Solid State Physics
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Lecture - 39
Electrical Properties of Metal (Contd.)

So, we will continue our discussion about the density of states. So, what we have seen that density of states this expression we got if we if we plot. So, this is the plot and below Fermi level all states are filled all states are filled. So now, I was discussing if temperature is not 0. So, then what will happen what about the about the density of states.

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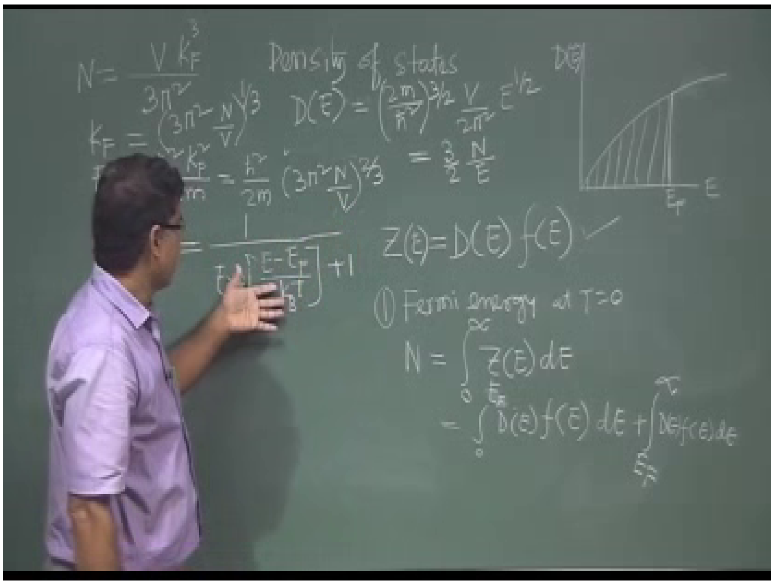

So, at finite temperature; so, density of states it is basically whatever density of states. So, that that is there; now that will be modified with the Fermi Dirac probability it is since its value is 1 to 0, its value is 1 to 0. So, if I multiply with this function. So, this function for temperature t equal to 0 it is one right it is one when E is less than E is less than Ef when E is greater than Ef. So, that is 0. So, there will not be any density of states because density of states its we are telling in terms of basically energy levels, but energy levels will be there since here I am telling it will be 0; that means here.

So, what about density of states that is equivalent equal to the density of electrons as I told this N we have considered in both way each electron will each electron will be in each orbit during the considering the degeneracy. So, that way this will be the distribution or density of states in general one can write this is the density of states even it is t equal to 0 this will be valid when t is not equal to 0, then also it is valid. So, in general this is the density of states one can write. So, I am writing in terms of Z; ZE equal to DE and fE. So, these are in general density of states for any temperature and this DE that is only valid for t equal to 0.

So, I think let us calculate something using this density of states let us be familiar with this. So, if I want to calculate say if I want to calculate the Fermi energy Fermi energy at t equal to 0. So, here we are considering t equal to 0. So, just using this density of states how to calculate; so, that is just to show you I am calculating. So, here Fermi energy I want to find out right. So, one thing is very clear to us that numbers of particles are constant in the system. So, if I intrigrate integrate this density of states. So, 0 to infinity energy 0 to infinity 0 to infinity; so, then it will give me all states that will be equal to the total our that will accommodate our all electrons. So, some states will be occupied some states will be unoccupied some states will be may be partially occupied whatever. So, in general if we consider.

So, I can write this density of states what is this number of states per unit energy. So, if I integrate with respect to E energy. So, I can write this right. So, so from here from here I can. So, how to proceed? So, this fine density of states now I integrate over the energy. So, all energy I have considered 0 to infinity that also there is the problem now I want to find out Fermi energy at t equal to 0. So, under this condition I can say that.

So, this I can write 0 to infinity ZE is DE and fE right and dE. So, DE is DE is this DE is this dE is this. Now here I can consider that since t equal to 0. So, I can this integration I can take in to 2 parts one is 0 to Fermi energy because I know at t equal to 0 up to Fermi energy all electrons; electrons are filled up and outside Fermi above Fermi energy. So, they are empty. So, here I can write DE and this fE is there fine keep it and then plus I can break it.

