**Classical Mechanics: From Newtonian to Lagrangian Formulation**

**Prof. Debmalya Banerjee**
**Department of Physics**
**Indian Institute of Technology, Kharagpur**

**Lecture – 03**
**Motion in resistive medium falling ball viscometry using thermal velocity**

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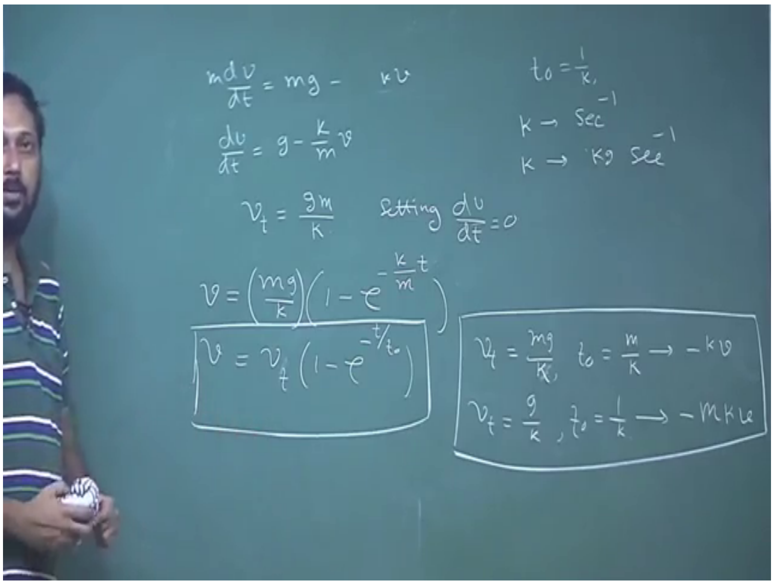


So if you recall we wrote this equation as m d v d t is equal to mg minus m k v vector. Here now here the drag force has been taken as force per unit mass. If we do not take this drag force as unit force per unit mass what do we get, we simply have m here. So, the equation becomes d v d t is equal to g minus k by m v. And the trickier I taught you is very recently is to set this equal to 0 in order to get terminal velocity. And we get a terminal velocity to be equal to g by gm by k by setting d v d t equal to 0. Also if we solve this full equation we get an expression of v to be equal to over minus k by m t.

So, you see not only the terminal velocity is modified, now if we write this as vt 1 minus e to the power minus t by t 0. We see that vt becomes m g by k and v 0 becomes m by k. So, if the same result, as the as we got from the previous date previous set of equation. Only k is replaced by k by m. And also note that when we have t 0 is equal to 1 by k; that means, the unit of k is given as second inverse, when I have t 0 is equal to m by k the unit of k is given by some kg or gram maybe kg second inverse. So, depending on if the data is given in a second inverse or kg per second inverse we have to use one of these 2 equations accordingly.

So, in one case we have we have to use vt equal to mg by k and t 0 equal to m by k. And the other case we have to use vt equal to g by k t 0 equal to 1 by k. And this is if the initial force is given as minus k v, and this is if the initial force is given as minus m k v . The final equation becomes same only thing the value of vt and t 0 if change.

Now, there is another type of very important, I mean there is a very important application for this type of equations and we will discuss one such application which is called the falling ball viscometry. That is used as a laboratory experiment also in industry people use it a lot in order to measure viscosity of a fluid. So, how it how does it work.

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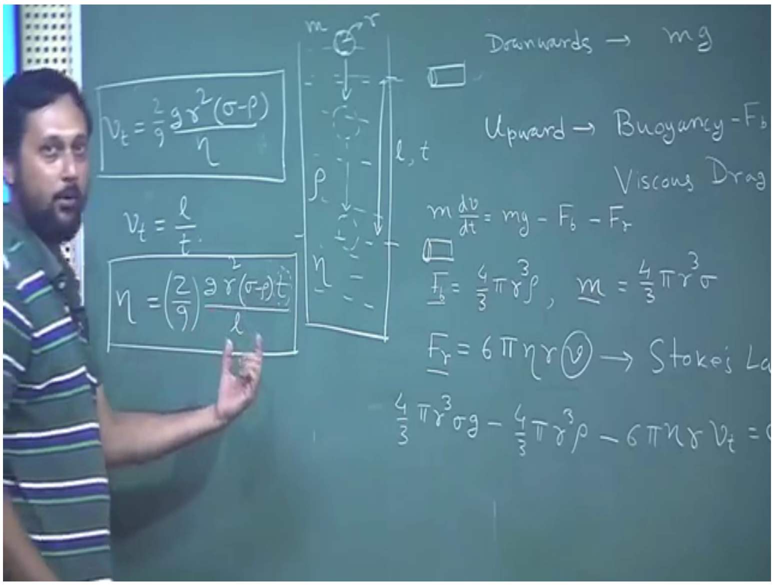


