Solid State Physics
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Lecture - 38
Electrical Properties of Metal (Contd.)

Today we will discuss about Density of States of three-dimensional electron gas.

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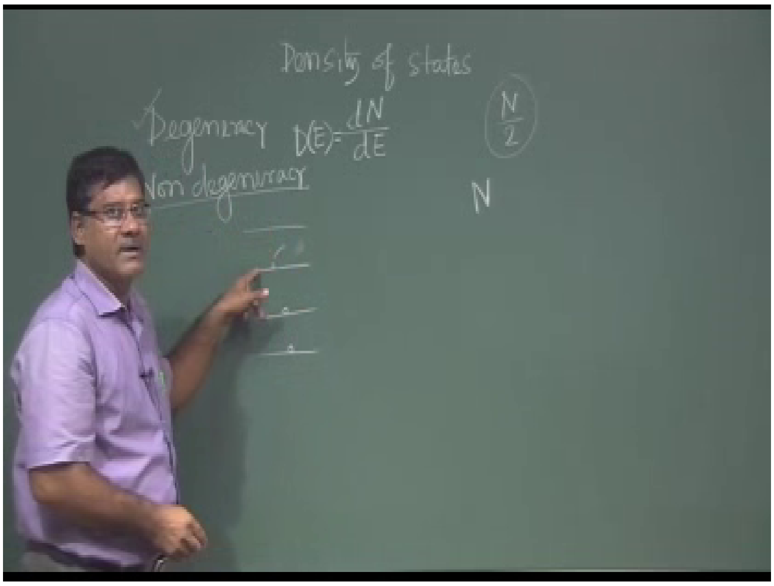

So, density of states; so density of states is defined as the number of orbitals per unit energy range number of orbitals; say it is N per unit energy. So, if N is number of orbitals, then dN by dE is defined as a density of states. So, what about N, N basically we have taken earlier that number of electrons in the system. So, total number of electrons in the system and number of all orbitals will be half of that right, because in each orbital 2 electron can stay one is pin up and another is pin down. So, basically N by 2 or number of orbitals, but if we consider that degeneracy; what is degeneracy and non degeneracy in quantum mechanics.

So, you have energy levels high electron states. So, this we tell this electronic state also. So, when here 2 number of electrons are staying, then this 2 electrons will have the same energy, but these 2 electron have different quantum numbers; otherwise they cannot sit in one energy level. So, depending on the quantum numbers; so this states of these 2 electrons are different, although energy are same. So, these we tell this degeneracy, you can take these 2 states, 2 energy 2 energy levels, but they have same energy. So, there you can tell this each energy level having one electrons; so that way here the number of electron and number orbital's. So, now, in that sense we tell the numbers of orbitals are n. So, their quantum number will be different for each electron and for each level; although energy may be same, when the energy of electrons are different in different states. So, then we tell this it is non degenerate ok.

So, degeneracy, number of degeneracy 2 fold here, 2 fold degeneracy means 2 states, 2 particle, 2 electron have 2 states, but same energy. So, degeneracy is 2 fold degeneracy, it will be 3 fold N fold degeneracy. So, in that sense here this number of states per unit energy. So, that is density of states. So, that is why here we have taken this number of orbitals are N. So, again we will not consider that in each state we will have 2 electron that way we will not consider. So, we will consider each state have, they are in different orbitals, but they may have the same energy the orbital may have the same energy.

So, that is degeneracy. So, why we take this degeneracy? Because under some external parameter this degeneracy is removed; so energy levels the same energy. Now after external, some applying external parameter. So, this degeneracy is removed, means the energy levels are separated like in case of if you consider magnetic material. So, electron in that case electron will have spin up and spin down. Now if you apply magnetic field; so this, then this same energy initially without magnetic field.

