Mathematical Methods in Physics-I

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Lecture – 47
Series and Sequence (Contd.)

Welcome back to our complex analysis course student. So, in the last class, we started something called series to let me remind you once again.

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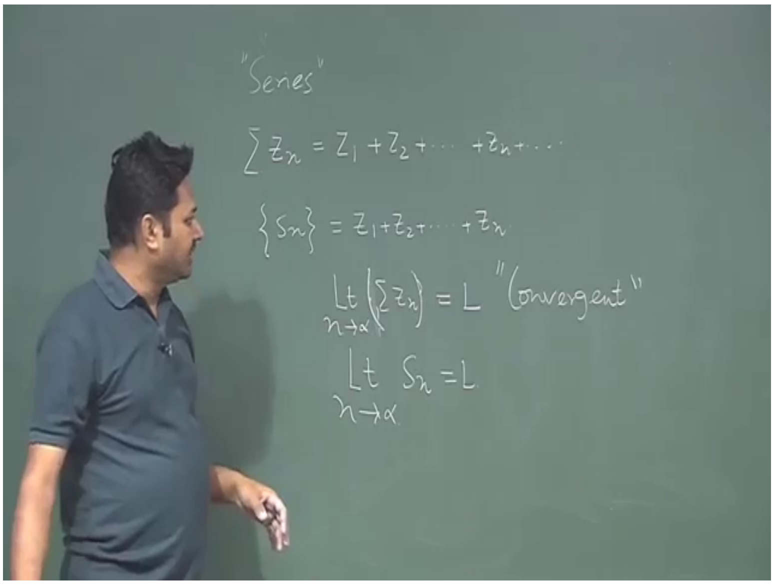

So, if we have this quantity then we called series; this is the sum over this Z 1, Z 2, Z 3; which has some complex number and we mention that it is nothing, but the Z 1, Z 2, Z 3 is a sequence of the partial sum this and since I mean last time, I mentioned it as this Z n. So, limit n tends to infinity sum over Z n; if it is goes to L some value; then we called this series is converging in the last class; maybe I miss; somehow I miss; this summation sign any way or in other word if this is L the partial sum whatever the partial sum; I have I; as I have mentioned this is just the submission over n.

If n tends to infinity this quantity goes to some value L this sum over this things last day maybe I mentioned only the Z n t is not the true and I just miss this summation sign. So, it should be the sum over this things if it is goes to L then I say that this series is converging; that means, I add different-different terms and when I add different-different terms I will get one of the addition. When I go to get the final value this value should be some value L then I called this is the convergence the series is convergent like the sequence here also I have the same thing, but only thing is that in that case my n th term should be goes to some value here the partial sum goes to some value partial sum means addition of that thing the difference between this; this is the basic difference between these 2 series and sequence.

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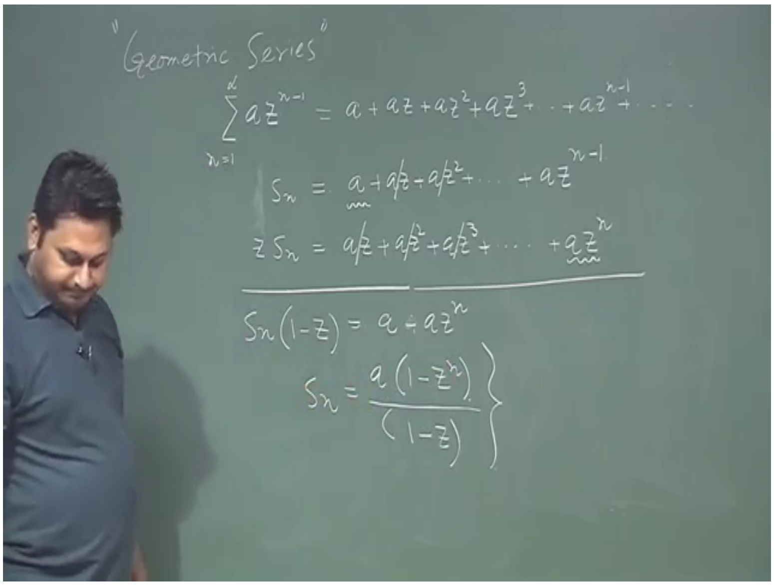

