**Solid State Physics**

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**Lecture - 45**
**Band Theory of Solids (Contd.)**

So, I will continue for this velocity of a electron in a band.

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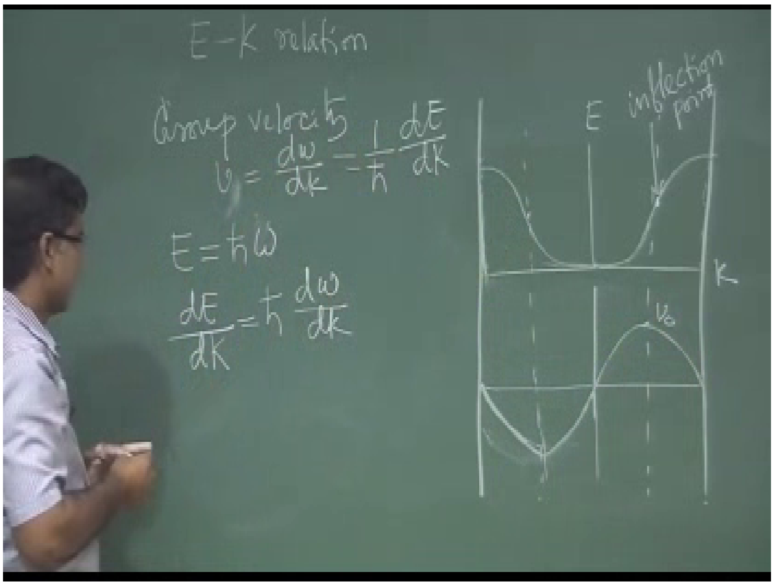


So, its changes like this and if you just see this is the now definition of velocity now if you take if you just consider the free electron not in a band. So, this what is the or. So, we have to know the energy of this electron free electron. So, energy of this free electron as a function of K if you know then we can find out the velocity.

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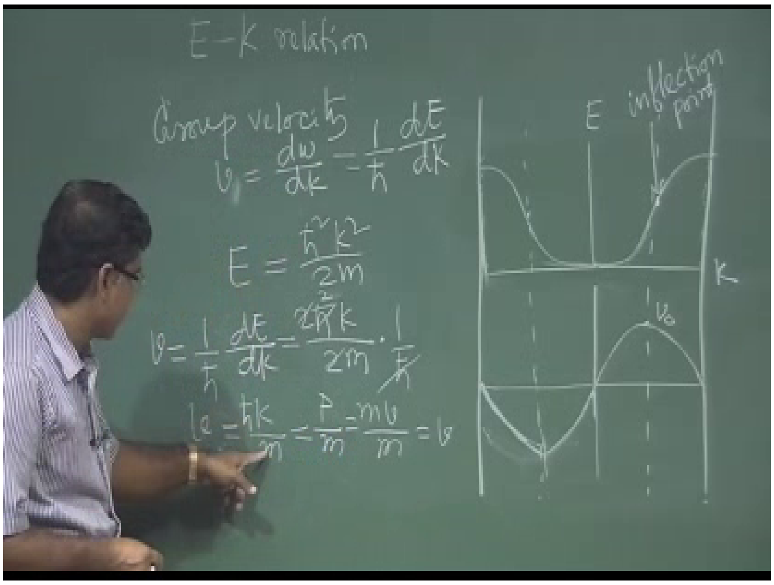


So, E equal to we know h cross square K square by 2 m right. So, dE by dK dE by dK equal to h cross square 2 h cross square k by 2 m right.

So, its. So, your velocity v equal to 1 by h cross dE by dK right. So, equal to; so dE by dK from here what I got, so 2 k 2 h cross square k by 2 m and 1 h cross is the f. So, this 2 2 going this h cross h cross is going. So, I am getting v equal to k by m right v equal to k by m. So, here I did mistakes. So, here basically h cross squared is there. So, one h cross 1, h cross is 1. So, one h cross will be there. So, h cross k by m right for free electron what is the h cross k that is P that is P momentum P; so P by m.