Let us assume there is a column of viscous liquid. So, there is some viscous liquid and what do you do we drop a small metallic sphere of mass m and radius r in this column of liquid. Now what happens is it as it starts falling through, because of this there is a viscous drag present it essentially attains a terminal velocity.

Now, to in order to get gain a more mathematical insight let us try to see what are the forces acting on this particular thing. Of course, there is a force downwards which is due to it is mass. Secondly, there is a force which acts upwards due to the buoyancy, we all know buoyancy right. Buoyancy is the force applied by the met the liquid displaced by this particular body and of course, there is a viscous drag force. So, if we write the downwards forces, this is simply mg and upward forces, one is the just a minute buoyancy and the other one is viscous drag we call this one as fb and this one as fr.

So, if we write the force equation, which will be m dv dt is equal to mg minus fb fr. Now what are the expression for this buoyancy forces and viscous drag. Buoyancy force the expression is pretty straight forward we all know it buoyancy force will be the volume of this particular object multiplied by the multiplied by the this one the density of this liquid.

So, fb will be 4 third by r cubed rho. So, let us assume rho is the viscosity of this liquid. Similarly, we can write m which is the mass of this small metallic balls as 4 third pi r cubed sigma. Sigma being the so rho is the viscosity, sorry. Rho is the density of liquid sigma is the density of solid. This solid the material of this ball it could be a steel ball it could be here any other hard sphere. And we also introduce or not introduce you know probably know that there is a viscosity. So, it is a viscosity eta we call the viscosity of liquid.

Now, where does this eta come in eta comes in to this term there are theories which tells you that fr will be 6 pi eta r v, v being the instantaneous velocity of the of this ball, ball is falling with a velocity v 6 pi is a constant r is the radius. And eta is the viscosity and this is the equation of viscous drag according to stokes law. So, the viscous drag is determined by stokes law and there is an expression for it please remember this expression is valid only for spherical object. And it is also a very idealistic equation in reality this 6 pi eta I mean 6 pi is not a 6 pi it generally there is a correction term associated with it.

But let us not go in to the all this technical details, let us keep it simple and if we now plug in this expression for f b f r and m in the left hand side. So, with essentially what we are planning to do is we are trying to get an expression for the terminal velocity for this particular case. So, what happens is for terminal velocity once again we set d v d t equal to0. So, the left hand side becomes 0, now if we do that and we put v t for terminal v because we are setting d v d t equal to 0. So, this equation becomes m. So, m will be 4 third pi r cubed sigma g minus 4 third pi r cubed rho minus 6 pi eta r vt equal to 0.

Now, once we simplify this, we get an expression for vt which is sorry 2 by 9 it will be 2 by 9 g, g r square sigma minus rho divided by eta. So, we have an expression for the terminal velocity, but that is not it. So, that essentially what we are planning to do is we are trying to calculate we are trying to estimate eta from this experiment. So, what happens is in reality, there will be marking in this on this tube which is separated by a distance l. Now there will be one camera here and one camera here.

So, once the ball starts falling of course, it will have sufficient length on this this marking. So, that by the time it reaches here, it falls it reaches it gains the terminal velocity. Please remember this experiment is will not be a valid experiment unless and until this sphere reaches sphere gets to it is terminal velocity assuming that we have enough length for the sphere to travel by the time it reaches, here it reaches the terminal velocity.