So, Ef to infinity right 0 to Ef and then Ef to infinity so at t equal to 0. So, there I should write DE fE fE dE, right fE DE. Now this distribution Fermi Dirac distribution it is telling me it is telling me at t equal to 0 at t equal to 0 within this Fermi energy Fermi level.

So, this will be one when E is less than Ef. So, this is one I have shown you and this is 0 this part is 0. So, so this is 0 means this integration will vanish and this I can remove this one right and here I have to replace this tE by DE by is here this part is say constant only E to the power.

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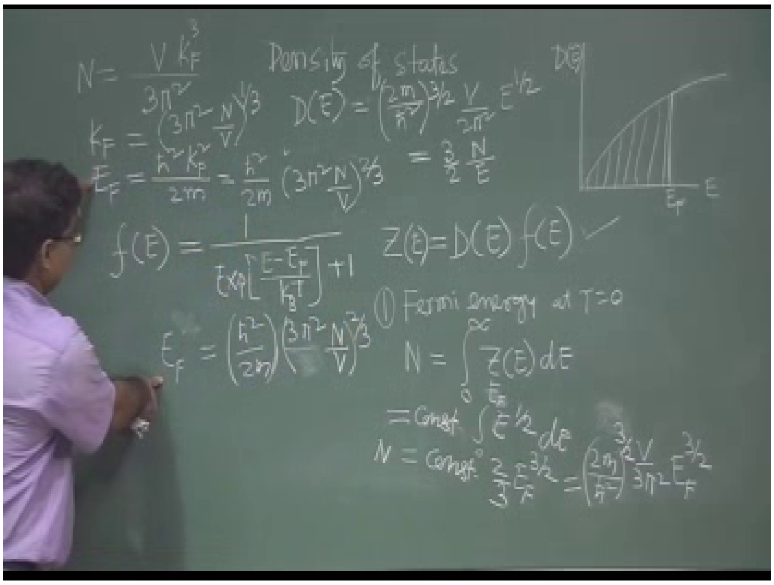

So, some constant so that is this I will write later on. So, here I have to integrate half E to the power half, right. So, this and DE is there DE is there. So, it will give me what it will give me this integration will give me this constant equal to constant into 2 the this one 3 by 2 E to the power 3 by 2 right after integration I will get this one get this; this constant is this constant is this right. Now 2 by 3 multiplied 2; 3 by 2 multiplied 3 by 2 multiplied I think it will be because 2 by 3 because E to the power 3 by 2 divided by 3 by 2. So, 2 by 3; so, 2 by 3 multiplied. So, 2 2 will go. So, it will be 3 V by 3 pi square and here E to the power E to the power 3 by 2 E to the power 3 by 2; so E to the power 3 by 2.

So, basically this density from here this one I am getting 2 m by h cross square then V by 3 pi square V by 3 pi square V by 3 pi square and then I have this E 3 by 2 right and if I put this limit. So, this E will be Ef this E will be Ef and this E will be Ef. So, you see N equal to exactly I got N equal to exactly I got what N I have written well. So, I wrote, but I have removed it. So, from there; so, this is N right. So, this is N. Now, we can see that you can write Ef 3 by 2 equal to h cross square by 2 m right 3 pi square by V. Now N is there. So, this N I can write N. So, I was writing in this form.

So, I will write in this form well now this will be 2 by. So, 3 by 2 I remove. So, this will be 2 by 3 or whole things will be 2 by 3 yes 2 by 3 Ef, sorry, I did mistake again N equal to 2 m h cross square V by 3 pi square Ef 3 by 2. So, here already 3 by 2 is there. So, that I have not written. So, that is why problem was coming. So, 3 by 2 I have not written that is why problem was coming. So, here; here 3 by 2 was there. So, it will be this and here V 3 pi square. So, 3 pi square by V N was there. So, that will have 2 by 3.

So, exactly it is same you see Ef h cross square by 2 m 3 pi square N by V 2 by 3. So, just according this is the way one has to one can calculate all things. So, thus it is very general method density of states is using density of states is very general method and one can one can calculate everything from density of states. So, this is the one example.

Another example I can I can tell you that if I want to find out suppose I am doing the simplest case just to make you familiar with how to use the density of states.

