Classical Mechanics: From Newtonian to Lagrangian Formulation

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Lecture - 01
Review of Newtonian mechanics

Hello and welcome to this course of classical mechanics. Now in this course we will be starting from basic, Newtonian mechanics. And look into different aspects of Newtonian mechanics starting from standard equations of motion. Then work energy resistance motion and registry medium. Slowly and slowly we will progress to moving coordinate systems central orbit rigid dynamics. And eventually we will go forward into lagrangian dynamics. So, in this entire course it will be a 30 hours' lecture, as you might already know and you have must you have seen the syllabus.

So, I will not bother to go into the details of that, but what I will start with is giving you a list of reference books. So, the first reference book which I would like to show you is this one, this is sorry the classical mechanics by Herbert Goldstein. It is not a very easy I mean it is not a very easy reading, but it is a wonderful book and I would request all of you to have one copy of each. Now this one it is theoretical mechanics by Spiegel. And it is a book which is which is full of problems every simple theories are also described as assume these are some small problems. And this is also very good book and I would request you also to have one copy of this. Then the other books which is strongly recommend is this short note book is a very thin book which is by a very famous very well-known reputed even in professor Amal Kumar Raychaudhuri, who has a professor in Calcutta university. And it is a collection of his class notes essentially and this is also very good books specially for the advanced topics.

And finally, there is this book which is classical mechanics and general properties of matter by S S Mighty and Devi Prasad Roy Chowdhury, which is primarily used in certain universities of West Bengal, but there are certain things which will be specially made which I will mention specially during the lecture. For example, certain chapters related to rocket motion and all which will be which you will find nowhere, but only in this book. So, as the class progresses slowly and slowly we will we will see the difficulty level will increase. And at certain times I might refer to one of these 4 books and maybe some other books which is also mentioned in the syllabus. And slowly and slowly we will go into the details of the subject.

Now, let us start with the review of classical mechanics review of basic Newtonian mechanics as you can call it. So, we know all know that the fundamental equation is F equal to m a.

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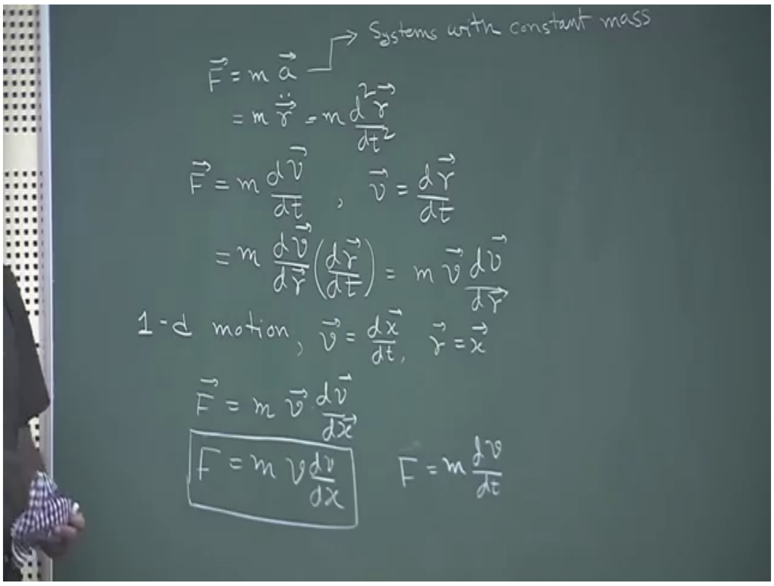

So, this is the fundamental equation which comes from Newton’s second law as a consequence of Newton’s second law. And this is the most fundamental equation of Newtonian mechanics. Now if we break this term we know that this is nothing, but r double dot which essentially means m d 2 r d t 2.

Now, please keep in mind that this equation is valid only for systems with constant mass. So, this equation is valid for systems with constant mass. If the mass of the system is also changing then this equation is not strictly valid, and we will have one additional term which will see when will be discussing rocket motion. Now for, now let us assume that the systems which we are considering in this class and few next few hours, where the systems are with constant mass. And we are we have this vector equation that F equal to m second derivative of the position vector. Now we can rearrange this equation slightly and we can write it as m d v d t where v is equal to d r d t. And then we can slightly rearrange it once again and we can write this as d r to d r d t.

