Solid State Physics
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Lecture - 35
Electrical Properties of Metal (Contd.)

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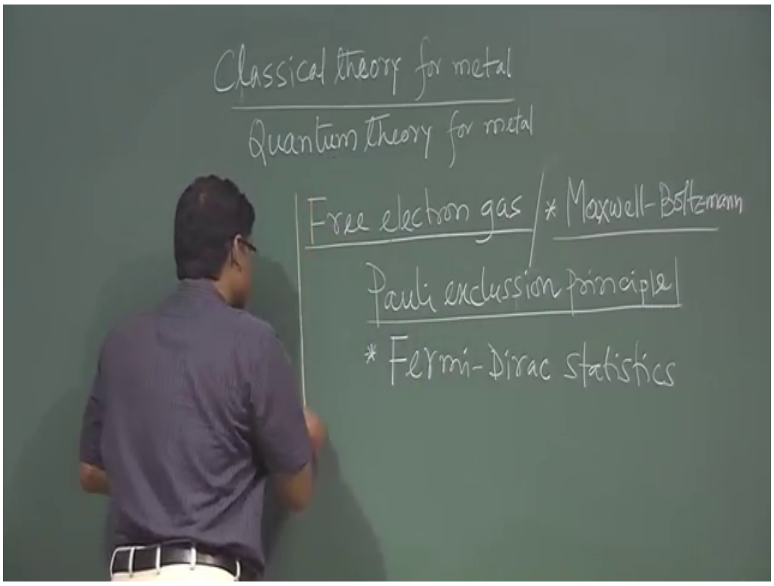

We have seen the classical theory for classical theory for metal to explain different properties. And we had seen this it is successful partially, but it has it has also remarkable failure. So, as I mentioned that we need to tilt this metal using the quantum theory quantum theory for metal. Basically the free electron gas it is true it is there in metal these free electrons are there.

So, in it may not be completely free it may have may have potential, but it is weak potential so, but main thing is that this free electron gas it is there in metal we can tilt the metal as a source of free electron gas. Now the failure of this classical theory is basically here this electrons it is considered like a gas atom material in gas form. Atoms was neutral whereas, the electron is have charge. So, that is the big difference and for this for free electron gas this it is classical because in this case the Maxwell -Boltzmann statistics was used.

This is basically classical statistics here in this case there is a distribution of there is a distribution of particle of electrons energy distribution or velocity distribution, whatever the distribution say energy distribution. So, that distribution is basically following the Maxwell- Boltzmann statistics it was taken that the distribution. In this case it may have this electron gas will have all sorts of energy it can take, it will be distributed among the continuous energy and statistic will tell this what is the how many particles are there with a particular energy or in a particular energy range e 2 e plus d, how many what is the how many particles are there.

That depends on temperature, but at 0 temperature if you consider 0 temperature then this all electron gas electrons we will try to go to the lowest energy level, lowest energy to minimize the energy of the system, thus this classical statistics allow these all electrons to go to a lowest energy. That was the problem because it is not allowed for electron is Pauli Exclusion Principle will not allow all electron to go to a one energy level right to have the one energy.

These you know this Pauli Exclusion Principle is very important and you have seen this in case I have when I thought you about the atom, these electrons are distributed among the orbits or orbital's it strictly follows the Pauli Exclusion Principle. This free electron gas there these electrons are treated just like gas particle and it can have all electrons can be put in a energy level or as many as electron one can put in energy level, that is not allowed because it is not gas particle it is electron.

It will not follow the Maxwell Boltzmann distribution statistics, it mainly that was the problem with this model, it is Fermi - Dirac Statistics is the right choice for this electron gas. So, that was the main difference between the classical theory and the quantum theory or free electron of metals so, but quantum approach, quantum mechanics or wave mechanics.

It has own language it is quite different from the classical mechanics, hopefully you are started to be familiar with these quantum mechanics, but I will try to use quantum mechanics from the basic.