What we can do is we can take to snapshots of this sphere passing this point and this point, which is separated by a distance l and then we can calculate the time t for which it took in order to reach from this point to this point. And then your vt will simply be l by t you got it. So, using this 2 cameras we measured the time and we already know the distance l. So, we get vt equal to l by t, and when we plug it back in here we get eta will be 2 by 9 g r square sigma minus rho times t by l out of this, expression 2 by 9 is a constant which has a fixed value l. We know g, we know r, we know a priory sigma and rho also we know all we need to do is we need to calculate this t or we need to measured this t at accurately enough. And essentially we have to give we have to if only if I can measured the value of t we can immediately get a value for eta. And this whole setup is called there is a name for it. It is called the falling ball viscometer.

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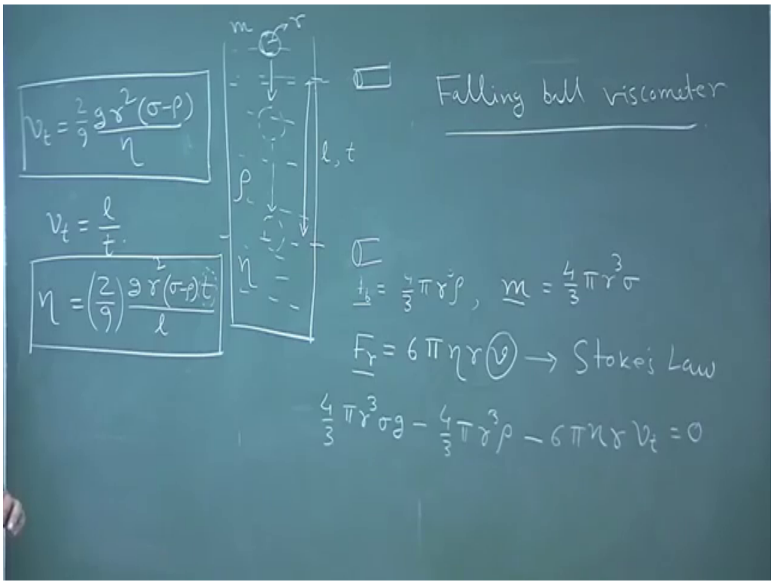

So, it is called a falling ball viscometer which is a. So, I will write it again. I think probably you would not see it properly I will just write it here falling ball viscometer. So, it is a very simple experimental setup, but which is pretty accurate we people use it in their laboratories in order to get a better first order approximation of first order estimator of this viscosity sorry viscosity eta of the fluid also industry people use it a lot.

So, now with this I just want to give you one more set of information, which is pressured drag.

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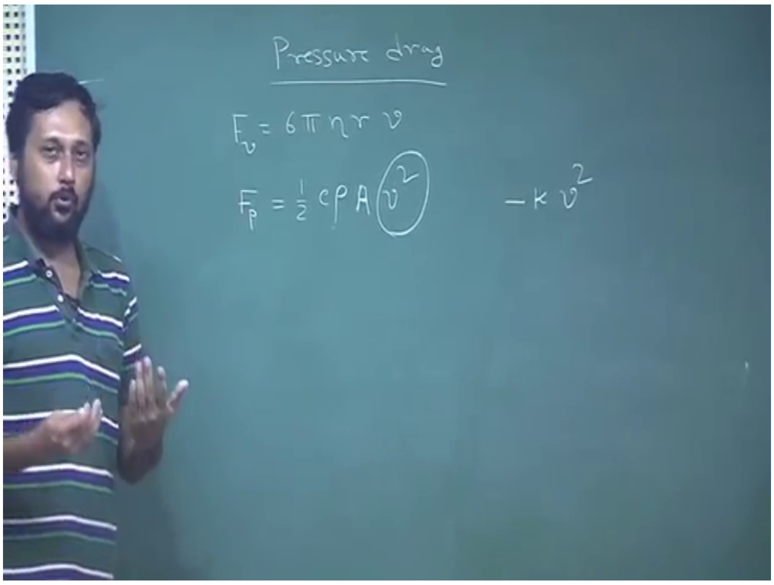


Now, viscous drag the expression for viscous drag was f is equal to 6 pi eta r v. Now this is valid for a very regular shaped object which is moving slowly enough inside the fluid, if the v has to be very small I mean not very small, but there are certain limits put on put on v in terms of the Reynolds number. And I am not going into the details of that, but as a as a thumb rule we can assume that if, if it is a well shaped small object moving with a limited speed. Then we can assume that this stokes law is a good approximation of the drag on this particular on this object and we call it the viscous drag.

