Classical Mechanics: From Newtonian to Lagrangian Formulation
Prof. Debmalya Banerjee
Department of Physics
Indian Institute of Technology Kharagpur

Lecture – 26
Moving Co-ordinate Systems – 4

We start this class by looking into some problems with which has vertical sorry horizontal motion involved in it.

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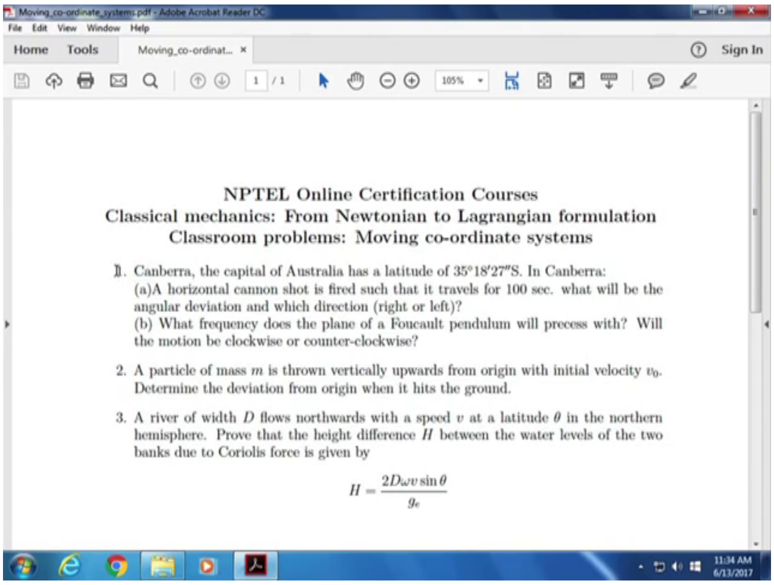

So, the first problem is number 1 Canberra the capital of Australia has a latitude of 35 degrees 17 minutes and 27 seconds. In Canberra, a horizontal cannon shot is fired such that it travels for 100 seconds, what will be the angular deviation and in which direction right or left.

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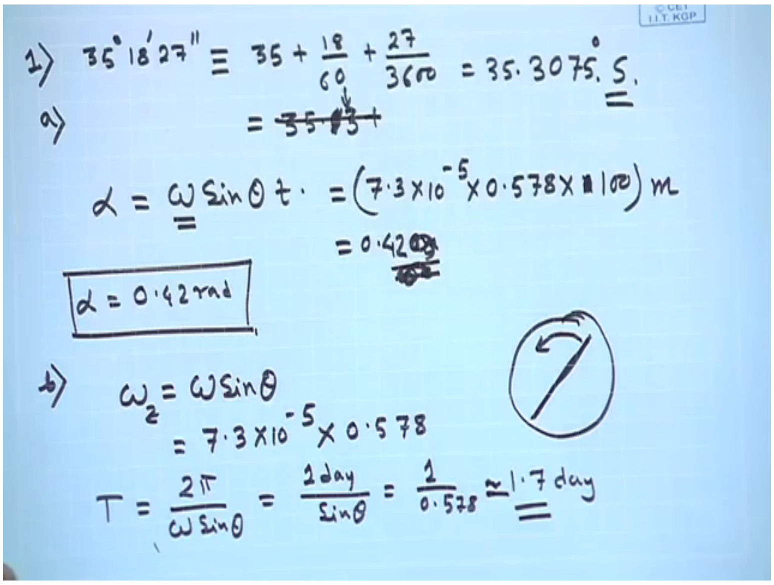

So, for this particular problem, what is given is the latitude which is 35 degrees 18 minutes and 27 seconds.

First of all what we need to do is we need to bring it to a decimal format which will be 35 plus 18 by 60 plus 27 by 3 6 0 0; now if you perform this it will be something like 35 point. So 18 by 60 so, it will be I will use my; I do not have a calculator, but I have a mobile which I can use as calculator. So, if I do that it will be 35 point sorry 35.3 for this particular term and 27 divided by 3600 plus 3 yeah. So, finally, it will be 35.3075 right and it is given as south that is very important.

