**Classical Mechanics: From Newtonian to Lagrangian Formulation**

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**Lecture - 04**
**System with variable mass – 1**

So, I will start by showing you repeat problem which we did in the last class.

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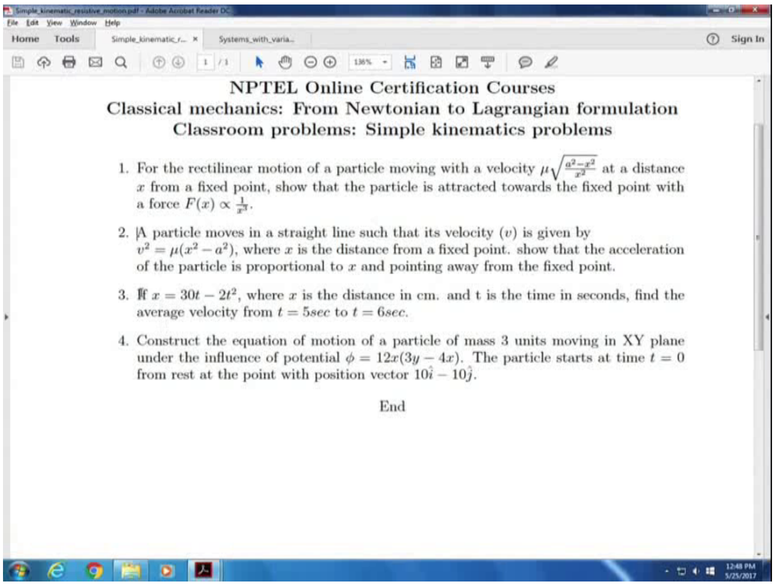


So, actually this particular problem, while doing it on board I made a small mistake. So, I will just correct it first ok.

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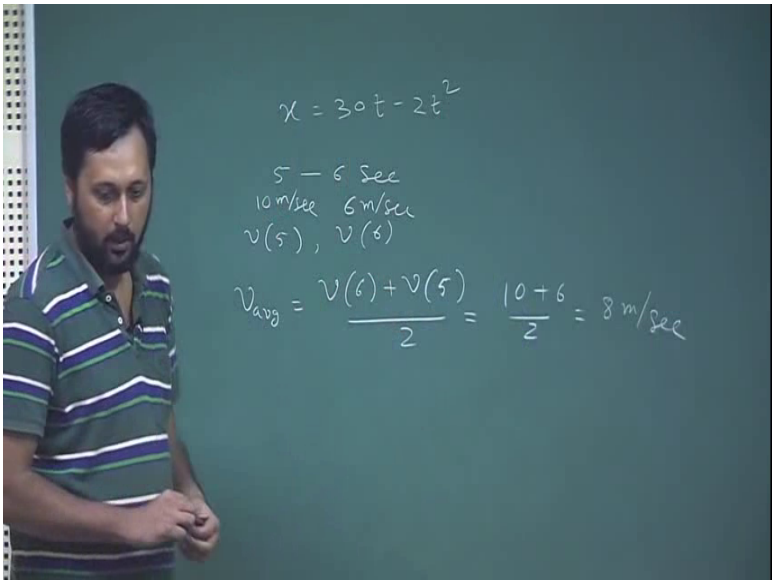


So, the description of the problem is x equal to 30 t minus 2 t square, and we have to calculate the average speed between 5 and 6 sec. So, what we did was we calculated v 5, we calculated v 6 and by mistake I took the difference of this 2 it will be simply the v average will be simply v 6 plus v 5 divided by 2. If you recall v 5 was 10, this was 10 meters per second assuming that the distance is given in the units of meters and this was 6 meters per second. So, your average speed will simply 10 plus 6 by 2 equal to 8 meters per second not 16 which we did in the last class where by mistake, I just subtracted one from the other very good.

So, with this we will lift to our next topic which is just a minute. So, we will discuss motion of systems with variable mass; what is the special about variable mass is ok.

