Solid State Physics
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Lecture - 62
Magnetic Property of Solid (Contd.)

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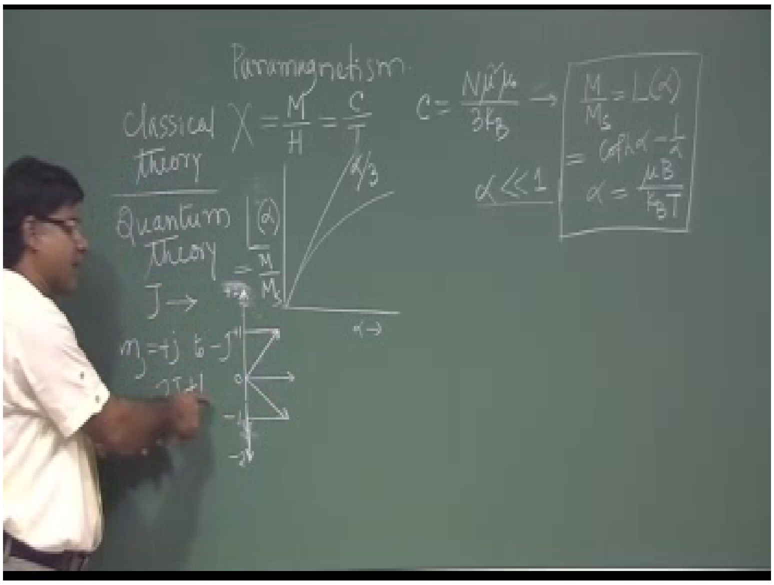

So, we discussed about the paramagnetic. So, susceptibility of paramagnetic material it is M by H, that we have expressed C by T, C is curie constant. And this law is called curie law and this C value basically C equal to N number of atoms per unit volume or number of dipole moment per unit volume, N mu square mu 0 divide by 3 KB divide by 3 KB.

So, that that expression will be see it for basically from that in general formula this M by MS equal to langevins function. So, langevins function was basically this form was cot hyperbolic alpha minus 1 by alpha where alpha was the mu B by KBT.

And this is general now, we assume that l is very less than 1, means low field high temperature in this condition we will get this expression. So, this expression is quite good agreement with the experimental result. So, paramagnetic material this it is basically observed at mainly at higher temperature. It is not observed at very low temperature.

But there are some materials complicated materials it is observed also at low temperature. So, that is the special case, but in general it is paramagnetism is the phenomena at generally at relatively higher temperature. So, under this condition one can get this and this expression this formula is quite good for most of the experimental result of paramagnetic material.

So, in general this as I mentioned at alpha this langevins function. So, this varies. This basically f is equal to M by ms; so it is varying like this. So, this alpha means this side it lower temperature this side it is higher temperature. So, at lower temperature it goes towards the saturation.

And at this condition, will be at basically the tangent. At higher temperature and this it is this curve phase this straight line is for l by 3. So, this langevins function is becomes l by 3 under this condition that, one can show from the expression of this langevins function.

So, these the expression we got from the classical theory, langevins theory that is basically langevins theory is classical theory here. So, in this theory, what we have used that paramagnetic gas movements can rotate freely in the in the material. So, that is all the assumption.

And now we will see the quantum theory for paramagnetism. And in quantum theory whatever here this main difference between this classical concept and this quantum concept is that, this rotation of this movement dipole moment in space as consider for paramagnetic gas, it has it can rotate it can rotate in any direction. It can, there is no restriction over it is free to rotate in it is own way.

But in quantum theory it is considered that, it is considered that quantum theory. So, there is a restriction on this free rotation. So, that the restriction comes basically from the quantization of the angular momentum: quantization of the angular momentum in space right. So, if this total angular momentum j, the total angular momentum j of this dipole moment or this angular momentum and corresponding dipole moment of this, whatever dipole we are considering for in case of paramagnetic gas these atoms having the permanent dipole moment. So, that dipole moment is it is corresponding to it is for this total angular momentum j; so then this j value j in space about a particular direction.