Now, the angular deviation alpha is given as, omega sin theta t omega is the angular velocity of earth's rotation, sin theta is the theta is a latitude which is given as 35.3075. So, all we need to do and t is given as 100 seconds. So, we have to put these values in. So, it will be seven points 3 into 10 to the power minus 5 into sin of this angle. So, once again I use my calculator here. So, it will be 0.578 into t is given as 100 seconds. So, the deviations and if we calculate this we will get this multiplied by 7.3 into 100 equal to divided by. So, it will be yeah 0.42 centimeters approximately right.

So, this is the angular or sorry 0.42, it is a dimensionless number. So, 0.42 will be the angular deviation; alpha equal to 0.42 radians this is the answer right. So, this is question number 1 a, question number 1 b is for the same place; that means, at Canberra which is given by 35.3075 then 3075 degrees south latitude, what will be the a frequency of a plane of Foucault pendulum. So, let is come to the question here what frequency does the plane of a Foucault pendulum will process with will the motion be clockwise or counter clockwise.

So, there is a second part of part I mean second part of this. So, we have to tell with whether the deviation will be towards the right or left, now it happened to be at the southern hemisphere this s means southern hemisphere and in southern hemisphere the deviation has to be left to the original direction. So, left is the right answer right. Now let is move to the second part what is the plane, what is the frequency of the plane or does the plane of a Foucault pendulum will presses with.

So, we know that the frequency is given by omega equal to or omega z, which is given by omega sin theta once again right. So, again we are not looking for the time period we are just looking for the frequency. So, we have to multiply 7.3 into 10 to the power minus 5, which is omega that has to be multiplied with 0.578. So, this will give us some number and now if I want to calculate the time period T which will be given by 2 pi by omega sin theta. Now recall that 2 pi by omega is 24 right. So, it will be or 2 pi by omega is 1 day essentially. So, it will be 1 day divided by sin theta, which will be 1 divided by 0.578 which will be once again I use my calculator ok.

So, 1 divided by 0.578 will be 1.78 day approximately. So, it will take almost yeah more than 1 and half days for the plane of Foucault pendulum to make a complete rotation. So, if we start here let say to begin with then the plane, again the plane will rotate whether it will rotate clockwise or anticlockwise. Now please recall in southern hemisphere for a person standing on southern hemisphere the earth seems to rotate clockwise right. So, the plane of the pendulum will rotate anticlockwise right. So, it will make a complete precision in 1.7 days right.

Now,. So, with this we move to our next topic, which is motion under I mean a deviation due to carioles force under vertical motion.

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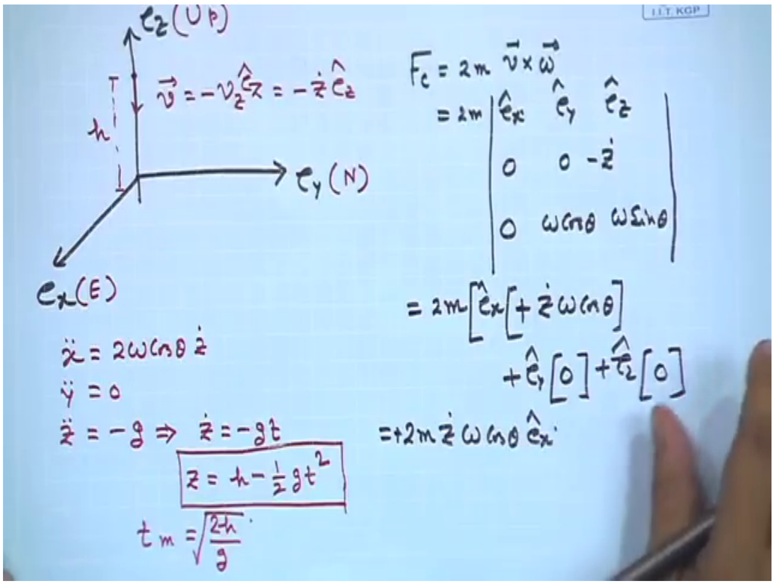

Let us consider this situation here we have ex ey ez, now if you recall ex is east, ey is north and this is up right. Now if this is the situation then if a particle is falling from rest at some height h from the horizon, then we can try modeling this by writing the velocity v vector is given by minus vz it will be ez, cap actually or we can simply write minus z dot ez cap this we can do right.

