Classical Mechanics: From Newtonian to Lagrangian Formulation
Prof. Debmalya Banerjee
Department of Physics
Indian Institute of Technology, Kharagpur

Lecture – 12
Central forces – 5

So, Kepler's laws, that is what we are going to discuss now, but before that as I have discussed, told it, I mean, I told you already we will look into the history of astronomy in mid 14th and sorry 15th and 16th century.

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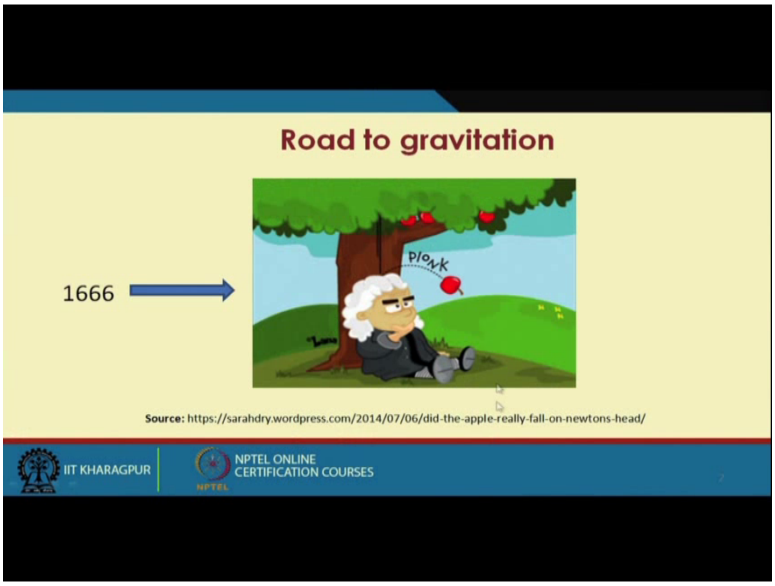

So, the famous apple that fell on Newtons, Sir Isaac Newtons said was somewhere in 1966, when Newton was a 24 years old student, of a, of mechanics in Cambridge University.

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But before this, there is a history of almost you know, 200 years, Nicolaus Copernicus born in 1473, was the first of this modern day astronomers, who came up with the first realistic model of solar system with sun at it is center now of course, this model was, I mean this model was far from being right, he tried out many different things, which eventually came. I mean he eventually came up with this model based on his observation of the stellar system and please remember that, this was the time before Galileo. Galileo came almost 100 years after him or more than 100 years after him.

And Galileo was the first to develop something like telescope. Even most likely there was a story that Galileo was not the first, who discovered telescope, but he was the first to discover, when to apply it in case for observation of the stellar system observation of stars and moons and their motion. So, all the modern day, I mean all the olden days Astro Normas, Astro astronomy, that was half observation and half myth. There was no very solid account of very solid physics, will rather solid knowledge, why the motion takes place in certain manner.

And there were lots of myths about stars myths, about movement of celestial systems myths, about you know how it can affect the fortune of one fortune of a nation fortune of the king. So, many of the eminent Astro astrophysicist. I as we could call them now, it in modern days, where associated with the royal coat, they were the royal astral lodger of some king, who has to tell the, suggest the king whether this is a good time to go for, I mean go for you know, what you call, you can go for a war or is it a good time, to you know to make a move in politics things like that.

Now, and that was there was a heavy influence of church in it. So, in Europe, specially the Roman, Roman Catholic church. They used to, you know used to have a huge influence on this astronomical, astrological observations and according to churchs believe, earth remains at the center of this universe, everything revolves around the earth. So, it was also a very intuitive knowledge on sitting on that day, because we all see that the sun comes up every morning in the east and goes down in the west. Moon comes up occasionally on the sky and goes down once again.

So, it is why, of course, it is true that earth is stationary and the everything, every other stellar objects are you know evolving around earth, but please remember, although they were as naked eye astronomers, they used to observe things very closely. So, before Copernicus, also they were Ptolemy and other astronomers and they were of course, contribution from Indian astronomers and Arabic astronomers, who had created a map of the stars. We could see with our naked eye and he, they could identify few planets as well.