So, it is quantized in such a way. So, it cannot take any direction, j cannot take any direction. It can take in space. It is quantized in space in such a way that the projection of this angular momentum on this particular direction it will be some integer value.

So, this we write this is mj this quantization quantum number magnetic material magnetic quantum number right mj. So, this will be such a way that mj will take value from plus j to minus j to minus j right. So, it will have 2 mj will take 2 j plus 1 number of value.

So, if mj equal to 2 M sorry j equal to 2. So, 5 mj will have 5 value. So, mj will be 2 plus 2 plus 1 0 minus 1 minus 2 right 5 value. So, that mj value is this projection of this j, j equal to 2 j equal to 2 projection on this direction it will be 0 projection of this j magnitude is same direction are different.

So, projection will be this is a plus 1. And this is another 1 plus 2. So, this will be minus 1 and this here projection here it will be minus 2. So; that means, here this j is directed along this direction along this direction this negative direction here it is positive direction. So, when equal to j equal to plus 2 or minus 2 this it is completely aligned.

This angular momentum this completely aligned along this this fixed direction. So, this fixed direction is generally we take when we apply magnetic field. So, this fixed direction is space this generally take the direction of the magnetic field. So, before applying magnetic field this magnetic moment of this atom, paramagnetic atom; so in space is it can orient only in this number 2 j plus 1. So, it cannot orient of infinite number means orient in all possible direction it is not possible. So, whatever considered for the classical theory.

So, in quantum theory mainly this is the quantization of this angular momentum in space that is considered and based on that this theory has been developed and got some similar result not same, but similar result so that I will discuss.

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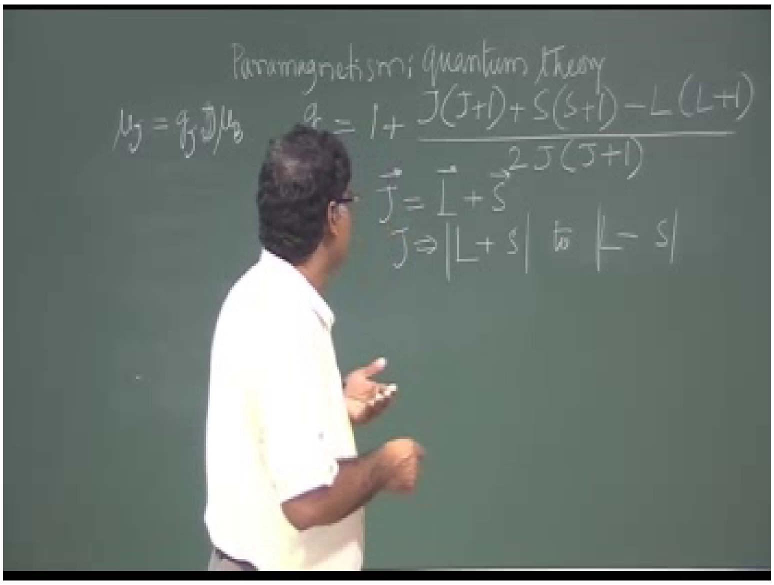

So, let me delete it if necessary I will write again; so paramagnetism basically quantum theory quantum theory. So here this that paramagnetic gas, we can take as a gas, but now this magnetic moment is not free to move in any direction in space. So, only it can take particular direction. So, that is specified by this mj, if that magnetic moment for magnetic moment for each atom magnetic moment for each atom is mu j say it is total angular momentum is j total angular momentum is j.