Now, if we calculate the Coriolis force or yeah; Coriolis force on this particle it will be twice m v cross omega right and once again if we break it into components it will be ex ey ez. Now for velocity the only component is z dot right and for omega the component is 0, omega cos theta omega sin theta right. So, if we continue we will get twice m ex caps, it will be minus z dot omega cos theta right plus ey cap which will be given by; So, 2 terms here yeah. So, it will be given by 0 because both will cancel this column is 0 and e z cap which is given by once again this and this is equal to 0. So, it will be once again equal to 0 right.

So, the only existing term will be twice m z dot omega cos theta, there is a minus ex cap right. So, we have already taken a minus here oh sorry. So, the there will be a minus here, but this will be a minus here. So, this will be a plus. So, this will be a plus right. So, I can write down the equations of motion in all the direction. So, x double dot will be 2 omega cos theta z dot y double dot is equal to 0 and z double dot is equal to minus g. So, these are the equations of motion which we need to solve.

Now, the for the last equation if we integrate it once we get z dot is equal to minus g t right and integrating once more we get z is equal to h minus half g t square. So, now we have included the initial condition that is started from a height h. So, if you put t equal to 0 you get equal you get h z equal to h right. So, if we include that here; also one thing we can do is we can find out the total time of flight, that the particle takes from coming falling from this point to that point and that is given by tm which is simply 2 h by g root right. So, we will just keep this for future.

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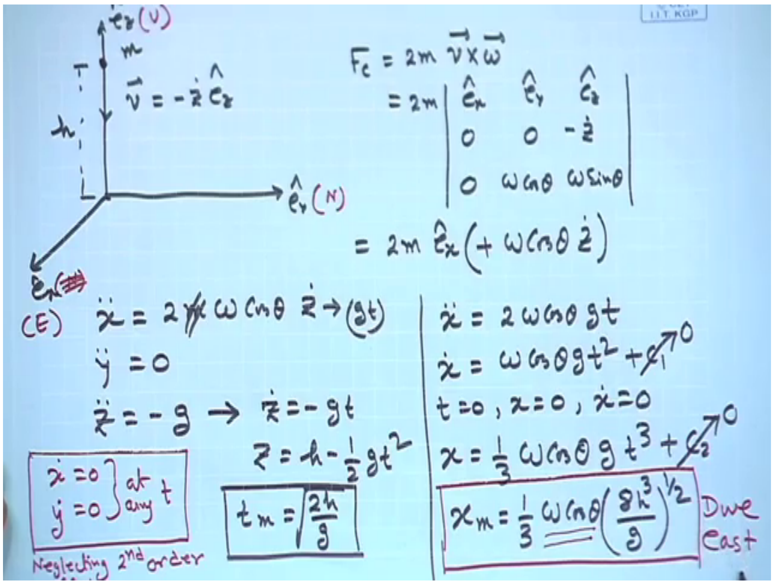

So, we have a particle which is moving which is falling from an height h, particle of mass m which is not very important, we will see once again we level this as ex ey and ez. So, we want to see how much this particle will deviate from. So, ideally in an ideal case if it starts exactly on z axis then it should fall at the origin. So, you want to see which direction and by what amount this particle will deviate due to the Coriolis force.

Now, as the particle is falling straight, we can assume the velocity v we can write it as minus z dot ez cap. So, this is the form of the velocity. So, the Coriolis force F c will be 2 m v cross omega, which is given by ex cap ey cap ez cap 0 0 minus z cap. So, we have included this minus here and we have 0 omega cos theta and omega sin theta right. So, if we do that we see finally, only the ex component will survive because we have a 0 column here. So, if we calculate ex it will be this 0 minus of minus so, there will be a plus omega cos theta z dot right.

So, if I write the force equations along 3 different axis, we get x double dot is equal to 2 m or m will cancel out because we are just writing the acceleration omega cos theta, 2 omega cos theta z dot y double dot is equal to 0 and z double dot is equal to minus g. Now for the last relation if we integrate it we will get z dot is equal to minus g t and z is equal to s h minus half g t square and from here we can calculate the maxi, the time of flight which is the time which it will take to reach the ground, if we put z equal to 0 we get t max which is 2 h by g right.

