Solid State Physics
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Lecture - 13
Crystal Structure (Contd.)

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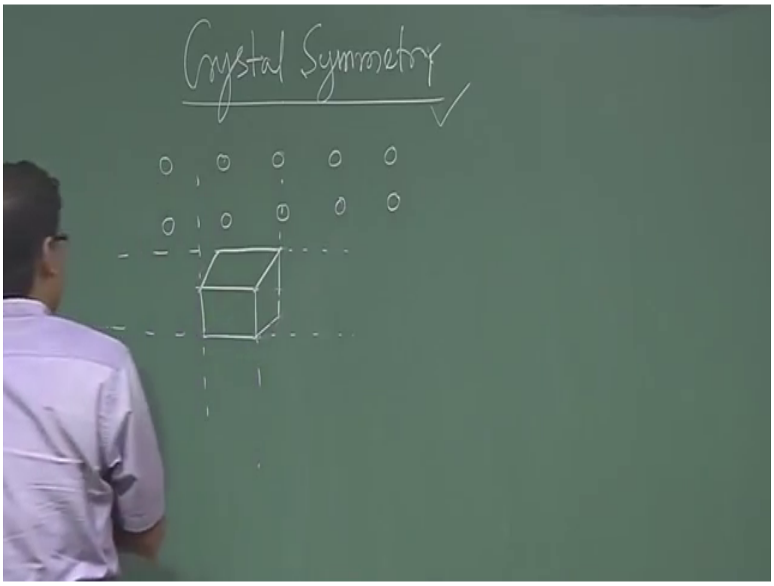

So, in this class, we will learn about the crystal symmetry. So, basically, crystal is have a puric nature, means lattice points are arranged, purically orderly right or unit cell. Unit cell are repeated in a space. So, this unit cell are repeated in means space in all direction, right. So, what I want to mean that crystal has periodic nature, either in terms of lattice point or in terms of unit cell.

So, whenever any system have periodicity, periodic in nature. So, it had some symmetry. So, crystal also have symmetry, because of this periodic nature. So, this, there are mainly four types of symmetric element or symmetric operation, one can apply on crystal. So, symmetric operation basically, when you operate on a system after operation, whatever change will occur whatever change will happens and then after that operation, it will come to it's original configuration. So, one cannot distinguish before operation and after operation.

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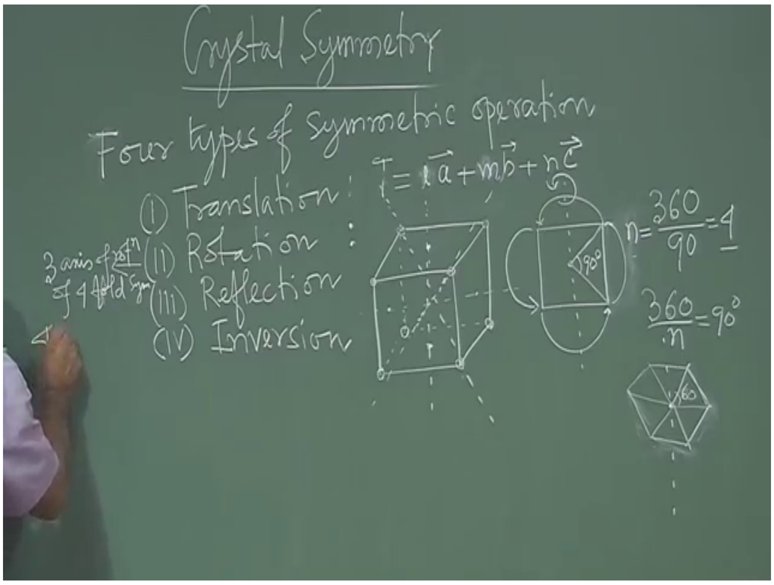

So, that is the symmetric operation. So, this mainly four types of symmetric operation, or seen in case of crystal structure, means this four type is, it is invariant with respect to this four symmetric operation. So, one is translation, second is rotation, third is reflection and forth is inversion. So, this four types of operation, symmetric operation, one can apply on the crystal and after operation, we can get the invariant position invariant configuration.

