Solid State Physics
Prof. Amal Kumar Das
Department of Physics
Indian Institute of Technology, Kharagpur

Lecture - 61
Magnetic Property of Solid (Contd.)

We will continue our study on magnetic properties of solids, I have discussed about the origin of diamagnetism and that expression we got of diamagnetism that is mu.

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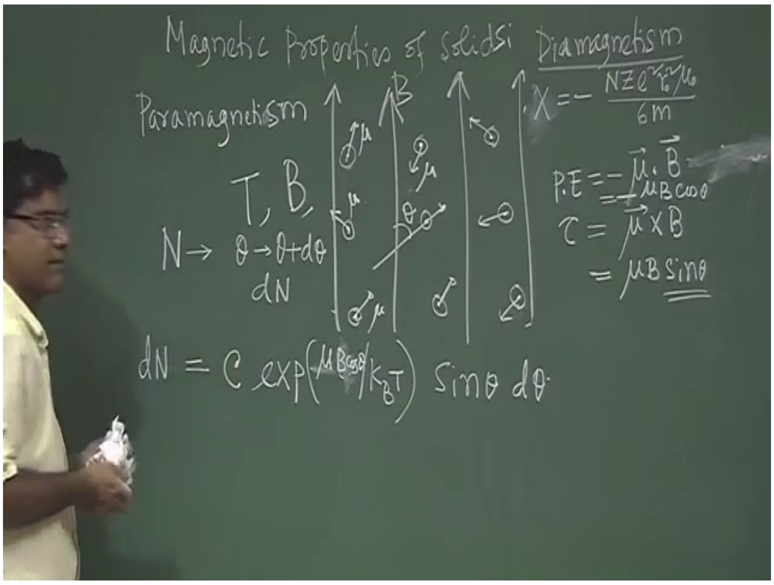

Let us write chi this chi equal to minus number of atoms per unit volume in each atom have atomic number z and then e square r 0 square electronic charge square then this is the radius of the atom as far as the radius of atom and then mu 0 divide by 6 m right mass of the electron, so next property next is the paramagnetic, ferromagnetic antiferromagnetic, ferrimagnetic.

These are the type of magnetism in solid which based on origin of this magnetism is basically origin of this is permanent dipole moment. So, permanent dipole moment mu it can be mu l, mu s, or mu j it depends on the value of l s j right, what if atom for atom it has permanent dipole moment that is based on total orbital angular momentum, total spin angular momentum and then from this to coupling of l s we get the total angular momentum that is j for that atom.

So, depending on this value we will get the magnetic moment for that atom, in general I am just writing mu it can be mu l mu s mu j that we will see that depends on the material which type of material and what are the atom, whether it is iron, it is cobalt or it is copper etcetera all right; however, the permanent dipole moment is there so, that material will show the property one of them.

We will discuss the paramagnetic material what is the reason of this paramagnetism that we will discus, dipole moment are there now depending the orientation of this dipole moment in response of magnetic field with field or without field based on that this para paramagnetism, ferromagnetism etcetera this comes right. Let us just concentrate on this paramagnetic property of solids obviously; these solids have the atoms sitting on lattice, this each atom is having the dipole moment they are randomly oriented.

Now Langevin basically, Langevin classical theory for paramagnetism, Langevin considered that this is a paramagnetic gas means in gas atoms are free to move in any direction they are free to move similarly here magnetic moment attest with each atom they are sitting at the lattice, but this moment can rotate freely in any direction. Thus it is called the paramagnetic gas all right there is no restriction on the rotation of the magnetic moment attest with the atom. This the assumption this the classical concept of the type of rotation of the magnetic moment and later on we will see that in case of quantum theory. So, this rotation it cannot be free it cannot rotate in all directions there will be restriction on this there will be it will be quantized.

That we will discuss later on let us discuss the classical theory, according to classical theory just it is assumed that this magnetic moment can rotate in any direction they are free to rotate like gas molecule or free to move anywhere in any direction. So, what will happen if I apply magnetic field in this material at a particular temperature if I apply magnetic field b at a particular temperature t the system is kept at a particular temperature then what will happen. So, this magnetic moment it has mu it is value is mu direction are different that is fine, but magnitude are same for the each moment.

So, you know that when it is under magnetic field you will it will gain it will have the Potential Energy equal to minus mu dot B and it will feel torque magnetic moment in a magnetic field it will feel torque and that is different by net mu cross B. If this mu if I take just one dipole moment, it makes angle theta with the direction of the magnetic field. Then this will be this I can write minus mu b cos theta probably cannot see I can write here.