Now, what we can do is we can substitute this with gt and please remember we cannot write minus gt here because we have already included the minus sign here. So, what we need to do is we need to write x double dot is equal to 2 omega cos theta gt, now integrating it once will give you omega cos theta g t square this 2 will cancel plus some constant c 1. Now please recall that at t equal to 0 x equal to 0 and x dot is also equal to 0. So, x equal to 0 means the initially it starts along above z axis only above the origin on z axis only. So, there is no x or y deviation, the second assumption essentially means that although ok.

So, now think of this situation we have already calculated the Coriolis force and we have seen that Coriolis force is acting along the east direction. Please remember that north sorry ex is sorry ex is east ey is north and ez is up right. So, the Coriolis force is acting along east direction. So, there will be a deviation along east, now this deviation along our force will cost a deviation along east; that means, there will be a velocity component along east as well. Now this velocity component along east will insert another force on in the y direction. So, there will be all the secondary effects. But because this force itself is too small it is proportional to omega cos theta. So, we are assuming that the secondary effects are vanishing non vanishing effect. So, we are setting x dot equal to 0, y dot equal to 0 at any t. So, this is a very important assumption we are making here and this is a valid assumption that means, we are neglecting the second order effect. So, this is a valid assumption given that the second order effects are actually very small. So, we can use that x dot equal to 0 which immediately sets c 1 equal to 0.

Similarly, we can integrate it once more to get one-third omega cos theta g t cubed plus some c 2 which we can also set equal to 0. So, the maximum deviation is when we put t equal to tm and we immediately see that this is nothing but omega cos theta. Now there will be a 2 h cubed by g cubed this will cancel out; oh sorry there is a root here sorry. So, if we include this and do the calculation, the final expression will be 8 h cubed by g yeah. So, there will be a g outside and there will be a g to the power 3 by 2 inside minus 3 by 2 so it will give you h to the power half right.

So, this is the maximum deviation and the maximum deviation is due east. So, this is the maximum deviation and this is due east right, because x axis if you recall is the eastern direction. So, this is the important result because we start from the top and we see that in the deviation is given due east and also please note that the expression, what is the final expression we got is omega cos theta, I mean it is proportional to omega cos theta. So, does not matter if we stay in the northern hemisphere or southern hemisphere it will give you a deviation which is due east right.

So, with this we also briefly introduced the effect of centrifugal acceleration that is the last topic of this chapter.

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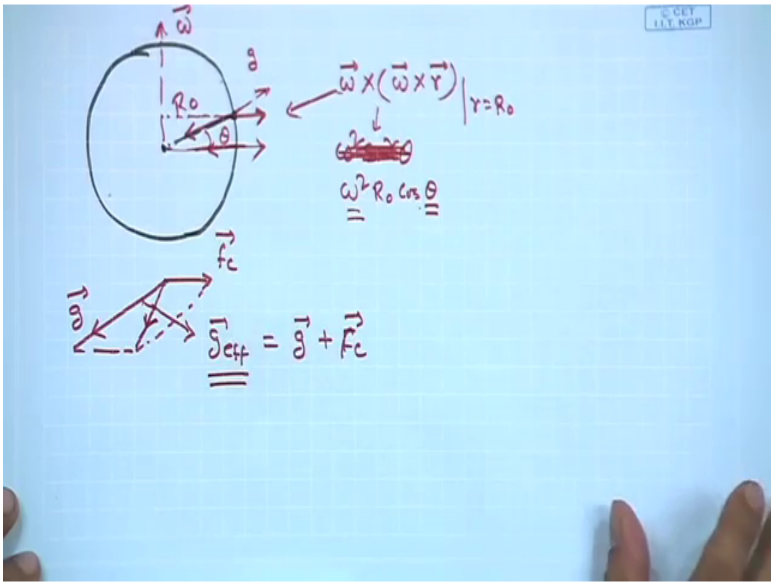

So, this is our earth any point on earth which is at the radius of earth is R 0, at any point the centrifugal acceleration which is given by omega cross omega cross r at any point r equal to R 0, I can you can carefully consider the components of omega and r is a radial vectors. So, the direction of r at this particular point is this. So, if you do this careful a calculation I mean if carefully check the sign of this vector, then it can be shown that this vector is just at any point it is along this particular direction.

