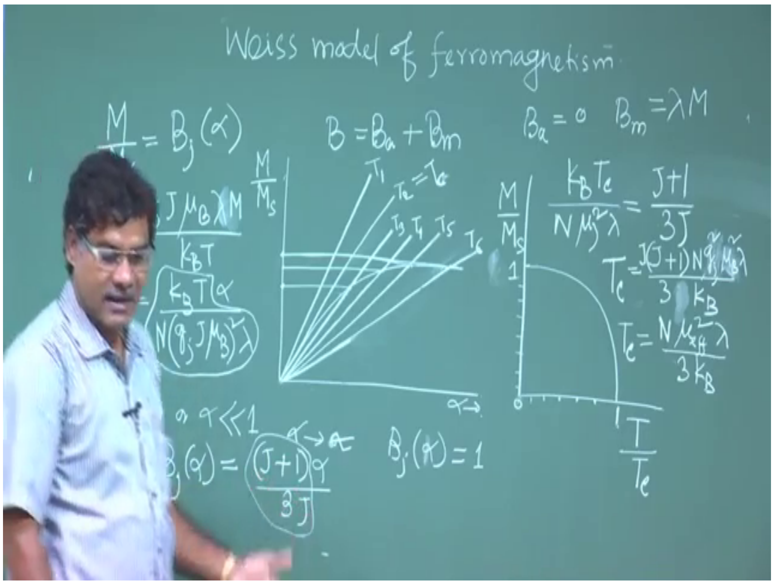

Mu be square and lambda. So, this j will go one j is there. So, in this case what? Yes, so this T c we got as a; so, these now T c equal to N, then this we write as a mu effective, you remember that p effective these the g j J plus 1 g j square mu B square, right.

So, that is basically you can write mu effective square, and this effective bohr magnify; effective bohr magneton number that is p effective earlier whatever we have discussed. So, they are the p effective was square root of j J plus 1 g j, right. So now, that I have included including mu B. So, this mu effective mu effective, and that that will be here it is mu effective square, lambda divided by 3 K B, right.

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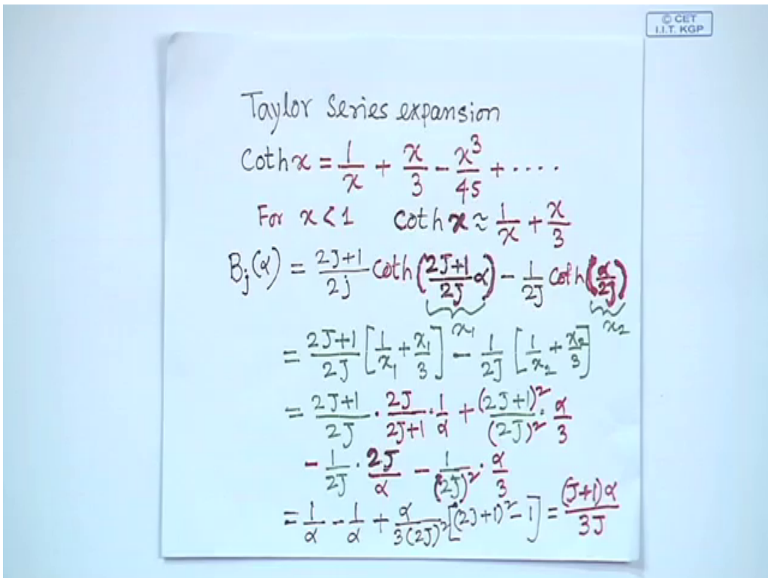

So, that is the T c. So now, here you can see T c is connected with this with this strength T c is connected with this strength. So, as I told you that I will show, how to calculate this? This for alpha is very, very less than 1. So, these the these the B j value will be this. So, that here I have just I have I have calculated. So, you have to take Taylor series, cot hyperbolic x right as I mentioned x is very, very less than. So, this one can take this now B j you see here just explicitly I have put this for all cot hyperbolic value and then from there I am getting j plus 1 by 3 J into alpha.

