Classical Mechanics: From Newtonian to Lagrangian Formulation
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Lecture – 25
Moving Co-ordinate Systems – 3

We have seen that in a moving coordinate system. The there is a pseudo force which comes with a term proportional to v cross omega v, being the speed of an object in the moving coordinate is measured in the moving coordinate system at omega is the velocity of rotation

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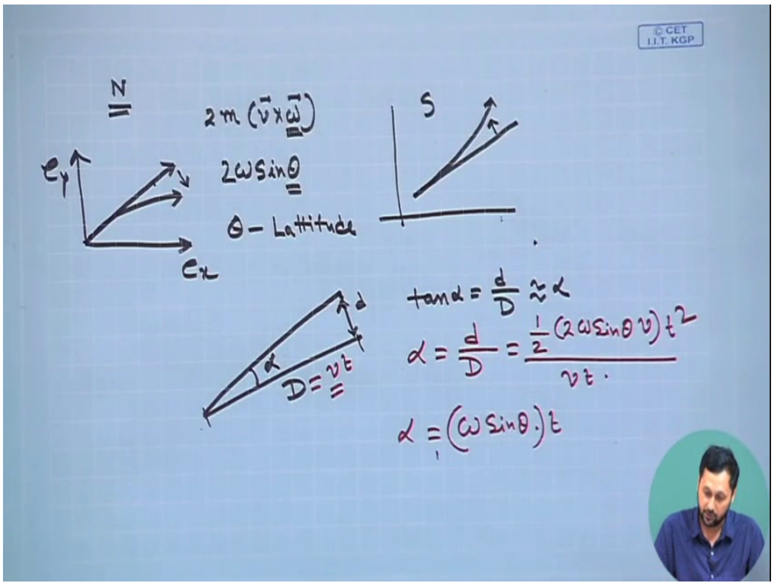

Now we have seen that this particular force in northern hemisphere, if an object is moving in a horizontal plane, that is the ex ey coordinate system in a northern hemisphere, this particular force which has the form 2 m v cross omega being the velocity of rotation earth's rotation in northern hemisphere. This force will make this particle this object to deviate in the direction in the right in the; you know, we will make it deviate to the right of it is original direction. Similarly, if we go to southern hemisphere then this force because the final form of the force will be something like we will have a term which is proportional to omega sin theta being the latitude.

So, as we go from northern hemisphere to southern hemisphere, this particular term will change sign. So, in northern hemisphere if the deviation is towards right of it is original path then in southern hemisphere the deviation will be to the left of it is original path.

Now we can also find out and find out, this the angle of this deviation, assuming that this was the path of original path which was we which the particle was intended to travel. And it end up going in this position instead of this position. So, the displacement is this and this is the angle alpha. Let us say so if the deviation is given by this distance t, then and this original path length is given by capital T, then we immediately see that tan alpha is equal to d by t.

Now, because this angular deviation is very small, typically then we can safely assume that this is approximately equal to alpha. So, we can write alpha is equal to d by t. Now if we go by this particular definition of alpha, then it can be easily shown that the. So, the acceleration will be a velocity will be a term proportional to omega sin theta. So, your final deviation in this direction will be half this is the acceleration term so it will be 2. So, there will be a factor of 2 coming up here, right. So, it will be 2 omega sin theta times the original velocity v in times T square. This is the deviation in the perpendicular direction, and the original velocity was v and if it is traveled for a time T then T is equal to vt assuming that there is no acceleration or deceleration in this particular direction.

So, it is a v is a linear well, I mean it is a uniform velocity we can take. So, we can write this. So, we see that the angular deviation from this calculation will be omega sin theta times t. So, from this, if we know the latitude theta we can calculate the angular deviation for a projectile in northern hemisphere or in southern hemisphere only differences in northern hemisphere. It will be towards the right of it is original direction in southern hemisphere is towards the left of it is original direction.

Now, there are this is one thing that the one affect of Coriolis force that we have discussed already, but there are more severe effects. For example, a cyclone what is the cyclone.

