Solid State Physics
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Lecture - 55
Thermal Properties of Solid (Contd.)

So, we are discussing about the Specific heat of solid.

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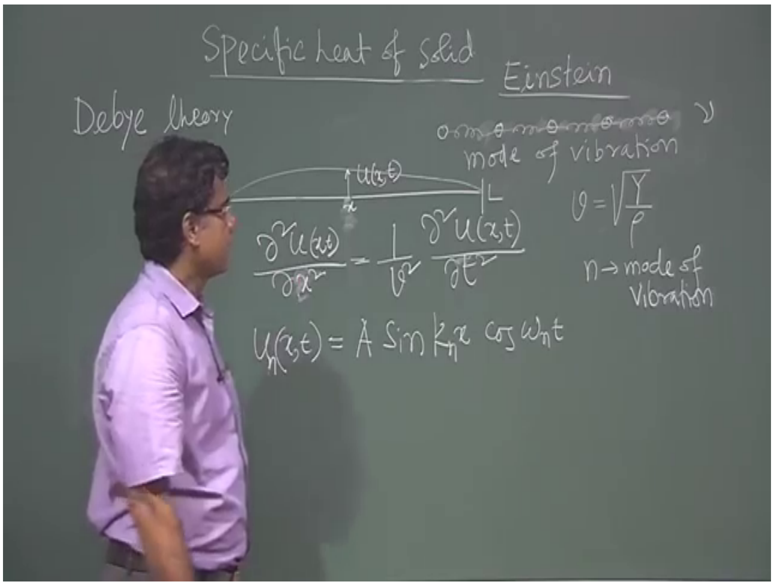

So, at height at high temperature it is nicely explained by dual embedded law as well as by young Einstein theory, but problem is to explain the lower temperature behavior of Cv specific heat. So, actually experimentally it is found that it is that Cv is basically proportional to T cube.

But in case of Einstein theory that we have seen that in case of Einstein in case of Einstein theory we have seen that Cv at low temperature it is proportional to. So, it is a exponentially dependent 1 by T square term is there, but it at low temperature towards 0. So, this expression terms dominate.

So, then Debye came forward to explain the low temperature behavior of specific heat. So, that is we tell this Debye theory. So, in case of Delonge petit law there this atoms in a crystal that was considered as a like gas particle. So, it can take energy from 0 to infinity and then that way it was treated and got the radiant which is only explained the high temperature behavior of specific heat.

Then Einstein what he assumed Einstein assumed that atoms in pistol it is not behaving like gas particle it is a, it is a, it is a situated at the at the lattice point or for lattice point itself is it is still can vibrate. So, as if in each lattice point there are atoms are there. So, in this case I Einstein concept that each lattice point or atoms vibrate with respect to it is equilibrium position.

But there is a each of vibrate each a vibrate and they vibrate independently they vibrate with same frequency like harmonic oscillator they vibrate with same frequency say nu as if this harmonic oscillator is oscillating vibrating and. So, they are. So, lattice points are atoms in the solid they are they are independent. So, they independently oscillate. So, that was the assumption of Einstein and he calculated the total energy of the system and from that he found the Cv which depends on exponential term at very low temperature.

So, then Debye realized that no this is not the correct assumption because lattice points are not they cannot they cannot oscillate independently, they are coupled with each other they are coupled with each other. So, like as if they are coupled with each other. So, like mass spring system.

So, they oscillate or vibrate as a whole a lattice as a whole they vibrate or they oscillate. So, that that was the assumption of Debye and they do not vibrate with just 1 frequency they may vibrate with many frequencies. So, each vibration as a whole each vibration is expressed in terms of mode of vibration.

So, each mode will have frequency so for different mode. So, mode is differentiated with in terms of their frequency. So, different mode will have different frequency. So, further we assumed further we assume that this basically elastic this vibration of the lattice it will generate elastic waves or elastic or in elastic wave will propagate through the lattice through the crystal. So, this lattice as a whole it will vibrates. So, it is vice versa basically.

So, he assumed that this wavelength of elastic waves generally it is higher than the lattice constant or distance between 2 lattice points and that is very small compared to the wave wavelength of the elastic wave. So, when elastic wave will propagate through this crystal. So, it will it will see this crystal as a continuum medium in it will not see the discreteness of the lattice points have atoms in the crystal.

So, he assumed that this elastic wave will see the system as a continuum system. So, this instead of this discrete area of lattice points he considered that this in case of elastic wave it is a it is a continuous medium. So, a 1 dimensional lattice array we can replace with a string. So, he considered basically string a wire under tension a wire under tension the string, now this steams such string is whatever crystal as whole this crystal is vibrating that so 1 dimensional crystal is vibrating as a whole. So, that is considered as if this string is vibrating.