Now, these were all tabulated. The tabulation was nice. Tabulation was accurate and based on their tabulation and based on the observation. Again there is a history behind it, I mean not that Nicolaus Copernicus, one single day came up with the idea that, no instead of earth sun is at the center of the solar system. It is not like that. So, he also you know went through all the available knowledge and he thought being you know, I mean logical person, he thought maybe church is wrong, maybe it is not the earth which is at the center of the system, but it is the sun, which is at the center of the system.

So, he came up with this new model of course, this model was not acclaimed heavily why, when by church. It was scrapped, but because Nicolaus Copernicus, he was also a clever man. He have to be a clever man, if you were you know, if you want to challenge a notion, which was there for centuries and which is directly backed by, you know the Roman Catholic church. So, he did not make much of much noise about it, but his contemporary astro astro astronomer Galileo Galilei, who discovered telescope well he probably he did not, there is a controversy related to that, but he used it at least, he was the first one to use it, to observe our solar system.

So, in a sense he experimentally verified the theory which Nicolaus Copernicus came up with that, it is not the sun which, sorry not the earth, which sits at the center of this universe or our solar system at least, but it is the sun, which sits at the center of the solar system. So, we probably, all know the story, that Galileo Galilei was under immense pressure, from the church and after few years, he was you know confined in his house, arrest almost he had to also suffer time in jail. So, he was under pressure to change his statement.

So, eventually he had to obey to what church is the, church said and he had to retreat, when he has retract his death statement, that earth is not at the center of this universe anyway, but all good work essentially pays off. So, after almost 100 years of his death, more than 100 years of his death, it was the first time, when church, Roman Catholic church officially agreed that Galileo was, I mean it was not a justified move to you know put pressure on Galileo and make him you know making with throw his statement and in 1992 else as late as 1992, Pope John Paul, two officially you know expressed, what you call.

So, he or officially condemned the move what church made in order to stop Galileo from publishing his results and Pope, benedict 2 in 2016, he declared Galileo, I mean he accepted Galileos contribution in astronomy, in a modern day astronomy and he considered him as one of the assets of the society. So, as we see all good work sooner or later it pays off.

Now, what happens after Galileo? Galileos time is almost like yes some overlap sorry, he did not have any overlap with Copernicus, but at the same time, 3 were 2 other very famous astronomers of their time, one is this Danish gentleman named Tycho Brahe, who was last of the naked eye astronomers.

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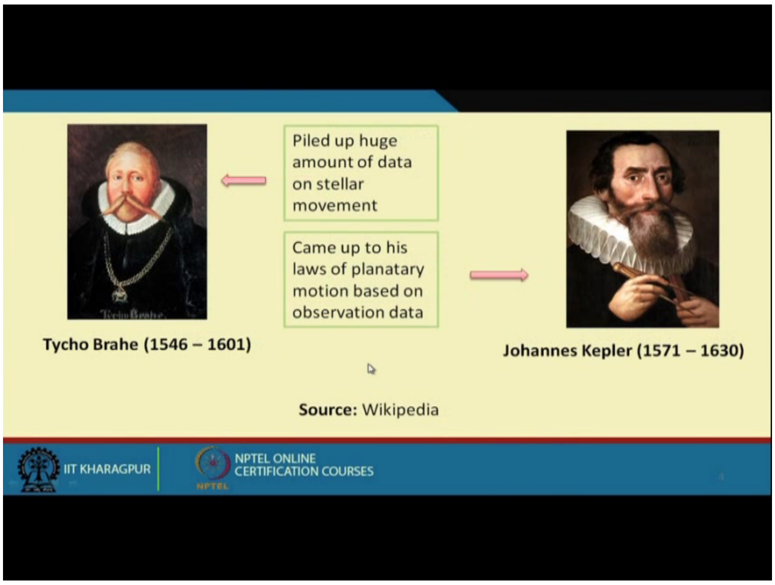

He did not believe in this telescopic concept, he were, he did it in the. So, called traditional way he started observing stars and their movements with the help of naked eye, but he maintained a very nice account of the thing, extremely well documented movement of stars and planets and moons, were found in his record.