Potential energy it will be minus mu b cos theta and this torque will be mu b sin theta, now my magnetic field magnitude direction are fixed say I have applied a particular magnetic field in a particular direction. Now this potential energy and torque it will depend on their orientation this theta see it varies with cos theta energy and this torque varies with sin theta right. It will be depending on this theta value or depending on the torque depending on the torque; this theta value will be decided. Magnetic fields apply torque on this dipole moment and dipole moments will try to orient along this magnetic field direction. So, from the above hand this system have some temperature because of this temperature it will try to disorder it, magnetic field is trying to take all the moment in it is own direction and, trying to order them in a in a at the direction of the magnetic field and on the other hand this thermal energy it will try to disorder it.

That decided by the Maxwell Boltzmann statistics Maxwell Boltzmann statistics, if your system, that is that factor is exponential minus E by K B T right, if system have energy E if something hold to it some this energy now this thermal energy will try to overcome it depending on this temperature now what will happen. This will decided the degree of disorderness due to temperature and this potential energy due to this magnetic field it will, depending on that it will decided the orderness of this magnetic moment. This E basically that minus, it will minus minus plus mu b cos theta.

This due to this thermal energy this is the factor which we will try to disorder this orientation, magnetic field applying torque and there the sin theta it will orientation will depend on the sin theta. If I have N number of moment or atoms per unit volume, out of N number of moment, how many will we oriented at angle theta to theta plus d theta in this range how many moment will be oriented in this range of angle d theta range of angle. So, starting from theta to theta plus this range this range in this range how many moment will be oriented, that will be definitely proportional to the, that say in this range you saw oriented out of this N d N number are oriented with this angle range d theta.

I can write, d N basically it will be proportional to due to magnetic field that is the this the factor sin theta and this due to thermal energy this is the another factor. These two factor competition between these 2 factor and then at equilibrium condition there will be some equilibrium at some angle it will be, this both factor are equally contribute, we will get the equilibrium and at that equilibrium condition, d N number of atoms or moments will be oriented at angle theta within these range d theta, I can write this proportional to this and then sin theta and then d theta right in this d theta range.

This proportional, I can take proportional at constant C, this the within these d theta range this the number of moments or atoms oriented, definitely orientation it can be, 0 to 180 degree. This theta can take value 0 to 180 degree, it can oriented along this or this opposite that is the limit, it is theta value can be 0 to 180. So, within this range all molecules all atoms or all moments will be there N number of moment's right.

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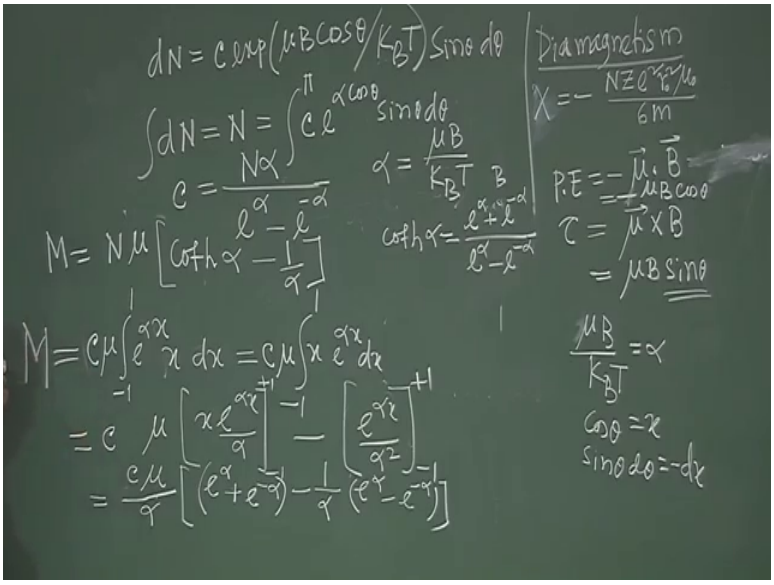

If I integrate the N right, that, d N equal to let me keep it because I need later on, this is exponential mu B cos theta by KBT sin theta d theta. Now, if I take all orientation 0 to pi orientation, this if I integrate what this angle, it has to be total number of atoms that is the condition and from that condition I can get the value of this C constant. This basically 0 to pi I have to integrate over this term.

