Solid State Physics
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Lecture - 26
Reciprocal Lattice (Contd.)

So, this we are discussing about the Reciprocal Lattice.

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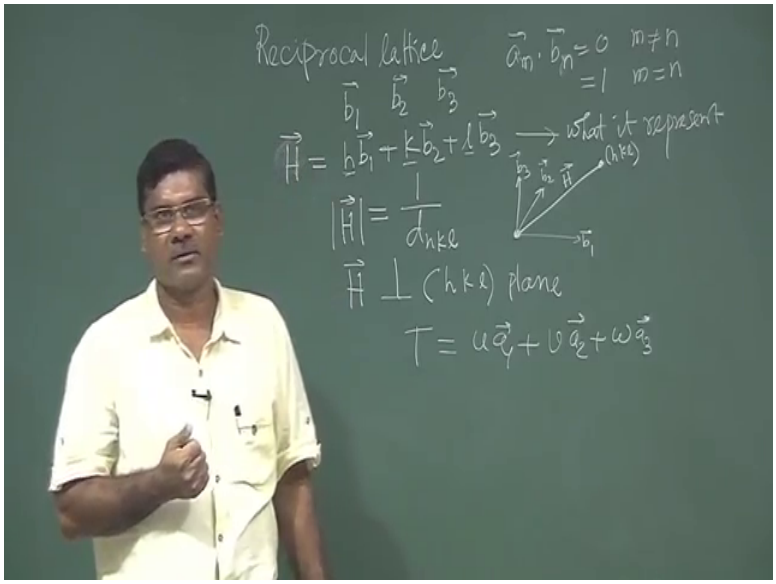

So, it has connection relation with the space lattice, crystal lattice, and we have defined the we have defined the axis system b 1 b 2 b 3 in connection with the a 1 a 2 a 3. So, also we have seen the relation important relation is a n dot b n b n equal to 0 1 when m is not equal to n and m equal to n right. And reciprocal vectors equivalent to the translational vector in space lattice. So, reciprocal vector H I wrote h b 1 plus k b 2 plus l b 3, right.

So, this as I told that just I have written taking miller indices so this is what is the what it represent. So, it basically we can show will show will see that this H it is magnitude will be it is magnitude will be just 1 by d h k l. Now if h is related like this if h follows this relation. Now, you can say that this h k l whatever have taken that that justified right. So, H written in this form so then h magnitude of h will be the reciprocal of the planer spacing of h k l planes right. So, this planer spacing is d h k l. So, that we have to prove

and then another it represent that from a b is a 1 b 1 b 2 a a 1 sorry b 1 b 2 b 3 this axis system it has origin.

So, this vector will represent this vector will represent if this is the origin for b 1 b 2 b 3 b 1 b 2 b 3 and this is a translational vector like we are telling reciprocal vector H right if it is coordinates this h k l. So, this which point, in this axis system these are the h k l point h k l point coordinate like in x y z coordinates system we represent a point with x y z coordinate similarly here b 1 b 2 b 3 in this axis system if this is a vector. So, this end of this vectors will be the point h k l right and this will be direction of this 1 it will be it is it will be it is basically perpendicular to the h k l plane.

So, H magnitude is this and it is direction H is direction will be perpendicular to the h k l plane it has to be perpendicular to h k l plane. Because that is why that is that is what you have seen in Laue spot. So, Laue spot each spot is basically 1 plane we are identifying that or designated each point with the h k l value. So, if you if you imagine that this Laue spot if they are the reciprocal lattice they are in reciprocal lattice. So, if I go from 1 point to another point. So, each point representing the lattice this lattice plane, this if I just draw taking origin just 1 point if I draw a straight line up to another point. So, that will be that that will be the direction of this plane right and it will be it is it is basically perpendicular to that plane.