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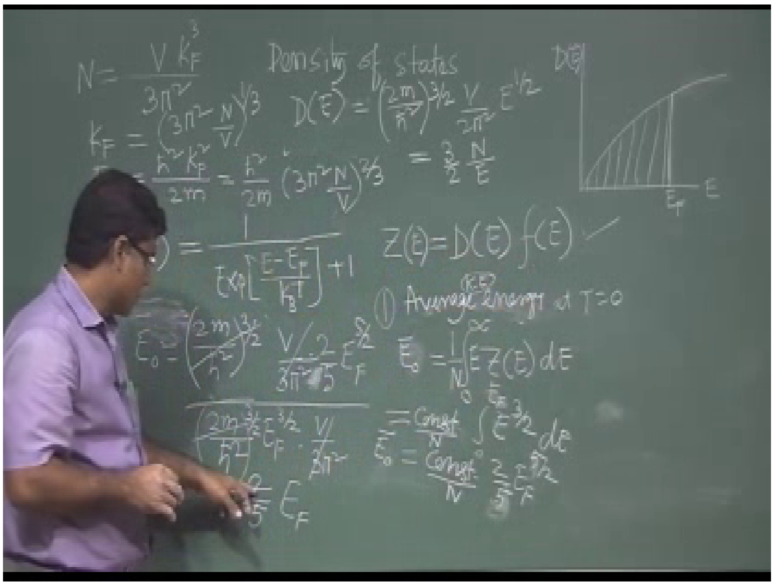

Now, if I find out the average energy; average energy average energy of particles or electrons in our system average energy of electrons. So, in that case; so, in our free electron system it is only kinetic energy it is only kinetic energy right it is only kinetic energy. So, average kinetic energy of free electron system our system if we want to calculate at t equal to 0 at t equal to 0. So, this is a simpler that is why I am calculating. So, again I have to for; I have to define. So, this is the density of states right this is the density of states general density of states now.

So, if I integrate this one if I integrate this one. So, it gives the total number of integrates from 0 to infinity energy it gives the total number of electrons in the system. So, now, I have to find out the total energy of all electrons they are staying in different energy levels. So, each one is having different energy. So, I have to sum up; that means, I have to integrate right. So, I have to integrate. So, I have to take energy, I have to take energy right energy E for this number of states for a particular energy E right and integrate over this de. So, from 0 to infinity; so, these will give me the total energy of the system of electron system having N electrons, right.

Now, I will divide by 1 by N right because in this energy range I know 0 to infinity our total number of electrons is N that I know already I have calculated now this then I can write this the this will be the average energy of our each electron average energy of our each electron. So, then I have to integrate. So, integration is similar only one E is there additional E is there; so, here what about this from this distribution E half was coming now another E is there. So, it will be 3 by 2, right, it will be 3 by 2 now integrate. So, everything will be fine only here 5 by 2; it will be 5 by 2, it will be 5 by 2 and it will be 2 by 5, 2 by 5, right; it will be 2 by 5. So, so this with this basically I can multiply what I can multiply I can multiply with earlier it was 2 by 3 and crossing 2 by 3 it was coming now I am multiplying 2 by 5. So, here basically; so, I have to I have to it is a; I think, I should I should do that. So, it is now 2 by 3. So, then it is not N it is E 0 average.

So, E 0 average what I will get E 0 average I will get. So, integration will be similar integration will be similar 0 to infinity I break see 0 to Ef and then another Ef to infinity because at t equal to 0 that that fE will control that one because this Ef to infinity that integration will vanish because function of E that is 0 for that case and below this Fermi level it is one. So, this is constant this constant and then this will be Ef 5 and then from there this constant is there. So, I can write E 0 E 0 bar equal to constant is what is this constant; constant is 2 m by h cross square 3 by 2 right V by V by 3 pi square V by 3 pi square.

So, that is the constant E that one we have included here and into 2 by 5; 2 by 5 into 2 by 5 into 2 by 5 E to the power Ef 5 by 2. So, this will be the average energy of each electron. So, for this if I put one thing is missing here this one by N that I have not considered. So, here this constant by N I have to write right this constant by N I have to write. So, this constant by n; so, 1 N will be there 1 N will come here 1 by N it is. So, this will be the form.