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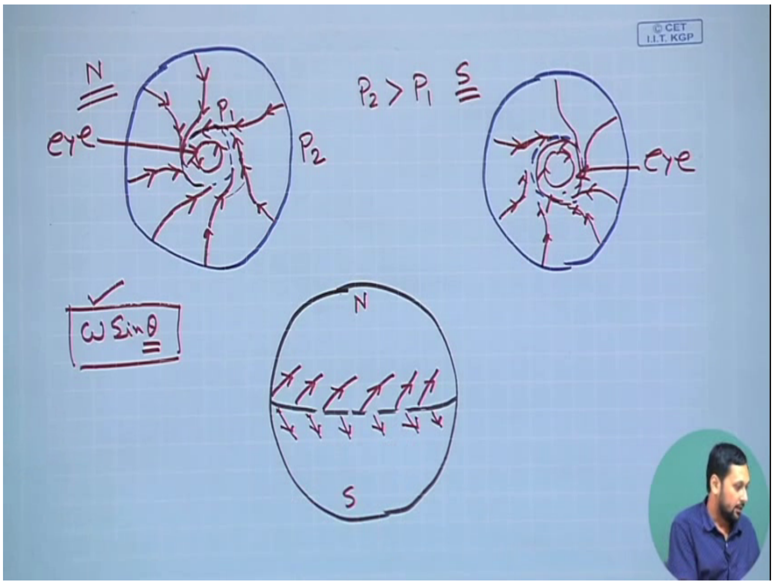

Cyclone is movement of air towards a low pressure zone from high pressure zone. So, let us say due to some temperature difference on earth surface or on the on the typically it happens from the ocean that there is a low pressure zone, which is you know, which is a void in the atmosphere and there is a high pressure zone surrounding it.

So, the air from this high pressure region. So, here let us the pressure is P 2. And here the pressure is P one and typically P 2 is greater than P 1. So, of course, the air particle would like to move into the low pressure zone. Now when it starts moving in initially let us say it is a linear path, but because of earth's rotation, there is a Coriolis acceleration on this and that that is why in the northern hemisphere the cyclone the path will be deviated to it is right.

So, in process the eye of the cyclone which is the central part, it will have a anti clockwise rotation. So, in effect it will have an anti-clockwise rotation in the eye of the cyclone. So, this is once again in the northern hemisphere. Once we move to southern hemisphere we will have a similar phenomenon, when we have a low pressure zone surrounded by a high pressure zone and the cyclone is created, but this time the air will the flow of air will be moving towards left of it is original direction due to Coriolis forces right. And what we will have is the eye of the cyclone will be moving in clockwise direction, right.

So, it once again this is the eye, but here the rotation will be in the clockwise direction whereas, in the so this is in the southern hemisphere and this is in the northern hemisphere. So, in northern hemisphere a cyclone will move into the anti-clockwise direction in southern hemisphere, it will move into clockwise direction also if you recall the familiar picture from your geography textbook, school life, geography textbook of monsoon. So, this is your earth this is our earth this is the equatorial plane.

Now, the standard picture for monsoon was in. So, equatorial plane is always at a higher temperature this side is at relatively lower temperature sorry, this is when the sun in the in the during the summer when sun moves to northern side apparently moves to northern side. So, there is a low pressure zone created here. So, the monsoon will start going like this. So, this is the direction for southwest monsoon right.

So, this is and also, similarly in the southern hemisphere during the winter time or i should not call it winter because in both in northern and southern hemisphere the winter season changes. So, this happens typically in the month of may, June, July in northern hemisphere. And in southern hemisphere in the months of September, October, December, the opposite exactly opposite thing happens. So, equatorial plane remains cooler compared to the compared to this side of earth and monsoon moves in, but this time because it will; right. So, it will start going in a straight direction, but it will move to left due to Corioli's forces, right.

So, these all these are due to the effect of Coriolis forces, they all this deviation also the ocean current is strongly driven by Coriolis forces on earth’s surface. So, these are all very well known effects well studied effects, but then and also you can really you will realize that as the Coriolis force is dependent on omega sin theta, theta being the latitude probably there are ways of measuring omega the earth’s rotation speed based on this formula it turns out that there is.