So, basically if I join that point with the axis of rotation this will be the extended direction of this particular vector. Now what is the effect? Recall, that the force due to gravity which is which will cause g the acceleration due to gravity is along this line. So, basically g vector acts along this line, now this force is there all that is also force which is omnipresent and the magnitude of this term will be omega square sin square theta being the latitude. So, I am leaving it up to you to prove this that omega cross omega cross r will have a magnitude of this and direction of this ok.

So, this I can so, sorry it will be omega square r cos theta I am sorry. So, it will be omega square R 0 cos theta not that right. So, if this is the case then this is a force which is present at any point of course, the magnitude will change depending on the value of theta right. So, if it is cos theta dependence then it will be 0 at the poles and maximum at the equator right. Now at equator does not have much of a problem because it for this one acts along this line this one the g acts along that line. So, they kind of cancel each other.

So, but at any point they are not in the equal direction, I mean equal or opposite sorry they are not in the opposite direction. So, the effective g at any point if this is my direction of g vector, this is my direction of this particular centrifugal acceleration which we will call fc. So, my effective g is a vector sum of this 2 right. So, this is my g effective which is a vector sum of g plus fc right. So, this is not. So, the g effective we as we see is not along the 4 line joining that particular point to the center of the earth, but which is a line which is slightly deviated from it; f course, this deviation will be small because it has a omega square term the this fc has omega squared term, but it is not 0 there is a considerable amount of deviation and this particular line is called the plumb line ok.

So, anyway this is what I wanted to express briefly. So, all I can tell you that, you can calculate in principle the change in magnitude of g at any point on earth, using this relation right. Now with this we move to the second problem of this problem, set the particle of mass m is thrown vertically upwards from origin with initial velocity v 0 determine the deviation from origin when it hits the ground, now it is a slight deviation of the problem which we have already performed instead of starting from a height h it is starting from the origin let we say.

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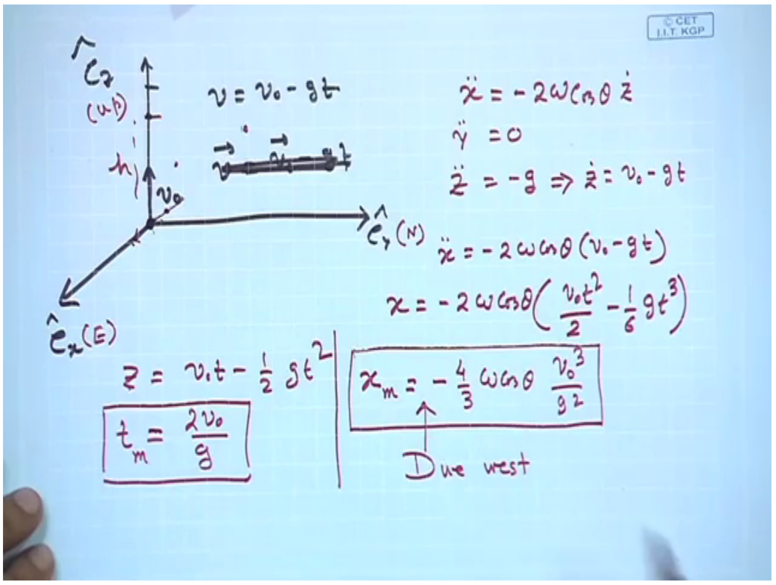

So, we have ex cap ey cap and ez cap. So, the particle of interest starts from this in a with a initial velocity v 0 in the vertical direction and then reaches a maximum height and then it comes back here right. So, once again we have to write the equation of motion, here please understand that v is given by v 0 minus gt right. So, I will not write a vector equation that will be bit confusing, the magnitude of v is at any point is given by v 0 minus gt right. So, as the velocity is in the upward direction what we can do is, we can once again calculate the coriolis forces and it can be shown I am leaving it to you for to do the exact calculation, this will be 2 omega cos theta z dot right y double dot as usual is equal to 0 and z double dot is equal to minus g.

Now, we see that because the motion is in the upward direction there is a negative sign. So, the force is not towards east, but towards west right. This is up now if we again substitute z dot which will be given by v 0 minus gt at any point keep taking this initial consideration into account. So, we get x double dot is equal to 2 omega cos theta v 0 minus gt right. So, after once again we have to integrate it twice and the final value will be I am just writing the final expression will be minus 2 omega cos theta v 0 t square by 2 minus 1 by 6 th g t cubed, right. So, this will be the final expression.