So, that is what this Laue spot we will try to explain right. So, from each plane is set of planes we are getting a Laue spot or will get Laue spot. So, that depends on the orientation of the lattice planes in this crystal system. So, depending on that orientation will get the distribution of the Laue spot and that each spot correspond to a plane and now we are seeing that the distance if I take a origin and if I just draw a line to up to any point.

So, it will represent the it will represent the normal to the corresponding to that plane and this magnitude of this line will be will be the reciprocal of that planar spacing. So, that is the necessity of this reciprocal lattice. So, in a plane or in a space what we are getting the diffraction pattern. So, that space is basically if I define that space with some other coordinate system axis system, where I can identify I can identify or I can represent each point with just with coordinate.

So, task will be easier then the explain the result in terms of the lattice plane. So, in terms of lattice plane as I mentioned that orientation etcetera so it is slightly difficult, but it will be easier if I can represent if I can explain the things, just using the coordinate as like translation vector as I mentioned u a 1 u plus v a 2 plus w a 3. So, here just these are giving the points and the coordinate of this points u v w is in space lattice now will tell this lattice space lattice is direct lattice.

So, because other one we have defined another lattice so that is called reciprocal lattice. So, just to differentiate these 2 so this space lattice will call the direct lattice. So, in direct lattice whatever the points we can designate those points using the just coordinate u v w, but planes we cannot define in simple way to define the planes we have to take find out the miller indices and then we can define. So, in terms of miller indices we have to define and so we have we have produced another type of lattice that is called reciprocal lattice where is the each point will represent the planes.

So, now in this lattice here we will be able to represent the planes just using the coordinate that coordinate as I told is that will be h k l. So, that is the advantage we will get using the reciprocal lattice to study the direct lattice. So, our aim is to study direct lattice, but this is the alternative this is the convenient way to deal with is crystal structure.

So, I think it is a clear what is reciprocal lattice and how it is defined what are the axis of these lattice and how the axis are related with the with the direct lattice because it cannot be arbitrary right. So, our aim is to explain the direct lattice crystal structure. So, we are forming alternate reciprocal lattice to analyse to analyse the diffraction pattern diffraction it is a diffraction result from where we will extract the parameters for real crystal.

Now, I as I have a told this I have to show that H is basically perpendicular to h k l plane and it is magnitude is just inverse of the planer spacing of h k l planes.

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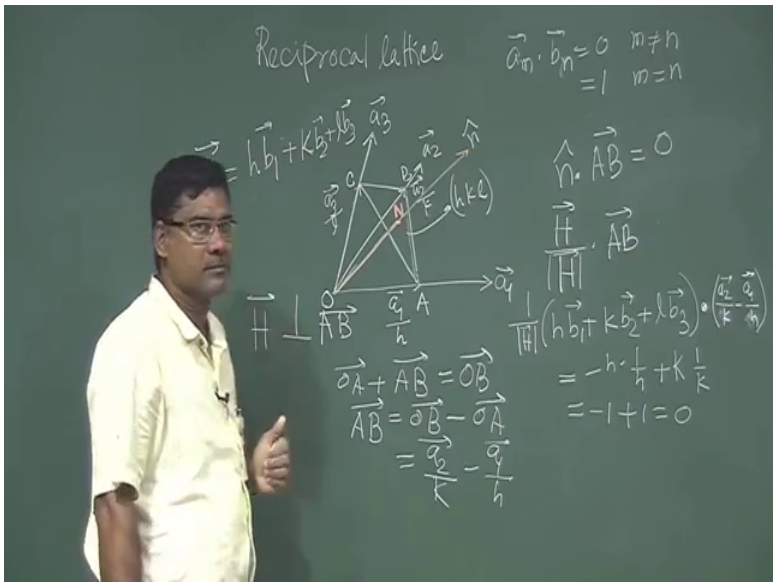

So, to show that 1 I have to. So, I will so let us take a axis system in direct lattice a 1 a 2 and a 3 right. So, in this axis system if I just draw a plane if I just draw a plane I think I should draw properly otherwise it will be I think this way if I draw. So, I think this is a plane a b c a a b c.