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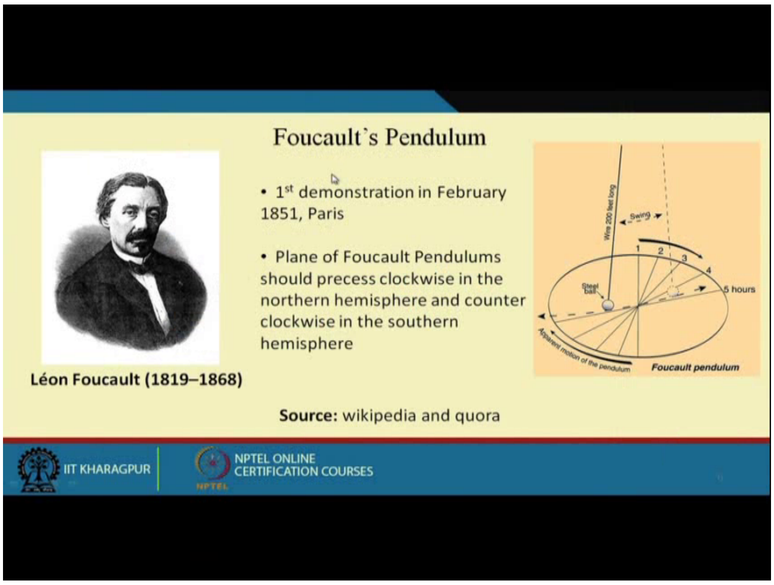

So, that was first done in by a French scientist named Leon Foucault do not go by the spelling the actual pronunciation will be Foucault and that was first demonstrated in 1951.

So, this he was a very famous scientist of his time, he not only this particular experiment, but he also did many other experiments we measured who he was the first one to measure speed of light and he was the one who first discovered where the systematic discovery of eddy current. So, in 1951 what he did was he hung a very long pendulum it was a simple pendulum with a metal bob, but the length of the thread was approximately 100s of feets, I do not remember the exact numbers, but it was hundreds of feets and it was done and the at the central observatory in Paris. So, the pendulum was hanging from the hanging from the roof of the observatory and the bob was somewhere close to the ground.

Now, when the pendulum was allowed a free swing for a sufficiently long time it was seen that the plane of the pendulum is moving in a very systematic manner in a clockwise motion. Now here we have this it is actually a cartoon picture it is not the exact values let us say, but let us assume that it is something close to the reality. So, in one hour initially the plane of the pendulum was along the swing of the pendulum one was along this line 1. So, let us say after one hour, it has moved from 0.1 to 0.2 after 1 more hour it has moved to 2 2 3. So, the plane of the plane of the pendulum was constantly swinging. So, this is a famous experiment of Foucault pendulum and Foucault used this experiment to first time to measure the rotational velocity of earth.

Now it is also known that Foucault pendulum will proceed in the clockwise direction in northern hemisphere, and anti clockwise direction in southern hemisphere. So, we can put all these things all this picture into a mathematical formulation and see if this is really true.

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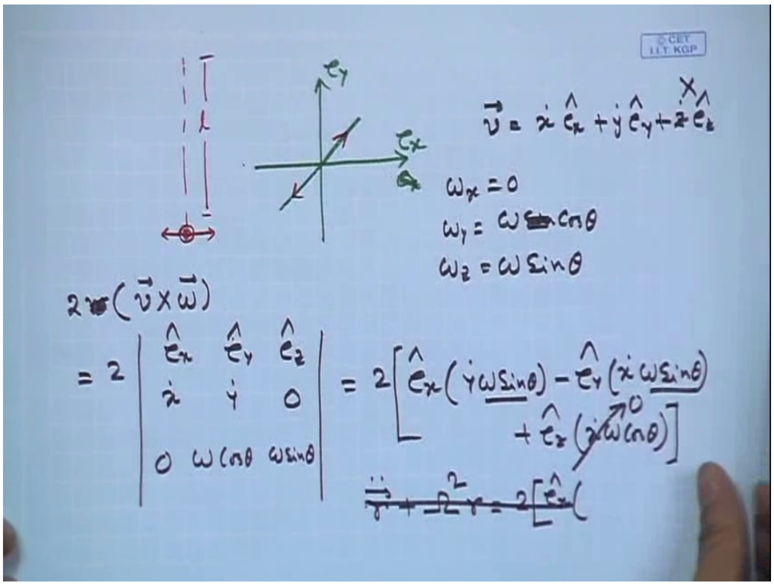

So, let us assume that this is a pendulum and this is the bob of the pendulum, this has to be really long this l which is the length of the pendulum has to be really long in order to demonstrate this effect. So, it is a simple pendulum and as we know that simple pendulums are simple only if the angular deviation is small, right. So, it is oscillating let us say.

