Solid State Physics
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Lecture - 54
Thermal Properties of Solid (Contd.)

So, we will continue our discussion about specific heat, which depend on the temperature especially at lower temperatures. So, how to explain Delong petit law could not explain.

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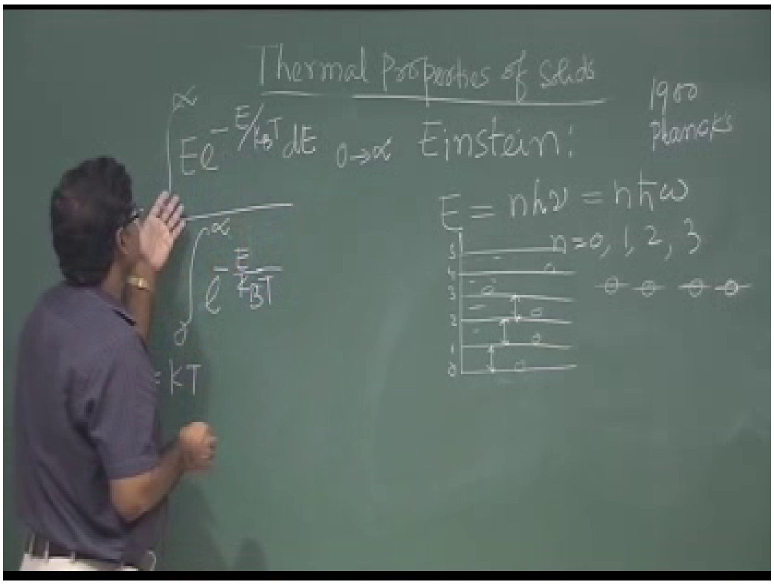

So, then Einstein came forward and he tried to calculate, he tried to calculate the specific heat for a for a solid. And he found that in this Delong petit law here. So, what we have considered that this is, this solid this lattice are there atoms are there. So, they behave like a harmonic oscillator. So, they oscillate vibrate and they are independent. They do not talk to each other and they can take any value 0 to they can take any value of energy 0 to infinity they can take any value.

So, but Einstein, he considered the similar model that the solid is made of atoms lattice and this each lattice point or atoms behave like a harmonic oscillator. And he considered that they oscillate, they vibrate independently. They do not they are not connected with the nearest neighbour. So, they can vibrate independently. And then only he considered that following the Planck's discovery that was the turning point of that is the beginning of the quantum mechanics right. So, these Planck's considered when he was trying to explain this blackbody radiation. Blackbody radiation and he considered that that oscillator harmonic oscillator it cannot take any value, between any value between 0 to infinity.

So, it can have only discrete it can take only discrete energy. So, and he tell that that E equal to basically energy of the oscillator it will be controlled by this relation, n h nu or n h cross omega; so harmonic oscillator. So, they will oscillate with this same frequency and, but they cannot take any energy. They can take only energy that is n equal to 0 1 2 3 so; that means, that that systems harmonic oscillator that systems are in a are having the systems are having discrete energy level discrete energy level. So, one can take tell this vibrational energy level if they vibrate. So, they can take. So, they are equally spaced they are equally spaced by h nu. This is 0 1 1 2 3 4 5.

So, this oscillator can have only this value not any other value like in between they cannot take. So, that is the additional criteria or assumption Einstein made over this Delong petit law. Whatever Delong petit considered the model is he considers same model. He considered same model, but only they considered that energy can vary 0 to infinity, but here Einstein considered that energy varies like this. So, this discreteness: so in 1900 this Planck or 1901, so, this Planck constant; Planck's distribution of energy. So, that is in that time.

So, this was the for oscillator harmonic oscillator. This was the suggestion came from the Planck's. And this was a great success to explain many things like this black body revision, photoelectric effect right. And then here also for specific heat so that can concept was taken by Einstein and try to explain these thermal properties. That is dependence temperature dependence of specific heat right.