So, the just is free electron right. So, P is mv P is nothing but mv divided by m it is a v is a v right. So, this is a general definition of a velocity of a electron whether it is in lattice whether it is in a band or it is free electron. So, whatever if you know the EK relation you can find out the velocity. Another aspect of here where what we have seen; so v equal to h cross k by m right. So, v is basically is proportional to linearly proportional to k right.

So, it is just linear function, see if I draw here. So, for free electron what was the a EK curve.

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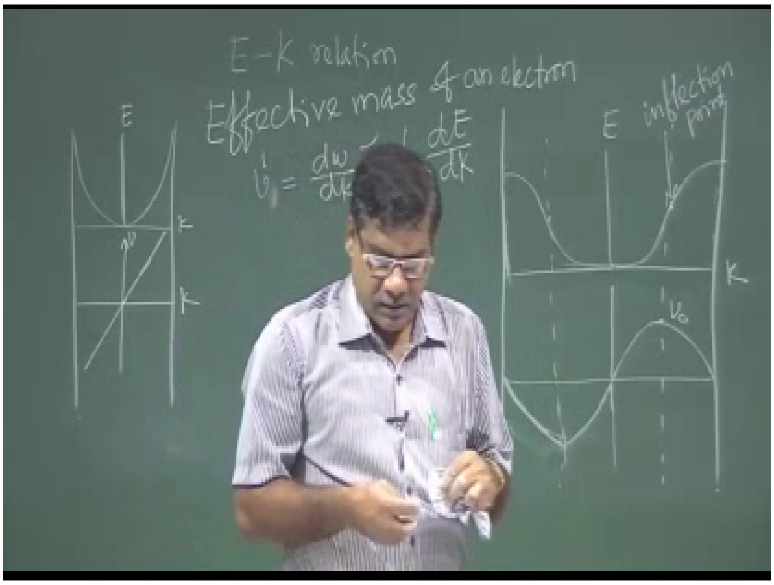


It was just parabolic curve right is EK it is EK this parabolic curve. So, now, corresponding velocity corresponding velocity; from here if you draw you can see this velocity here this it just slope is changing continuously, there is no inflection point there is no inflection point is it is changing. So, it is changing it is changing it is changing right. So, linearly it is changing that is what here it is linearly changing.

So, it is see quite different it is quite different for free in case of free electron just velocity is linearly changing it k this is this axis is velocity is, but here is when electron is not free it is in a band, then its variation is quite different. So, it is important to notice that the that the velocity it is it depends on this relation EK relation for that electron, that electron in which situation it is in which an environment it is. So, that will decide that environment will decide that EK relation and corresponding velocity also follow that relation.

So, another parameter is very important that is effective mass of a electron when it is in band ok.

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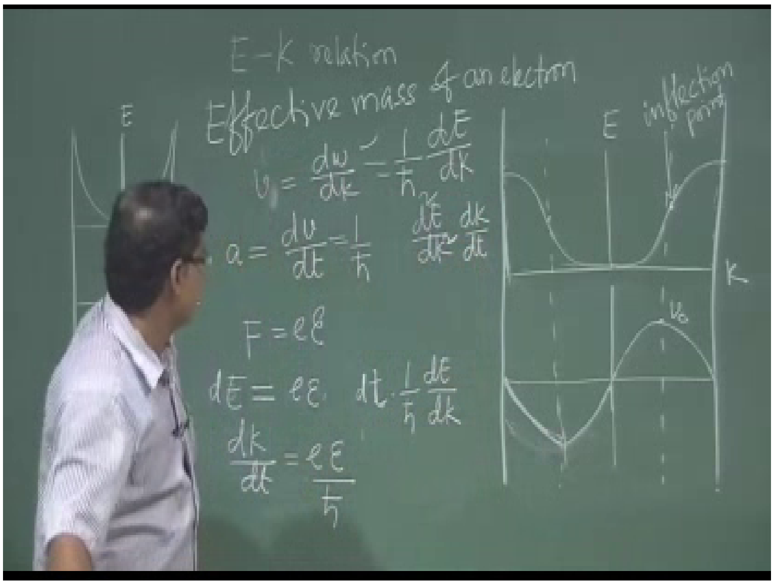


So, effective mass of electron here also if you will find out the definition of this effective mass, acceleration if you a electron is there if you just apply electric field. So, it will get force and it will be accelerated. So, when acceleration is basically change of velocity with time change of velocity with time right.