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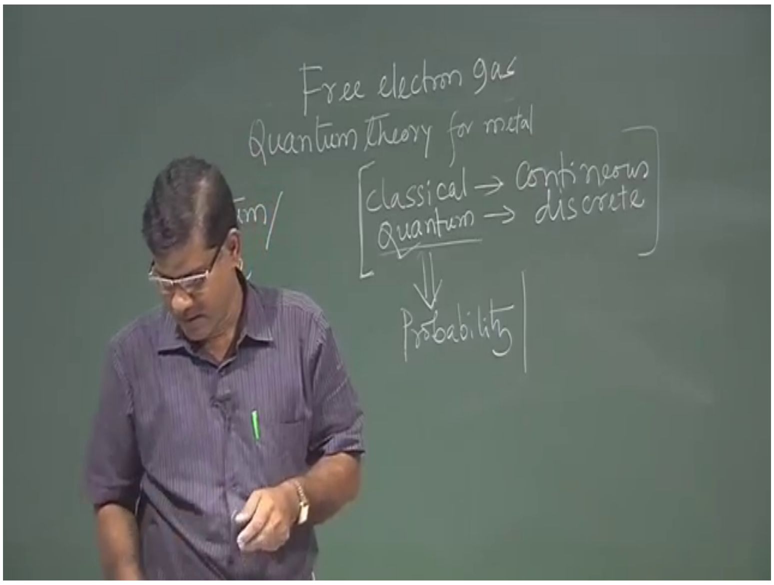

Let us just tell me few words about the quantum mechanics or wave mechanics you see this and classical mechanics main difference is that this classical mechanics is basically one can tell continuous energy is continuous any energy particle can take any velocity particle take can take etcetera, continuous and quantum mechanics is basically discrete.

You have seen from both theory that electron or orbits cannot take any energy it can take only particular specific energy and that is the main one difference and another in case of quantum mechanics, momentum and position of a particle we cannot determine simultaneously. If you determine one precisely, uncertainty will come to another one so, but in classical mechanics it is, one can determine position as well as momentum accurately simultaneously.

These are the things are there right and this quantum mechanics is basically deal with probability and that one has to find the probability of staying a particle at this position or to have the momentum some value. So, that is the probability it is not always it is not only that value it will have that is only that value that is accurate value, but we can only find the probability under this situation this particle can have position in this place or it can have the momentum of this value or energy of this value etcetera. It is a probability; it is not what has to find out the probability.

Now, for free electron gas now our topic is free electron gas, how to apply quantum mechanics, that we have to see, this is as I told this quantum mechanics or wave mechanics we tell wave mechanics, wave mechanics and, this light is a wave or X- ray is a wave, light or X- ray.

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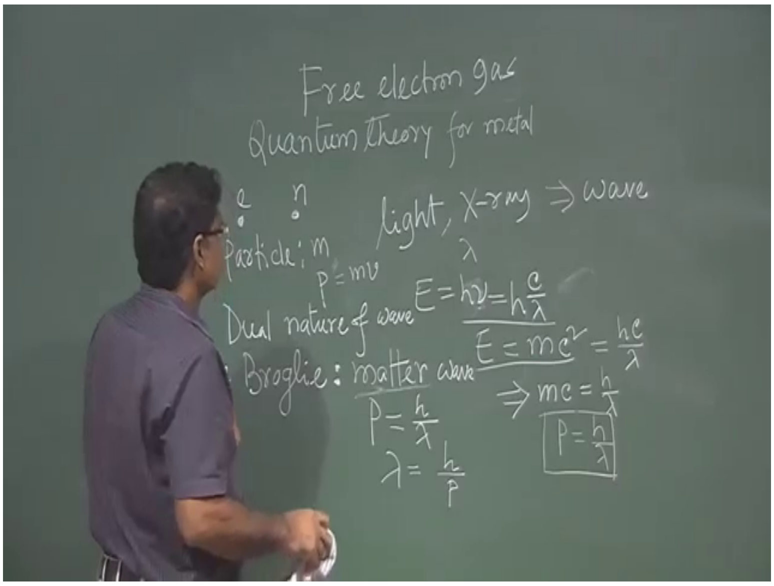

In general electromagnetic wave electromagnetic radiation, it is known as a wave form wave and it is expressed in terms of wavelength wave means it has wavelength right and particle either one electron or one gas molecule. These are this basically we will consider them as a particle they are particle, electron is a particle, neutron is a particle, proton is a particle, atom is a particle, right neutron is a, so it can be electron it can be neutron it can be proton, it is a particle. Particle means it will have mass, it will have momentum, P momentum like basically in mv.