Now, if you look at this term here this is nothing, but u v. So, this can be written as m v d v d r. Now if we consider motion in one, one dimension one d motion then your v is nothing, but d x d t and r is nothing, but x. And this entire equation vector equation can be reduced to m v d v d x which can also be written as F m v d v d x is sorry, there is no vector sign here. So, this equation is the scalar equivalent of this same equation in one direction one dimensional motion when the motion is only allowed x direction.

Now, in this form this equation, if you compare it with the standard F equal to m d v dt, where sorry where v is the velocity along x direction. So, this is simply arrangement of this this equation here, but this equation in this particular form is very useful as will see very soon. Now let us try to see let us try to review some fundamentals of classical mechanics Newtonian mechanics by using this form. And please keep in mind that will be using systems with constant mass. So, I will just leave this line here I am not remove it. Now let us assume that one particle of mass m which is again a system with constant mass is moving from point 1 to point 2. And the work done which is can be written as w 1 to is as we all know from the fundamental principles of mechanics is F dot d r.

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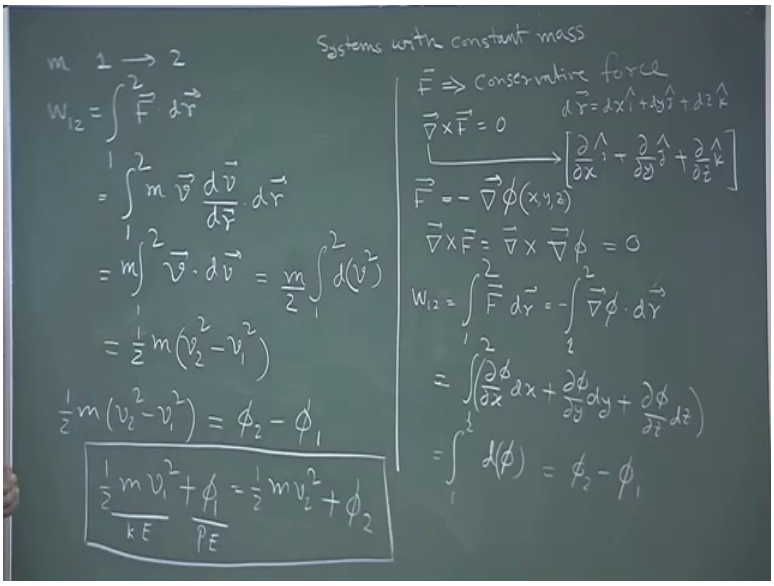

Now, once we use the modified form of this force F, we can rearrange this to write F sorry. So, essentially we replace F with d r dot d r. So, this will be nothing, but m v dot d v, which is nothing, but half sorry m integration 1 to 2, d of v square sorry. There will be a factor of 2 coming here, because of this change in notation. So, it will give us half m v 2 square minus v 1 square.

Now, if we consider at the same time, that this force field F is the conservative force field, then what we get? Then, according to the definition of conservative force field, we know that for any conservative force field called of F is equal to 0. I am I am sure that you are familiar with this particular operator, it is the it is the delta operator and it is written as del del x, i cap plus del del y j cap plus del del set k cap. So, it is essentially a vector operator. And this particular operation is the curl operation which is a cross product between this operator and this force filed F.

Now, this will be true if and only if force F can be written it written as gradient of some scalar field phi. Now this is the definition of conservative force field and this conservative this equation is valid only if F can be represented as gradient of some potential function phi. This negative sign essentially imp implies that the force increases in the direction where phi degrees. So, this has some other implications which will be discussing later on this negative sign now. For now, if this is the case then curl of F is curl of grad of phi and from our vector calculus, we know that curl of grad of something some scalar field will always be equal to 0. So, if this is the case then this will be a valid equation.

So, now if we go back to this work done w 1 2 which will be equal to F dot d r, we can write this as integration or w 1 2 as integration 1 2 2 F dot d r, which will be 1 to 2 minus grad phi dot d r. Now this d r is any arbitrary displacement, arbitrary small displacement vector in this entire xyz coordinate system. So, d r has is general form of d r equal to d x i cap plus d y j cap plus d z k cap. I will just write it once again it might not be.

So, d r equal to d x i cap plus d y j cap plus d z k cap. So, this equation this integration, if you if you commute the if you complete this dot product between grad phi and d r then we are we will essentially get integration 1 to 2, del del phi del is d x plus del phi del y d y plus del phi del z d z. And this whole thing from the fundamentals of differential calculus this entire term is nothing, but integration 1 to 2 d phi, phi being a function of x and y and z.

