Solid State Physics

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Lecture - 25
Reciprocal Lattice

So, in this class, we will discuss about the reciprocal lattice.

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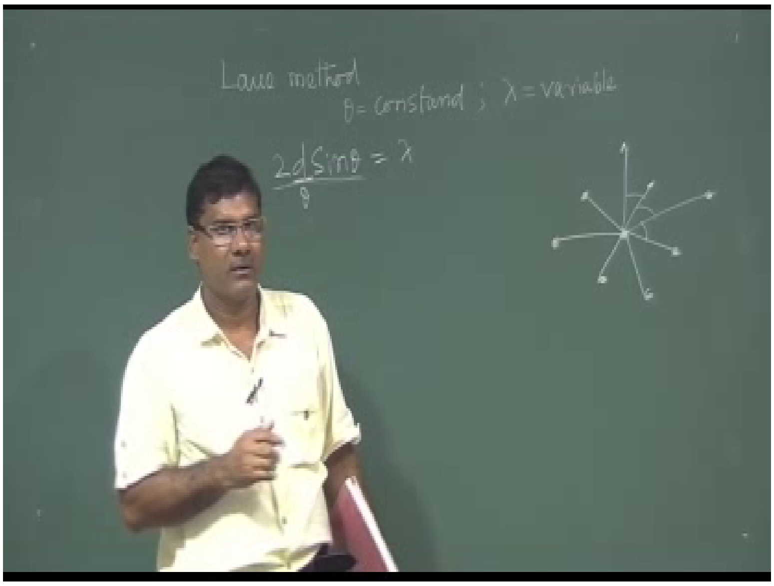

As I mentioned, the diffraction, diffraction spot basically if I take a Laue spot so on a photographic plate. So, they show the symmetry the crystal. So, this Laue spots are of the photographic plate this are they are if we look them they will they have some symmetry as a similar type of arrangement of Laue spot to the direct lattice in case of direct lattice; what about the lattice point arrange. So, in photographic plate the Laue spots are also arranged with some order and each point is basically for consider as a point for a plane.

So, planes of real crystal are converted into points on diffraction space. So, that is basically is called is called the case space also that we will discuss later on. So, so that space where we can think of similar type of lattice as we have seen space lattice. So, in in diffraction space also they are. So, that is we can gets that type of arrangement of points where each point will represent the plane of crystal. So, that planes or that lattice is basically called reciprocal lattice.

So, why it is called reciprocal lattice and how it is important that basically we will discuss in this class. So, so you have say a 1, a 2, a 3 axis in space lattice or Bravais lattice or real crystal. So, earlier I used a, b, c, but now I am using there then as a, a 1, a 2, a 3 because I have to set another axis system. So this, a 1, a 2, a 3 in space lattice, they are if it is; this a 1; this is a 2 and this is a 3 in space lattice. So, they will form. So, they will form a unit cell they will form a unit cell. So, I have I need space. So, this too has to be parallel its model is yes. So, this model is it is done, but it is not perfect one anyway.

So, this is the unit cell this may be like in unit cell. So, here this if I defined; if I defined another set of axis say b 1, b 2, b 3. So, this definition if it is like that; so b 1; let me write b 3 first b 3 equal to a cross, a 1 cross, a 2, a 1 cross a 2 by volume of this unit cell. So, what does it mean what is the relation between this; this a 1, a 2, a 3 axis and b 1, b 2, b 3 axis, if we define the axis relation between this axis like this. So, you can see this b 3 is. So, what is a 1, a 2, a 1 cross a 2? So, this is the area this is the area of this plane right. So, this is the area this plane area this; this represent area I think this basically area.

Now, now what is the volume then? So, if it is area and this direction of this direction of this will be that b 3 direction. So, it will be normal b 3 will be normal to a 1 and a 2 right because this a cross a 1 cross b 2. So, the direction will be the normal to the normal to the to this plane. So, so if b 3 is equal to this then this b 3 direction will be normal to this plane. So, let us draw the normal to this plane. So, the normal; this is direction of b 3, it is the b 3 direction, right.

So, from the definition you can you can get the direction of b 3 and what is the magnitude of this b 3 that let us check. So, what is volume? Volume is basically v equal to v equal to I can write this area into height that will be the volume of this unit cell right area of this one into multiplied by this height. So, that will be the volume. So, just here we are writing this area of what I can define if it is o; o, it is a, it is b and it is c.