So, what happens? So, this was the classical model right. Dulong petit used this one. Now in case of when Einstein considered this energy; so average energy. So, this one can calculate the same way, but it was concept this energy distribution that was cons that concept was this classical concept. Maxwell Boltzmann statistics was used and Einstein again, he argued that this oscillator fixed in the in space oscillators and with respect to this fixed position they are oscillating.

Now they have location they are sitting at a particular position. So, this oscillator can be identified. Each oscillator can be identified. So, they are distinguishable. Since, they are distinguishable so one can use classical statistics Maxwell Boltzmann statistics. And there is the difference you see classical and this quantum. In case of quantum statistics, they are these particles are indistinguishable. Like in electrons free electron theory there we have considered Fermi Dirac statistics because there you cannot identify electrons. So, all electrons are same, similar here all oscillators also similar. One can apply the others quantum statistics. Because we may considered that they are oscillating identically they are we cannot distinguish them.

So, that was not considered by Einstein. Einstein argued that since they have location, they are located in the particular position with respect to that it is oscillating. So, we can identify each oscillator. So, they are distinguishable. So, classical statistics is applicable. So, that is why they he considered the Maxwell Boltzmann statistics. Means this is the distinction, this distribution was considered. But that means, he used quantum oscillator means energy cannot take any value. So, it will take discrete energy value. So, that is why it is quantum oscillator, but he follows the classical statistics. So, that is what the Einstein took into consideration.

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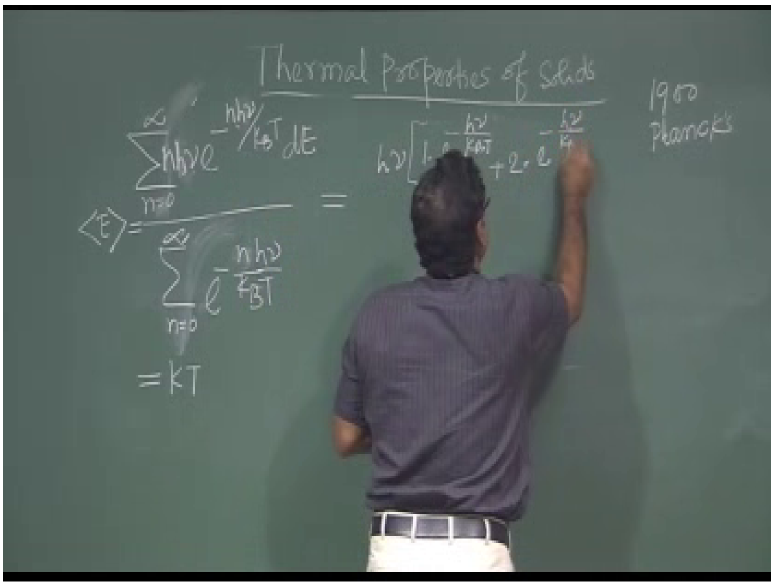

So that means, this sets these we can just replace, these we can just replace with this continuous integration. Just we can replace with the summation. So, this E is basically is h n nu. And this will be n h nu. And here it will nh nu. And this integration will be replaced by summation. So, summation this for n, n equal to 0 to infinity so that is what considered same for a energy one can find out from here. And he considered that this n number of particles is there. So, they are oscillating. So, it has 3. So, in 3 dimension means it each particle we will have 3 degrees of freedom. So he considered equivalent one dimensional oscillator. So, that is number is 3 n.

So, he considered 3 n number of one dimensional oscillator, but solid is 3 dimensional. So, it can oscillate along this, it can oscillate along this, it can oscillate along this. So we can make it into 3 one dimensional oscillators. So, that is what he did. So, this is whatever here we are getting energy of each for each one dimensional oscillators. So, this we have to evaluate now. So, where how to do that? So, this you can do. Just take I think what we can do. So, this we can write. So, m equal to 0 to infinity. So, for n equal to 0 this will be this term will be 0, right? For n equal to 1, I will get n equal to 1.