After having that; so, next thing is special series we called is something very important the geometric series geometric series geometric series is something like this; a z to the power n; n is say one to if it is infinity then infinity. So, then this quantity is if I expand it will be a plus a z plus a z square plus; a z cube and so on if I do the partial sum up to n S n is the partial sum, then S n will be equal to a plus a z plus a z square plus a z to the power n minus 1; I am taking only the n number of term and then I am adding this 2 and I called this as S n which is my partial sum it is called the partial sum because I am not adding entire infinite number of terms rather I am adding only n minus n; n number of terms this is the first term second term and this is the n term that is why the power is n minus 1.

Well how do now calculate this S n; there is a standard way to calculate and that is if I know multiplied Z with S n; then I have a z the right hand side a z square a z cube and finally, I have a z to the power of n; this quantity I have now next I just to; I just subtract from this to this when I subtract, then I will have S n 1 minus Z when I subtract then what happened this quantity and this quantity within cancel out this quantity this quantity cancel out all this quantity cancel out except this quantity and this quantity .

So, if I do; then I will have a plus a; a minus Z to the power n this things. So, what is my S n then S n is if I take a common one minus Z to the power n divided by 1 minus Z this is the result of partial sum. So, I have a series of this if I have a partial sum this then I have this quantity the summation is this quantity. Now the special this is this very special series with geometric series after having this.

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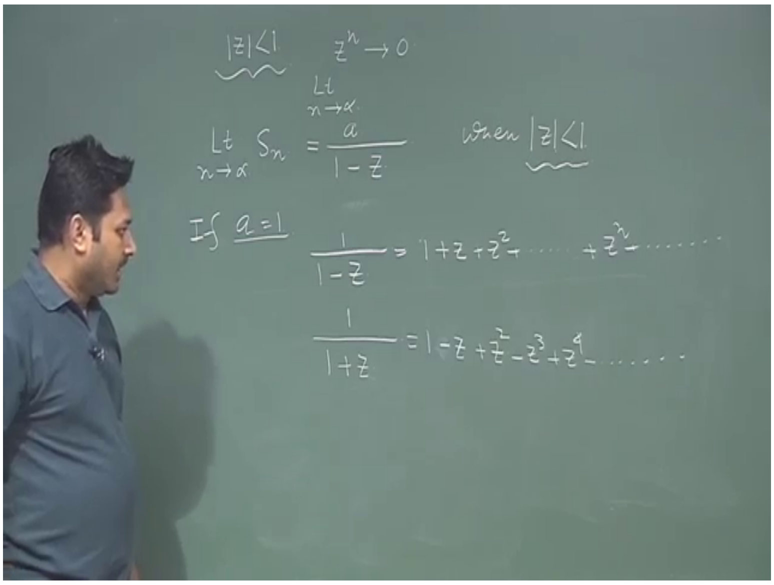

I can now put one special condition what happened when mod of Z is less than 1 when mod of Z is less than 1; then Z to the power n is vanished when limit n tends to infinity. So, this partial sum limit n tends to infinity S n is equal to a divided by 1 minus z.

When mod of Z is less than 1; that means, if mod of Z somehow less than 1, then I can able to calculate the infinite number of sum this is the partial sum I know up to n term I can calculate, but if n tends to infinity and mod of Z less than 1, then I can calculate something which is interesting and this value is this a divided by 1 minus Z and this value is the summation or the addition of this n num n tends to infinity terms and this value if a equal to 1 if a equal to 1, then we can write 1 minus Z is equal to a equal to 1. So, my right hand side a is 1; a z is Z a z square is Z square and so, on this.

So, I can have the entire term; that means, the series I can have which is going to infinity and the value is this now you can change the sign here you can also find another series this by changing the sign. So, Z is now become minus z. So, it will be minus Z plus Z square will remain same minus Z cube plus Z to the power 4 and so on all the autumn will have a negative sign and all the even term will have a positive sign. So, we have another series if I change this quantity; so, now, after having this knowledge of geometric series let us go to some example and problems which is important here.