Now, Jonas Kepler, he was a German in by birth. So, he by the way Tycho Brahe, although he was a Danish scientist, Danish nobleman, essentially he moved to Prague and that time Prague was the capital of the you know, almost the intellectual capital of Europe. So, all the big names in literature, in history, in mathematics, they all went I mean, they tried to went, to Prague that was their final destination.

So, that is where Kepler and Tycho Brahe met essentially. So, Tycho Brahe has his, had his own observatory, which was some distance, which was set up at some distance from Prague and Kepler, went there as an apprentice to work with it. So, very soon he found out. There is a lot of potential, in the data which Tycho Brahe had or by the way Tycho Brahe also had, his own model of the universe or the solar system, which in a sense was almost right. He said, sun remains at the center of the solar system, with all other planets revolving around sun, but earth stays outside and you know the whole thing moves around earth. So, he again, once again, he did not want to go directly against the church, but what he did was, he was trying to, you know make a bridge between the philosophy and science; our religion or science without antagonizing both of the sides.

So, anyway. So, his contribution, his main contribution to this, to this movement of movement towards gravitational theory of Newton was that, his documentation he was very good with his documentation power and then came Jonas Kepler, sometime in mid or early 19, early 1600, he essentially gathered, you know he essentially gathered all the information, he could find from the work of Tycho Brahe and Galileo and of course, Copernicus.

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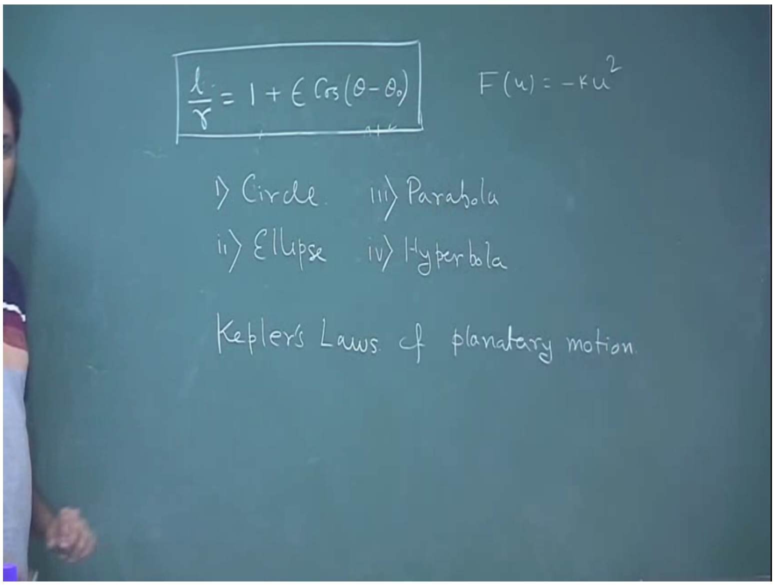

Copernicus was the first to propose this model and tried to combine them in three laws and then he came up with the Kepler's laws, famous Kepler's laws of planetary motion. What are this laws, we will see in a moment.

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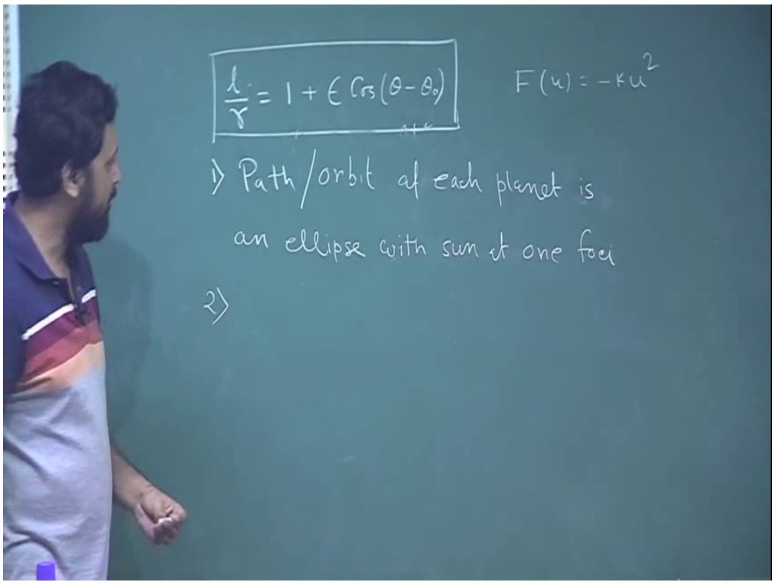