So, this series I think these are very important, know this trigonometric hyperbolic function and their relation and the Taylor series.

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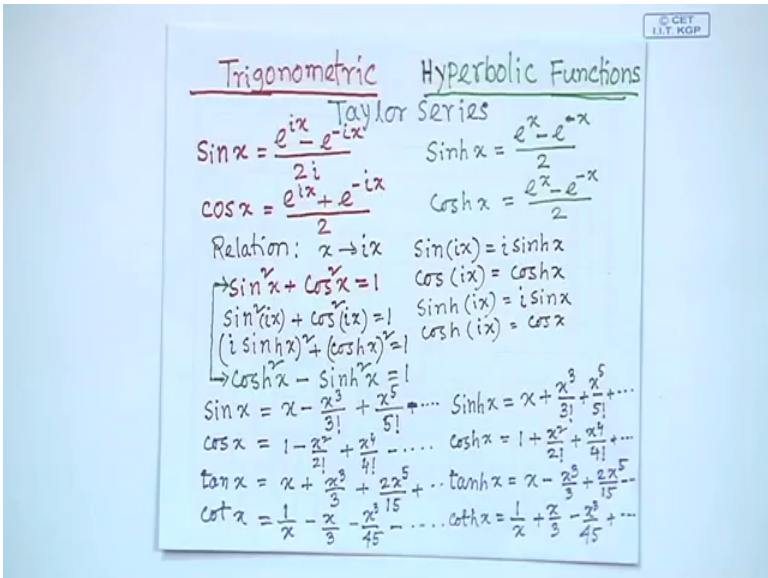

So, it is very simple you are familiar with the trigonometric one, but we are not much familiar with the hyperbolic function, but here you know just i. So, if you know sin x cos x sin hyperbolic x or cos hyperbolic x, you can find out if you replace x by i x if you replace x by i x, then from sin cos, you can find out sin hyperbolic, cos hyperbolic. So, and they are in that case to satisfy this 2-exponential form. So, this sin i x is equal to i sin hyperbolic x cos i x equal to cos hyperbolic x.

So, reverse way sin hyperbolic i x equal to i sin x cos hyperbolic i x equal to cos x. So, if you know the sin cos tan that relation, sin cos if we know then will go know this tan cot sec and cosec. And there if you replace x by i x, and you just remember this sin i x equal to i sin hyperbolic x, cos i x equal to cos hyperbolic x. And reverse way sin hyperbolic i x equal to i sin x cos hyperbolic i x equal to cos x, it will just remember this part. So, from. So, trigonometric function you can easily get the hyperbolic function. And the these are the Taylor expansion Taylor series Taylor expansion right sin x equal to this. So, if you replace this x by i x we will get sin hyperbolic x, you just check it.

So, similarly cos x is this. So, cos hyperbolic x we will get if you replace x by i x. So, similarly tan x cot x again if you replace x by i x you will get the cot tan hyperbolic x cot hyperbolic x. So, this generally frequently we use in solid state physics. So, just I showed you. So now, I got this relation where T c and this lambda, interaction energy constant. So, they are related. So, T c depends on this material T c depend on this material, and in that material how strongly they are interacting this origin of this molecular field how this dipoles are interacting. So, this depending it depends on the strength of that interaction, which is expressed by lambda it is metal dependent.

So now if you find out the magnetic susceptibility. So, I think here I will use magnetic susceptibility means, at higher temperatures say at alpha equal to very, very alpha is less than 1 alpha is less than 1, in this condition so alpha is less than 1 in this condition if you find out yes. So, here we have this relation; here we have this relation. So, here I cannot see magnetic field, but anyway.

So, I think if I start from here if alpha is, but alpha here I am missing that is why. So, originally alpha this B, right?