Please remember this phi here is a function of x and y and z. So, d phi is del phi del x d x plus del phi del x d y plus del phi del z d z. So, then this equation the integration will essentially give us phi 2 minus phi 1. Now let us compare this 2 we do not get this. We know now if we compare this 2 w 1 2 expressions we have got from both sides. So, essentially we what we get is half m v 2 square minus v 1 square equal to phi 2 minus phi 1. And rearrangement essentially gives half m v 1 square plus phi 1 equal to half m v 2 square plus phi 2.

And this is our family expression of conservation of total energy. This part is the kinetic energy. This is the potential energy of the particle at point 1. Similarly, this is the potential energy kinetic energy of the particle and potential energy of the particle at point 2. So, we see when we when a system with constant mass is moving we in a conservative force field then the total mechanical energy at point 1. And point 2 remains the same which is the very well-known principle we just derived it using this slightly different form of the force F.

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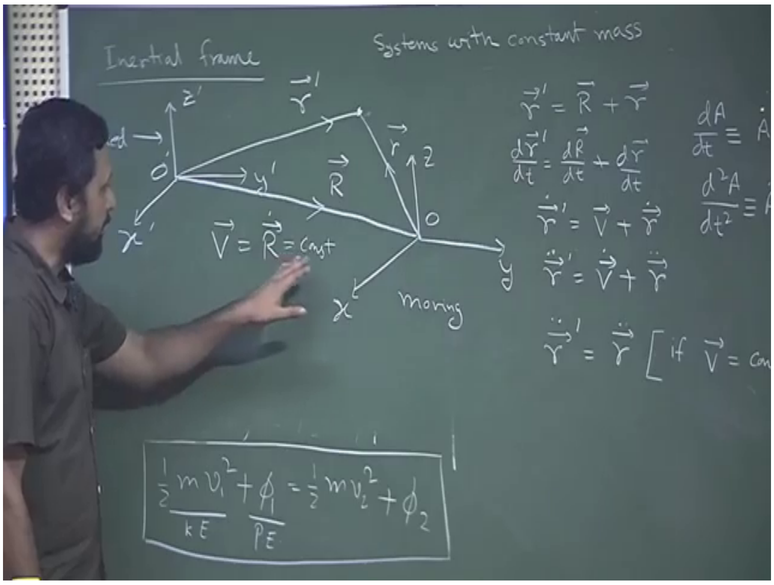

So, this is one thing I wanted to discuss, next we would like to define what is meant by an inner inertial frame.

Now, in inertial frame is a frame of reference in which or let us say let us defined it in this way we have 2 inertial frames. And we tried to we tried to construct the equation of motion of a particle moving in a conservative force field or any 4 field that that is to say in both the frames. So, we have a framework one, with coordinate systems let us say o prime, x prime, y prime, z prime. And let us say this is the particle of our interest and this particular coordinate system this particle has a vector position vector as r prime assuming there is another coordinate system which is given as o x y z.

The same position same particle has position vector which is described as r in this particular system. So, we have one frame which is x prime y prime z prime another frame which is xyz, we have the position vector of same particle please remember the particle remains the same, but the position the reference frames are different. So, in one reference frame this is given as r prime in another reference frame this is given as r. And let us assume, let us say that the origin of this 2 frames are connected by this vector capital R. So, from the triangular law of vector addition, we can immediately write an equation which is r prime is equal to R plus bar, small r prime is equal to capital R plus small r.

Which comes from the triangular law of vector addition, now if you take time derivative by the way time derivative this operator is equivalent of placing a dot. Let us say if d a d t then is equivalent to saying a dot. So, if you take time derivative of this equation we get d t equal to d r d t plus d r d t. Now assume that this frame is fixed and this frame is moving. So, this is the frame which is fixed in space. And this is the frame which is moving with respect to this frame. So, the velocity of this frame the movement of this frame is given by or I will just write it as capital V is equal to r dot right. Going back to this equation we can write r prime equal to v plus r dot.

Now, if we take the second derivative, then we have r double dot sorry r double dot prime is equal to v dot plus r double dot double dot is equivalent to take taking 2 time derivatives. Now this equation if we just remembered the fundamental form of Newtonian equation which is m d 2 r or rather m r double dot is equal to F. Now if we recall this is the fundamental form of Newtonian fundamental equation ah force equation of Newtonian mechanics. Then if we have a velocity v which is not a constant then definitely, we have r double prime dot sorry r prime double dot is not equal to r double dot.