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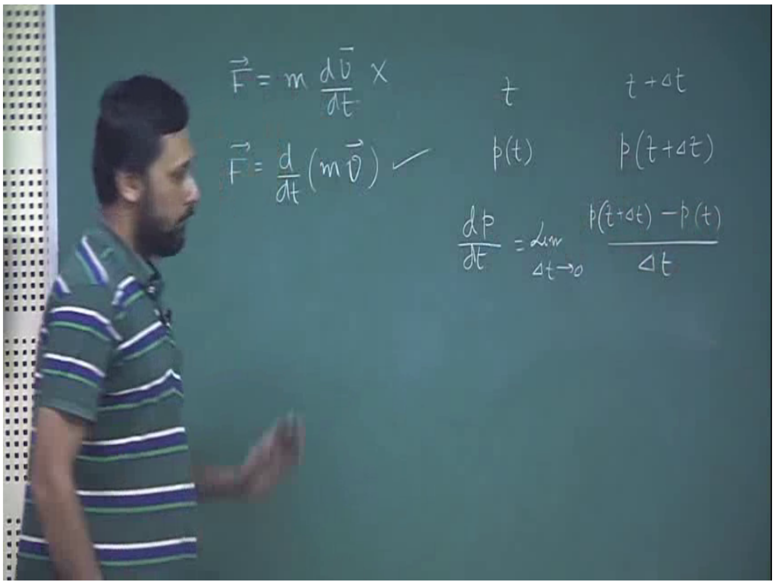


So, we have written this equation a number of times that F is equal to m dv dt; which is correct if the mass of the system is constant, but assume that mass of the system is not a constant this equation in this particular form is not correct. But please note that that does not mean that Newton’s second law is not correct, Newton’s second law is very very very correct. Only this is if we go by the exact statement second law, it says an external force which is proportional to the rate of change of momentum. So, the actual form to form Newton’s second law is F equal to d dt of m v, not m dv, because m is constant for most of the time we take it out of the equation we just write m dv.

But, when the mass of the system is also changing we cannot use this equation anymore, but we have to rewind back to this equation. Or even better, what we need to do is in order to get to the equation of the motion of a particular system which where the mass is changing where to slowly and step by step we are to construct the change in momentum at 2 given time steps: one is time t and one is time t plus delta t assuming the delta t is small. So, essentially we compute p of t, we compute p of t plus delta t then we compute dp dt by taking limit delta t goes to 0, t, t plus delta t minus p of t divided by delta t.

So, this will be our general treatment for system with variable mass. So, one of the most common system that we study in a at the textbook level.

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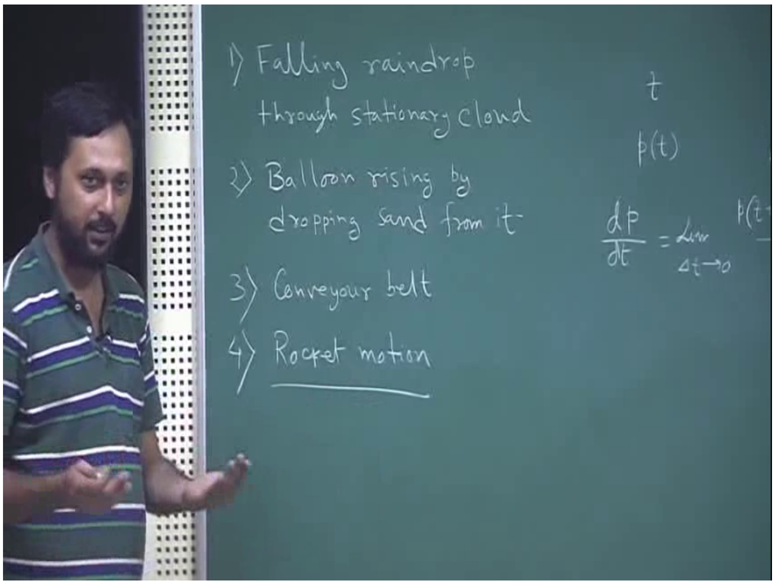


Where the mass changes the function of time is falling rain drop; falling rain drop through stationary cloud. So, this is one, second system which is also very common that is balloon raising by dropping sand from it. Third system which we are going to study is the oh sorry convey our belt; so and the forth and probably the most important system which we are going to study is rocket motion. So, rocket motion is a rather elaborate treatment, because here the we have the first establish the equation of motion then we have to study the cases with or without gravity and there are many outcome that will. So, once we solve the equation we will see many interesting things coming out. So, we keep this for the last.