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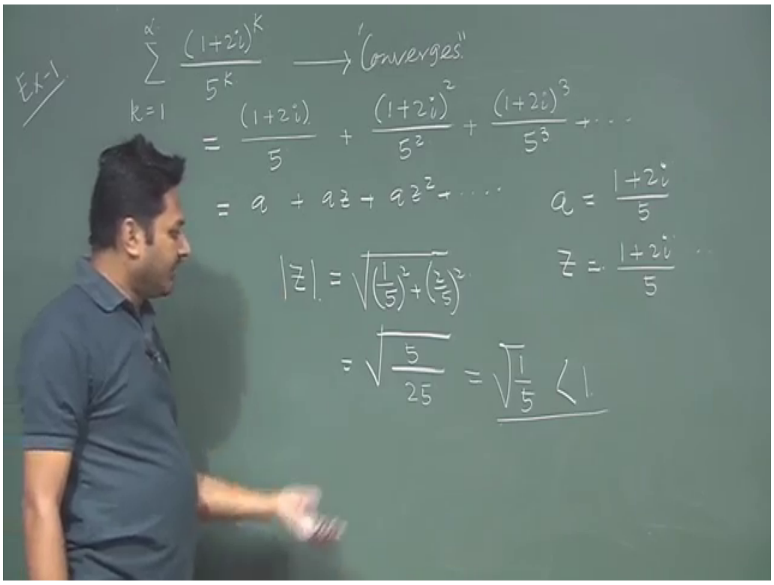

So, let me give you one or let us try to put some problem. So, what we find that a series S n is converging to this value is converging to this value under the condition limit n tends to infinity when mod of Z is less than one ; that means, if a series is given to you first need to find out what is geometric series is given to you; need to find it what is the value of a what is the value of this Z and then if you calculate then you find what is the summation at s tends to infinity if this value is not going to infinity then you will say that this summation goes to some converging this is some we have some convergence here.

But also we need to figure out whether the giving Z is less than one or not mod of Z is less than one then only we can see the result is something like that. So, let me let me show you few examples. So, things will be then clear example 1. So, the series that is given to us is something like this k is 1 to infinity 1 plus 2 i whole to the power k divided by 5 to the power k; we have this series in our hand and the question is whether this series is converging or not whether this series is converging or not. So, let us write what is the value of this series. So, first term is one plus 2 i divided by 5; let me erase this.

Second term is one plus 2 i square divided by 5 square third term is 1 plus 2 i cube divided by 5 cube and so on. Now if I try to correlate this things with our geometric series which is a plus a z plus a z square and so on; then readily we find that here a is one plus 2 i divided by 5 which is a complex number and Z is the same value Z is 1 plus 2 i by 5. Now once I know for this series infinite series once I know what is the value of Z, then the very first thing you need to check that the mod of Z is less than one or not if it is less than one then it should go to a convergence otherwise not.

So, we need to find out this; what is the mod of these things. So, mod of Z first thing mod of Z is how much root over of 1 by 5 square plus 2 by 5 square or root over of 25 and it is 1 and it is 4; it is 5. So, it is root over of 1 by 5. So, root over of 1 by 5 is; obviously, less than 1. So, my first conclusion is that this; this series is converging in nature. So, it will converge that is for sure this we will converges; the next point next thing is that what should be the value first the question is whether it is converging or not I said it is converging because I find the value of Z is less than one which is the first thing I need to check after having that I know it is converging.