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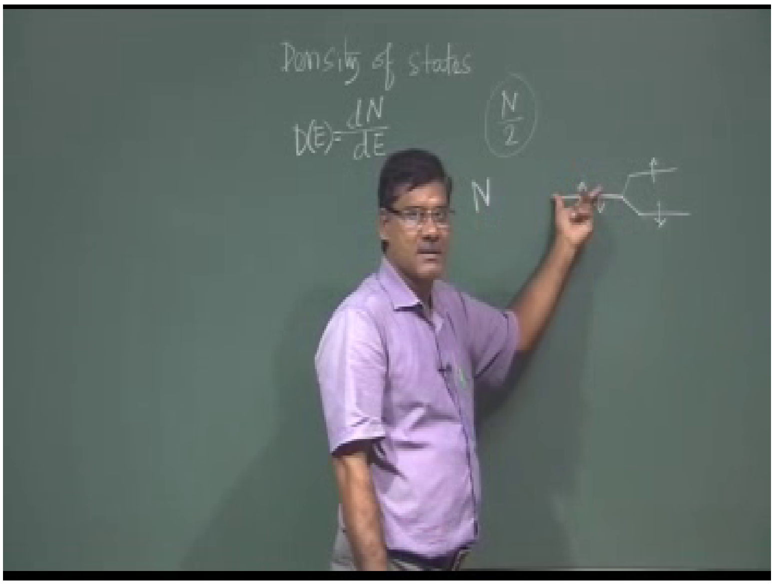

Now they will be splitted. So, their separation will be. So, initially it was the energy or spin up and spin down, if we apply magnetic field, this energy level splitted into 2, and this spin up and spin down are separated now. So, they will tell degeneracy is removed. now we will have this separate energy, and of course, separate state, but here we have same energy, but separate state. So, here we are considering 2 orbitals, having same energy and having 2 spins of electrons.

So, I have to go back, this I need N number of orbitals.

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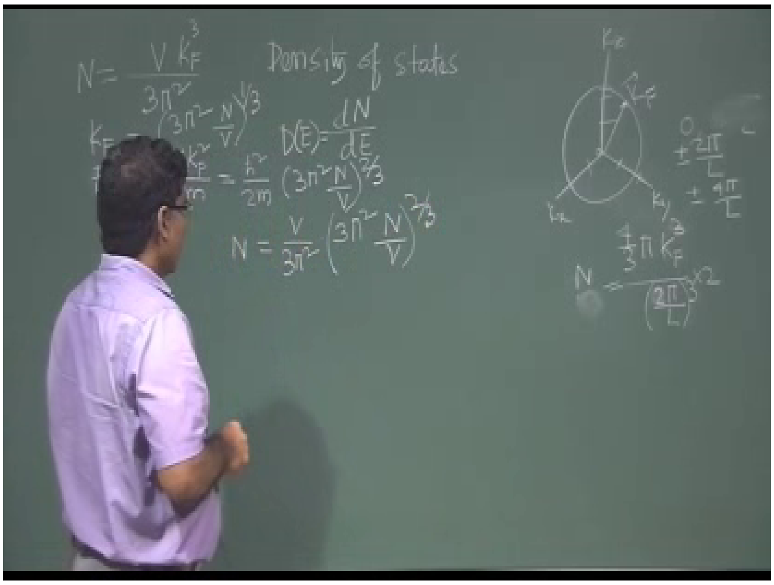

So, that we remember that we considered the Fermi sphere of in case space and kx ky kz, they can take value this 0 plus minus 2 pi by L 4 pi by L etcetera right, 0 plus minus 2 pi by L plus minus 4 pi by L right, and this all kx ky kz can vary, can have these values overall 2 N pi by L kx ky kz, can take 2 L pi by 2 N pi by L. So, here in this case, this volume of this sphere is 4 by 3 pi k square not square cube, and the smallest volume it is a, if it is origin then 2 pi 4 pi etcetera, 2 pi 4 pi. So, smallest volume will be just with length of 2 pi by L. So, 2 pi by L cube. So, in this volume, you will get one set of triplets set of k value kx ky kz, ok.

So, this will give as the total number of orbitals in this sphere. now if this sphere this is kf, this is the Fermi surface, this sphere surface is Fermi surface, then all electrons are inside the sphere right. So, if you put kf, then you can write, this is the total number of orbitals. So, from here one can calculate N right. So, just I will write this value. So, N will get basically here, let me write N, you will get what is the, what you will get, this will give 8 8, this for this 8. So, it will go. So, it will be basically and pi cube 3 pi square, you will get 3 pi square and half, you will get V V L cube; that is V basically. So, L cube that is V kf kf cube you will get.