So, we are I think you are quite familiar with the string vibration, then it is it can expressed in terms of wave equation right. So, this say 0 and this length of this string is L. So, at a point x at the point x if you disturb it by if you disturb it by ux as a function of u as a function of t that is the just displacement say these are just displacement of the of the string disturbance of the string at point x.

So, this disturbance this is uxd. So, u as a function of x so disturbance we express at any points on the string and at any time t. So, if u is the disturbance of the string. So, 1 can; 1 can set differential equation of differential equation or wave equation or string vibration. So, this that is well known wave equation that is valid this equation is valid for any o f it can be electromagnetic wave it can be sound wave. So, sound wave is basically elastic wave.

So, d 2 u this function of xt . So, dt square sorry dx square equal to 1 by d square, d 2 u dxt by d 2 dt square; so this wave equation. So, v is the velocity of wave v is the velocity wave velocity of wave through the string through the string. So v generally v, generally square root of Y by rho.

So, Y is the young modulus of this medium and rho is the density see in case of string. So, mass per unit length in case of three dimensional material. So, this mass per unit volume it is taken mass per unit volume. So, this is the wave equation now if this string is if this string is fixed at a at one end then if just vibrate. So, then you will get this basically traveling wave the solution will be basically can be the traveling wave, but when this both end are fixed then this wave will be confined between these 2 end.

So, basically this wave will reflect from this other side. So, when come back here. So, it is reflected. So, they will form standing wave or stationary with. So, does this for this equation wave equation, so under our boundary condition, so this solution will be stationary or standing wave. So, this solution you know standing wave solution this x is a function of i. So, this I am writing u n equal to A sin knx cos omega nt.

So, n is basically mode of vibration n is basically mode of vibration mode of vibration. So, this it will vibrate with different mode. So, that that mode is basically by n n equal to 1 2 3 etcetera that we will see later on.

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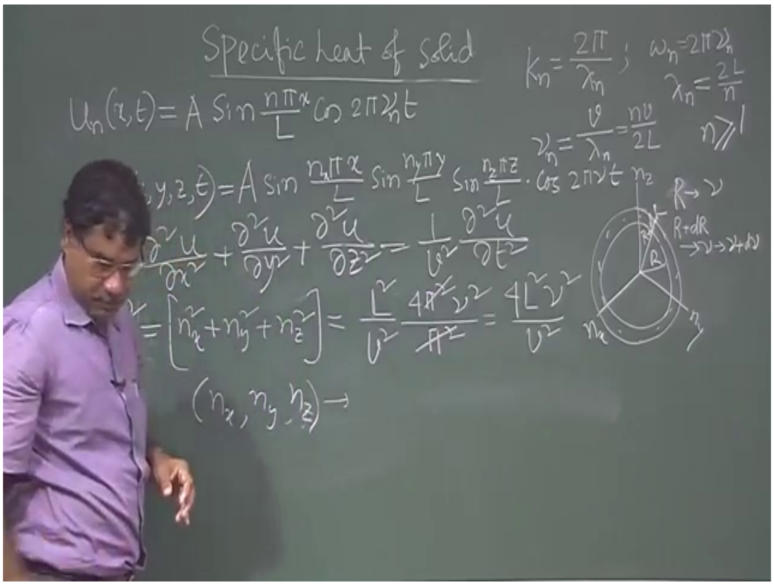

So, where this Kn is basically 2 pi by lambda n 2 pi by lambda n is the wavelength of nth mode and omega n is omega n is equal to 2 pi nu 2 pi nu n n is frequency and omega n is the angular frequency of nth mode.

So, this solution has to satisfy this boundary condition because at x equal to L this all the time this displacement has to be 0 means u n or nth mode x equal l and at any time t this has to be 0. So, when it will be 0 this cos omega nt for any time it cannot be 0, a cannot be 0 then all the time it will be 0. So, only this part has to be 0 sin kn x has to be 0.

So, sin this condition is telling sin knx has to be 0, when x equal to L when x equal to L. So, this has to be 0. So, we actually from here kn. So, k knl it has to be n pi it has to be n pi sin n pi is l 0 n equal to n equal to 0 n equal to 1 n equal 2 etcetera, but later on I will show you that n cannot be 0 it has to be greater than equal to 1. So, that we will see so condition this boundary condition is giving us that knL has to be equal to n pi.

So, this kn is 2 pi by lambda n kn is 2 pi by lambda n into L equal to n pi. So, from here you are getting lambda n equal to lambda n equal to lambda n. So, pi will go pi will go. So, lambda n will be equal to 2 L by n it will be 2 L by n right. So, here if n equal to 0 then lambda n is infinity. So, that is not allowed lambda n lambda has to be within this length no it is not correct.