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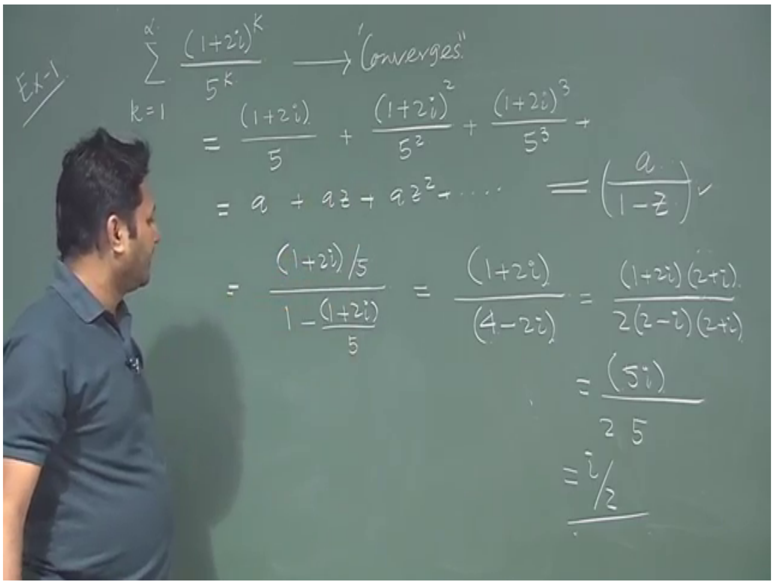

And I also know what is the value of that and value is this series the value is a divided by 1 minus Z; right.

This is the value I already have this is the summation value of the summation I have. So, a is now known Z is now known. So, just I put this. So, what is the value a is 1 plus 2 i by 5 and Z is 1 minus 1 plus 2 i by 5. So, it will be one plus 2 i divided by this 5 and this 5 will going to cancel out. So, 5 minus 1 it seems to be 4 minus 2 i seems to be 4 minus 2 i. So, 5 minus 1 it is 4 and minus 2 i and this 5 and this 5 will going to cancel out. So, we will have something like this. So, I need to simplify I need to simplify this a bit. So, it is 1 plus 2 i the denominator if I take 2 common. So, it will be 2 minus i; I multiply with 2 plus i and after I multiply 2 plus i.

Then it is 1 into 2. So, it is 1 into 2 to and 2 into minus i. So, minus 2; so, cancel out this, then 2; 2 5 i 2 to 4 i and 1. So, it seems to be 5 i in the up stair and the down stair; I have 2 multiplied by a plus b into a minus b again; I will put this formula. So, it will be a square plus b square. So, a square is 1 a square is 4. So, it seems to be 5. So, the result seems to be i by 2. So, this series first of all this series will going to converge because my Z value is less than 1 mod of Z is less than 1; first I conclude that second thing is that what should be the value where it should converge and I find that if I add on this; this quantity infinite with the infinite number I will have the summation and this submission is this.

So, it will; the total sum of these quantities will be just I by 2. So, let us go to few other problems other examples; next example we have is.

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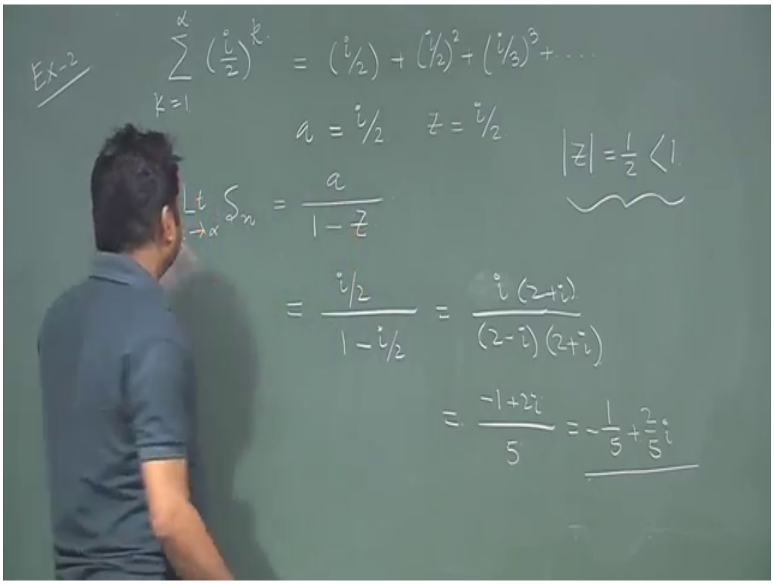

So, example 2 summation of I by 2 hold to the power of k k is moving from one to infinity again a similar kind of problem that we addressed just before, but let us try to do that. So, if I expand these things. So, my first term is I by 2 my second term is I by 2 square third term is I by 3 cube and so on. So, my a here; obviously, is I by 2 and my Z here is also I by 2.