So, mu j is basically what is the value mu j, mu j is gj ga. I think I should write capital this total j I should write capital j. So, gjj mu b right gj gjj mu b mu b is Bohr magneton and j is the total angular momentum of this of this paramagnetic atom right. And gj, gj is lande g factor it is 1 plus j, j plus 1 plus s, s plus 1 minus l, l plus 1 divide by 2 j, j plus 1, where j, j equal to j equal to this L plus S right. Vectorial from vectorial model from vectorial sum one can get. So, it is magnitude basically varies j value varies. It have this value L plus s to L minus S. So, I think this I have discussed earlier; and how to get L total angular momentum orbital angular momentum, how to get L, if 2 electrons are there L 1 L 2.

So, similar way L 1 plus L 2 to L 1 minus L 2 that will be the total orbital angular momentum. So, if 3 L 1 L 2 L 3 are there. So, first you have to find out the resultant of L 1 and L 2, then what are the resultant you will get 2 3 values of l then with each one again you have to add this L 3. Similarly, for s one also s also one has to one has to find out this total spin angular momentum.

Now, this paramagnetic atom of paramagnetic gas this atoms are having this this magnetic moment permanent dipole moment right.

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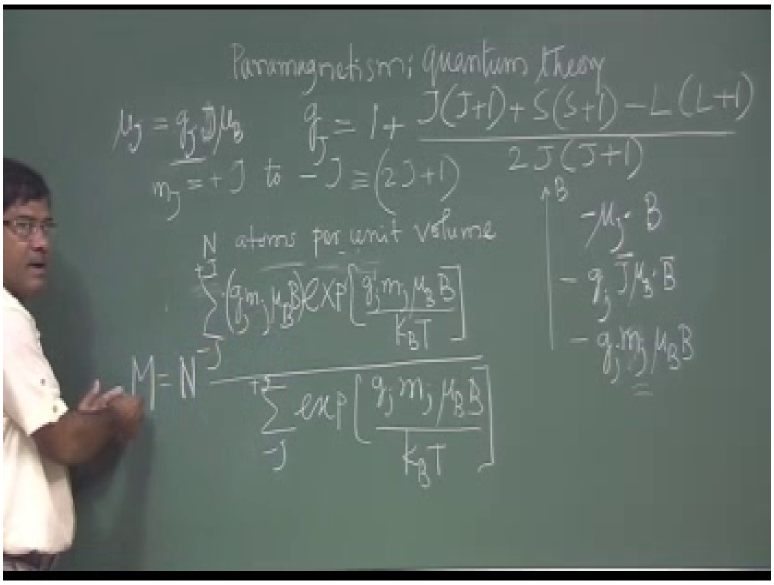

Now this magnetic moment will have orientation in space is mj, mj equal to plus j 2 minus j right. So, total 2 j plus 1 orientation is possible orientation is possible.

So, to find out the magnetic moment, what is magnetic moment? Of each atom one can find out. So, how one can find out? So, one can find out the average. So, if I consider, if I consider that N number of magnetic moment or atoms per unit volume per unit volume. So, when I will apply. So, when I want to find out this one?

So, magnetization if I want to find out magnetization. So, magnetization: now magnetic moment total dipole moment per unit volume along the magnetic field direction that is the definition of magnetization along the magnetic field direction. So, I have to apply magnetic field b. Now before applying magnetic field this was the decided states, because the j depending on depending on j this energy levels this atom will have energy levels right.

Now, before applying magnetic field each j value there will be 2 j plus 1 degenerate states. That is because of the space quantization. Now if I apply magnetic field. Now these space quantization this degenerative will be removed. So, energy levels will be what about degeneration energy levels will be splitted. So, this splitted this separation will depend on this lande g-factor, this also called splitting factor right. Spectroscopic splitting factor it decides the magnitude of the splitting of the energy levels on application of the magnetic field.

So, here will get 2 j plus 1 number of energy levels right. Now what will be the energy of those levels, that is basically have mu j if you have mu j. So, energy will be mu j dot b energy will be mu j dot b. So, minus sign this is basically potential energy. These states are going up or down. So, it is energy will change that is basically potential energy will change. So, that mu j dot b so that we can write mu j equal to this, so minus gj, j mu b dot b.