So, here it is lambda n will be in infinity. So, that has no meaning. So, that does we avoid this n equal to 0. So, we consider that n will be greater than equal to 1. So, then we will see that this omega n equal to 2 pi nu n. So, basically nu n equal to nu n equal to velocity by velocity by wavelength this lambda n.

So, this velocity by wavelength so lambda n equal to lambda n equal to L by n, so 2 L by n so it will be nv by 2 L. So, basically here you can see this wave length n equal to 1 wavelength lambda 1 it is 2 L and nu n it will be v by 2 L. So, for first mode for first mode lambda 1 nu 1 for second mode n equal to 2 it will lambda n lambda 2 will be L and nu n means nu 2 will be 2 v by 2 L means p by L.

So, we get we will get different mode of vibration having different frequency and wave levels. So, here this length is fixed as well as this velocity is constant because it depends on as I showed that n modulus of the material and the density of this material. So, v is fixed v is constant. So, velocity of waves in the in the string is constant.

So, ultimately what we are getting. So, I think I mean leave it the solution we are getting u n function of xt equal to function of xt equal to a sin n pi by n pi by this kn, what is the kn value kn value is 2 pi by lambda n lambda n is 2 ln. So, n pi by L right n pi by L n pi by L cos 2 pi nu n t and here I have to put x.

So, this for 1 dimensional case this is the wave function for nth mode. So, from here you can see basically in this case you will get in this case you will get frequency as a function of frequency you will get equidistant you will get equidistant frequency for different mode for different mode. So, one can find out density of density of modes density of mode z nu density of modes.

So, one can one can find out. So, that is basically dn by d nu density of states it will be it is basically dn by number of modes will be in the range of frequency d nu. So, if you get density of states at different frequencies and for each frequency what is the average energy average energy of the mode of the mode then you will get you will get the energy for that mode energy for that mode and then you can integrate for almost.

So, let us do it for three dimension because our system is lattice the crystal is three dimensional object. So, this wave function whatever we got. So, that can be this for 1 dimension that can be extended for three dimension, that can be extended for three dimension. So, like then u say u you can write n or x. So, n I will I will avoid now so u as a function of xyzt so in three dimension.

So, this function will be standing wave and this 1 can write a sin n pi x by L. So, this I am putting in this x in x or along this x x axis, then sin n y pi y by L, then sin n z pi z by L and then your cos 2 pi nu t. So, that will be a solution for three dimension for three dimension.

And if you so this solution power of which wave equation the three dimensional wave equation that is basically d 2 u as a function of this xyzt, d 2 u by dx square plus d 2 u by d dy square, plus d 2 u by dz square equal to 1 by v square, d 2 u by dt square. So, that is the wave equation for three dimension and corresponding solution is this.

So, now if you put this solution here it has to satisfy this equation then only it will be our solution. So, here we have considered that this material is a this crystal the solid we have taken as a cube as a cube. So, inside length is L and there this the phase of the cubes is fixed of 1 dimensional string. So, both end wire fixed. So, in under that condition we got this solution.

Similarly, similar solution we have consider for three dimension. So, condition has to be similar so here this. So, this all phases of the cube all phases of the cube are fixed. So, under that condition this will be the solution and this has to satisfy this equation see if we put if you put here. So, what will get. So, n x pi L square you will get twice differentiation you have to do.

So, basically you will get n x square pi square L square n y square pi square L square right equal to. So, here you will get n x square take common pi square by l square, plus n y square, plus n z square, keep l 2 other side one by v square twice differentiation if we do 2 4 pi square nu square here. So, if I take this pi square by L square this side. So, it will be L square by pi square right to be L square by pi square. So, pi square will go.

So, ultimately you will get 4 L square nu square by v square. So, basically what you are getting you are getting n x square plus n y square plus n z square equal to this here L is constant or for cube for a particular mode nu is fixed for a particular mode nu is fixed and velocity is constant in this medium.

So, basically for a particular mode so this square of this n x square plus n y square plus n z square equal to constant equal to constant for a particular mode of having frequency nu. So, for different nu basically for different nu value for means for different mode you will get different sets of value of n x n y and nz.

So, basically n x, n y, n z a set value of a set of these will represent a will represent a mode will represent a mode under this constant under this constant. So, this I can write as a R square. So, in this n x, n y, n z axis system; so you will get a basically sphere of radius R you will get a sphere of radius R.

This radius R for a particular frequency R R for a particular frequency R so between R plus Dr. So, we will get those mode in the range of nu 2 nu 2 nu plus d nu nu 2 nu plus d nu. So, in this frequency range will get number of modes will get number of modes within this spherical shell of radius this is r and this radius is R plus dR.