I can take this mu let me write here, mu B by KBT that is equal to alpha, this I can write C e to the power alpha cos theta sin theta d theta, where alpha equals to this and then if I will take cos theta equal to x, then sin theta d theta equal to minus d x , cos to sin minus and then this N equal to C then I am getting x cos theta, this is 1 to minus 1 and exponential alpha x and then sin theta d theta is equal to minus x minus d x. So, negative sign is there, instead of putting negative sign I can put this minus and this is plus, this will give me C by alpha e to the power alpha x equal to plus 1 minus e to the power minus alpha.

This I had to put in bracket, what I am getting, I am getting the C value equal to N alpha by this, let me write it here I think that I need, ultimately what I got C value equal to N alpha divide by e to the power alpha minus e to the power minus alpha, where alpha equal to mu B by KBT, that is the c value constant under that condition that within all direction 0 to 5 direction all dipole moment will be there. Now, what I want to find out the magnetization along the direction of field, what will be the, this the mu, this the B, this the theta. So, component of this along this fill direction will be mu cos theta, each moment will have component along this field direction that is mu cos theta.

Contribution in magnetization along the field direction, that is the definition of the magnetization basically magnetic moment per unit volume along the in which direction along the magnetic field direction. So, along the field direction that is the magnetic moment, for which dipole now which are having the angle theta, how many dipole moments are at this angle now d N number at this angle within this d theta range right. So, each will have this, if I multiply d N with this mu cos theta, that will be the magnetic moment for this d N number of dipole moment along the field direction.

That means, here I have to multiply this, what is this value mu cos theta I have to multiply mu cos theta say additionally I have to put here mu cos theta. Now, I have to integrate for different theta, this again this I have to integrate 0 to pi, then all dipole moment in different angle, what is the component along this magnetic field direction that I will get, for all N atoms per unit volume in magnetic moment per unit volume so, that is nothing, but the magnetization, then this will be my magnetization.

I have to evaluate this integration, this is basically M, M equal to and, let us keep C this one and let me put that I had mu here also I can keep out mu, c mu, cos theta sin theta d theta and then here I will get, 0 to pi e to the power alpha cos theta and then sin theta or cos theta sin theta d theta.

Now again you take this cos theta equal to x, this will be x cos theta that will be x and then sin theta d theta will be minus d x, x value was here, minus it will be pi to 0, pi is minus 1, 0 is 1 as we have done earlier. This integration we have to do, this I replaced with d x right, integration by parts, if you take integration of parts, C mu and then integration of parts you know this I have to integration of u d v equal to u v minus integration of v d u right. If you know how to do that, basically x 1 has to consider as u x, this I think I should write before, there is a procedure for doing this.

So, x e to the power alpha x d x, this as if this u x is u and this is d v, integration by parts will give you C mu and then mu v x mu v, e to the power alpha x by alpha right and then minus 1 to plus 1 minus integration of minus 1 to plus b d u, b is alpha x by alpha d u is d x and this will give me alpha square if I do integration of this alpha square and then this will minus 1 to plus 1. So, what you will get, you will get C mu I can take alpha common, I think C it is divide by alpha and then what I am getting here, I am getting e to the power alpha and then e to the power minus alpha and x is minus 1, I will get plus e to the power minus alpha right alpha I had taken out.

This will give me this value and this minus here I will get 1 by alpha because this 1 alpha I have taken out, e to the power alpha minus e to the power minus alpha. If I put value of C N alpha, by alpha, alpha will go, c if I put, n mu will be there, if I put value of c what I will get N mu this alpha will go right and then divide by this, this will go it will be 1, minus 1 alpha. So, what I will get N mu minus 1 alpha this last one and here I will get e to the power alpha plus e to the power minus alpha divide by e to the power alpha minus e to the power minus alpha. So, this is nothing, but cot hyperbolic alpha, where cot hyperbolic basically alpha equal to e to the power alpha plus e to the power minus alpha divided by e to the power alpha minus e to the power minus alpha right, this the magnetization for that.

This is the magnetization of the paramagnetic material or paramagnetic gas whatever we have considered, here you see now this alpha content each row B, this M we write M equal to and N mu.