So, when it is in lattice. So, or it is just for general. So, this velocity is this. So, I can write 1 by h cross d by dt and that is dE by dK right. So, here I can write d 2 E dK squares then dK by dt. So, this with respect to dk you will get this. So, acceleration is 1 by h cross d 2 E by dk square dk dt. So, this when EK relation is known we know then we can find out d 2 E by dk square, but what is this dk by dt what is dk by dt that we have to find out. So, if we apply the just electric field for a short time dt that electron initially that electron velocity is v, now we have applied electric field e. So, it will fill force of how much F equal to e that electric field right.

Now, that electron will gain energy that will be equal to the work done by the external electric field on that electron right. So, work done is? Work done is basically that is that will be a energy E equal to force into due to the that force what is the distance is covers. So, your force is this and now its velocity is v and electric field was applied for short time dT. So, it will travel distance v dT. So, force into that this distance that is v dT for short time it is applied. So, that will be the distance and this will be the energy, so this also small energy. So, de equal to this right.

So, when we apply electric field on a electron moving with velocity v for short time dt. So, what will be the change of its energy what energy it will gain? So, d energy it will gain. So, that right. So, it is not I think time. So, you should write this. So, from here one can. So, one can find out this that dk by dt because this e is what is this E? Energy; energy E is h cross omega e is h cross omega I think this yes. So, this v I think here is I have to here. So, vdt. So, v is by definition it is this. So, I can replace v by 1 by h cross square h cross dE by dK right.

So, what we are getting. So, dE equal to this dE. So, basically dt by dk. So, just if you take this side dk by dt right. So, del E by del E is one. So, dk by you will get dk by dt equal to e electric field 1 by h cross dk by dt 1 by h cross right is it correct yes I think. So, dk by dt equal to this by h cross right; so this from here when we are getting this. So, now, this acceleration for a that we have seen that is in terms of this here we have written now we have to find out dk dt. So, we got this dk dt.

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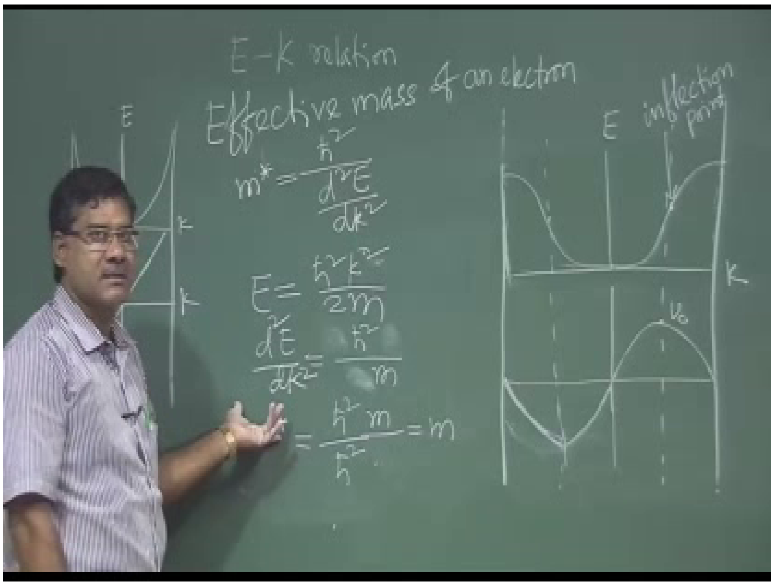


Now, I can replace this dk dt; now I can replace this dk dt by this term epsilon e e electric field divided by h cross right.

So, we are getting basically acceleration you are getting a equal to e this now 1 by h cross square 2 e by dt square dk square ok. So, for any particle it can be electron free electron. So, what is. So, what is the force F equal to or we can write ma right that is the Newton’s second law ma.

And in this case this force is if we apply electric field on a electron. So, it will field force how much e for the any charge particle free particle. So, a charge particle of mass m if we apply this force, then its acceleration will be a equal to this by m right. Now if we compare this 2 then I can write this 1 by m equal to 1 by h cross square d 2 E by dk square right.