So, this translation you know that as we have discussed that crystal lattice, one can generate from using this translational vector. This T equal to what I wrote, I think n l m 1 can write n a plus m. So, l m n. So, taking all possible value of l m n one can one can generate lattice points in space. So, so this lattice itself is defined that this environment or surroundings of each lattice point is identical.

So, from, in crystal, one lattice point to another lattice point, one can go using this translation of vector. So, when it will go there. So, you will get the same environment. So, whether this, whether any translation or any shifting from one position to other position of a point, it, you are cannot distinguish. So, it is crystal had that symmetry and basically crystal lattice or lattice points are generated using this translation vector. So, it will have this symmetric, is obvious, it is very obvious, but this other rotational reflection inversion that one can check whether, how; this crystal shows this symmetry.

So, rotational basically, in crystal, if you choose, a choose an axis and with respect to that axis, if this crystal, this is rotated. So, after rotation, if you get the same configuration; if you get in variant position configuration, then we will tell that this crystal have rotational symmetry and that axis will be called axis of rotation. So, if I take this a, say cubic crystal of. So, this is the cubic crystal, this unit cell eight corner 8 lattice point.

Now, if I choose an axis. So, the center of the phase and perpendicular to the phase, is passing to the opposite phases. So, the middle point of the phases. So, now, with respect to this axis, if I rotate this crystal, if I rotate this crystal; so, what will happen? So, just if I draw just 2 dimension, just if I draw this phase, if we make it square. So, this space I have taken and this is the axis of rotation. So, it is basically this axis and this other part is attached to this phase right. So, when we are, this phase will rotate. So, whole things will rotate.

So, that way if we imagine; so, what will happen with respect to this axis. So, so when you will rotate, when you will rotate, you see this corner will come here. So, this corner will come here, after rotation of 90 degree, after rotation of 90 degree. Similarly, so, it will. So, you cannot then, after this rotation of 90 degree. So, it will be invariant. So, what about the, now we are seeing. So, after 90 degree rotation with respect to this axis, with respect to this axis, basically, you will have the same configuration. So, one cannot distinguish.

So, similarly, another 90 degree, it will come in again, in same configuration and then another 90 degree, then another 90 degree. So, 360 degree. So, complete rotation 360 degree. Now, in this case, in this cubic crystal with respect to this axis. So, basically, after 90 degree, we are getting the inherent position, same configuration. So, so n equal to 360 by 90 that is 4.

So, this four basically, it is called, this fourfold, this fourfold axis. So, what does it mean with respect to this rotation axis, this we can, we can rotate this q and with it, it comes in four primes in symmetric position, in variant position. So, if we know this is fourfold symmetry, it has fourfold symmetry then what is the angle that you can find out 360 by n. So, it is fourfold symmetry. So, after 90 degree, we will get the symmetry, in case of hexagonal, in case of hexagonal 1 2 3 4, 1 2 3 4 5 7 8.

So, this is a hexagonal, we know this hexagonal bravais lattice, where basically a b c, a and b are same, but c are different and two angle are same, 90 degree with this one, but one angle is 120 degree. So, if you rotate this crystal with respect to this axis. So, dotted line I am giving, because this is perpendicular to the plane. So, then after 60 degree, this 60, then another 60, then another 60. So, 1 2 3 4 5 6. So, it will have 6 fold, this axis will be called 6 fold axis and; that means, n equal to 6. So, after each 60 degree rotation, you will get the symmetry, you will get the invariant position.

So, this is the one, has to choose proper rotational axis, axis of rotation and then with respect to that axis of rotation, what is the, how many fold of symmetry exist for that system, that crystal, that one has to find out. So, in case of cubic crystal as I. So, it has rotational symmetry, this crystal has rotational symmetry, not rotational symmetry, it has fourfold symmetry. So, degree of symmetry depends on, if it is just one fold symmetry means, after complete 1 3 60 degree rotation, you will get the symmetry. So, that is obvious always, this happens most of the case, it happens.