Yes hopefully I have written correctly. So, so kf 3 pi square N by V, it will be this fine. this is total number of N electrons, and then kf, kf will be equal to 3 pi square N by V right, and then corresponding energy Fermi energy, one can find out Fermi energy, one can find out ef equal to h cross square kf square divided by 2 m. So, kf square you know. So, it will be h cross square by 2 m 3 pi square N by V N by V, then it is square. So, 2 third ok.

So, for now I have N, say N is equal to put kf here. So, V by 3 pi square kf cube. So, this is kf. So, 3 pi square N by V to for the power square 2 by 3 right. So, so V by 3 pi square V by 3 pi square is and, but I am missing one part that should be 3 pi square N by V one third that is correct and that N h cross square 2 m; that is also correct; e also correct and kf is this. So, if that way this is also correct fine.

Now, actually if I put kf. So, this N equal to V 3 pi square and kf is 3 pi square N by V one third. So, this is cube right. So, then it should be like this, then it should be like this. So, this, so this we have this, will come this N equal to n, but that is not my aim actually. I want, I understand what is the problem actually in terms of energy I want to get N. So, I have to use this one. So, from here I am getting that E equal to this.

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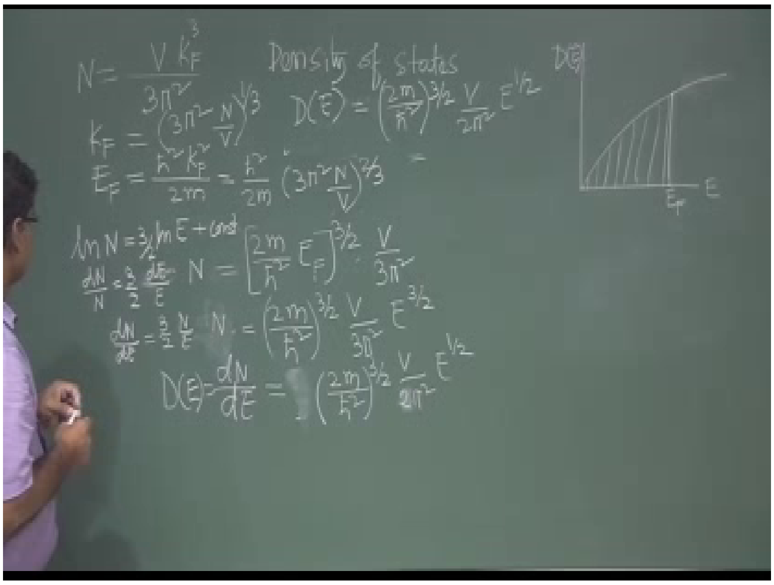

So, N equal to N equal to, so 2 m by h cross square 2 m by h cross square, and then this ef is there, but let me write ef and then 3 by 2 3 by 2, then V by 3 pi square V by 3 pi square; that is the expression for N. So, 3 by 2 it is N 3 by 2.

Now, what I want to do. I want to find out density of states, this is the dE equal to dN by de. So, this dN by dE for that. So, what about the; so now, N is total number of electron in the system, and that we are taking as a total number of states, total number of orbitals as I mention this considering the degeneracy. So, this if I consider this, I think I should write N equal to 2 m by h cross square 3 by 2 then V by 3 pi square, then this e to the power 3 by 2. So, I removed this suffix f, because I want to find out in general number of number of states at different energy. So, here just we have used the condition that here, we have used the condition that, this inside the Fermi sphere all electrons will be there, but if I take in general energy, if the energy is higher than the Fermi level.

So, I do not have any problem, because my total number of states, total number of electrons, it is (Refer Time: 20:31) it will remain same now. So, automatically it will calm that this up to which energy that all electrons will be there so that we can generalize using this f. So, then from here you can take differentiation of this. So, it will give you 2 by this is 3 by 2 3 by 2 this term 2 m h cross square 3 by 2 and this term V by 3 pi square e to the power half e to the power half right. So, this 3 will go, and then actually I can write this 2. So, this is the expression for density of states, and you can see the density of states it varies with square root of energy. So, if I plot what I will get.