Mod of Z is equal to half which is less than 1 so; that means, the first thing I conclude that this series will going to converge the first conclusion is that this series will going to converge. So, after conv; so, the if this is will converge this is the first conclusion second thing is that what should be the value and I know that my partial sum at L n tends to infinity when mod of Z is less than 1 is simply a divided by 1 minus Z; we are using this formula last time also here; we will going to use the same thing.

So, it will be I by 2 divided by 1 minus I by 2 this. So, it will be I by 2 or 2 will cancel out. So, it will be just 2 minus i multiply 2 plus i here and then multiply it 2 minus i to make it a plus i b form. So, the up stair it is minus 1 plus 2 i and the down stair it is 5. So, the sum will converges to value something like this.

If I going to add this term of to infinite number then my summation should be something like this more examples.

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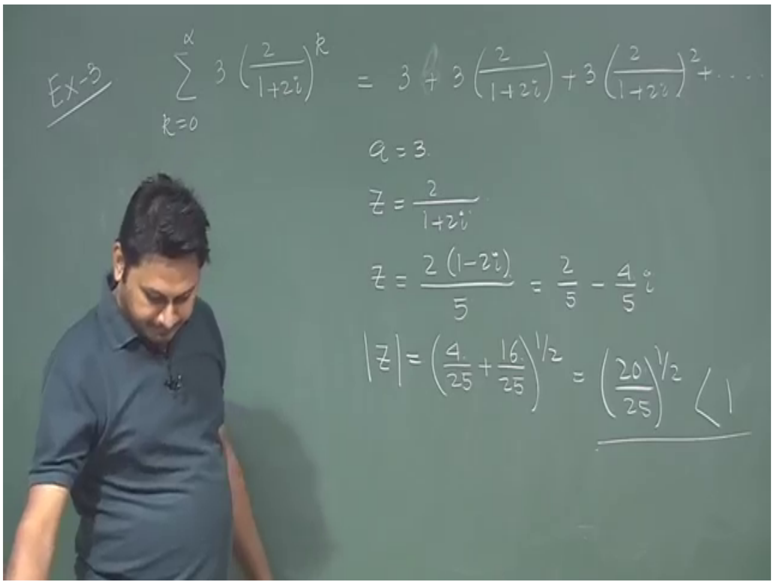

Example is important because when you do this kind of examples and then you will going to understand how this things is working. So, next example 3 and example 3 gives me something like this k 0 to infinity 3 into 2 by 1 plus 2 i whole to the power of k again we have one series in our hand and now in the similar way we need to figure out what should be the value of the what is the value of this partial sum with the condition k 2 is infinity and first of all we need to find out whether this series is converging or not. So, let me do that.

So, if I explain expand these things. So, it will be just 3 first term is k equal to 0 mind it. So, first time will be 3 second term will be 3 multiplied by 2 1 plus 2 i third term is 3 2 divided by 1 plus 2 i square and so on so; obviously, here a is my 3 and Z is 2 divided by 1 plus 2 i this quantity. So, the first thing first we need to find out mod of Z is less than one or not. So, I need to find out the modulus some these things. So, in order to do that; I need to modify this slightly. So, 2 i need to multiply 1 plus 1 minus 2 i and it should be something like 5 because one plus 2 i multiplied by one minus 2 i. So, 1 square plus 2 square it will be 5. So, I will have this. So, it will be 2 by 5 minus 4 by 5 i mod of Z is 4 by 25 plus 16 by 25 whole to the power of half.

It is 20 divided by 25 whole to the power half. So, since this quantity is less than 1 so; obviously, this quantity whole to the power half is less than 1 so; obviously, this series again we find is converging because my mod of Z is less than 1.