So, let us look at into the Kepler’s three laws one by one, first law is movement or path, that time orbit, the term orbit was not coined, but let us use orbit, because we use, we will be using this term, lot orbit of each planet is an ellipse with sun, at one, one of the foci ellipse has 2. So, two focal points sun has to be at one of them. So, it is called a sun at one of the foci. So, this is the first law, second law is the line or rather I will instead of. So, you can find the statements anywhere. So, instead of writing them on board, what we are going to do is I am just drawing, going to give you a brief picture.

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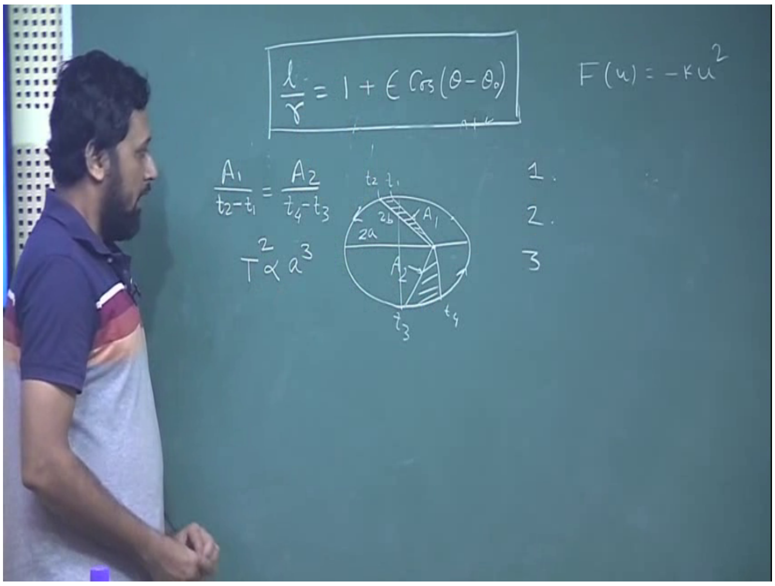

So, what he said was if this is the orbit of a planet, it is, it has to be an ellipse with sun at one of it is center that is the first law one of his foci, second law the line that joins the planet to the sun, sweeps out equal area in equal interval of time. So, let us say, if it is a time t 1, it is here and time t 2 it is here, if this is the area, if the swapped area checked out area is A then and in another time t 3 and t 4. This area is let us say this one is A 1 and this 1 is A 2, then his second law was A 1 divided by t 2 minus t 1 is equal to A 2 divided by t 4 minus t 3. So, the thing is moving in this direction. So, t 4 is greater than t 3 and t 2 is greater than t 1.

So, it sweeps out equal area, in equal interval of time so; that means, aerial velocity is constant. So, that is the second law. Now, the statement of the third law is a bit tricky, if the planet has a time period of t time period, means if it starts from this particular point, it is also around the sun and comes back to the same point in time t, then this time t has a relation with the length of semi major axis of the ellipse. Now, we all know that ellipse has two axis, one semi major and one major axis and one minor axis.

So, the length of this major axis is taken as 2, whereas, that of the minor axis is taken as 2 b. So, the semi major axis is this length essentially. So, the statement of this law was T square is proportional to a cubed. So, length of the semi major axis is proportional to the cube root of or sorry, cube root of the length of the semi major axis is proportional to the square of the time period. So, that is the third law.