So, in today's class we plan to take this three cases one by one and show you how to construct the equation of motion in each of the cases and we will take illustrative examples of how to on how to solve each of this type of the problem. So, we will first start with the rain drop to a stationary cloud.

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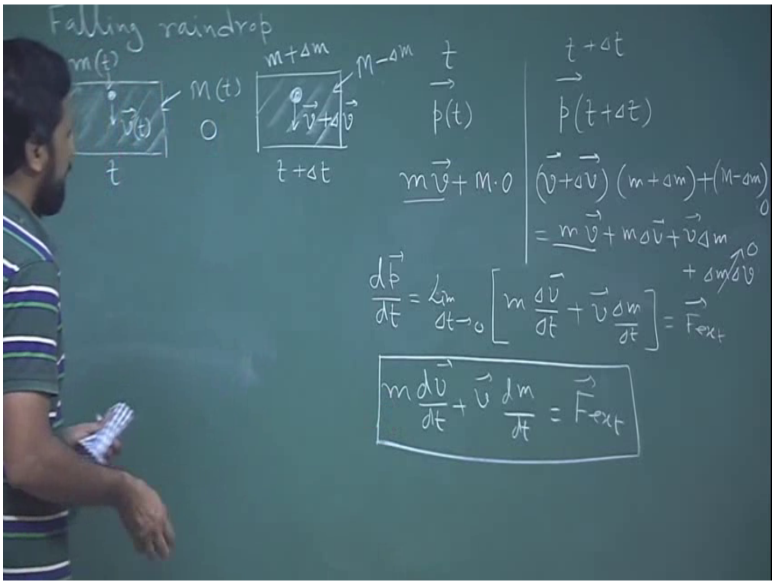


Now, what happens here, let us say this is a cloud which has a mass of capital M why I am putting a t here I will explain in a soon, and inside that there is a rain drop which is falling let us say under the action of gravity and the mass of the rain drop is m which is also a function of time; write slightly bigger for your convince. So, let us say this is also m which is a function of time right. Now what happens is as the rain drop falls to this cloud the it absorbs moisture from the cloud and it gains it is size and it is mass. So, that is why this mass is a function of time.

Similarly, because there is there are mass conserve towards the mass conservation laws valid. So, the mass it gains the mass this cloud will loose, but once we construct the equation we will see that in equation only there this term will appear not this term how. Because this is stationary cloud the velocity of this falling rain we can assume that to be v which is also a function of time, but the velocity of this cloud because it is a stationary cloud it is 0. So, we this is the situation at time t, now what happens at the time t plus delta t? So, we have a velocity which has gone up from v to v plus delta v, a mass which has gone up mass of the rain drop which has gone up from m to m plus delta m and the cloud mass has gone down to m minus delta. So, this a capital M this is a small m if we. So, this is a time t, this is at a time t plus delta t. So, what are the momentum p and p plus what are the momentum of this system at time t and time t plus delta t we first have to construct that. So, momentum at this time is m v, please remember the v is the vector.

So, we will just keep writing vector here plus m into 0, because the cloud is not moving. Now for this time point the net momentum is v plus delta v into again vector, m plus delta m once again m minus delta m which will be multiplied by 0. So, essentially this is m v plus m delta v plus v delta m, plus delta m delta v. So, we try to construct the rate of change of momentum by taking subtracting this from this, and dividing the whole this by delta t and taking this limit. So, this will be dp dt please remember there are vector quantities dp dt, and we will see that immediately this term and this term will cancel out this term we can neglect because it is a multiplication of 2 infinitely small numbers it is. So, that is why we can ignore this term.