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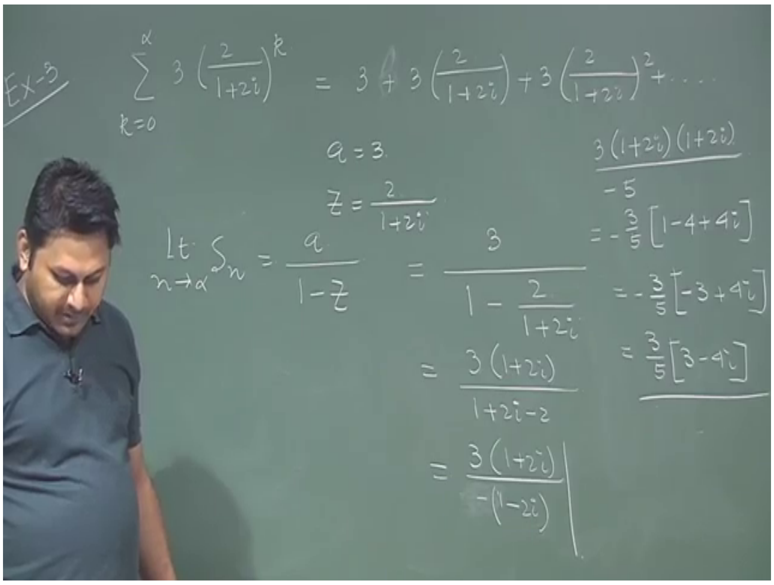

Now, the next part which is; so, sum S n with limit partial sum n tends to infinity; I know it is a divided by 1 minus z. So, I will do that thing its quite mechanical, but this good to practice this kind of things this. So, it is 3 multiplied by 1 plus 2 i is coming from down stair and then it is 1 plus 2 i minus up to. So, I will have 3 1 plus 2 i and then I have minus 1 take minus sign outside minus 1 minus 2 i this quantity.

So, it is 3 let me check everything is correct or not a is 3 Z is Z 2 by one plus 2 i, then a 1 minus Z is my solution or my result of partial sum at instance infinity win mod of Z is less than 1, we find that mod of Z is less than 1, then it is a divided by 1 minus Z; Z I find it is 2 divided by this. So, if I now calculate I find 1 minus 2 this. So, this is 1 plus 2 i minus 2 and this term will go up. So, it is 1 minus 2. So, 3 1 minus 2 i divided by minus common. So, I will get this; now I need to simplify that. So, let me do that also. So, it is 1 plus 2 i multiplied by 1 minus 1 plus 2 i once again and in the down stair I have minus 5.

This multiplied by this I have minus 5 it sums minus 3 by 5 and inside I have square of this. So, 1 minus 4 plus 4 i, minus 3 by 5 multiplied by minus 3 plus 4 i; if I take minus common it will be 3 by 5; 3 minus 4 i; something like this something like this if I am not doing any mistake algebraic mistake, it will be something like this 3; this the summation will be something like this. So, let us check any other problems are there or not if not then we will go to more important think. So, so far we know how to calculate the partial sum of a given series and first thing we need to do that for that particular series we need to know what is the value mod value of Z if the Z for this geometric series for if this Z if I find that this mod of Z is less than 1; then we can find out what is the value of the partial sum under intense to infinity.

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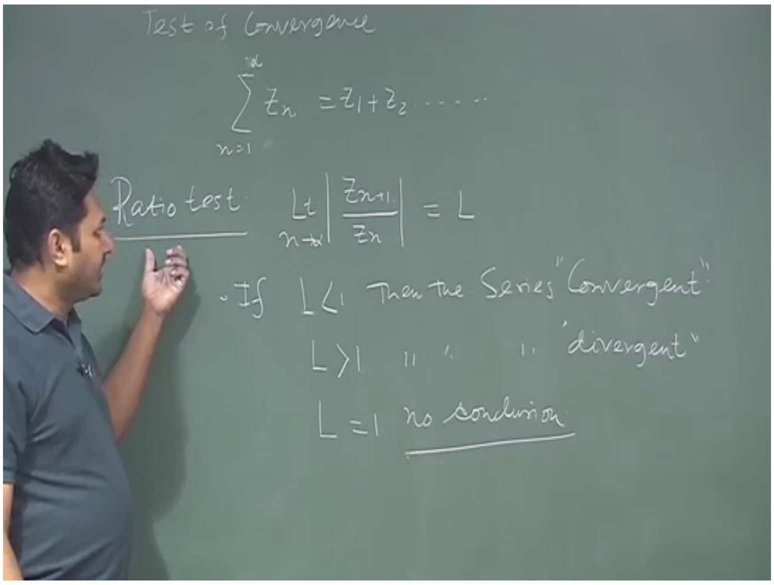