But it is a twofold, threefold, fourfold. This as many number of fold of symmetry are there. So, that are basically, it it tells that, this system are more symmetric. It has highest degree of higher degree of symmetry. So, another thing, this is not only the one axis, with respect to that crystal, have cubic crystal, have absolutely rotational symmetry. So, there are other axis also, one has to find out so; obviously, we can see this through these two surface. Another two, these two surface, if I take the axis, if I take the axis. So, that also, it will be four folded axis of rotation, fourfold axis of rotation.

Similarly, you have this, other two phases, this phases and this phase. So, through this 2 phases also, through the middle of this, these two phases, if you draw the axis, it will draw the axis. If you draw the axis basically, you will get, that will also this similar that same similar, rotational axis having fourfold symmetry.

So, basically, for about this, about the axis passing through the center of the phase. So, this cubic crystal have 3 axis, 3 axis of rotation. About this axis, passing through the center of the phases. So, other axis also possible, say if I take axis with respect to that body diagonal, you know I take the axis with respect to body diagonal. So, it will have also, it can, it will have the rotational symmetry. So, with respect to this axis if you rotate. So, one can see that after 120 degree rotation, it will come to the, original configuration.

So, this is also rotational axis, Now, it has how many fold, this I think, it will have three fold symmetry. It will have threefold symmetry, because n 360 by 120 . So, it is the n equal to 3. So, you will have threefold. So, here this is four fold symmetry, 3 axis of rotation of fourfold and then you will have this type of axis passing through the diagonal, along the diagonal. So, how many; you can have this one. So, one and then this and this other side, which one I think this 1, this 2, then similarly, this and this other side, this and the other side.

So, this four starting from forth is diagonally opposite, this lattice point, if you connect. So, it will be body diagonal and you will have 4, obviously, this 4 point. So, you will have basically, 4 axis of rotation, of threefold symmetry. Similarly, whether any other rotational axis exist or not one has to find out. Yes, if you take, if you take the, if you take the axis passing, through the middle of the edge passing, these are the edge, 8 edges are there, 8 edges are there.

So, passing through the edge, middle of the edge, opposite edge of the opposite edge of the cube. So, opposite edge is, this one and opposite to this another (Refer Time: 22:09). So, through middle, if you take the axis, through middle, if you take the axis, say will get you, will get the another rotational axis. Now, with this rotational axis. So, if you rotate by 90 degree, no you will not get the same configuration, but if you rotate by 180 degree. So, this phase will come down and this will come up, but both phases are same. So, you will get invariant position.

So, you will have. So, it can be treated as a axis of symmetry rotational, axis of symmetry and it has twofold symmetry. It has twofold symmetry and you can draw, you can have this type of from this you, how many you can have. So, through these two edge, one through these 2 edge 1 and through these 2 edge, this and, this opposite, this 1 3, this and opposite this 1 4, this and opposite this one, I think this 1 5 and this and opposite one. So, that will be 6.

So, you will have 6, you will have 6 axis of rotation of twofold symmetry right, twofold symmetry. So, rotational symmetry, what is rotational symmetry? It may have meaningful symmetry. So, for a particular crystal system, we have to find out the first axis and with respect to that axis, we have to see the, how many fold symmetry are there.

So, here just I have taken one example of cubic, cubic say, cubic crystal and I am trying to find out this in details. I am trying to find out in details.

So, I think these are the axis of rotation and no other axis for this rotational axis, for this cubic system. Similarly, reflection also with respect to reflection 1, can see whether the crystal is having symmetry or not, crystal, some crystal may have symmetry; some crystal may not have symmetry. So, with respect to this, let me draw a cube.