Now, this is basically j dot b: so the quantization- so quantization along this magnetic field direction; so this will be gj j- so this we basically write- so j dot b. So, along b direction what is the j component right. So, there j component is mj. So, we can write mj mu B, B right. So, these will be the potential energy or the splitting of the energy levels depending on the mj value for different mj value right; so will get different energy. So, that is basically the energy of different levels.

So, 2 j plus 1 number of energy levels have been different energy will get, having different energy will get gj mj mu b, b; so mj having 2 j plus 1 number. So, now, now this system will have. So, I have identical atoms right all atoms are having this mu j dipole moment right when it will have mu j dipole moment. So, there are 5 not 5 2 j plus 1 number of energy levels. So, these atoms will be distributed among this energy levels right. So, that is the Boltzmann statistics s will tell us that how they are distributed among this energy.

So, this energy specified by this magnetic field right. Because of magnetic field these energy levels. Now because of kinetic energy, because of thermal energy, so these atoms paramagnetic atoms how they will distributed among this energy levels. So, that is you know that spectral Boltzmann factor, so e minus exponential e minus energy by kT right. So, this is very important statistics factor which this E in this in our case what is this is the e energy right.

So, I can write exponential minus, minus plus gj mj mu B, B right by KBT; so for one atom for one moment, the factor probability to occupy a state right. So, I have total. So, and what will be the energy of this one; so this probability of that atom to have this one energy level. And what is the energy is gj minus gj m sorry yes mj mu B, B. So, that is the energy. And that is the probability.

So, now I have to take summation sum over minus j to plus j right; because mj value can take minus value to plus j. So, I have to take sum over. So, I will get basically, for all energy states average this energy I will get. So, then I have to divide by this total probability j to plus j plus j exponential gj mj mu B, B by KBT; so the total probability. So, I have divided by this. So, this will give me average dipole moment of this of the atom along the field direction right.

So, now my magnetization M will be equal to this is the average of one atom right. So, if I have N number of atoms per volume. So, I multiply with N. So, then that is the magnetization. So, magnetic moment total magnetic moment per unit volume along the magnetic direction. So, here we have considered this term has come to get the component mj along the magnetic field direction. And this temperature these factors are there right so this magnetization. So, that we have to evaluate. So, now, whether I should do the steps or I should leave for you well.

So, this here basically magnetization, I got now these the expression. Now if you consider 2 cases one is as we have consider earlier alpha is very greater than 1, and alpha is very greater less than 1. So, if we consider this term is very greater than this term, means no just opposite. This term is very less than this term means low field and high temperature.

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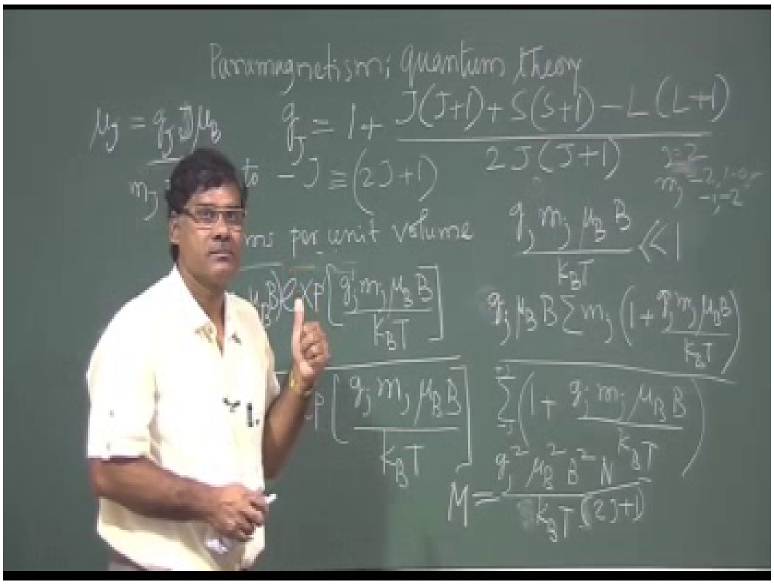

If you consider; that means, gj mj mu B, B this is by KBT if it is very less than 1. So, earlier this this type of things we have done. So, this will be this exponential term it will give you 1 plus x right, e to the power x, x is very less than 1. So, I can write 1 plus x this 1, 1 plus x. So, sum over minus j to j that is here. So, basically I can take I can take gj mu B. Because the j is basically it is variation of mj. So, just I have to keep mj inside.