So, essentially what we are left with is m delta v delta t. So, just a minute, they will be a limit delta t goes to 0, m delta v delta t plus v delta m. So, if please remember this is equal to if external according to Newton's second law. So, the final equation we get is m dv dt plus v dm dt equal to F external and if we start from the standard equation without doing all these elaborate procedure, if we simply write Newton's second law in this in it is original form equal to F external and take derivative of this we will essentially get the same equation.

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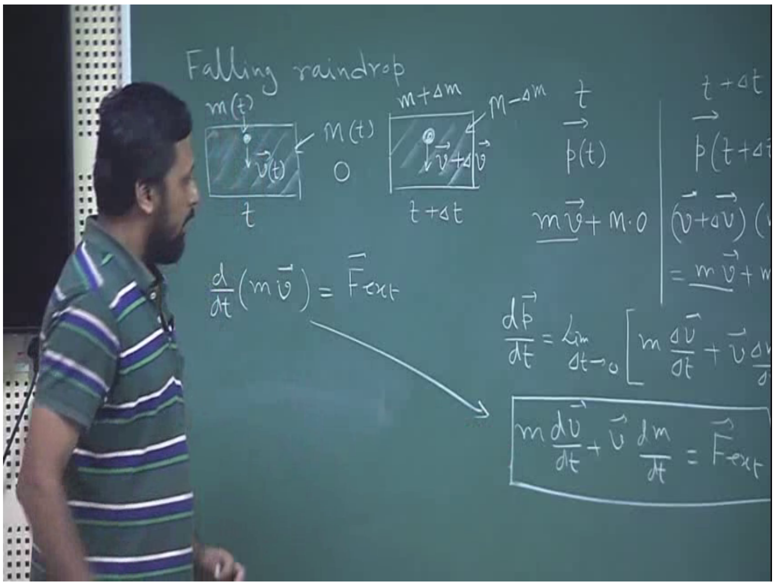


So, these and these will essentially give you the same equation, but I will tell you this is this one to one correspondence between this I mean. So, basically if I take a time derivative of this this will essentially be m dv dt plus d b m dt equal to F external this is identical to this equation, but please do not try to generalize this, because in this particular case this might be there is a one to one correspondence between these and that, but we will see very soon that in next few examples this correspondence does not hold. So, it is better in order to get to the equation of motion of the system with variable mass it is always better that we construct this momentum at a time t and momentum at t plus delta t, and then we take this difference and then divide by t take this limit essentially get to this equation. So, this is a much little bit elaborate, but it is rather physically it is rather correct way of getting to the equation option ok.

So, right now we are not focussing on solving it I will do that when we solving the example. But please remember that we have to put expression for v external sorry f external and also we have to put starting functional forms for v m dt. So, do that, but now let us go to the second example which we are going to discuss. So, what we will do is that I will just keep this final equation for future reference.

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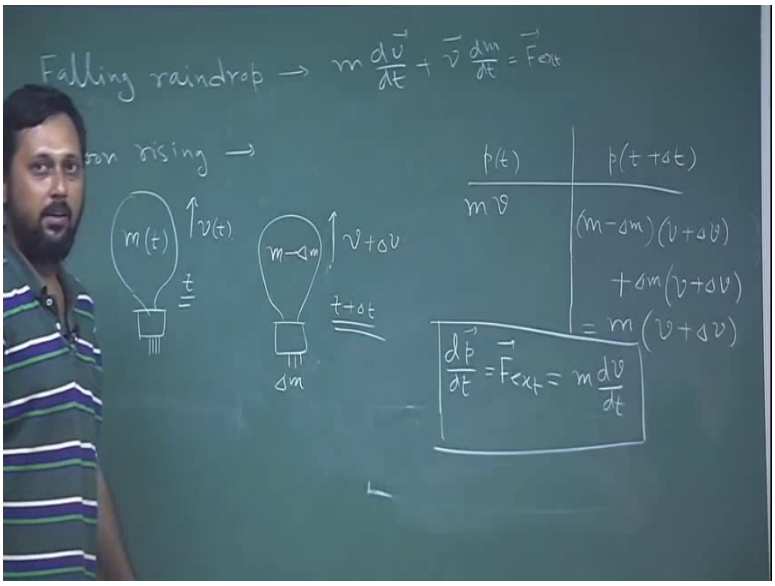