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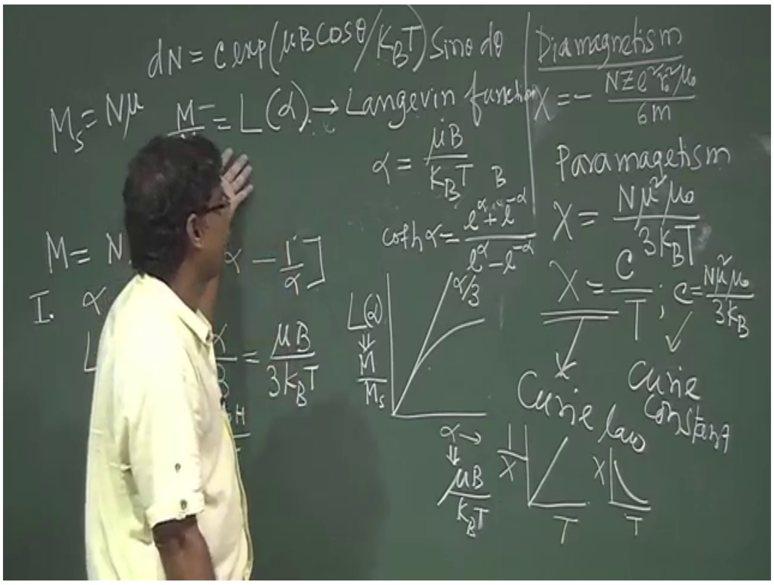

What is N mu I can write M s saturation magnetization because all N number of moments are aligned in the magnetic field direction this the N mu, M s equal to N mu and this yes this I do not need hopefully. M I am getting, M by M s from here I am getting I am writing L alpha this called the Langevin function.

If you take approximation, Langevin function, here if I draw the curve this L alpha it generally varies like this and this is a function of alpha and this is nothing, but this equal to M by M s, it is a, magnetization basically magnetization is will vary like this. So, this the what is alpha is mu B by KBT now if I take if I consider this case 1 is that it is magnetic field is low and temperature is high at higher temperature and lower magnetic field, alpha will be very very less than 1 right it is less than one means B is small it is higher temperature lower field under this approximation one can show that this langevin function will gives us a approximately it will gives us alpha by 3, you can calculate just take the series e to the power alpha e to the power x is 1 plus x plus x square by factorial 2 right plus l cube by factor l 3.

So, just take and put here and you will find out that this under this condition it will be alpha by 3, this calculation you should do this very simple just put the this value of the alpha e to the power alpha and you will get it. So, mu is what is that mu B, mu B by KBT and this 3 is there, under this condition M I am getting m I will get N mu so; that means, N mu square B is mu 0 H by 3 KBT. This 3 KBT, from here one can find out chi equal to for one can find out for paramagnetic material paramagnetism.

So, you will get chi equal to N by H that will give you N number of moment or atoms per unit volume mu square the permanent dipole moment is mu then mu 0 divide by 3 KBT. Now here you can see this the susceptible at higher temperature and lower field, there it is chi will vary it is not independent of temperature it depends on temperature, how it depends is chi equal to it is proportional to one by T inversely proportional to one by T and that we write some constant C where C equal to N mu square mu 0 divide by, this is permeability absolute permeability for here right here to 3 KB.

This C is called curie this is called the curie constant because dipole moment of each atom dipole moment attest with each atom that is constant right for a particular type of atom N number of that density of the material that is also constant it does not depend on field or other things right KB everything here constant right. So, that C is constant this is called curie constant and this x equal to C by T that is called the curie law of paramagnetic material. It is varies like this, at higher temperature basically it is the alpha by 3 it is varies with alpha by 3.

This M s know M by M s equal to alpha by 3, it is the straight this the curve for alpha by 3 right, paramagnetism it varies following this curve right, if you plot this chi 1 by chi by T, you will get straight line. So, this is the variation of the inverse of chi with temperature in case of paramagnetic material or if you plot chi by T, chi versus T, that is this is like hyperbolic kind of things, this will be hyperbolic this variation will be like this.

If you get this type of features of susceptibility, then it is this metal is paramagnetic, I think I will stop here this based on classical theory of the classical Langevin theory. So, same thing also I think in next class I will do for the even the quantum theory, then we will get the I will show that get the similar result, in case of diamagnetism also one can apply this quantum theory, but you will get exactly the same result. So, that is why I have not done that one, but for this I will do for quantum theory also, that is very interesting we get the similar, but in modified ways, so that is the that gives more information about the about the paramagnetism of the of the material, I will discuss in the in the next class.

Thank you for your attention.