So, if now we; so, some test to convergence test of convergence. So, far we know that when geometric series when Z mod of Z is less than 1 we have, but apart from that there are few test we can think of if we have summation of these 2 1 to infinity; this sum I have Z 1 Z 2. So, I can do one thing called ratio test. So, there is your test is I will calculate Z n plus 1 divided by n limit n tends to infinity n plus 1 th term divided by n, I will calculate the limit say the value of this things is L the conclusion is if L is less than one then the series is convergent if L is greater than 1, then the series divergence then the series is divergent in nature L greater than one or L maybe infinity.

Now, if L is equal to 1. So, we have no conclusion we have no conclusion we cannot say it is divergent or convergent. So, this is called the ratio test also we can have one test which is something like this which is called the root test and say limit n tends to infinity.

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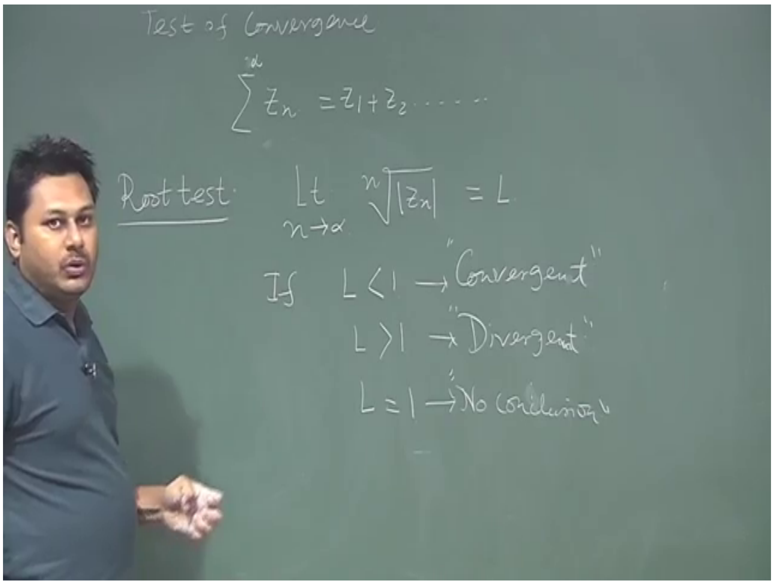

If I have the square root of this n th root of this quantity; if it is L, then if L is less than one it is convergence; L is greater than one or infinity it is divergent and L is equal to one no conclusion. So, in the next class, we will show few series like this and try to find out for this particular series if I apply the root test or ratio test then it is possible to find out whether these things are convergence or not apart from that we will also introduce something very interesting called the power series and for this power series; it is important to note that when I expand a particular function in terms of power or in terms of polynomial, then what should be the condition that we should keep in our mind in expanding that particular thing. So, that we have a convergence expansion. So, these 2 important thing will going to cover in the next class.

So, up to this; we will learn how the series and sequence are defined in complex analysis case and also find out how we can find whether the series is convergence or not in the next class we will go further and try to find out a power series something called power series and convergence of that and then we will go to the next level and trying to find out how a Taylor series or McLaren series can be applied in this complex analysis at a function can be expanded in this particular series all this things will be I mean discuss in the next class. So, with this; so, let me conclude the class here; thank you for your attention. So, see you in the next class.

Thanks.