So, area of o a b c; area of o a b c; area of o a b c right into height; height of this unit cell right; so, what will be the height of the unit cell? So, the height of the unit cell will be if we just take this projection of this a 3 on this normal. So, that will be the height that will be the height.

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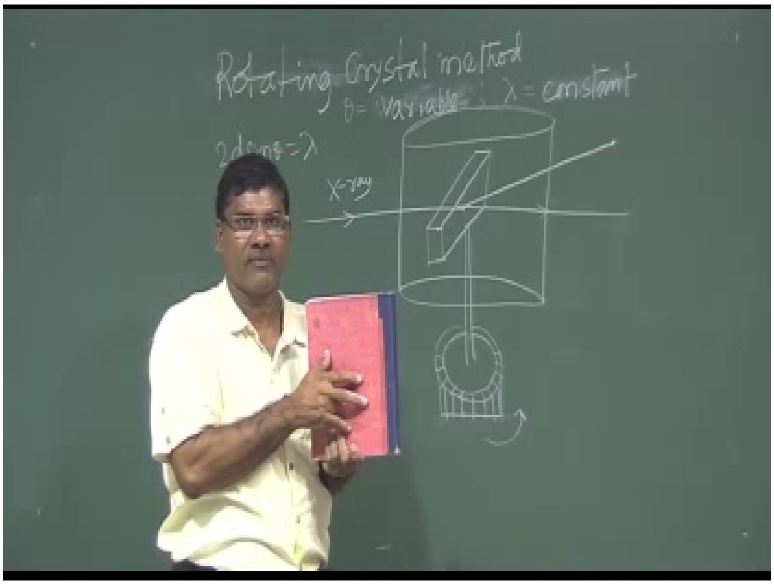

So, these heights if it is say p; so, height will be o p, so area into height o p right. So, b 3 is basically you can write b 3 equal to or magnitude of b 3 magnitude of b 3 magnitude of b 3 will be. So, this, this, area, area, area of o a b c; o a b c divided by area of o a b c into o p right height is o p. So, we are getting basically this one equal to this one equal to 1 by o p right we are getting 1 by o p.

So,. So, what is o p o p is basically height of this unit cell and it is and this it is it is it is also it is normal to this plane o a b c and it is a along the b 3 direction. So, this direction this, this is basically height of which plane if you see this, this is the plane this is the parallel plane of this one this is the parallel plane of this one right. So, which plane it is it is if 0 0 1 plane, right; 0 0 1 plane, this is 0 0 1 plane and this height the 0 0 1 plane and this height whatever this is nothing but the spacing between this spacing that spacing between these 2 planes right.

So, distance between these 2 planes right. So, that is nothing but the planer distance. So, this o p is basically planer distance of 0 0 1 planes. So, this is one plane and distance from this plane to this plane; they are parallel plane. So, that the lattice spacing basically. So, this o p is nothing but o p is nothing but is planar spacing of 0 0 1 of 0 0 1, right. So, so b 3 I got its magnitude of b 3 is got this d 1 0; sorry 0 1 1.

So, similarly we can show we can show that in case of in case of b 1 in case of a b 2. So, that will be the planer spacing the magnitude will be planer spacing of this parallel to this cut this b axis. So, which one this is a axis this 2 a axis and b axis will be this; it is this plane it is this plane and another plane is this. So, that will be distant between this 2 plane. So, same way you can write d 0 1 0 and b 1 will be 1 by d 1 0 0, right.

So, just we have derived one from here the same way one can see that is the magnitude of b 1, b 2, b 3. So, we defined we defined vector this 3 axis like this. So, this will be a 3 cross a 1 by volume right and other one will be a 1, b 1 will be equal to a 2 cross a 3 by v right. So, a 1, a 2, a 3 this is the axis for real crystal for space lattice right.

So, now I defined I 3 more axis b 1, b 2, b 3 taking this definition, right. So, if I take this definition then what I what I see that this is the magnitude of this b 1, b 2, b 3 is nothing but the reciprocal of reciprocal of the planar spacing right. So, so these 3 axis, it is basically reciprocal of some planar spacing. So, now, using this b 1, b 2, b 3 using this axis if we define if we produce points lattice points then it will form a lattice if we produce many points in points. So, it will form a lattice. So, that lattice is basically called the reciprocal lattice right taking this a 1, a 2, a 3; this 3 axis, we can just using translational vector using translational vector right here we can produce points in space.