Now, if we look at this pendulum, from this we take a top view of this pendulum we basically project it on the standard x y axis system. Then we see that it is let us say it is oscillating in this particular manner. So, it is going in and coming back in the my coordinate system. Let us level it ex or rather x let us call it e x and ey, because we are using this same notation through and through right.

Now, what is small means the displacement is such that the z deviation of the pendulum can be neglected? So, what we can do is we can safely write the velocity v of the of this pendulum bob as x dot e x cap plus y dot e y cap, we can neglect there will be a term of z dot e z cap which we can safely neglect assuming that it is a small deviation. So, the pendulum will be we can assume that pendulum the bob of pendulum only oscillates in x and y plane.

Now, once again omega has 3 components omega x equal to 0 in the current construction we are using omega y equal to so this is omega. Yeah, so omega y will be omega sin theta no yet this will be cos theta, sorry omega y is omega cos theta and omega z is omega sin theta right.

Now, using this if we try to calculate the Coriolis force which will be 2 m times or Coriolis acceleration. So, we can get rid of this m 2 m 2 times v cross omega. Then that will be there is a 2 here ex cap e y cap as cap x dot y dot 0. We are taking z dot equal to 0 and it will be 0 omega cos theta and omega sin theta. So, this will be 2 times x cap y dot omega sin theta. And for e y we will have so this first term will be 0. It will be y cap x dot omega sin theta, right. And then there will be an as term which will be equal to x dot omega cos theta, right.

Now, as we are assuming that the plane of pendulum is confined in the x y direction once again we can just neglect this term. So, including this we can write the equation of motion for this pendulum as r double dot plus omega square r equal to 2 ex cap, sorry we can actually what we can do is we can just take omega sin theta out from these 2 terms and we can write this as ok.

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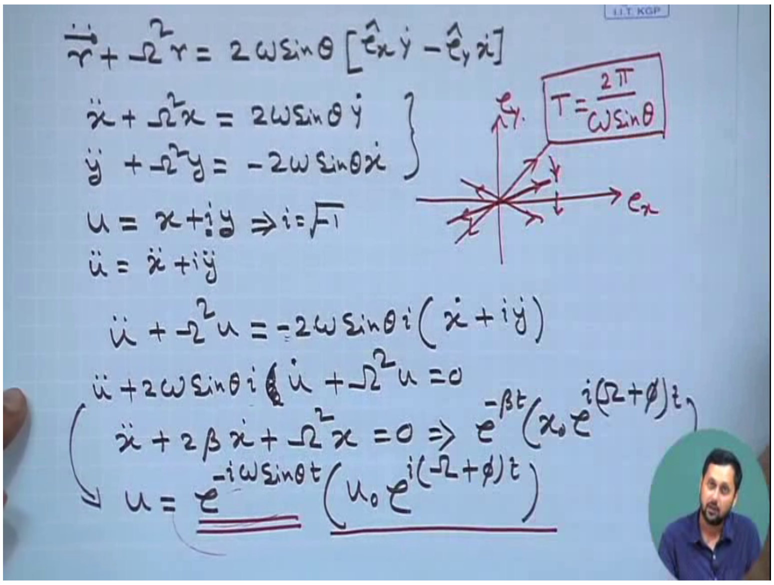

I just write it again r double dot plus omega square r is equal to 2 omega sin theta e x cap y dot minus ey cap x dot, right.

Now, what is r? R is a vector which is confined in this my plane. So, R, now if we resolve this equation in the component form. Then we immediately see it will be x double dot plus by the way. This omega is the frequency capital omega is the frequency of oscillation of this pendulum. So, it will be 2 omega sin theta y dot y double dot plus omega square y is equal to minus 2 omega sin theta x dot, right. In order to solve this set of coupled equation, what we do is we introduce a new parameter a complex quantity which is x plus i y, right.