Now, to second equation second case, that is balloon raising; what happens is a balloon generally a balloon generally goes upwards in through atmosphere because of it is buoyancy forces. Balloon is a basically it is a container it is a very this world container which has lots of light gasses inside typically helium and that is why it has a huge buoyancy forces that is enough to provide an upward lift to that to the whole system. So, it cannot only if the balloon can go up, but it can also carry a certain amount of mass in terms of gondola and if somebody is sitting in that, but typically what happens is as the atmosphere as the balloon goes up in the atmosphere, the air becomes thinner and thinner and the buoyancy forces decreases.

So, at some point the balloon will reach a stand still where the buoyance force and the downwards gravitational force they are exactly like exactly balancing each other. So, the balloon will be stationary now to overcome such situations, we also have people generally carry a bag of sand in that. Now typically the sand or bag of sand or extra weight and it is what people does is typically drop the sand. So, essentially the mass reduces gravitational pull reduces and balloon starts moving upwards. Now what is it might be very risky to drop a big bag of sand because it does sometimes people who are dropping it there not sure where it is going to land it might land on some yon it might land on some house. So, typically it is done in a smoother way. So, there is a small hole in the; they make small hole in the bag and the bag looses sand in I mean it is it looses sand in a slow manner.

So, that is why nobody is getting hurt and also the balloon, but the mass of the balloon reduces or balloon starts moving upwards slowly. So, let us assume our situation where we have a balloon with initial mass m which is once again a function of t, and let us say it starts loosing sand at a given rate. So, let us say in time t the velocity of the balloon is given as v of t and in a later time t plus delta t that is the time t; at time t plus delta t once again we have m minus delta m because delta m amount of sand has gone down and the velocity is increased from v to v plus delta v ok.

So, once again we try to construct the momentum p of t and p of t plus delta t. Now once we do that we immediately see that the momentum initially was simply m v and the momentum at time t plus delta t is m minus delta m into v plus delta v plus delta m times v plus delta v why this why the second term here? Because the sand which gets released from the balloon at the point of release it also has the exact same velocity of that of balloon. So, in this case the mass is released at the same related speed as the original body I hope you are getting the point.

So, the sand got release from the balloon in the slow manner, as the sand particle releases the moment when the sand particle releases the balloon body, it has the exact same velocity. So, that is why essentially if we look into this equation, this two terms cancel out which leave us with v plus delta v that is it. So, if we construct dp dt from here which will be equal to F external according to Newton's second law, which what will get is the difference between these 2 divided by time and if we do that we simply get m dv dt. So, this is our final equation. So, you see if you start with start by writing p as m v and take this time derivative and then you write m dv dt plus v dm dt will be fooled. So, this is not right in this particular case.

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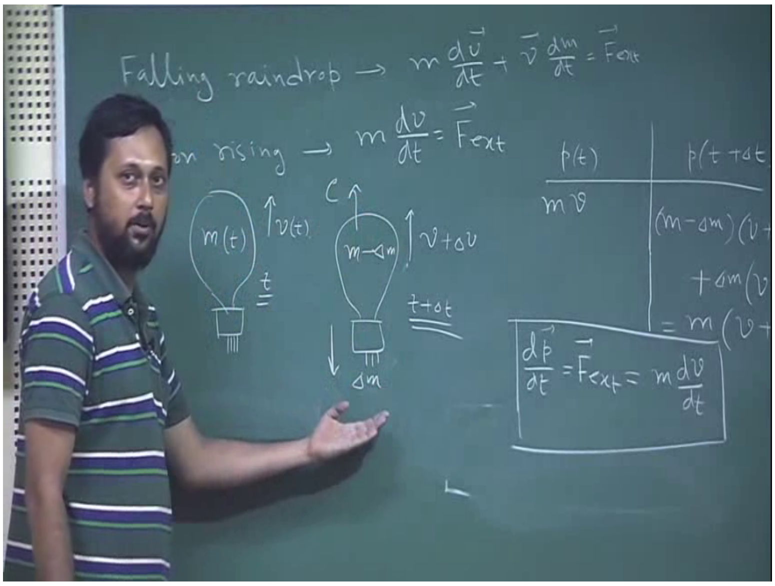