So, gj mu b gj mu b I can take out from the summation, gj mu b even I can take b. And then sum over this. Here mj and here I am writing 1 plus x hence gj, mj mu B, B divide by KBT. Similarly, here I can write 1 plus sum over 1 plus gj mj mu B, B by KBT right.

So, from here I am getting this is there. So, one means sum over mj. This is one term this sum over mj plus this and here I will get mj square. You know here I will get mj square right. So, sum over mj is 0 because minus j equal to minus j to plus j right minus j to plus j. So, like you remember, if it is mj equal to j equal to 2 j equal to 2. So, mj will be minus j; so plus j to minus j, so 2 1 0 minus 0 minus 1 minus 2 right. So, summation sum over of mj will be 0.

So, this term mj, this term will not will be 0. So, other term I will get other term. So, I can write gj square mu b square right. And then I am getting b squar sum H value is here b square by KBT. And here what I am getting? This summation over mj, this part will be 0.

Now, sum over 1 minus j to plus j right; so sum over one this will give 2 j plus 1 this will give 2 j plus 1. So, below I will get 2 j, sorry below I will get 2 j plus 1. So, this will be the under this condition. So, I have to put N, I have to put N here. So, then this will be the M right. This will be the M. So one mistake, I have done I do not know where I have done.

So, it will not be b square it will be b; so yes. So, that mistake is here. So, here this it will not be b, this not what is this gj, gj mj mu b. So, here I am not finding out the energy right. I am finding out the average magnetic moment along the magnetic field direction right. So, average mu j basically average mu j along the field direction.

So, this is the gj j mu b. So, j I have replaced by mj because along the magnetic field direction mj mu b. So, this b will not be there, this b will not be if I put b, then I am finding out the average energy. So, it is not average energy because magnetization is the is not average energy is the average magnetic moment along the magnetic field direction. So, this basically mj mu j average mu j this we are finding out average mu j. So, that is why this b square has come. So, this b square will not be there. So, it will be b it will be b.

So, what I got? I got basically I can write here. So, under these assumption right.

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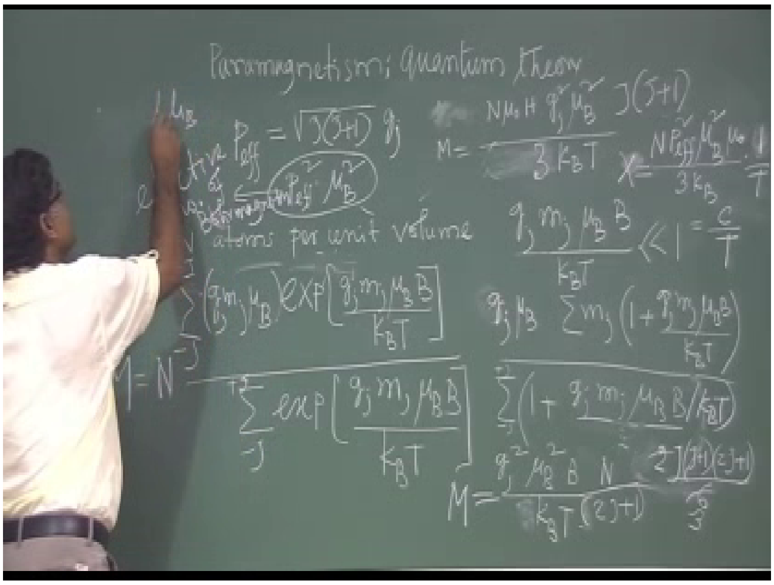

So, I got M equal to gj, I got N. This b I can write mu 0 H mu 0 H. And then I can write gj mu b gj square mu b square right. Divide by 2 j plus 1 KBT; so gj mu b.