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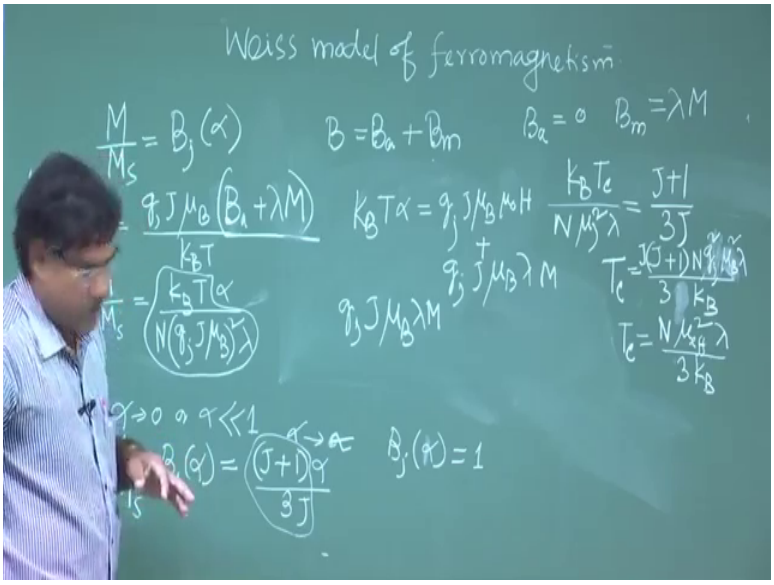

Now, if I consider this B a plus B m this term is now not B a is not equal to 0 in this condition. So, you will get the B m I will replaced by B m I will replaced by lambda M. So, it will give me 2 term one is yes. So, I need space I think I will. So, here what I want to get. So, I will get 2 term yes. So, from here I can write K B T alpha equal to g j J mu B right, into B a; so, B a I can write mu 0 h B I can write mu 0 H, and then plus another term I will get g j J mu B lambda M, right.

So, these 2 term. So, actually I think yeah, I am not using the right one. I will j plus 1 have derived I think I should use some relation M s; M by M S that and then alpha I am getting this one, but we are anyway. I think what I should do this the alpha term and here from here also I can find out, what I want to do I want to find out susceptibility. So, here g j J mu B lambda M, lambda M equal to so if I think I can where is my alpha? Alpha is here. So, I have to put, I got.

So, I think yes; I think this have to start from here. So, this what you want to find out? You want to find out magnetic susceptibility. So, from here alpha is already there. So, I do not need to I do not need to break it. So, what I should do? So, these alpha is this when we are applying magnetic field, now we have this M by M by M S equal to this. So, at alpha is very, very less than 1.

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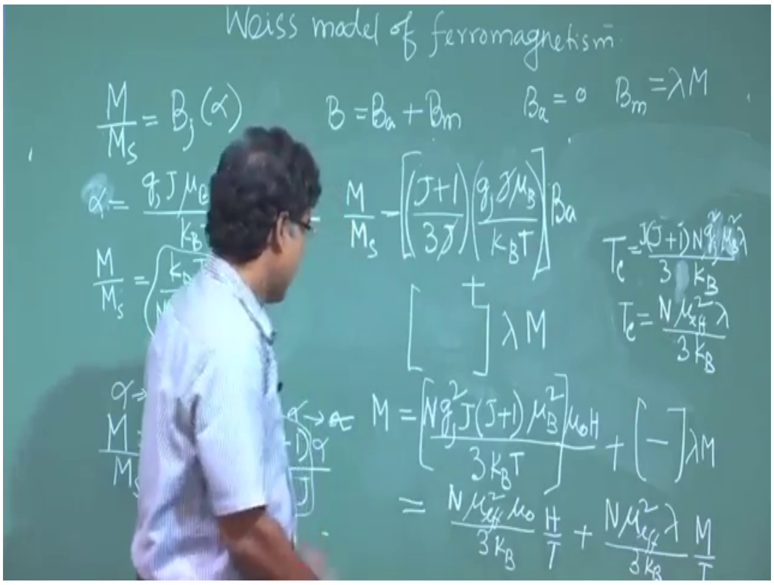

So, this M by M S. So, M by M S is B j J. So, that will be j plus 1 by 3 J yes, I think this into alpha. So, that I have to use this alpha. So, here I will get g j J mu B by K B T. So, this is another term. So, I think I have to see I can remove this all to see remove this all. So, then it has 2 term B a and another term I will get plus the same term; this same term, I will get here then lambda M right. So, lambda M.