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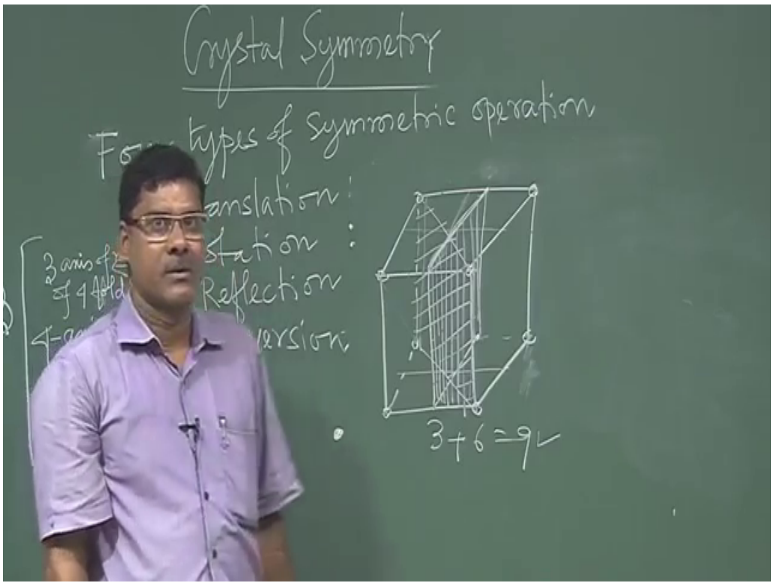

So, for reflection you need like rotational symmetry. One has to choose axis, a line for reflection 1, has to choose plane. So, it is with respect to that plane, that is basically, mirror plane with respect to that plane, whether crystal has symmetry or not.

So, if I take a plane, I think, this not if I take a plane, if I take this plane. So, with respect to this plane, you can see this half. So, this point will corresponds to this point right. This point will corresponds to this point, reflection from this plane. Similarly, you can see this point. So, this of exactly same to the other; so, what does it mean. So, I have taken cube and then you have this type of things, lattice point arrangement, I have taken one mirror plane and I am telling that this, so, this point is equivalent to this point.

So, but to see the symmetry, if I tell the symmetry that cube has this reflection symmetry with respect to this kind of plane; so, from this information itself, I can generate this lattice point, I can find out the lattice point, position of the lattice point, position of the atom. So, that is why this symmetry basically, helps to determine the internal structure of the crystal, how atoms are arranged just if one know the symmetry of that crystal. So, one can know about the details, distribution of the, of the atoms are lattice point in that crystal.

So, that is the importance of the symmetry of the crystal. So, similarly, see this just this plane, we have taken say. So, similarly, this way also one can choose plane. So, how many planes one can choose this way? So, so that will give basically, three planes of symmetry, this parallel to phase 3 plane, this three plane of symmetry, because two phase giving, one plane. So, six phases are there. So, we can draw this type of 3 planes right, mirror planes; so, you will have 3 planes of symmetry and if you take the plane along the diagonal. So, this. So, with respect to. So, this with respect to this diagonal plane. So, this side one half and this you see, this reflected one will be then another half right, reflected one will be another half.

So, this is one plane. Similarly, you can draw this other one plane and from this, from here again, you can draw this and this it is 4. So, basically opposite phases from opposite phases, you are getting 2. So, 6 phases. So, from each phase, you will get 2 each parallel phases, you will get 2. So, 3 set of parallel phases in cube. So, you will get 6. So, 6 planes, 6 diagonal planes, you will see 6 diagonal plane, you will see. So, earlier we have seen 3 and for this diagonal plane, this 6 total 9.

So, I will discuss in next class, I will discuss about inversion. So, just remember. So, from axis, from rotation, we have got 13 axis of rotation symmetry, here we got 9 plane of symmetry, mirror plane of symmetry, then rotation, from rotation, I can show for this cube, it will be just one point at the body center, that is the only inversion point, with respect to that, this cube crystal will have the symmetry. So, that is why, this to a 13 9 22, pass this one. So, cube crystal have 23 symmetry element have, symmetry element. These are at rotation axis of rotation plane and this point, these are called symmetry element. So, cube crystal have 23 symmetry element. So, I will continue in next class.

Thank you.