So, it will intersect a 1 axis by a 1 by h right that we are seen earlier. So, b 1 it will it will be this intersect will be a 2 by k and this intersect will be a 3 by 3 by l. So, this magnitude will be a 1 by h this magnitude will be a 2 by k and this magnitude will be a 3 by l right. So, this is (Refer Time: 15:31) then this is an h k l plane if it is h k l plane. So, this is a h k l plane. So, h k l plane it intersect at a 1 a 2 a 3 axis by this.

So, this is the origin now let us take a normal on this plane. So, I will use say different colour. So, this is the normal on this plane this is the normal on this plane. So, this is the, say N is the normal on this plane. So, this is the, of this normal on this plane if it is O N. So, if N is normal on this a b c plane and this is if this normal. So, normal on a plane normal on a plane if I represent with this 1 if represent H equal to h b 1 plus k b 2 k b 2 and l b 3 if I take the direction normal on this plane, if I take this 1 because whether this now I I am going to prove whether it is it is normal on this plane or not that I want to verify right.

So, this is the normal on this plane. So, I am taking this is vector H this. So, if it is normal on this plane. So, it will be and A B, B C, C A they are on this plane. So, it will be

it. So, this H has to be normal on this A B also or B C also or C A also right. So, if then if I can show that H is normal or is perpendicular to A B. If I show that H is perpendicular to AB then it is I can tell that is H or other a 1 has to. So, 1 other line also one can show that this is perpendicular. So, then it is confirmed that this H is perpendicular to this plane. So, here A B, if I take vector O A and this is A B.

So, from here I can write O A plus A B right O A plus A B equal to O B equal to O B equal to O B. So, A B equal to A B equal to O B minus OA right. Now what is OB? OB is basically a 2 by k O B is basically a 2 by k and what is O A, OA a 1 by h so minus a 1 by h. So if in this direction unit vector is n this along this normal direction if it is unit vector is n right. So, n dot, n n dot a b n dot A B will be n dot A B what it will give me n dot a b has to be there perpendicular if they are perpendicular cos theta 90 degree cos theta 90 degree.

So, it will be it has to 0 if it is 0 then this direction is basically perpendicular to this on a b at least before telling this on this plane. So, n n n is n is H by magnitude of H. So, that is the unit direction right dot A B is this an H is H is this we have taken right. So, this H I can write h b 1 plus k b 2 plus l b 3, this and then dot product of A B then dot product of A B what is A B a 2 a 2 by h minus a 1 by k sorry it is k and it is h and that magnitude of this 1 is there. Now these equal to here just see this relation. So, b 1 dot a 2 0 m is not equal to n, b 1 dot a 1 it will be 1, right.

So, I will get minus h b 1 dot a 1 is 1. So, h 1 by h here then b 2 b 2 a 2 1 b 2 dot a 2 1 so I will get plus k dot it is 1 1 by k and then b 3 a 2 0 b 3 dot a 1 0 right. So, here equal to minus 1 plus 1 equal to 0. So, H is normal to A B when if I take h equal to h b 1 plus k b 2 plus l b 3 then only then only it is 0 otherwise not.

So, these say I justified. So, I proved that this reciprocal vector it will be in terms of is coordinate in terms of h k l. So, for similarly you can you can show similarly you can show this it is it is h is perpendicular to a b h is perpendicular to a b h is perpendicular to b c you can show or H is perpendicular to A C just same way you can. So, that means, H, if this is normal to 2 2 vector on this plane. So, one can tell that is the normal to that plane which contain this 2 vectors.

So, similarly one can show this other 1 or 2 and then when I can say that h in this form H in this form this is very important this just I wrote after writing I am just to proving H in

this form it is normal to the h k l plane h k l plane. So, that I have to now I will show that the magnitude of this h is nothing but 1 by d h k l is nothing but d h k l.