So, this if I plot density of states, this is energy and this is density of states. So, what you will get, you will get basically parabola, you will get this type of curve. So, let me write up to this, to this, this. So, it is there it has, it is there. So, it is there, states are there, but say this point, this energy ef. So, density of states that is a, it will, you will have learnt this Fermi level also, but these are Fermi energy; so this part, so this part is basically occupied by the electrons. So, this density of states varies yes. So, now we see that that this, whatever this density of states, and here we have plotted it up to this. So, all electrons are just filled this, below the Fermi level. So, this is the Fermi level, Fermi energy level.

So, this is basically situation of down state and temperature at t equal to 0, absolute temperature is 0. So, the ground state system so, far whatever we have discussed and there electron is distributed like this, and this density of states I have to keep note here, say density of states dE, what we got that is that equal to 2 m by h cross square 3 by 2 V by 2 pi square e half e half. So, here this you can see this I can write in terms of, basically see, if you see this ef is h cross square by 2 m 3 pi square N by V right. So, it just it just from here, it just opposite to, I think here. Also I can say that N by here N and this is. So, if I want to write in terms of in terms of N and this Fermi energy. You can see N by N by ef N by ef N by ef, what it will give N by ef. So, this term will go. no I think I have to, basically what other way one can do this one, I can write ln N equal to not this term, this term ln N equal to 3 by 2 ln E 3 by 2 ln E ln E, and this term is constant this term is constant ok.

So, this if I differentiate. So, you know the dN by N log is there equal to 3 by 2 dE by e right and this constant. So, it will go. So, from here you can get dN by dE that is nothing but density of states equal to 3 by 2 N by e N by e; so this way also one can express the density of sates. So, that is why from here I was trying to get this expression. So, this also one can get 3 by 2 N e, ok.

So, if it is. So, I think now I can remove this part, because I have kept the note of this. So, here density of states we got now this density of states that expression is very useful for our different calculation. now here whatever things we have distribution of electrons in orbitals, that is at low temperature t equal to 0, absolute temp the temperature t equal to 0, and it is in ground state distribution, but what will happen when temperatures is t is not equal to 0.

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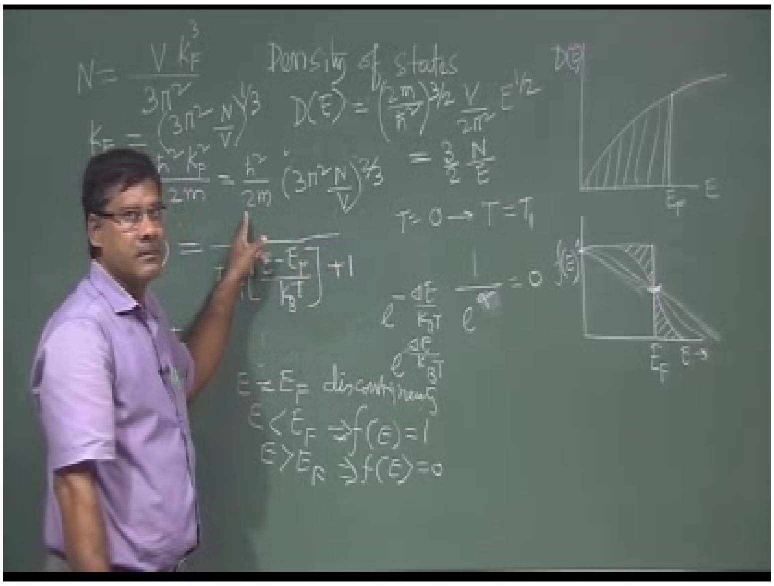

What will happen; then how the electrons will be distributed in the orbitals that are the question. So, here this these density of states or this expression whatever we got there, we assumed that following the Pauli exclusion principle starting from the lowest level their filled up to Fermi level. So, that is fine for t equal to 0, but for t is not equal to 0, there is a temperature. So, what will happen when I will increase the temperature of free electron gas from t equal to 0 to t equal to some temperature; say t 1 what will happen. So, this answer is, for this answers we have to consider the Fermi-Dirac statistics, Fermi-Dirac distribution; that is the basically probability, what is the probability of an electron to stay at a particular energy state when temperature is increased to t equal to 0 to t equal to some temperature.