So, this mass generally we called effective mass we call effective mass. So, here effective mass what I got m star effective mass we defined by this. So, h cross square by d 2 E by dk square. So, these are again general definition of mass whether electron is in is free or it is in lattice just check it. So, you know that free electron for free electron I have to know EK and a EK relations. So, that was the h cross square k square by 2 m right. So, what will be d 2 E by dk square? So, dE by dK it will be 2 h cross square into k by 2 m 2 m again differentiate right. So, this k will remove 2 will go. So, h cross square by right.

Now, if you put here. So, m star; m star equal to h cross squared by this; that means, by this h cross squared by m. So, multiply is equal to m you see. So, for free electron whatever this mass effective mass it is the same as its original mass. So, it is very interesting that is this long time it is known to us that is mass of electron is well known and its constant right it has some definite value particular value, but when that electron is in a band then its mass how it is it change.

So, interestingly you can see that this is the definition of mass and d 2 E by dk square what it is? For this curve d 2 E by dk square it is nothing but the curvature; curvature of this curve what is curvature this it is curvature right.

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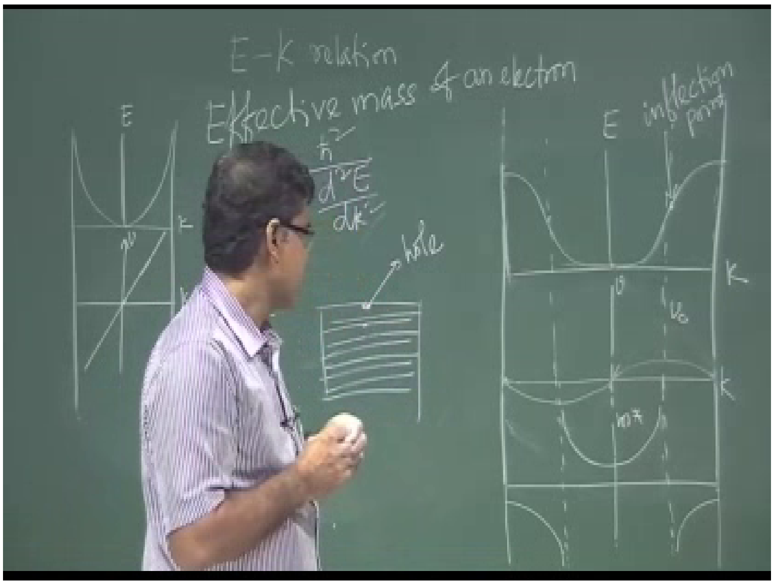


This curvature radius r; so generally curvature is inversely proportional to this radius; this radius is higher curvature is smaller radius is smaller, radius is smaller smaller curvature is higher radius is higher curvature is smaller curvature is smaller. So, here in this curve there is 2 curvature one is this, another is this and the same represent by this is a constant forget, so this curvature, this curvature. So, here if I draw whether I can let us see yes let us see.

So, let me draw is slightly smaller size is this one slightly smaller size then I can. So, just I draw like this, I draw like this the just for space I need space. So, that is why. So, this is the inflection point under this inflection point. So, for mass if I draw this is k axis this is k axis these are k axis right and mass I am changing this way this is the mass effective mass, this is the effective this is the velocity right this is the energy and in this case what will happen is star is star. So, this is the curvature right.

So, here basically and you will get this type of curve, you will get this type of curve you will get this it has to be symmetric this type of curve and at impression it is just discontinue and then this is this curvature and this is the another curvature. So, just opposite to that, and you will get yeah you will get this right you will get. So, it is you can think from here you will see this type of change of affective mass we will see what you will see. So, it is here it is positive mass is positive; see it is at the edge of this delusion at the edge of the delusion or tough level energy levels of band. So, this mass is negative this mass is negative right.

But in middle of the band in middle of the band this mass is positive at the edge it is negative; so before inflation in inflection point is a positive and after inflection point. So, this inflection point is the turning point of the curvature you know. So, this is very important. So, before this inflection point and after inflection point these things are these change are different in case of velocity in case of mass also. So, here negative mass, so when electron in a band. So, its mass can be negative when that electron is in a, this is the band energy levels are there when electron search on the top of the energy band. So, its mass can be the negative right.