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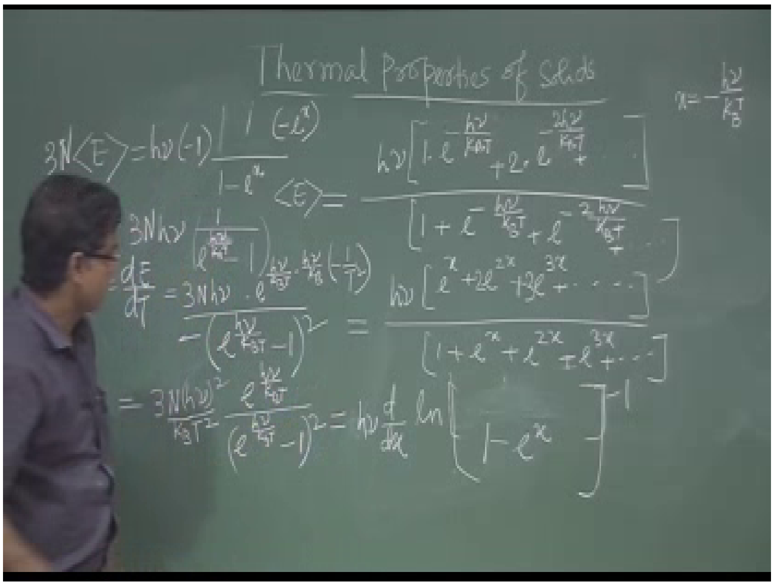

So, h nu; I can take out h nu. I can take out. And then I can write term 1 into e to the power minus h nu by kt kbt right; plus 2 into e to the power minus h nu by kbt plus etcetera. So, infinite series divided by here what I will get? I will get for n equal to 0 this is 1. So, I will get 1 plus e to the power minus h nu by kbt plus e to the power minus 2 h nu by kbt. So, if I replace h nu by kbt. So, if I take x equal to minus h nu by kbt. So, I can rise write this series as a h nu h nu, then this e to the power x plus e to the 2 e to the power 2 e to the power 2 x.

So, here it has to be 2 plus e plus 3 e to the power 3 x divided by divided by, I can take 1 plus 2 to the power x plus e to the power 2 x plus e to the power 3 x right. So this in principle I can write like d by dx, d by dx h nu I can keep here. So, this I can write d by dx log 1 plus e to the power x plus e to the power 2 x. So, log x. So, if I differentiate it will be 1 by x right? It will be 1 by x right; so n d by dx of x. Now whatever inside is there right. So I will get this term 1 by x, 1 by this, 1 by log n.

So, differentiate 1 by this. So, I will get this denominator. And then this dx will apply on this. So, I will get here it is e to the power x. So, it will remain e to the power x, then here e to the power 2 x. It will be there and then 2 x it will give 2; so 2 e to the 2 x. So, I will get this term. So, this in principle I can write like this. This you can express this way and this is a series, infinite series. And this will give you just if it do it. So, this will give you 1 minus x if x is. So, here I think we have to know is, it is 1 minus e to the power x. It gives basically 1 by 1 minus e to the power x right.

So, that series will give this. Now if you differentiate it with respect to x. So, what we will get; so, this basically this again ln. So, this I can write this minus 1 right? Minus 1. So, this if I differentiate, I think I can write this side; so this average energy of each one dimensional oscillator. So, this I will get E equal to h nu. And then if I saw this, ln this minus 1 means minus 1 may come. So, minus 1 this minus 1 will come; so ln of this one. So, what we will get 1 by x so 1 by this. 1 by this, I will get 1 by this 1 by 1 minus. And then if I differentiate this part.

So, it will give minus e to the power x, it will give minus e to the power x right. So, this term is like this. So, it is minus, minus plus h nu, e to the power x. So, this equivalently I can write 1 by e to the power x if I take down. So, it will give me 1 minus x minus 1 right. So, this is my energy. This is the energy of one dimensional oscillator average energy of one dimensional oscillator. Now I have to find out total energy. So, total energy E will be so multiplied by 3 n right. 3 n number of 1 minus dimensional. So, E will be n 3 n 3 n. So, now Cv will be de by dt at constant dt is there on that.