Now, z if I integrate we will get v 0 t minus half square right, now from here what we can get is once again a total time of flight by setting z equal to 0 it is a quadratic. So, it will have 2 roots 1 for t equal to 0 will have z equal to 0 and once for 2 v 0 by g just directly from this relation z equal to 0 and you will get t m is equal to 2 v 0 by g. So, when we put this into this relation to get a total maximum deviation, what we get is minus 4 by 3rd omega cos theta v 0 cubed by g square right, so this is the final answer and because of the minus sign this deviation is due west.

Now this is a very interesting observation because if you recall in the previous problem what we did, the deviation was due east and now the deviation is due west. Now in the previous problem we started from this point and the particle came hit the ground not here, but somewhere along x axis right, but in this case when we start from here the particle comes back and hits the ground somewhere in the y, somewhere along minus x axis. Why it is so I am just leaving on to you.

Now, before we finish what we can do is we can quickly look up into this problem once more, look at this particular expression for xm which was the case when the particle was dropped from a certain height. So, by the way if you find out the maximum height h and reduce this expression in terms of that, it will be exactly you will find that it will be exactly 4 times this expression there with the minus sign here.

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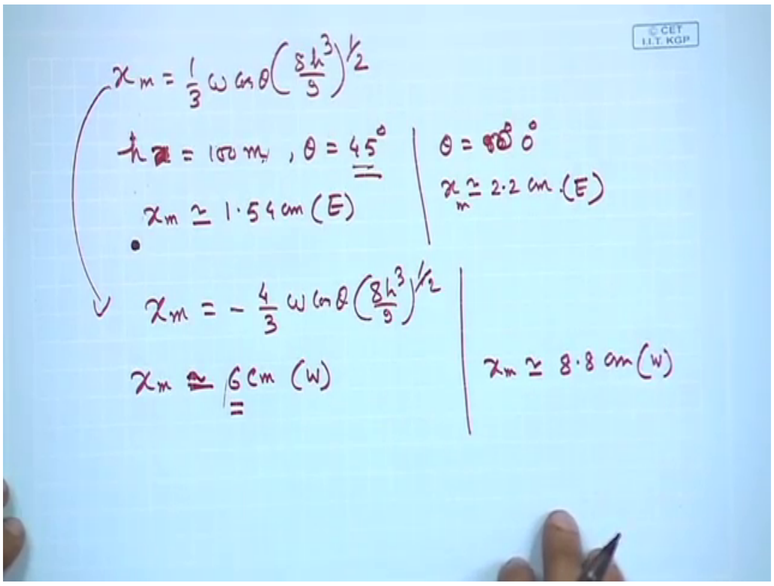

So, let is examine this expression then we will get a field for this expression also, very quickly xm is equal to one third omega cos theta 8 h cubed by g to the power half.

So, if we put say x is equal to 100 meters sorry h is equal to 100 meters, what are the values of this deviation we get. So, for theta equal to 45 degrees, does not matter if it is north or south because it is a cos theta dependence, we will get xm is approximately 1.54

centimeters and for the same height theta equal to 90 degree, means we will get sorry theta equal to 0 degree, where we will get maximum deviation we will get x is equal to xm is equal to 2.2 centimeters.

So, you see that the deviations are small when we are compared to a height of 100 meters the deviation is small, but now consider the case when the object will be deviated from a height of few kilometers, then these numbers are not small so these are significant numbers. Now as I said when we are throwing the object this particular case, when we are throwing the object from this one from this point and it comes back this it is not only due west, but it will be exactly the same expression multiplied by 4. So, in this particular case xm will be minus 4 by 3rd omega cos theta 8 h cubed by g to the power half.

We have to substitute, we have to find and we have to replace v 0 by h. So, in that case if we use the same number x m will be so this will be towards east. So, it will be almost 6 centimeters towards west and in this case it will be xm approximately 8.8, approximately 9 centimeter towards west. So, you see these numbers are not very insignificant anymore. So, if we can throw a stone at up to a height of 100 meters, when it comes back it deviates by 6 centimeters at 45 degree latitude and that 0 degree latitude the deviation is almost 9 centimeter. So, these numbers are not very insignificant with this we end this class. So, we will see you in the next class.

Thank you.