It is particle when we tell particle, particle have mass when we tell wave do not have mass, but it has wavelength right. First time it was that Plank mentioned about the about the quantization in terms of some, this is some plank constant. So, energy of that quanta this is h nu, this is the energy, that electron have this light or X- ray it is wave if it is frequency is nu then or wavelength is lambda then light velocity c, c by lambda, that the frequency nu, h c by lambda that will be the energy of what, energy of photon.

Light it is well known as a wave it has wavelength, but this light also it can be treated as this whole light whole wave and it can be treated as a the propagation of the quanta, that is called photon, is the propagation of the photon, it is a discrete nature of this wave, that is basically called the photon, it is just like as if it is giving flavor of the particle.

This wave or light it can have a particle nature, if you consider it then only you can write that energy of this is not wave energy of that then photon, part of the wave, it is as it is quantized, make it quanta, or many quantum size of the portion and that we tell this photon and this photon each photon will have energy E equal to h nu or h c lambda alright and that and from Einstein's mass energy relation.

So, E equal to m c square, if any particle is moving with velocity with light velocity with velocity c, it is energy will be m c square, from here if you can see that I can write this equal to h c by lambda and this is given m c from here I can write this m c equal to h by lambda. So, m c mass into velocity this is momentum right, I can write this P equal to h by lambda.

This with light one can hear, one can see that, one can get the momentum of that light that is free, as if this wave also has some mass that feelings it is giving that feelings waves also have mass. So, it is mass have only this for particle right when we consider particle, it is we accept that it has mass. When it is when in case of wave we accept that it has wavelength, but in case of wave it will have the momentum, it will have the mass, so that is not in general it is not accepted, but it is the case from here.

These 2 both these 2 are well this one and this one, I can write E equal to h nu and then this, these are the this 2 relations are very famous and if we just go ahead with these 2 energy for wave electromagnetic radiation say for light, we can we reach here. So, as I told momentum is basically is it is the parameter for particle and wavelength you say parameter for wave, but in case of wave light here we are saying they have relation P equal to h by lambda as if from here we can tell that wave has dual nature, it has wave nature as well as it has particle nature.

So, duality dual nature of wave right, now, De Broglie it is extend this he extended this concept or matter De Broglie extended this concept for matter. He mentioned that matter of material or particle may have also the wave nature, that is called really matter wave or particle behave also like a wave it has particle nature also it has wave nature. So, just extending this concept or particle De Broglie mentioned this about the matter wave and it will follow this relation.

If matters have the momentum P then wave associated with this matter will be this wavelength will be lambda and they will be related like this. So, that is why this lambda is called the De Broglie wavelength of matter. Now, this free electron gas, electron is particle now electron is particle, electron can be treated as a free electron it can be treated as a wave also right, it may have it will have the wavelength lambda equal to h by P.

That means, for this free electron gas we can use wave mechanics or that is quantum mechanics as I mentioned this quantum mechanics is called also wave mechanics. So, it is wave mechanics, either it is quanta or it is wave, quanta means one term, quanta from quanta means this it is particle quanta quantized quanta discreteness it is not continuous. Thus here I just try to justify that free electron gas it can be treated as a wave, wave mechanics that is quantum mechanics can be used for this electron free electron gas.

Now free electron it is in a inside metal and I mentioned earlier that electron are completely free in inside the metal and we can take the potential energy of electron E 0 inside the metal.