So, this now, I can write in terms of x, I can take out and put here. So, that is the x E here it was minus. So, there is minus, minus plus. So, it will be plus, so e to the power h nu by kbt, right. So, T is there here. So, I have to differentiate it. So, what I will get? I will get 3 n h nu that is there. And here I have to differentiate. So, this part I have to put under the d by dt. So, T is there. So, these like 1 by x right. It is 1 by so whatever this function of T right. So, if we differentiate it. So, it is always a minus 1 by x square kind of things. So, this will give me minus 1 by. So, minus will come here.

So, e to the power h nu by kbt minus 1 whole square and 1 minus sign is there. So, this I can put here anyway. So, I will put now this part is done. 1 by is done. Now these parts differentiate with respect to tdt, T dt. So, it will give me e to power h nu by kbt and then this part. So, again here 1 by T is there. So, it will give me h nu by kb and then minus 1 by T square right. So, if you just differentiate. So, this you will get if you differentiate this you will get. So, ultimately what I am getting. So, minus, minus it will go.

So, I am getting 3 n h, h nu square h nu square by kb right? T square and then I am getting e to the power h nu by kbt. This one divided by h nu by kbt minus 1 whole square. So, I do not need this part. So, this will be the Cv. So, this I can write here. I can multiply with 1 kb.

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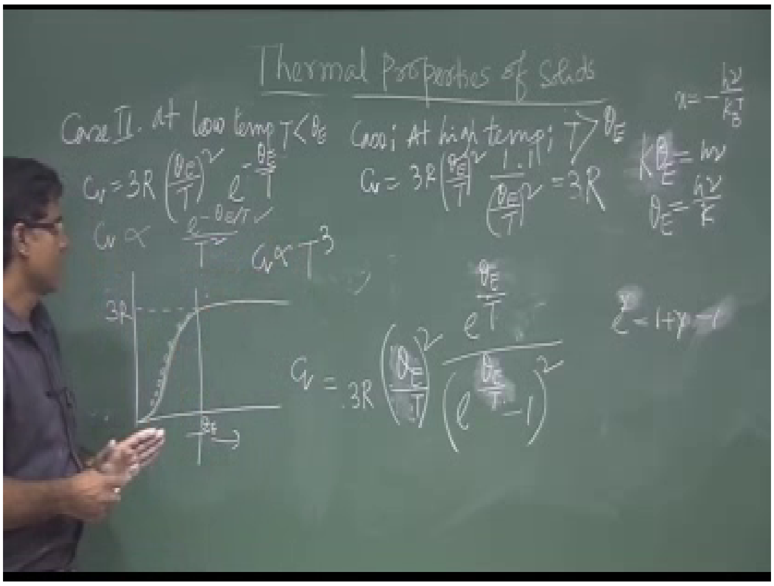

So, this will be kb square and if I multiply with one kb. So, n kb it is R. So, I can write basically this will be 3 R, and then h nu by kbt whole square right. And then e to the power h nu by kbt divided by e to the power h nu by kbt minus 1 whole square.

Now, if I take 2 part one is for low temperature and this is for high temperature. So, case one, if it is we take case one, this at high temperature. So, basically in physics in science there is no things like high temperature, low temperature. Always you need some reference. So, low temperature means below of hot temperature, I will call it low temperature. Below of hot temperatures above of hot temperature, I will tell this high temperature. So, we need all these reference otherwise there is the meaning of high temperature low temperature so that one can define. So, if I defined here so h nu by kb.

So, that is the temperature. So, this that k the T equal to h nu kt equal to h nu. So, this is the temperature is such a temperature, if I defined by theta E Einstein temperatures kt. So, k theta E equal to has will be equal to h nu. So, then this theta E is my reference temperature h nu by k. So, this is called Einstein temperature; so the temperature below. So, temperature here at high temperature means temperature is greater than theta E. So, then we tell it is high temperature. So, theta is defined by like this. So, then here I can write here this one, theta E by d square.