So, if we go back here. So, far balloon raising the equation is simply m dv dt is equal to F external. Now what is this external force? External force is essentially the fight between upward thrust c and downward gravitational pull. So, we will again we will give an example of this particular topic particular problem and then we will see what are the details of this upward thrust and the downward forces, but from now we will just remove everything and we will just keep the final equation.

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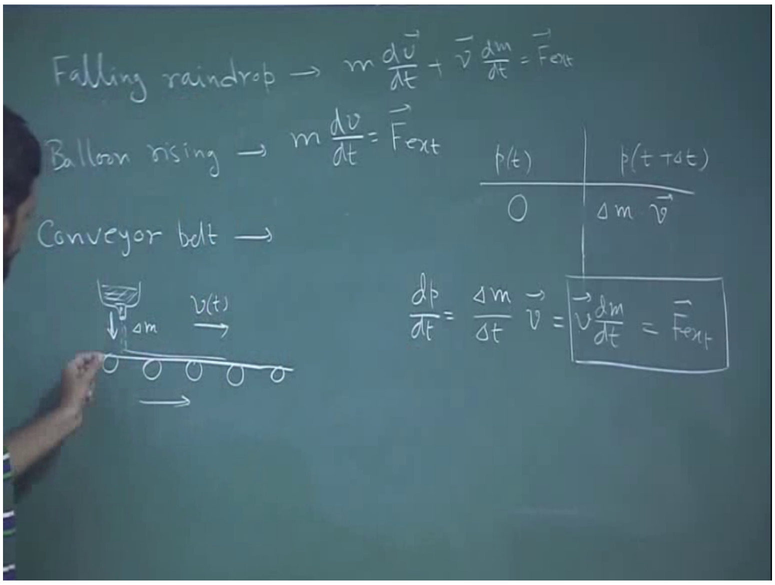


So, the third topic what we are going to discuss is topic of conveyor belt. Now what happens in the conveyor belt let us say there is a belt which runs on we are assuming that it runs on frictionless proeglerce, let us assume there is a stationary. Hooper: a Hooper is essentially the sorry stationary hopper is essentially the object which is used at different construction sites which are used for mixing cement and sand together, and let us say this particular hopper is dropping sand on this conveyor belt which moves along, the sand then once it get drops on the conveyor belt it moves along in this direction with a velocity v of t.

So, now, if a amount of sand delta m is dropped in the time interval of delta t before the net momentum at time t before the drop took place, the net momentum is nothing but 0 net momentum I mean the net momentum in this particular direction, because sand is coming from a direction sand is dropped from a direction which is the when it is falling on this conveyor, it has a velocity which is in this direction which is perpendicular to this direction. So, there is no component of velocity in this particular direction.

So, the initial momentum is 0 and the final momentum at time t plus delta t is delta m times v of t oh sorry simply v not v of t right. So, now, we once again construct a equation. So, your dp dt will be limit t goes to 0 delta t times v, which will be simply m sorry v dm dt which will be F external. So, we see that the equation of a sand falling on conveyor belt and moving towards along the conveyor belt after falling is F external equal to v times dm dt. So, once again this is very different from this initial equation we wrote here. So, initial equation it seems like apparently we just started taking derivative of the p term just writing it as p equal to m v and taking derivative, we got this equation not like not that. So, we have to follow a step by step construction to get the correct physical insight in order to get to the right equation.