And its velocity when it is here its velocity decreasing here. So, when you are applying electric field and because of that this electron are moving right. So, when. So, in this region here its velocity zero then it is increasing, when it is reaching the some energy level some places and then it is its velocity is decreased. So, why velocity decreased suddenly? So that means, we can think 2 ways one is. So, this electron are moving influence of external electric field right. So, it is moving its velocity will increase and then there is a acceleration right.

Now, suddenly after this point its velocity is decreasing. So, its velocity is decreasing means some de acceleration is working on it. If it is accelerated its velocity will increase then it is decreasing now there is a de acceleration, there is a force on opposites side on that electron. So, you can consider that why suddenly this force is on opposite side. So, if we think that when electron are moving in this in top band, then if we think that electrons charge becoming positive, electron charge is negative now we can think that electron charge becoming positive. When electron charge becoming positive then due to that electric field it will fill a force in opposite direction right.

Whatever the force it was enjoying because of its negative energy negative charge, so the direction of the force now if it becomes suddenly become positive charge then it will be it will fill the opposite force. So, its velocity will decrease right. So, this may be one case that may be can explain this velocity decreasing and becoming zero when it is reaching at the top from here to top. So, this is one case second case. So, this positive charge of electron becoming having the positive charge becoming positive charge is it possible?

Yes it is possible. So, that is called basically hole. So, that is why is called basically hole hole is having same electric charge whatever electron have, but it is positive charge ok.

So, these are giving us impression or hints about the existence of hole right in solid similarly other way one can think. So, when it is in upper band right upper band say its mass is negative. So, you can think that, that electron is moving under a electric field. So, its negative charge is negative charge. So, then it is moving with some force in one direction, now when suddenly if this mass is whatever this mass was there now when it is moving on the top band top of the band then it is from positive mass to it is going towards negative mass right.

So, then also acceleration can be opposite side because m that acceleration is. So, it can fill because this force is ma right acceleration and this mass. So, this now if mass become (Refer Time: 26:59) negative, then force will act now minus ma kind of things. So, mass this force will act on this electron in opposite direction due to the negative mass. So, electron have basically negative charge electron have basically positive right positive mass.

So, but in upper band in upper band we are getting that electron whatever here charge that is as if it is positive charge and yes. So, when electron are here. So, it is as if this electron mass is negative right electron is here with negative charge. So, it is negative charge. So, its mass is negative right mass is negative. So, electron here it has negative charge and negative mass here. So, now, absence of this electron absence of this electron we can we can think that vacancy you can think that vacancy absence of this electron having negative charge and negative mass absence of this we can think this one is positive charge and positive mass.

So, this one is nothing but the hole absence of electron is nothing but the that act as a hole that is positive charge and positive mass. So, electron here is negative charge and it become the negative mass, now absent of such from this top of the band generally hole we found we found at the top of the band. So, absent of that electron having negative charge and negative mass. So, that is equivalent to a hole which will have positive charge and positive mass. So, these are very interesting. So, this from a Kronig Penney model. So, lot of mathematics determinant you have to do, but whatever the condition we got this set of expression from the we extracted in rich physics and very interesting also one can one can explain the band explain the electrical property of different material whether it is insulator it is semiconductor it is metal and also it can give the concept of it can give hints at the concept of a hole right .

And this effective mass is ma equal to this one. So, yeah. So, from here this basically EK relation you know I have shown this for electron free electron that is m star equal to m. So, this is the general definition. So, for free electron whatever the energy expression E equal to h cross square k square by 2 m. So, if you replace this m by m star effective mass. So, this equation is valid for others also right it is not only valid for free electron; it is valid for other electrons also. Whether electrons in band of a other if you know the EK relation of that electron. So, you can find out the energy ok.

So, these are very interesting I think there are many things we discuss about the energy about the band and this band contains the how many Orbital's how electrons are distributed among them, how when it will show the show as a metal behavior or semiconducting behavior or insulating behavior right and velocity mass they are quite different from the free electron.

So now, I will stop here. So, basically band structure using the band structures. So, we can classify different material. Next I will teach you about this semiconductor crystal, semiconductor physics. So, how one can define semiconductor and what are the types of semiconductor, so that I will discuss in next class.

Thank you for your attention.