But if you have a situation where this velocity is a constant, then we immediately see this term cancels out and we have equal to so; that means, if this frame this particular frame is moving with a constant velocity with respect to this frame then the equation of motion of this particular particle whether we construct it is a con construct this equation with reference to this frame or that frame, it will remain invariant because in the second derivative of the position vector it remains the same, but if the velocity is not a constant then what will have is an additional term which is v dot the velocity dot. So, this happens if and only if v is a constant. So, by definition and inertial frame is a frame which is either at rest like this one rest as in rest with respect to the rest of the universe.

We are assuming that everything is static and we have a frame, which is also static with respect to the surrounding and this is called an inertial frame. Now if the frame is moving with a constant velocity with respect to a fixed frame that is also can be called an inertial frame because as we have already seen the equation of motions with respect to that particular frame will also b invariant. So, so far we have learnt 2 things one is the slightly different form of the Newtonian equation and from there we have derived the fundamental principle of conservation of energy conservation of total mechanical energy it secondary. We have learnt what is an inertial frame and we have seen that the equation of motion of a particle remains in variant in an inertial frame, irrespective of the velocity of the frame if the velocity is a constant does not matter, how faster. Or how slow it is moving inertial frame remains an inertial frame.

But if the velocity is not constant, then we have something called a non-inertial frame of more. And then we have to reconstruct the equation of motion we will be taking that up in a later chapter. So, what will do now, we will give you some examples of examples of different types of forces one is F equal to constant.

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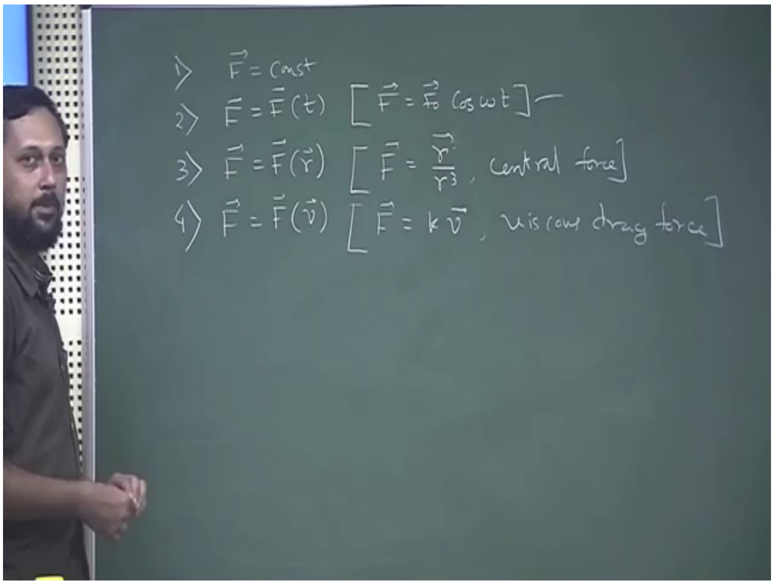

So, this is called a constant force, constant force field. So, there is no specific example given, but for example, gravitational pull you can assume near to the earth gravitational acceleration is constant an object that is an example of a constant force, then we have F equal to F of t and example is let us say F is equal to F 0 cos omega t.

So, that this is a periodic force with changes the magnitude of the force changes with time. Thirdly we can have something like F of r for example, F is equal to r cubed sorry r by r cubed. So, this is a central force. So, we will have separate chapter all central forces. And we will discuss this force field in a would details. And finally, we can also have a force which is proportional to the velocity and example of this is the viscous drag force. So, this is an example of and this particular force is the registry force. For example, if partically if I throw this chock through the air in this room this will have instead of so alongside the gravitational pull it will also have a force field due to the presence of air in this room. And that force will be proportional to the velocity of this particular object.

So, this is called the viscous drag force. So, all these types of forces we will have. So, this one we will not immediately discuss this type of forces constant forces, we will take examples of constant forces in in different for different context. This particular force field the periodic force will keep it for the end, if we have time and we can discuss the details of small oscillation we will go into that this one is a central force, as I fed there is a separate chapter dedicated to this type of forces. And this is the viscous drag force which will be discussing in the very next chapter and also there are some other examples other mathematical examples, but I think that can be done towards the end of this chapter of review of Newtonian mechanics which will have one more classes or I think 2 more classes on this. And after that we can have a detailed discussion on the examples.

Thank you.