So, first law says, it is an ellipse, second law says the areal velocity is constant and third law says, this now please remember that, these three laws were purely observational at that time he observed things he not only, he is he relied not only on his observation, but the observation his resisters has made his contemporary other scientists like Tycho Brahe has made, he or analyzed all this data and he made turned he came up with 3 laws, which later on was proved by Sir Isaac Newton.

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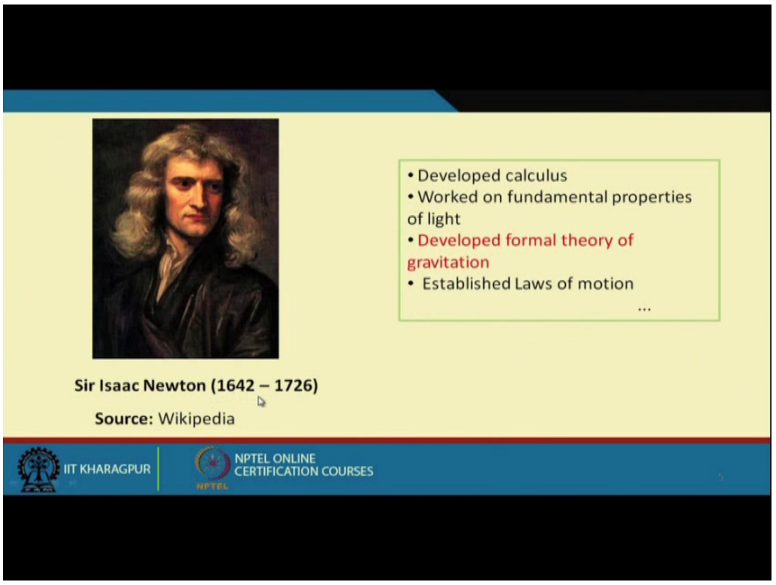

Now, what happened is now, it is a, it might be a coincidence, but if you look back Galileo Galilei was died in 1642, same year Sir Isaac Newton was born of course, in a different country. Galileo was from Italy and Sir Isaac Newton is from Britain. Great Britain, he was born and probably we cannot really summarize, the contribution of Sir Isaac Newton in one slide. We in one page of a presentation, it is not possible,

because he is the man who developed modern Calculus, he is the man, who worked out fundamental properties of light, we all know that experiment of Newton, during we perform even today, in our laboratory, he developed the, he established the laws of motion, we all know that Newtonian laws of motion, law first, law second, law third we just still I mean, which basically, you know is the fundamental of modern day mechanics and also he developed the theory of gravitation based on, the which was the one of the prime motivation, for that was, this three Kepler's laws, anyway.

So, with that can wait later, I mean, how this happened and how this development was particularly made that we can, we cannot go in details of this, in this particular class, but what we can do essentially, we can try to prove the three Kepler's laws, first law, second law, third law, assuming that the force between the sun and the planet is of this nature.

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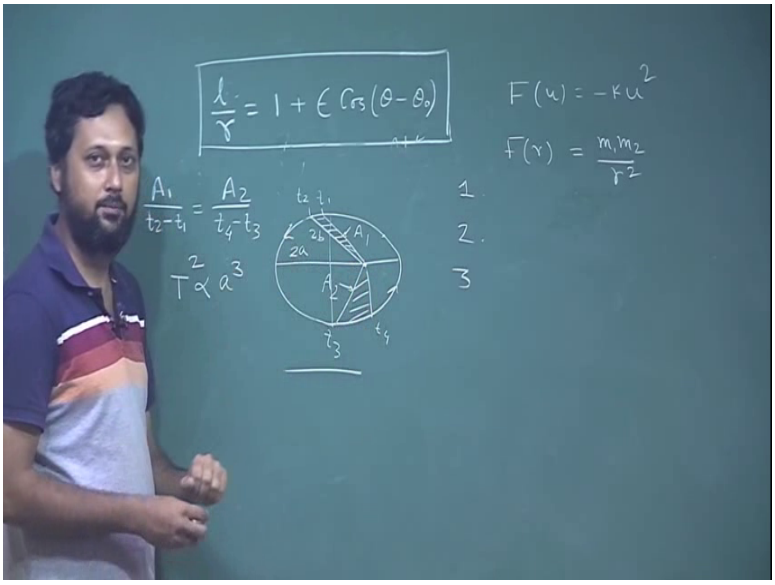

Now, this is a contribution of Sir Isaac Newton k. We all know that, this law exactly is f of r is m 1 m 2 by r square. We all know that and this law, came more than a 100 years later of this. After this law, after this Kepler's law, Kepler's first law, second law and third law.