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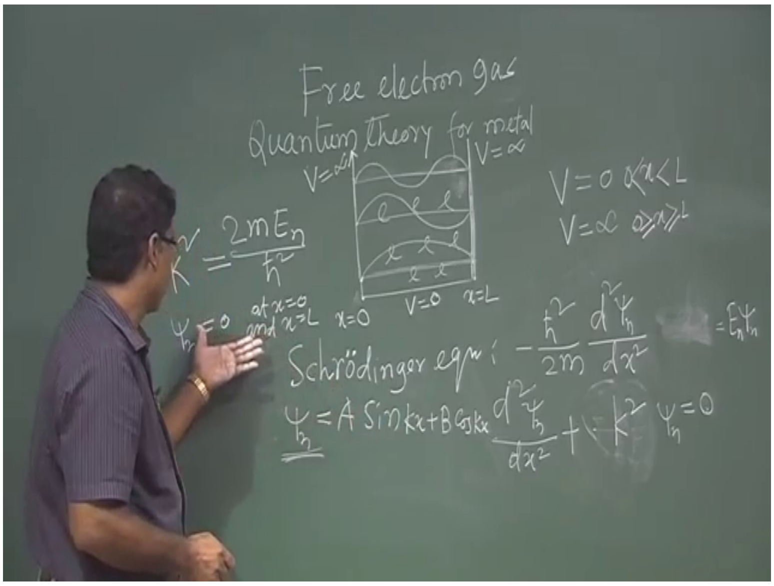

So, just consider a as if this inside metal electrons are there, inside this potential is 0, if I consider that if it is in a box it will be 3 dimensional box right, but for simplicity we can consider the just one dimension and then we can extend for 3 dimension, one dimensional box.

Here if I put the coordinate here x equal to 0 and here x equal to L this is the dimension of the box in one side, one dimensional case if I consider, electrons are there. This potential energy it is 0 when x is less than L and greater than 0 and potential energy is very high. So, for convenience we really take infinity when it is just outside of this box or it is just on the surface of the box.

X means when x is greater than equal to l or less than equal to 0, this we have, when we consider because we mention that because electrons are in metal and that electrons at even at room temperature they do not come out from the metal and simultaneously we had seen these electron are free inside the metal. So, that is why it is potential it is may be small or0, but it cannot come out; that means, that on the surface it is it will have the high potential, it will act as a potential barrier.

This is the realistic situation for electron gas for electrons in metal now that we are just considering that now electron is in a box one dimensional box it is length is L and this outside, this h is a here this potential is very high is infinity outside here also potential is very high infinity whereas, inside within this length potential is V equal to 0 in this inch, thus the we have taken a here you can take this just analogy whatever the we have real system.

So, compared to in comparison to that we have taken for calculation for the purpose of calculation, we have taken this box model and initially we have taken one dimension or you can extend this for 3 dimension and now we have to use the this Schrodinger equation. Wave mechanics quantum mechanics see it is based on the Schrodinger equation this is a Schrodinger equation right what is that Schrodinger equation that is minus h cross square by 2 m, d 2 psi n by d x square right, plus potential energy equal to total energy right is this total energy is for n-th this n we have, I will tell you for n-th state energy of n-th state.

Here we have considered the potential energy 0 inside the box, we are treating the system inside the box, then your equation become d 2 psi n by d x square equal to, minus is there I can take 2 m E divided by h cross square, here also because psi n is there. Whatever b x I wrote, here also you have to put psi this is for 0, this I can write in psi n equal to 0.

This is the, I have written Schrodinger equation in general then I have to adopt that equation for my system and this is the equation. If I take this as a one parameter 2 mEn by h cross square equal to some K square, what is K square that we will see, I can write then this is a k square psi n right now this see this I do not know you must be familiar with the spin max system when it is oscillating. If you set equations it comes like this right and it is solution is joint solution one can write psi n equal to A Sin kx plus B cos kx right. This is the general solution for this Schrodinger for this equation, this is basically now it is the solution for our system.

Now, for quantum mechanics everything this psi is a wave function which contains all information about that system. Now, one has to extract all information from this wave function and wave function we have to adopt with our system putting the all conditions right. So, what are the conditions here that we have to see here this particle cannot go outside electron cannot go outside. We will not talk about the electron in quantum mechanics it is not electron it is a wave function.

We are telling wave function means it is whatever it contains all the information about this system, instead of telling electron we will tell the wave function of this system free electron gas now, electron cannot go out means here we will tell psi n cannot go out. So, always it will end here start here and then same here see when we it is basically waves, it is it can be like this wave this function wave function psi it can vary like this.

So, here we have to put the condition because we have to be realistic right, psi n will be 0 at x equal to 0 and at x equal to l, this is the condition boundary condition. So, this is the general solution now we have to apply this boundary condition and find out we have to make more particular for our system.

I will stop here. We will continue in next class.

Thank you.