So, the just if easily one can show that this I have to show this is what is the magnitude of this o n right this is the normal of this plane and what is the magnitude of this O n and that is nothing but the magnitude of that is are that is that is nothing but the is the magnitude of the or this planar spacing of this plane right h k l plane. So, basically I can right I do not need this one I can.

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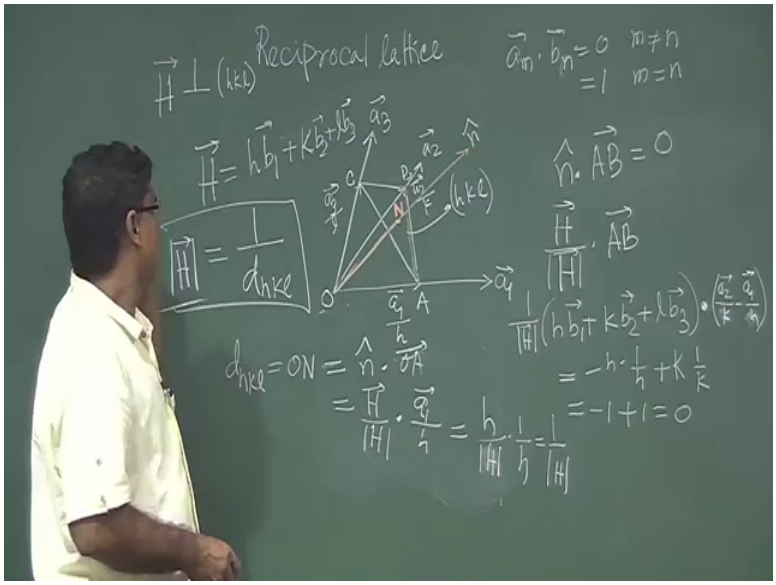

So, what I need that I want to get the value of ON ON and this is known to me that this is this is known to me that this is d h k l right this is d h k l because as earlier I have mentioned in earlier class. So, this plane from origin at origin we can we can consider always we consider this parallel plane of this one. So, if you draw a normal from origin to this plane. So, that is nothing but the planar spacing. So, this ON is nothing but the d h k l right. So, this O N 1 can get this taking the projection of this O A O N this right

So, projection, if this is the unit vectors. So, just take dot product of O A; dot product of O A in this direction if you take so that will be this O N right. So, you can here replace this again this n is nothing but this one n is nothing but this H by H magnitude right H by H magnitude and then O A is O A is that a 1 by h right. So, as if here I have whatever this term I got anyway. So, I will do it. So, from there itself I could tell anyway. So, this what it is giving me so this H vector is basically this 1 h b 1 plus k b 2 plus l b 3 dot a 1. So,

you know following this 1 only m equal to n will give 1 otherwise. So, a 1 dot a 1 means it will be this term will exist other will be 0. So, I will get basically here h b 1 dot a 1, that is a 1 that is 1 and by h is there here right and I have this.

Let me write this say this 1 by h. So, b 1 dot a 1 is 1 and other term will give 0. So, this basically you are getting 1 by magnitude of h. So, d h k l is equal to 1 by magnitude of h. So, h magnitude of h is nothing but 1 by d h k l. So, here what will representing reciprocal vector this is reciprocal vector and we have to write in this form where this coordinate is x y x y z value is h k l by x y z means x y z coordinate system if designate a point by x y z similarly in reciprocal lattice. So, this in b 1 b 2 b 3 axis in that axis system, point will be represented by h k l.

And this vector is perpendicular to the plane h k l magnitude of this vector is magnitude of this vector is means inverse reciprocal of the of the planar spacing h k l planer spacing. And if I draw a this vector from the origin. So, end of this vector end of this vector this coordinate of the end of the vector will be h k l. So, this all this information point (Refer Time: 33:15) by this.

So, I will stop here.

Thank you for your kind attention.