But it turned out that these laws were developed, keeping in mind the observation which Kepler had and based on which Kepler came with these three laws. So, Newton was a student in Cambridge and he was a student of mechanics to begin with and then by the time he, I mean he is the self as far as I understand, he was a self-taught mathematician. He first derived the Calculus, I mean he, modern day Calculus, he started working on them, he started reading the works of previous bath finding astrologers, astronomers like Copernicus and Tycho Brahe, he was going through their journals and at that time, he realized, he was, I mean then he realized that there must be some kind of an universal law, which is actually essentially, you know making this I mean basically, which drives his whole universe and that is when the apple.

Then he realized the pull, the force, which makes the apple coming down from a tree is the same force, which makes the moon revolves around the earth and is the same force with same force that makes the earth to evolve around sun and also he came up with a very important concept, which will be discussing very soon, that see it is a two body problem. So, if you look into his relation the force is proportional to m 1 m 2 and r square.

Now, for this entire discussion so, far we are assuming that the force center is a stationary object. We have never discussed about the motion of this force center. He also came up with the idea that it is not, nothing is stationary in this world. So, even if sun is very heavy and earth is very small, compared to sun, the sun on earth is as equal to the equivalent opposite, to the force of that earth extends on sun.

So, if earth is moving then sun is also moving. So, it is not the distance r, what he realized is not exactly the distance that joins the two masses, but it is the motion it is the locus, I mean sorry, it should not be measured from setting origin on one particular mass, but we have to set the origin to a point, which does not move with respect to both of this mass, then came the concept of center of mass.

So, that way he is also the pioneer of modern day rigid body dynamics, which deals with the center of mass of a body and will see, will see later in this course, even in this in the scope of discussion on central forces itself that, this r is strictly speaking is not the position vector that joins these two lines, taking the origin on this particulars, when a fixed mass, there is nothing called a fixed mass, it is basically, the origin at which the center of mass lies, will do that in a moment.

But now, let us try to use this particular force law and try to derive there or try to prove three Kepler’s laws, to prove the first law. We do not have to do much, because we have already seen that under the action of inverse square law of force, the closed or I mean only possible orbits, a circle ellipse parabola and hyperbola and out of those only circle and ellipse are closed orbit.

Now, again when we will be going slightly advance into this, we will see that circle is a very special case of ellipse. Ellipse is the most general case of a closed orbit. So, the first law, which says the motion of a planet around sun is an ellipse with sun at one of the center of center, sorry, sun at one of the foci is correct.

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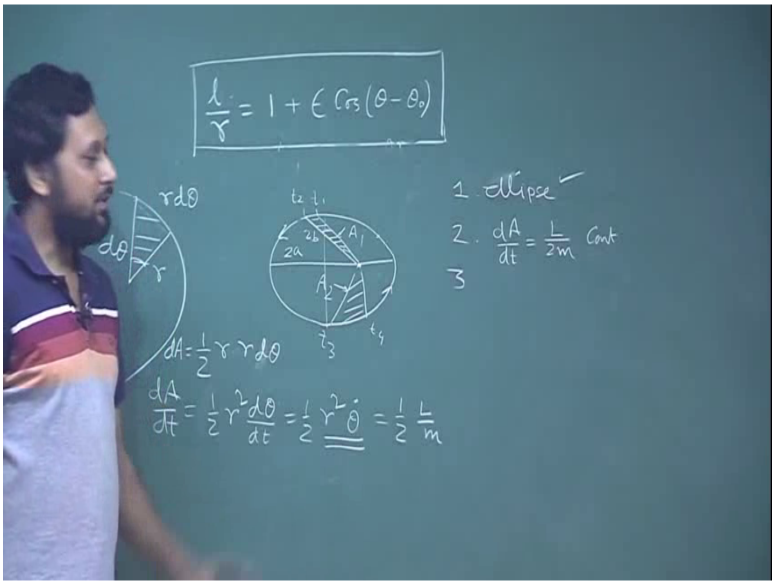