But when the velocity becomes higher and higher, then from we essentially enter into the regime of turbulence flow and there we have an expression. So, we call it f v, this is for viscous drag and we have a pressure drag which is a half c rho a v square rho being the density of the liquid a is the cross section of the object, which is moving the effective cross section v and c is something called a shape factor, which is related to the geometrical shape of the object, but let us not going in to go in to the all the details, but what is more important is v square.

So, essentially we can represent this force in our situation as minus k v square. So, in certain occasions you might see in in the problems or in in certain books that some of the in some of the treatment force the resistive force has also an also been taken as minus k v square. And you will immediately know that this is also nothing. So, what why I am trying to tell you this because you need to know that, this is not something very impractical it is just the just that people are working in the pressure drag regime.

So, let us not go in to the details of this things and let us try to solve some problems.

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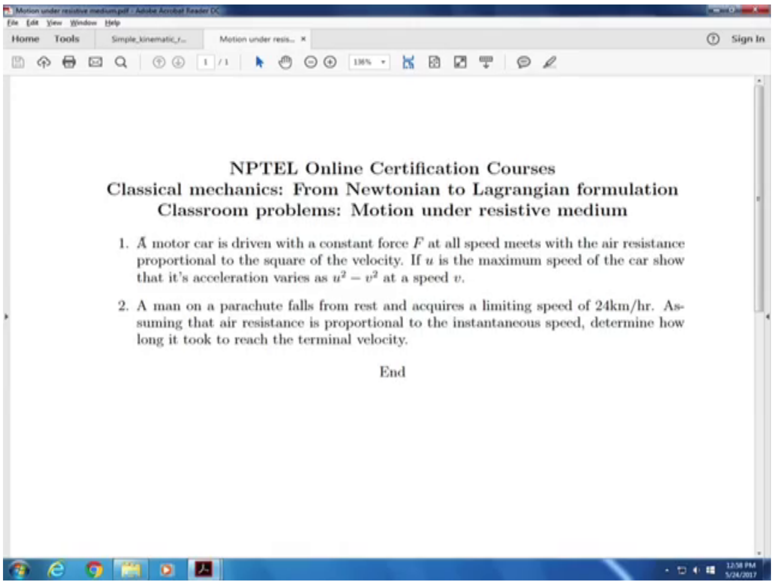


First problem. We have 2 problems for today's class. First problem is a motor car is driven with a constant force f at all speed meets with their registers proportional to the square of the velocity. If u is the maximum speed of the car, show that it is it is acceleration varies us u square minus v square at a speed v. So, here the force which with which the car is driven is a constant force and it is given as f.

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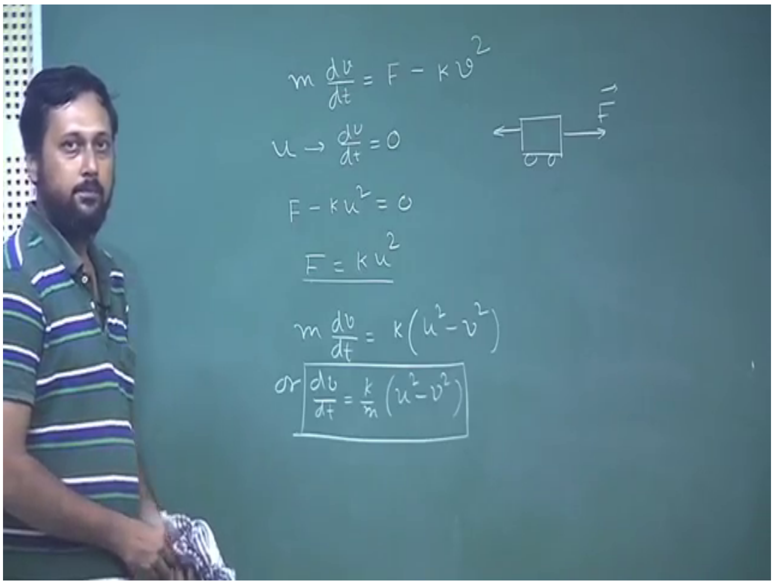


So, if we write the equation of motion it will be m dv d t equal to F minus k v, k v because sorry k v square.