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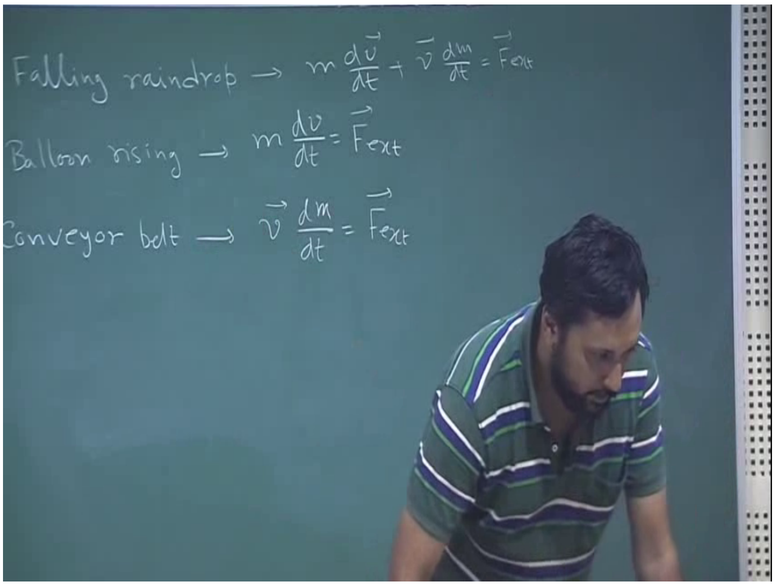


So, we write it once again the final equation for conveyor belt is v dm dt equal to f external. So, we have discussed all these three theory of this behind this three equations one for the falling rain drop other one for balloon raising and third one is far conveyor belt.

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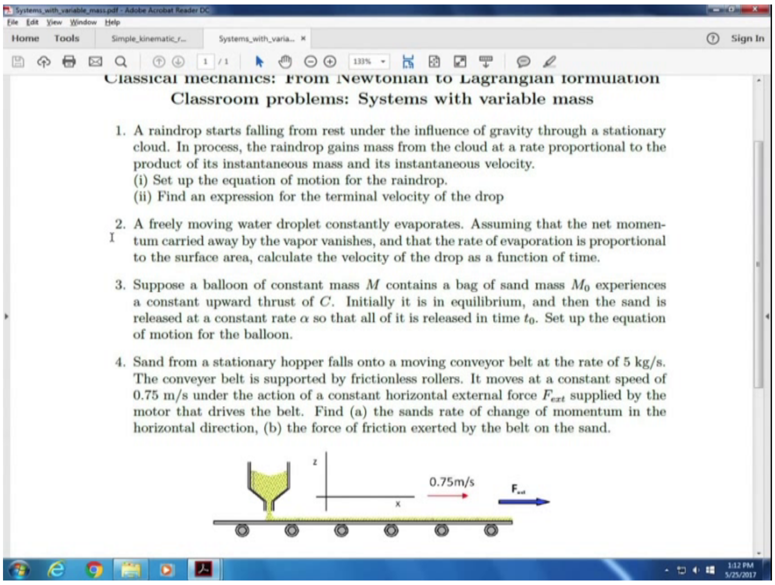

Classical mechanics: From Newtonian to Lagrangian formulation

Classroom problems: Systems with variable mass

1. A raindrop starts falling from rest under the influence of gravity through a stationary cloud. In process, the raindrop gains mass from the cloud at a rate proportional to the product of its instantaneous mass and its instantaneous velocity.
(i) Set up the equation of motion for the raindrop.
(ii) Find an expression for the terminal velocity of the drop

2. A freely moving water droplet constantly evaporates. Assuming that the net momentum carried away by the vapor vanishes, and that the rate of evaporation is proportional to the surface area, calculate the velocity of the drop as a function of time.

3. Suppose a balloon of constant mass $M$ contains a bag of sand mass $M_0$ experiences a constant upward thrust of $C$. Initially it is in equilibrium, and then the sand is released at a constant rate $\alpha$ so that all of it is released in time $t_0$. Set up the equation of motion for the balloon.