So, they are lattice point of space lattice right. So, what translation vector we use. So, t equal to u a 1 plus v a 2 plus w a 3 right. So, earlier I used a 1 a b c now just a 1, a 2, a 3 just I mentioned earlier. So, so using this translation vector we can produce all lattice points, right. Similarly using this axis we can produce points in space. So, it will give another lattice and this that lattice is different from this lattice and that lattice is called reciprocal lattice, right.

So, for this also we need translational type of vector. So, this we tell this is called reciprocal vector it is not translational vector, but is similar if this reciprocal vector say if it is h then we can write h b 1 plus k b 2 plus l b 3, right.

Similar to this right here I have considered h k l as a as a coordinate right; so, here whatever u v w; so, here h k l. So, these h k l is same as the miller indices this is h k l are the basically h k l; h k l are miller indices miller indices. So, why should I call why should I take this h k l, right? So, one can you can show that this a nothing but miller indices I can I can take other others say p q r other coordinate, but the p q r later on you can show that is nothing but it is miller indices. So, directly I have written as a miller I have used miller indices.

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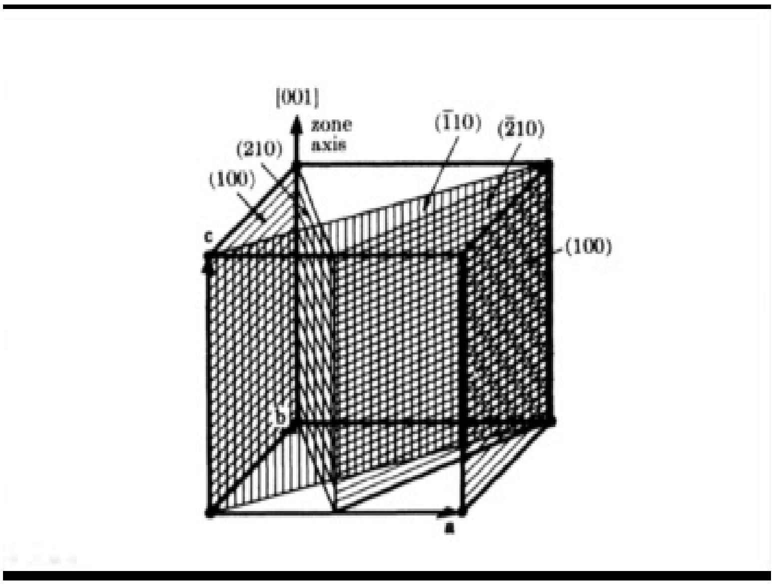

So, we will find out later on so, but one can guess also that. So, from here you are seeing that basically direction is this b 1 direction b 2 direction b 3 direction, it is the directions are in this case; it will be; it will not be along the a 1 axis. So, that I cannot say, but if you take orthogonal axis system like cubic orthorhombic and tetragonal where this axis are orthogonal a 1, a 2, a 3 orthogonal then we can show that b 1, b 2, b 3 also they are orthogonal and a 1 will be parallel to b 1, a 2 will be parallel to b 2 and a 3 will be parallel to b 3.

So, this is the reciprocal vector we tell this reciprocal vector we define like this where h k l are miller indices. So, not necessary to take directly miller indices other parameter also we can take, but ultimately we will see later on that they will be nothing but the miller indices, right.

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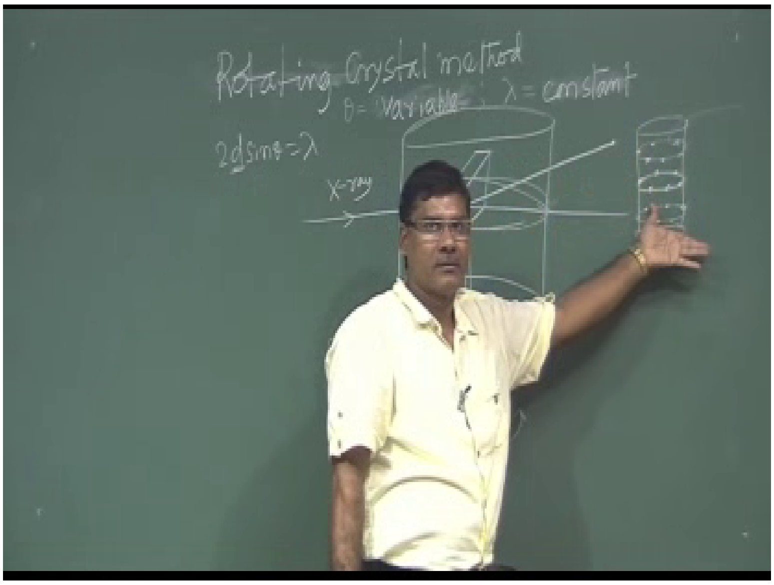