So, because k is the proportionality constant because it is given that the resistive force is proportional to the square of the velocity again. Please remember that because motor car is an object which moves relatively faster compared to a you know raindrop falling rain drop or sphere falling in the viscometer, motor car has a fast I mean much higher velocity. That is why we are working in the pressured drag regime and we have the viscous for or the force resistive force. It has minus kv square. Now if u is the maximum velocity possible, now understand the situation is let us say this is our motor car and it is moving under the influence of a constant force F which is in this direction it comes from it is engine.

Now, air resistance will try to slow it down. And essentially these 2 forces will come back and after is what happens is, because this force F is a constant, we have to at some point we will reach a maximum force u r sorry maximum velocity u beyond which the car cannot move. And that u will be reached if d v d t equal to 0. Now when this happens we immediately see that, F if d v d t is equal to 0 then F minus k u square is equal to 0. And that essentially means k is equal to or rather F is equal to k u square ok.

Now, if we move back to the original equation, and put this expression for F over here we see that our equation becomes m d v d t equal to k u square minus v square. Or d v d t is equal to k by m u square minus v square. So, we got out desired result, this is our acceleration which is proportional to u square minus v square at all speed v with the proportionality constant k by n right. So, we have solved this problem.

Let us move to the next problem. The next problem is a paratrooper falling from breast and acquires a limiting speed of 24 kilometers per hour. Assuming that the air resistance is proportional to the instantaneous speed determine how long it took to reach the terminal velocity. So, to in order to solve this problem.

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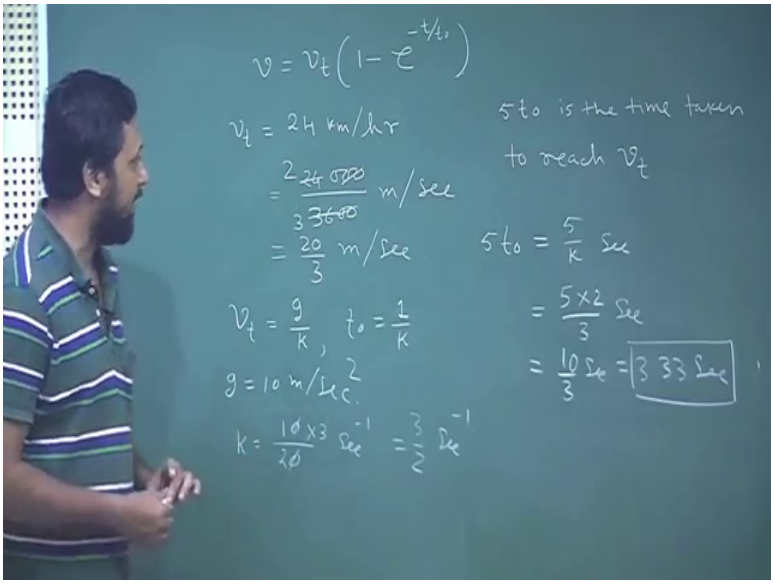


We immediately recall that v is given as v t 1 minus e to the power minus t by t 0 right. That was the expression we used irrespective of whether we took kv for the force of minus a minus k v, for the force of minus m k v for the force that was the final expression only thing is v t and t 0 we will have slightly different form.

Now, it is given that vt is equal to 24 kilometer per hour, which will essentially reduced to 24000 by 36000 meter per second 2 (Refer Time: 23:02) 2 3. So, it will be 20 by 3 meter per second. So, this is our terminal velocity.

Now, what we need to find out essentially is 5 t 0. As I have discussed just sometime back that 5 t 0 is the time taken to reach vt. So, if we assume this particular expression that vt equal to g by k in that case t 0 will be simply equal to 1 by k and if we take gt equal to equal to vt equal to gm by k. So, this will be equal to m by k. So, all we need to do is we need to calculate this k or k by m whichever way you prefer it. It will essentially give you the same answer.

Let us do it with simply k, because it will look better nothing else we know that g is equal to 10 meter we are assuming that g is equal to 10 meter per second square it could be 9.8 meter per second square, but just for simplicity I am taking 10 meters per second square. So, that will give you k equal to this is 20 by 3. So, it will be 10 divided by 20 into 3 second inverse. So, essentially it will be 3 by 2 second inverse and we need to compute 5 t 0.