4. Sand from a stationary hopper falls onto a moving conveyor belt at the rate of 5 kg/s. The conveyer belt is supported by frictionless rollers. It moves at a constant speed of 0.75 m/s under the action of a constant horizontal external force $F_{ext}$ supplied by the motor that drives the belt. Find (a) the sands rate of change of momentum in the horizontal direction, (b) the force of friction exerted by the belt on the sand.



Now, let us look at look into the problems I have collected some of the problems for you. So, the first one let us start with the first one a rain drop starts falling from rest under the influence of gravity through a stationary cloud in process the rain drop gains mass from the cloud, at a rate proportional to the product of it is instantaneous mass and it is instantaneous velocity. The question is first we have to set up the equation of motion for the rain drop and then we have to find the next expression for the terminal velocity, that is very nice because here we do not have any resistive forces still we could get terminal velocity.

So, let us try to solve this on board. In order to do that let us first try to construct the equation of motion. So, we know that for m dot dv dt as v dm dt is equal to F external and here F external is m g, ok.

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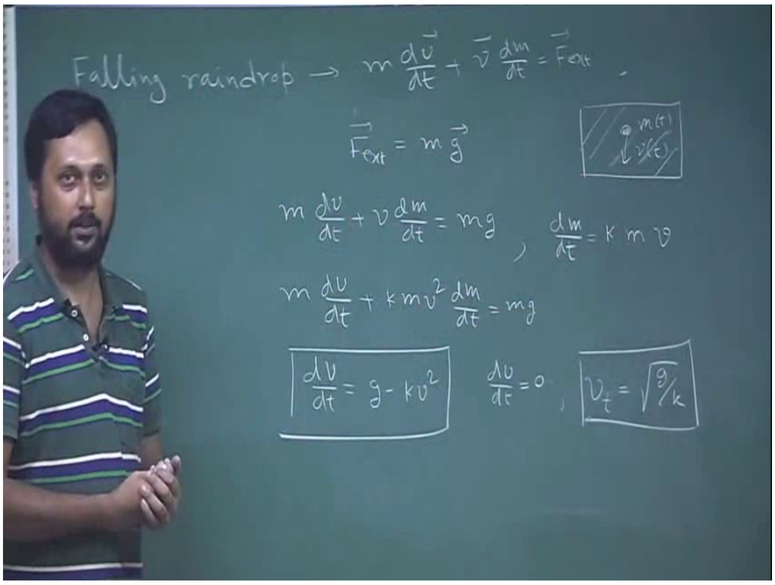


Assuming that we are measuring again once again we have to understand this we have this stationary cloud, the rain drop is falling with a mass m t and a velocity v t now where is our origin? Here our if we place our origin down here then there is a negative term coming, but let us assume for this problem let us say the origin is somewhere up here, we do not care where exactly it is, because we are not planning to give you an exact you know measure of the distance, we are just trying to solve construct the equation and get an idea of the terminal velocity. So, we assume that it is up here. So, this g remains positive.

So, the final equation without if we remove the vector will simply be m dv dt plus v dm dt equal to m g now the second information which is given that dm dt the rate of change of mass for this particular rain drop is proportional to instantaneous mass a product of instantaneous mass and instantaneous velocity and we can take this as some constant k times m times v plug it back to this equation see we have to get we have to have some model for this mass change. We will take up another example and see it could be proportional to instantaneous volume it could proportional to instantaneous surface area in this case it is proportional to instantaneous mass times instantaneous velocity. Unless and until we have a specific model for dm d t, we cannot solve this type of problems ok.

So, if we plug this in we have m dv dt plus k m v square dm dt equal to m g simplifying we have dv dt equal to g minus k v square. If this is the equation of motion we are looking for and second part is we have to get to the terminal velocity and we see the terminal velocity all though there is resistive force here, because of this mass change is the function of velocity this itself velocity and mass. So, this term it works as if it is an upward force it is working against the gravity.

So, in order to get to the terminal velocity once again we put dv dt equal to 0 we said that, and we immediately get v t is equal to root g by k. But, here we have to remember this k has nothing to do with area distance, it is a proportionality constant we have used. And we have to have some additional information in order to get this proportionality constant fine.