So, this law is immediately verified, because we have a inverse square force law acting between these two and the only closed orbit in an inverse square force law is a ellipse circle is a very special case of ellipse that will see later. So, first law is immediately verified, second law says, it sweeps out equal area, in equal interval of time. Now, in terms of Calculus, this law essentially means. So, the first law is ellipse which we immediately verify, second law is dA dt equal to constant sweeping out equal area in equal interval of time, essentially means that in the words of Calculus areal velocity is constant; that means, it is equivalent of saying dA dt is equal to constant, a being this area.

Now, let us try to elaborate this. I am just taking a segment of any conic section or any orbit, which is stressed out under the action of a central force, let us say, this is my r and this is my dr which is r d theta and this is my d theta. So, the area is half into r into r d theta. So, da is equal to half into r into r d theta, which is nothing, but half r square d theta.

So, if we now, try to find out the aerial velocity in differential form, we immediately see that dA dt is d theta dt equal to half r square theta dot and what is r square theta dot r square theta dot for any central orbit any central orbit does not matter if it is an inverse square law or not r square. Theta dot is a constant, which is given by L by m. So, immediately see that dA dt is L by twice m L by twice m which is a constant.

Now, comes the third law and the statement of third law is T square, proportional to a cubed how do we define t t is the real, t is the time period, which could also be equal to radial velocity anyway.

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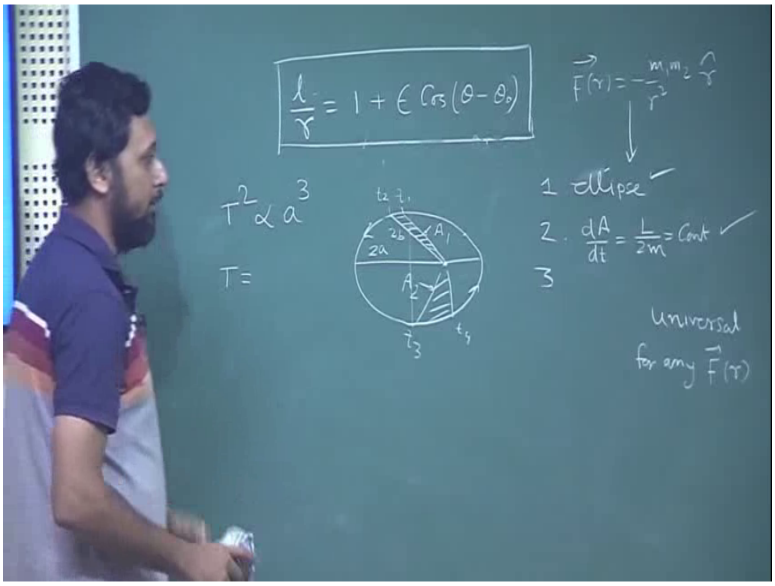

So, it will take little longer to derive this equation and we also have to use this equation at in part. So, will just keep it for the next class, but for now, what we see is we have already verified two laws out of the three laws taking force F of r to be minus 1 by r square or rather m 1 m 2 by r square minus sin comes, because of there is a sorry, write this and r.

So, minus sin comes, because these are gravitational forces, attractive force, it will also be equally applicable for electrostatic attraction or electrostatic repulsion. So, for electrostatic attraction or repulsion the path of a charged particle will be a conic section. Now, what is important is the first law is valid only for this particular force law whereas, the second law is universal for any Fr, any central force will have dA dt equal to l by 2 m, which is equal to a constant.

So, out of three Newtons law, in out of three laws of Kepler, he did not know that he found out some law, which will be universal to all central orbit and without surprise that is also followed by our planetary, the solar system. So, with this, we stop here today, this particular class and in the next class we will start deriving this particular equation.

Thank you.