So, I think I am little bit hurry. So, that is why I am doing mistake. So, from here what we have to find out this, this part 0 and this I have g gj square plus into mu b square into b right. Now and inverse there find, but here what I have not taken this summation of mj square; so here this term summation of mj square. So, summation of mj square it is basically mj equal to j to minus value 2 j plus 1 term. So, summation it will be j square plus j minus 1 whole square plus j minus 2 whole square right and so since it is square. So, minus j square is j square. So, it will be 2. So, this will give plus 2 into j square plus j minus 1 whole square plus j minus 2 whole square; so up to one term up to 1.

So, one square where it basically one square plus 2 square plus 3 square plus 4 square up to j square. So, this summation of this one, it gives j plus 1 2 j plus 1. So, this summation of this series will give j and 2 1 2 will be there that 2 is coming as I told this j square and this minus j square; so minus j square. So, this j square plus j square so that of for each term it will come 2. So, 2 sorry I think I should. So, this k bt I should write KBT here. So, these will give me j 2 into j, j plus 1 2 j plus 1, right.

So, these summations of the series divide by 6 here. So, this is the summation one square plus 2 square plus 3 square up to plus j square j, j plus 1 2 j plus 1 by 6. So, this will come basically 3. So, that part I have to take that part I have to take; so j, j plus 1 2 j plus 1. So, this 2 j plus 1 will go and 3 are there. So, 3 yes now I got the right one.

So, here P effective something I am writing P effective equal to square root of j, j plus 1 j j plus 1 and gj; now, P effective square mu b square. So, if you see earlier what was mu in case of this paramagnetic, this classical theory; so this is what about mu we took. So, now that is coming in terms of like this. So, from here we can see. So, this I can write N then P effective P effective mu square, mu b square mu b square. And then mu 0 divide by 3 KB, 1 by T. I am writing one by T.

So, now this I can write as a I can write as a C by T where C is this C is this. So, earlier whatever C was there. So, it is similar term where only difference this P effective. So, these called effective Bohr magnetron. This is called effective number of Bohr magnetron, b o h r Bohr magnetron right.

And this chi value will be H, I have not written here H I have not written here. So, H I have to take out this side. So, this I have to write chi equal to this M by is. So, this chi equal to this C by T. So, C is this where effective Bohr magnetron is there. Effective Bohr magnetron number of Bohr magnetron is this one. So, this is very important because it gives this effective number of Bohr magnetron, because you see this we write mu b right l mu b or ml mu b. So, either it was earlier it was multiple of mu b 1 mu b 2 mu b 3 mu b right. These are the magnetic moment of a of a dipole.

Now, it will not be till this that number is not like this now it is different. So, that number 1 2 3 whatever times of mu b. So, now, that number is this. So, that is call the effective Bohr number of Bohr magnetron. So, this at higher temperature lower field. So, you got the similar expression, where this is a curie law, but only some difference from classical theory whatever you got this similar expression you got.

But there it is some difference here effective Bohr magnetron. So, this where it gives very important information about the material paramagnetic material whether the dipole moment, in dipole moment this total angular momentum j contribute or only orbital angular momentum contribute or only spin magnetic spin angular momentum contribute.

So, these information can be get got from this from this expression, where as in case of in case of this classical theory that information. Directly it was missing, but in directly one can say from there also how about now explicitly, here we can get about the these basically will tell us that is whether this index coupling in that material is strong or not. Whether it is weak or it is not there only spin can contribute or only orbit can contribute or this total momentum can contribute.

So, I will stop here then I will continue in next class.

Thank you.