So this, I think Ef is h cross square 2 by N that is fine. So, whether I can make it more simplified. So, if we put this value of N value of n, but unfortunately it is not there value of N what is the value of N value of N is Ef 2 m by h cross square 3 by 2 2 m by h cross square 3 by 2. So, N is coming divide by this. So, this term and this term will be same. So, this; so, I am putting the value of m. So, let me do explicitly otherwise I can do mistake. So, this N equal to N equal to a is 2 m by h cross square into Ef. So, this part then 3 by 2, 3 by 2 this also 3 by 2 this also 3 by 2 and then now it is removed. So, 3 pi square by V. So, it will be V by 3 pi square V by 3 pi square, ok.

So, from here what we are getting equal to. So, this will go this will go. So, 2 by 5 will be there 2 by 5 2 by 5 and then I will get Ef; Ef 3 by 2 Ef 5 by 2 Ef. So, this is also very important expression that average energy of the electrons free electron gas. So, its a at absolute temperature t equal to 0; it is two-fifth of Fermi energy what about Fermi energy. So, average energy will be two-fifth of that Fermi energy. So, this also it is I did mistake in here it is telling three-fifth my no I think it is two-fifth that is; so two-fifth of the electron Fermi energy.

So, this way one can use the density of states and calculate everything calculate everything. So, that is why in solid state physics density of states is very important if you know the density of states how it is related with the energy density of states how it is related with the energy means energy means at different energy what is the number of density what is the number of states. And if you know the relation how it is varies so that if you know for a particular system you can calculate everything.

So, you as you as you remember that that whatever. So, far we have done. So, everything just developed the quantum theory this is the mathematics. And we are getting some expression everything just the just the method in quantum theory we use. And now our main purpose is for doing this main purpose is what is the main purpose.

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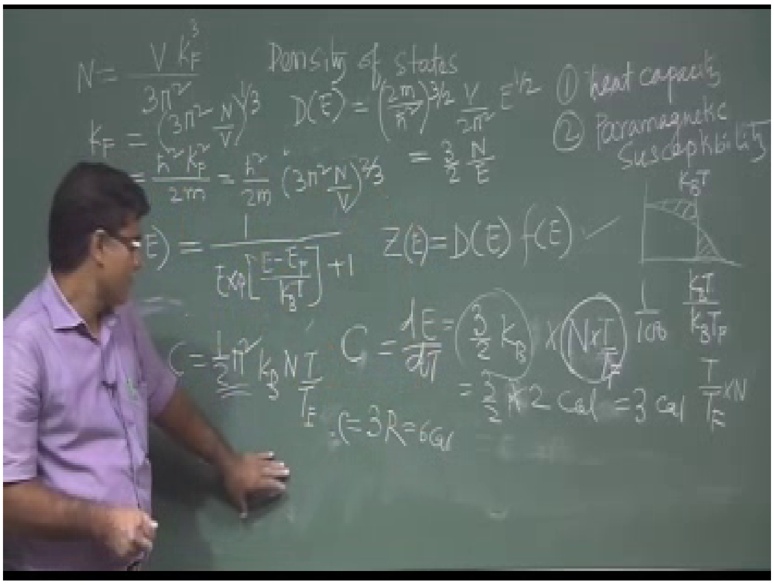

That free electron theory basically classical theory it could not explain it could not explain thermal heat capacity it could not explain heat capacity heat capacity right capacity it could not explain the paramagnetic susceptibility, right. So, this is our. So, what about here you have developed using that one whether we can whether we can now explain the experimental result what was the experimental result I think I need space.

So, I finished these things; so, this heat capacity heat capacity for any classical theory basically uses the Maxwell Bujmen's statistics Maxwell Bujmen distribution. So, that was the mistake basically. So, one has to for electron system one has to Fermi deduct statistics. So, classical concept and quantum concept here this main part is this distribution whether it is following the Maxwell Bujmen distribution or Fermi deduct distribution ok.

So, in classical theory that is Maxwell Bujmen distribution was considered. So, heat capacity what is this heat capacity to increase unit temperature what amount of heat we require right. So, when we give heat to a system when it is temperature change by t. So, then classically we have seen that because it is field electron will have kinetic energy. So, each electron will have kinetic energy 3 by 2 kbt, right; 3 by 2, 3 by 2, kbt. So, that was the classically it tells that or this kinetic theory from kinetic theory that is the standard result. So, that was applied for free electron gas it is as if that it is gas; gas probably its bf has gas particles bfs. So, it will have this 3 by 2 kbt is energy kinetic energy thermal energy basically.