So, this sphere and with these thickness of the sphere spherical shell thickness dR. So, here whatever the points each point will represent a mode having frequency nu 2 nu plus v nu. So, how many number of steps modes will be there. So, that one can find out that one can find out. So, what is the volume of this spherical shell of this shell? So, this volume of this shell I can I think I do not need this anymore.

So, some volume of this of this shell will be 4 pi R square that is the surface area of this surface area of the sphere and yes and then volume will be into dR into dR.

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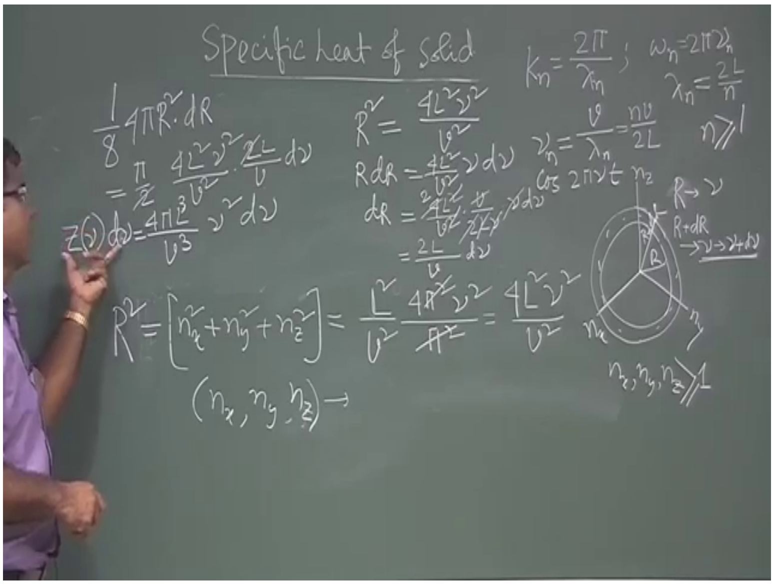

So, that will be the volume of this. So, in this volume, we will get the total number of modes having the frequency between in this range. So, basically n x n y n z like n is greater than equal to 1 similarly n x n y n z n x n y n z is also 1 has to use value n x n y n z nz it is also greater than equal to 1.

So, if you see this sphere it has 8 octet it has 8 part. So, only 1 part 1 of this 8 8 part this that will be in the what I should tell that that will be that part will have only the positive n x n y n z value right. So, you have axis system right x y z see if you consider a sphere; so only this part, positive n x positive n y positive n z. So, only this part will have the positive n x n y n z value right. So, in this half x n x will be negative, in this half n y will be negative n x will be negative in this half n y will be negative.

So, similarly if you go below all will be negative; 7 7 part will have the negative value 1 or for both or 3 will have negative value only all will have positive value in this part. So, that is why the steps will get number of total steps will give this 1 8 of this 1. So, in the frequency range of nu 2 nu plus d nu.

So, here what we have that. So, I have to, this R basically it is a it is a related with the nu. So, R square what we have taken R square equal to 4 L square, 4 L square, nu square by v square. So, between R 2 R plus dR. So, that is corresponds to between nu 2 nu plus d nu right. So, differentiated R dR you get equal to 4 L square by nu square d square nu d nu nu d nu.

So, from here you can find out the dR equal to 4 L square d square and divided by 1 by r. So, R is basically 2 pi nu by v. So, 1 by R it will be v 2 L nu and then your nu d nu is there. So, this nu nu will go. So, here I will get 4 that was 1 by R 1 by R. So, R is this 2 pi nu into v. So, here will get this v this square will go, this L this square will go this 2 it will be 2. So, I will get 2 L by v 2 L by v d nu 2 L by v d nu.

So, this is representing basically density of density of states as a function of. So, let me. So, this will be equal to this will be equal to or pi by 2 pi by 2 R square is that 1 4 l square nu square by v square and dR is I found 2 L, 2 L, by v d nu. So, from here you are getting what you are getting this 2 2 will go I will get 4 pi 4 pi L cube 4 pi L cube 4 pi L cube 4 pi L cube then I am getting by v cube velocity cube nu square d nu.

So, this basically density of states z nu d nu in the in the range of frequency d nu from nu to nu plus d nu in that range, total it is total number of total number of mode in that total number of mode in this volume will be this will be this all right this frequency range varies from nu to nu plus d nu. So, that will be the total mode between these 2 frequency range.

So, if we express this z is a z is a density z is a density of modes. So, then within this range what will be the total what will be the total number of modes. So, that one can write, this one can write this.

So, I think here, I will stop here and then I will continue the calculation in next class.

Thank you for your attention.