So, 5 t 0 is equal to t 0 so; that means, t 0 is equal to 1 by k, 5 t 0 is 5 by k which will be 5 into 2 by 3 second which will be 10 by 3 seconds equal to 3.33 seconds. So, the final answer is 3.33 seconds a paratrooper, who reaches a terminal velocity of 24 kilometers per hour it takes 3.33 seconds to reach the terminal velocity. So, immediately know that although it looks I mean it is not that straight forward. Because this viscous drag is not very a very straight forward phenomena. Sometime we have a more I mean more critical expression for the terminal velocity.

We are working on a very simplistic model here. Please remember always remember that the actual physical situation could be lot more complicated we are just assuming that there is no you know flow in air and there is no turbulence in the air which is not true there is a flow there is a turbulence. So, all these terms it comes into account, but even without considering all this we immediately see that, it is a very short time it might take little longer might take even shorter, but almost immediately after opening of the parachute a paratrooper reaches the terminal velocity. And that is why even after landing from a distance of an height of you know sometimes 10s of kilometers they do not broke their break their bones. So, this is essentially it and also I would like to touch up on one particular topic which is a projectile motion under air resistance, I will not go in to the mathematical details of this because this is light slightly too complicated and it is probably not needed at this level.

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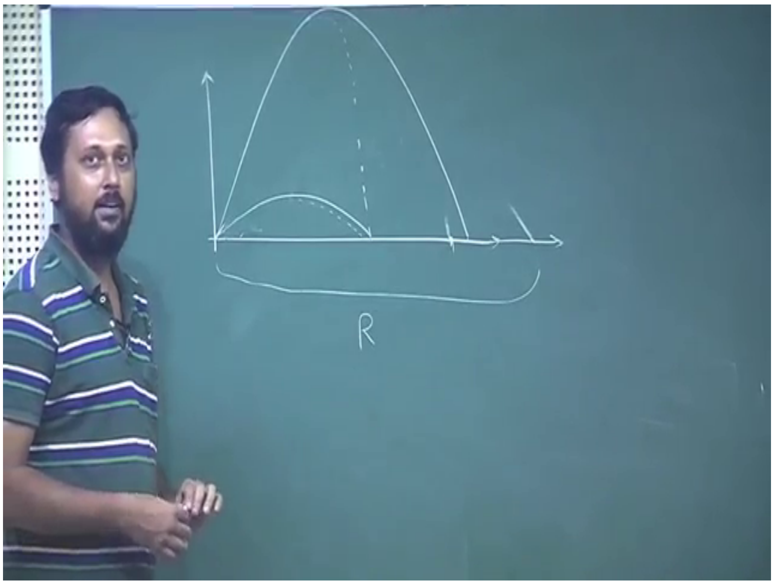


But I will just give you a very basic brief description of what happens. So, what happens when we fire a projectile, let us there is no air resistance nothing and we all know that a projectile essentially takes a parabolic path. And the range of the projectile this is called a range of this projectile depends on the initial velocity v 0. And the angle theta we know that and we have all solved and we know that by optimization of this range, we get theta equal to 45 degree is the optimal angle for which the projectile will have maximum of range.

Now, what happens is in case there is air resistance present, what happens is a projectile will fall short of it is desired range. And what is more important is if we fired the I mean. So, this all this things can be solved automatically, but I am not going into the as I said I am not going into the details of the equation, what I am more concerned about to give you a over all description physical description of what happens. What is very important is, if we fired the projectile higher as in if the angle theta is somewhere in the range of 60 or of 70 degree the effect will be more prominent, let us say for 60 degree if this is the desired range then with air resistance it will fall much shorter, but where as if we fired the projectile at an angle of 20 degree, if this is the range with air resistance the range will hardly change.

What I am trying to tell us as we go higher and higher up in this angle, the effect of air resistance is more and more on the project elegants. And that is very quantitatively understood because you know as we reach higher there are more scope of this particular object which is falling to reach terminal velocity. If you are not reaching high enough the terminal velocity might not be reached. Now all these things can be looked upon mathematically more, mathematically, but we are not going into this. So, with this discussion we are closing the topic of motion in resistive medium. Next class onwards what we are going to do is we are taking up their problem of variable mass.

Thank you.