Now how many electrons are there N number of electrons are there right. So, I will multiply with n. So, total energy. So, each electron will have this will gain this energy. So, total energy gain by this system is this or we need this amount of energy to increase by this temperature t right. So, so that is the energy we need to increase by temperature t. So, DE by dt that is basically heat capacity heat capacity C and if we take specific amount of methyl one mole or so, molar specific heat or per gram if you calculate this specific heat. So, in general it is heat capacity. So, this C is equal to DE by dt equal to this right 3 by 2 N kb if you get take one gram mole right one gram mole material. So, it will have Avogadro number of atoms. Generally, metal atom it is seen that on the average it contribute one electron in free gas o in free electron one electron per atom we get as a free electron.

So, then it will be na Avagadro number. So, this 3 by 2 r right gas constant. So, this gas constant this r is 2 calorie 2 calorie 2 calorie, right. So, it gives 3 calorie right per gram per temperature that is there now you remember that for Glom Petit law. So, this specific heat basically incase of material specific heat is basically 3 r. It is around 6 calorie, right. So, that is that that for any material this is same and here whatever here we have considered that is for contribution from electrons.

So, in case of metal whatever specific heat or heat capacity I will see compared to the insulator if I take insulator it will not have any free electron. So, electronic contribution will in specific heat will be 0. So, that will be only this contribution sorry it will be six calorie only this contribution now if I take metal. So, I will have this contribution this that is there additionally I will get I should get this in confusing 3 calorie.

Additionally I will get this one. So, for metal it should be fifty percent higher than the insulator heat capacity right, but experimentally it is seen that it is very very small contribution of free electron in specific heat, it is a what about here we are getting nearly getting. So, it is one hundredth of this one; 100th of this clearly experimentally seen and also it is seen that it is in this case, it is independent of temperature, but for experimental it is seen; it is not independent of temperature it depend on temperature. So, that classically we cannot explain that one.

So, from here we can explain because you see problem is here problem is this N; N is I told that classically it is considered that all electron will have the energy right, but quantum from here as I mentioned this statistically it is telling that when you will increase temperature increase temperature. So, this distribution that is that was telling. So, it is. So, depending on this temperature those will kbt energy that kbt energy and that energy depending on t. So, it will remove the electron from the near the near the Fermi energy. So, it will not affect the other electron. So, only this part of electron because of this temperature will get kinetic energy right. So, this compared to. So, this will compare value with Fermi energy Fermi energy as I mentioned that this temperature kt and for Fermi energy kbtf right.

So, this with compared to this temperature this temperature whether it is small; how small it is etcetera depending on that one can one can get from here that this ratio t by tf that is that will tell you the it will give the fraction of the electrons total electron N will have the kinetic energy. So, well this factor we use. So, here it is not this many all not all electron only this into this fraction. So, this number of electrons will get kinetic energy will get thermally excited. So, here what about C; we are getting what about C here we have calculated. So, with that here I am getting additionally t by f that is because of this distribution and only electrons near the Fermi energy Fermi surface it will be excited. So, here I am getting this C. Now it is it depends on temperature right it depends on temperature t and also this value whatever I got. Now it is t by t f it is t by tf. So, this is very small right.

Generally if in case of room temperature if t is in room temperature and this for generally typical Fermi energy that is say 3 electron volt if it is 3 electron volt. So, this temperature is 3 thousand more than 3000. So, 3 hundred divided by 3000. So, it is a just small fraction of the total contribution classical contribution. So, that is why its value electronic contribution in this capacity heat capacity will be very very small as well as it will depend on temperature.

So, these are just phynominocally qualitatively I explain you, but using this density of states one can one can calculate and get the correct factor. So, these dependents will come these dependent this part will come only these factors slightly it will be modified. So, that value I can tell you exactly half pi square if you use this density of states. So, this C it comes half pi square half pi square kb all other things same t by tf. So, just this factor 3 by 2 factor is slightly modified it is around nine that is 4.5 here it is 1.5; it is 4.5 anyway this factor is there, but using this density of states exactly you can get the get this result which main fact is this; this factor is very small and it depends on temperature.

So, similarly paramagnetic susceptibility also; I can show you as I told classically it is not explained, but using this quantum theory, we can explain exactly whatever experimentally found. But now I will not do that when I will teach you magnetism, so that time I will teach you. So, I will stop here.

Thank you for your attention.