So, now, important fact is that. So, from here we can see that b 1; b 1 is b 1 is the direction, b 1 direction is perpendicular, this is the perpendicular symbol right, this is the b 1 is perpendicular to this plane. So, on this plane a 2 and a 3 vectors are there. So, so if b 1 is perpendicular on this plane. So, it is perpendicular to a 2 as well a 3, right.

So, b 1 is perpendicular to a 2 and a 3 right a 2 and a 3, then b 2 is perpendicular to b 2 is perpendicular to on this plane, right. So, it is perpendicular to a 1 and a 3 right and b 3 is perpendicular to a 1 and a 2, right. So, so these axis are perpendicular to these are axis perpendicular to the; so, the axis of real crystal and then we can we can see that if they are perpendicular. So, we can we can get some relation that b 1 dot a 1 b 1 dot a 1 I cannot say I cannot say, but I can say b 1 dot a 2 equal to b 1 dot dot product basically is 0, right.

Similarly, b 2 dot a 1 equal to b 2 dot a 3 equal to 0 b 3 dot a 1 equal to b 3 dot a 2 equal to 0, right. So, in general in general I can say in general I can say a m dot b n equal to 0 equal to 0 when m is not equal to n right when m is not equal to n. So, so for what about the other one what about the other one this when this m and n are same like b 1 a 1 dot a 1 b 2 dot a 2 b 3 dot a 3. So, what about that one? So, this say if I take b 3 b 3 dot a 3 b 3 dot a 3.

So, what will be that? So, b 3 is basically b 3 is magnitude along this axis b 3 axis right and magnitude is what is 1 by d 0; sorry, this I did mistake. So, it will be 0 b 3 will be 0 0

1 yes. So, so it is b 3 magnitude is 1 by d 0 0 1 1 by d 0 0 1 is nothing but o p right its nothing but this o p magnitude o p right and so, here I will not. So, basically magnitude of b 3 and magnitude of a 3, then this cause some angle between these 2 angle between these 2, right. So, so if I take the magnitude of b 3 that is o p and now the unit vector along this b 3 now if we take the dot product of this a 3 on this unit vector along this direction. So, that is basically projection of a 3 right projection of a 3 on this.

So, this dot product is basically this o p then magnitude of this one and the projection of this. So, projection of a 3 along this direction; so, projection of a 3 along this direction what is that that is. So, this a projection of this; this will be basically I can. So, this a 3. So, this will be o p. So, I did mistake. So, b 3 magnitude of b 3 is magnitude of b 3 is 1 by o p. So, I have erased it I erase it. So, it is 1 by o p because o p is this d 0 0 1, right d 0 0 1. So, this 1 by o p o p is d 0 0 1 right. So, into the projection of a 3 along this direction that is projection of this along this direction is o p.

So, this basically equal to 1 by o p into o p. So, is equal to 1. So, so I can write that this b 3 dot a 3 that is one similarly one can show a 1 dot b 1, a 2 dot b 1, a 2 dot b 2 will be equal to 1. So, this will be equal to 1 when m equal to n. So, this is very important relation and which follows the reciprocal which follows the reciprocal lattice in connection with the space lattice.

So, here whatever you have done. So, you have defined axis system b 1, b 2, b 3. So, this is the axis system of reciprocal lattice why it is reciprocal as I defined because this is reciprocal lattice this axis system are related with the planer spacing of the space lattice and the relation is inverse is reciprocal with each other. So, that is why it is reciprocal and relation between the axis systems of both. They follow this and also this translation vector produced can produce the space lattice space point. Similarly, these reciprocal vector this is the reciprocal vector it can produce the similar lattice which is basically reciprocal lattice.

So, I will stop here. I will continue in next class.

Thank you for your attention.