Here I can write this one, theta E by t. Here also I can define theta E by t. Now when theta is very greater than theta e, the theta is very greater than theta e, sorry T is very greater than theta E. Then T is very, very greater than theta E right. So, this term is very small. Theta E by T will be very, very small. So, it is like e to the power x or say some parameter e to the power say theta, e to the power y you take y equal to theta E by a. So, y is very, very small. So, then we can expand it 1 plus x to the basically 1 plus x square etcetera.

So, I can neglect higher term right. I can neglect higher term this minus 1. So, it is basically x. It will give basically x right. So, this for lower temperature Cv equal to 3 Rs theta E by T square right. And here it is giving me, oh sorry, oh I have written y. So, I should write y. So, y square, below y square means theta E by T square. So, I am getting 1 by, theta E by, T whole square right. So, these 2 will go. And this term is there, so this again e to the power x so 1 plus. So, this approximately this one can take 1.

Because this because 1 plus, y plus y square etcetera. So, since theta is, if theta is very greater than theta E. So, one can write this term is like one. So basically you are getting. So, 1. So, this will give you this will give you equal to 3 R if theta is very, very greater than theta E. So, I will get Cv equal to 3 R. So, that is nothing but Delong petit law so at high temperature. So, we are getting the result as same result as the result of Delong petit right. So, second you can think of that. So, this also I do not need. So, this was the expression Cv equal to this right.

So, now, for case 2 for low temperature, at low temperature; so low temperature means theta T is very, very less than theta E right. Now Cv will be equal to 3 R. Now we cannot neglect this this term is the higher. So, theta E by T square, now this theta E by T that is greater than 1 is higher term right. So, it is compared to compared to 1, it is higher right. So, I can neglect 1. So, it will be square of this. And then this one is there. So, it will be 1 by this. So, it I can get e to the power minus theta E by T. I can get e to the power minus theta E by T right. Or this lower temperature at lower temperature this.

So, here what I am seeing this theta E by T is very, very high term is higher term right, because this theta, T is smaller, so this thing higher. So, at very very low temperature, when temperature will be very torched temperature tends to 0. So, this term this exponential term will dominate right. Then this term because T is very, very small. So here in general it is Cv is proportional to e to the power minus theta E by T square right. Theta E by T exponential term and this T square right.

Now, for when the low temperature; so this term exponential term dominate, because if it is e to the power minus x if x is very high. So, it will fall very sharply, it will fall very sharply. So, here Cv curve if temperature T. So, this Cv curve, as I experimentally whatever we have observe is the experimental this is the 3 R. That is fine. So, this is the theta E. Say this is the theta E, this is the theta E. One can find out theta E. So, this is the theta E. So, below theta E; so this below theta it is falling like this. Now at very, very low temperature at very, very low temperature this falls exponentially it falls exponentially right it falls exponentially, because this term is very, very small.

That means, when you are increasing temperatures. It is increasing like this. So, Einstein whatever the result got it can explain the behaviour of low temperature, but it is not fully satisfied. Because it is value was slightly this if it is the experimental result. So, Einstein's result was slightly higher of that one, like this. So, this result it is could not fit with the experimental data completely but partial is. So, partial it is successful. So, actually this curve, experimental curve, it can be fitted with the T cube law. This called T cube law. So, it is varies like this T cube curve.

So, Cv is proportional to this T cube. If so then this peep the experimental curve. So, but this here it is the T cube. It is a exponential term is dominating. So, in case of Einstein this at lower temperature, this if follows the exponential curve, but experimental curve demands the T cube curve. It is falls following the T cube. So, then then this ex Einstein model is not fully successful to explain the Cv as a function of temperature especially at lower temperature. So, then Debye came forward to explain this Cv behaviour at lower temperature. And he found the T cube dependent of the Cv.

So that we will discuss in next class, I will stop here.

Thank you for your attention.