Now, what we can do is we can multiply the second equation. So, if we do that. So, if this is u then u double dot will be x double dot plus i y double dot now we can multiply the second equation by i, i being the complex. So, i equal to root over of minus 1. So, if we multiply this with the second equation and add, then we will see it will be u double dot plus omega squared u equal to 2 omega sin theta. Now there will be a minus sign coming here and i coming here it will be x dot plus i y dot.

So, if we take this i minus, i inside the first term will be minus i x dot which we will get if, i multiply this with i and the second term will be i square will to the will be minus 1 and there is a minus here. So, the second term will be i y dot which we get here. So, rearranging we can write this as u double dot plus 2 omega sin theta i sorry times u plus u dot plus omega square u equal to 0.

Now, if you recall if now if we go back just give me a second i will just bring it in here right. Now if you recall this equation x double dot plus 2 beta x dot plus omega square x equal to 0 which is the standard equation for a damped harmonic oscillator what is the solution the solution to this equation from your this is this was part of your even plus 2 syllabus nowadays. So, the solution of this will be e to the power minus beta T x 0 e to the power i omega plus phi some phase T ok.

So, this part is the oscillatory part which gives you the standard oscillation of x, x direction and x 0 is the amplitude, phi is a phase term which has to be determined from the initial condition. It is not terribly important because finally, this represents an oscillation in the x and there is a damping term. Now if we compare these 2 equations the second term looks exactly like this particular term except that there is a i here. So, what we can do is for this also we can write a solution in this familiar form by replacing beta with omega sin theta i, if we do that we get solution of the form e to the power minus i omega sin theta T u 0 e to the power i plus phi t.

Now, this part the lower part once again, it just represents the oscillation of the pendulum in this my plane. So, this part actually represents the oscillation in this my axis system. Now what this what does this additional term imply. Now if you compare with this standard solution for damped oscillation e to the power minus bt is just a damping term which will gradu which will cause in great gradual dumpee damping of this amplitude, but here because of this presence of this i here, this is not a damping term anymore; this is also an oscillating term.

So, this oscillating term actually this term actually represents an oscillation of this whole thing; that means, oscillation of this pendulum plane pent the plane of the pendulum in a particular direction and this oscillation so; that means, the introduction of the i mean having this term into the solution means, the whole thing will oscillate so; that means, the plane of the pendulum will also oscillate with time. So, this is the oscillation of the pen plane and this plane itself will also oscillate in a particular direction with a time period T which will be given by 2 pi by omega sin theta, right.

So, we see that the plane of the pendulum is not fixed. If we go back to this figure now we see that the plane of the pendulum is not fixed as was demonstrated originally in the Foucault pendulum experiment and it also rotates in a certain direction. And now if you recall if you come back to this formula, if we can somehow measure the plane the rotation of this rotation of the plane as a function of time then we can from this we can find out this time period from this we can calculate omega knowing that theta is the latitude

So, this is a very simple i mean not a very simple experiment to demonstrate because you have to make this pendulum with very good accuracy in a sense it has to be a really good support it has to have a really good supporting system, it has to have a very long thread, but all in all if we can do that the Foucault pendulum experiment can provide us a value of omega.

Now, the question is in which direction the plane of pendulum will precise. Now to answer this question let me let me try to explain this. So, this is our earth it is not; it is the best thing I can find in my office to demonstrate. So, what happens is earths rotates. So, this is my north northern direction this is my southern direction. So, earth rotates from west to east that is why sun rises to the rises in east and sets in west right. So, this is how the earth rotates.

Now, during the rotation of the earth now. Let us assume that you have set up a Foucault pendulum experiment exactly on the pole exactly on north pole for an observer or north pole the rota or observer in the northern hemisphere the rotation of the earth is in clockwise direction sorry rotation of the earth will be in counterclockwise direction, but for a observer in the southern hemisphere the rotation of the earth is clockwise direction please think of it.