So, then M I will get from here if you simplify this one. So, this J, J will go. And then I have j plus 1 g j mu B. So, I will get and then if I multiply M S with this. So, M S is N g j J mu B; I multiplied this M s. So, M equal to this. Now here then what I am getting you see that j plus 1. So, let me write here j J plus 1, and then I am getting mu B was there; again, one is mu B is there. So, mu B was there some another mu B; this square g j square, right, g j square divided by 3 K B T, right. And then B a means it is mu 0 H, and then plus the same term I will get same term up to this up to this I will get same term; except this lambda I mean instead of this this will be lambda M this will be lambda M, this this one lambda M.

So, here you can see, this mu 0 this term; I can write N mu effective square, mu 0 by 3 K B h by T h by T, and plus N mu effective square in this here mu 0 here lambda divided by l lambda, divided by 3 K B and M, M by T right. Here I will get M by T.

Now, this here we can see this one is T c; so, this I can replace by T c, this I can replace by T c, and this one as you remember.

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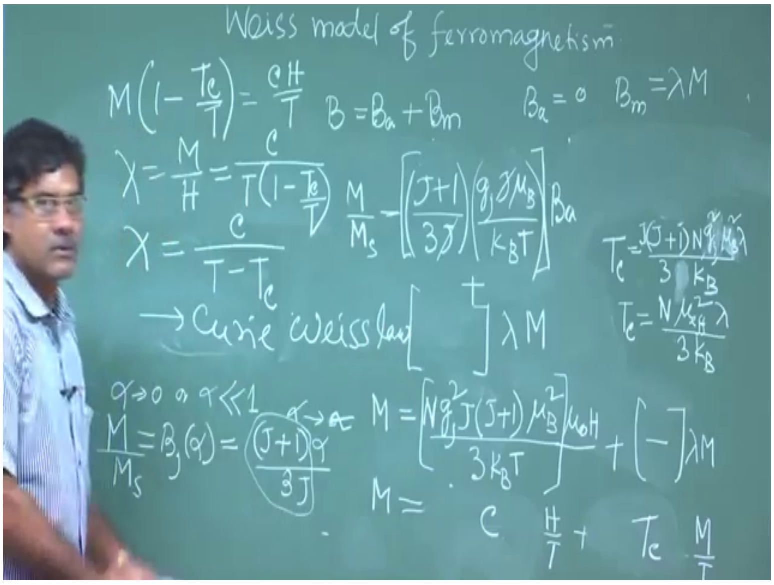

This is basically c curie constant; this is c curie constant if you see earlier records. So, you will see this will be curie constant c, and then this is my M. So, M is this side also. So, what I am getting? So, M this one M so if I take M, I think, I should write here. Let me write here. So, I am getting M 1 minus T c by T c by t, but something wrong or what an effective T c by T fine T c by T equal to C H by T.

Now, chi is M by H, right. Equal to c by T into 1 minus T c by T right. So, chi equal to c by T minus T c. So, this expression if found for taking this approximation alpha is very, very less than 1 means T is very, very high. So, this is valid for T is very, very high; at least T is greater than T c. T is less than T c it is not; so, it is for paramagnetic region it is valid.

So, this is called the curie Weiss law. So, this basically it explain the paramagnetic phase of ferromagnetic material. So, these very famous relation. So, that is the curie Weiss law. So, I think that is all; yes. So, I will stop here. So, I will continue in next class.

Thank you.