So, this Fermi; So, to know that distribution that is the to know how to know that distribution at higher temperature, then this Fermi-Dirac statistics Fermi-Dirac probability, Fermi-Dirac distribution is the valid one and that is 1 by exponential e minus ef by kt KBT Boltzmann constant plus 1. So, that is the Fermi-Dirac distribution. So, if temperature exist for the system, it is not equal to 0, then following this distribution one has to find out the electrons in different energy levels. So, from here you can see that when t equal to 0, when t equal to 0. So, this is infinity right. So, this denominator is infinity. So, 1 by infinity is 0.

So, when t equal to 0 this f 1 is 1 f f e is 1. So, for what you are getting for t equal to 0 f thus distribution f equal to 1 right. so; that means, is it correct, yes when it is 0. So, that is infinity this is 0; so this infinity exponential infinity. So, this will be 1 right now when t is not equal to 0 that will come. So, for t equal to 0 when. So, then f e is 1 fine. Now if I consider for t equal to 0 and e is equal to ef e is equal to ef, so this. So, 0 here it is 0 by 0, so it its undefined right. So, it is the signature of disconnectivity, it is a signature of a disconnectivity curve will discontinue for such case, and when e is not equal to ef e is less than ef below the Fermi level. So, it is negative right it is negative some value right. So, it is like this exponential be like say e to the power minus del E by kbt right.

And when e is greater than ef e is greater than ef right. So, this is greater than ef. So, this will be positive quantity so not minus. So, del e by kbt. So, in this case for t equal to 0, for t equal to 0 this is infinity. So, this is infinity. So, I can write this 1 one by e to the power del e by 0 means infinity. So, this term will be infinity. So, 1 by infinity will be 0 1 by infinity will be 0. So, this term will be 0 plus here 1 is there minus 1 by 1. So, in this case in this case, this fe you will get 1 right, and for other case for other case it will be 0 for other case, because this plus right.

So, this will be infinity; so this infinity plus 1 sorry. So, for t equal to 0 this is infinity. So, e to the power infinity is infinity plus 1 infinity 1 by infinity is 0. So, this part will be for this case fe will be 0. So, this is telling at t equal to 0, I think this right, not right statement I think this is the right one. So, here you can see that when temperature below not temperature this energy is below Fermi level. So, probability of filling the this orbitals with electron are 1; that is the maximum probability, and this when it is above the Fermi level. So, this it is probability zero; that means, there will not be electron above the Fermi level.

So, you can one can draw this distribution it is like this. So, this is basically e and this is Fermi distribution. So, maximum value is 1, maximum value is 1, and here. So, as I told that when e equal to ef there is a discontinuity when e equal to ef, there is a discontinuity. So, this is discontinuity, now when temperature increase. So, it is not t equal to 0 t have some value. So, what will happen from here you can show that this this curve will be like this. For other temperature curve will be like this, I think it is not. So, these are higher temperature higher temperature. So, basically this part is telling that this electron will be removed from here and occupied in this place. So, from here easily you can explain this so far this one.

So, all the time for any temperature when e equal to ef this is 0 this is 0. So, exponential 0 is 1, so 1 plus 1 2; so by 2. So, here this value this is basically this probability fe is half all that time for all energy. So, this important fact this important fact, and if temperature is such that. So, depending on temperature, if temperature is; so what we had seen the electron, when increase the temperature electron are ex going to the excited level (Refer Time: 39:43) from where near the Fermi surface.

So, temperature is higher and higher, and then it is going deeper and deeper. So, this energy when energy when temperature such that electron will be exerted from here also so lowest energy level; so in that temperature. So, this basically we write tf Fermi temperature and KBT, this energy will be such that this, it will be equal to ef; so when this temperature such that when kb tf is equal to ef Fermi energy. So, this temperature is called Fermi energy.

So, these are basically we take as a reference when temperature, whatever temperature we take. So, it is greater than tb tf or it is less than tf if it is less than tf, means electron will not be excited from here. So, it will be this side when it is greater than t tf. So, electron will exceed from here also. So, it will go like this also right, like this also. So, these are the significance.

So, I will stop here. I will continue in next class.

Thank you for your attention.