Now, if now you set up a Foucault pendulum experiment let us say exactly on the pole. So, what happens the pendulum is rotating and underneath the earth the pendulum is oscillating in this plane and underneath the earth is also rotating. Now for an observer standing on earth he or she cannot feel the rotation of the earth, because it is difficult it is almost impossible i mean not difficult, but it is impossible to fathom the rotation of the of a frame standing on that particular frame because we are so used to the rotation we are all rotating all the time with us, but we are used. So, much used to this particular rotation that we do not feel it anymore.

So, unless and until there is some other measurement device we cannot fathom the rotation of the earth. So, for an observers sitting on the next to the pendulum standing next to the pendulum for him or her, it will be the plane of the pendulum rotating in the opposite direction of earth. Now for northern hemisphere people the plane of pendulum rotates in the direction opposite to the earth's direction for northern hemisphere people earth rotates in a anti clockwise manner.

So, the plane of pendulum will precise in clockwise manner for people in southern hemisphere the earth's rotation is in clockwise direction and plane of plane of precision, i mean plane of rotation of the Foucault's pendulum will be in the counterclockwise direction.

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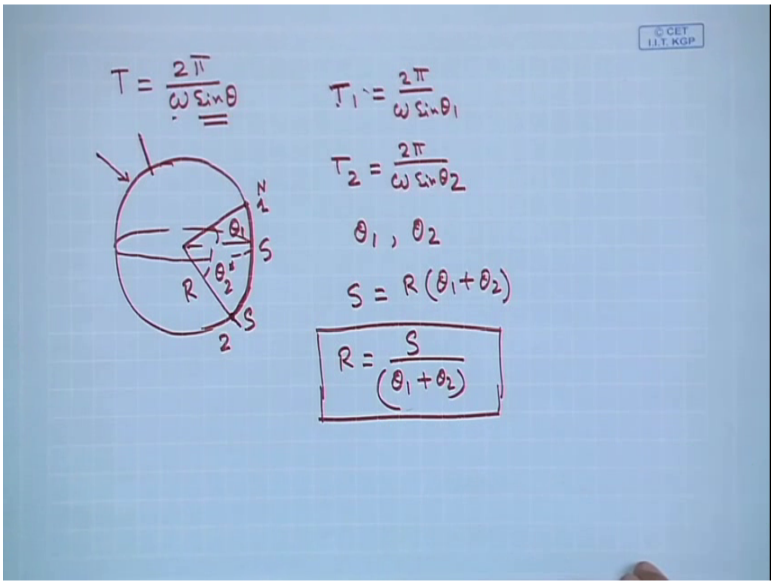

If we now come back to this formula once again it is 2 pi by omega sin theta think of it, it is again it is also apparent from this equation because it is dependent on sin theta being the latitude. So, if I go from northern hemisphere to southern hemisphere my theta changes sign. So, this is my equatorial plane this is how you measure your theta. So, this is a theta for northern hemisphere and this is another value of theta dash that southern hemisphere. So, in this case sin theta will change sign. So, that is why the plane of rotation will be in the opposite direction.

Now, think of a situation where you have 2 time periods T 1 measured at some location. Let us say in northern hemisphere which will give you a value of let us say it is theta 1 and this is theta 2 theta 1 2 2 pi by omega sin theta 1, and you have another magnitude or another value of T 2 which is measured at southern hemisphere with this particular value right.

Now, if you know the distance along earth the exact distance along earth between these 2 places, which is along the arc length which is given by let us say s then from so from these 2 experiments T 1 and T 2, we can immediately calculate theta 1 and theta 2. Now if you also know the late lateral distance between these 2 points 0.1 and 0.2 on along the earth's surface then this experimental result itself can be used to calculate the radius of the earth because this s will be nothing, but r into theta 1 plus theta 2.

So, it is Foucault I mean it is just a toy model, I mean it is a never it is never easy to set up a Foucault pendulum to measure earths the radius of earth, but in principle this because this is not very accurate to begin with, we will get values of T 1 and T 2 which are not very accurate, but at least in on theoretical basis on theoretical consideration we can say that earths radius can be measured from a Foucault pendulum experiment. So, this is how it is used and this is how people, have I mean people have developed science over the years.

So, with this we will move to our next topic which is the question of horizontal drop. So, what happens if we drop something from a height or we throw a stone then when it comes back will it hit back the same position or not, surface or it will hit at a different position we will take up in the next class.

Thank you.