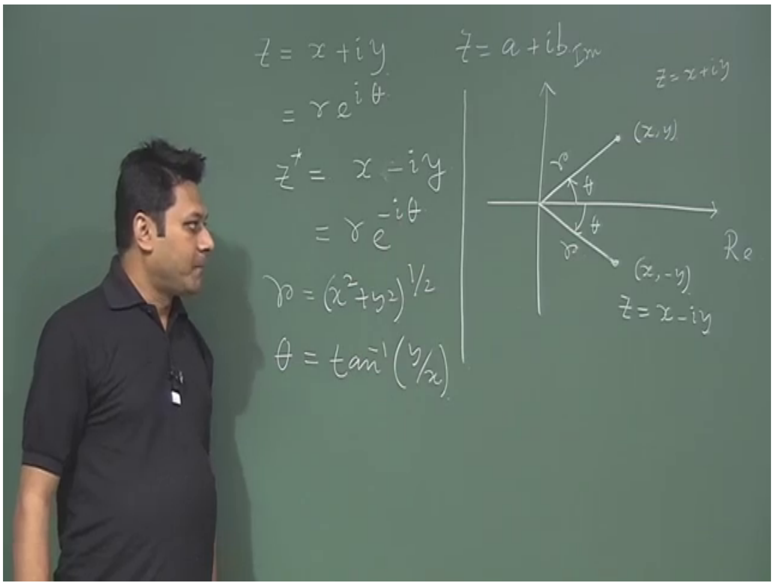
Mathematical Methods in Physics-I
Prof. Samudra Roy
Department of Physics
Indian Institute of Technology, Kharagpur

Lecture – 31
de Moivre's Theorem

Welcome student to these mathematical methods one course, where now we are in the process of learning the complex analysis. In the last class, we learned a few things of complex variable and we show that a complex variable Z can be represented in terms of this.

(Refer Slide Time: 00:34)



I call it variable that is why I put x and y last, here also I write something like a plus ib which is the value which is the value fixed value, but it is changing may change that is why I call it is a variable, the same thing can be represented in terms of polar coordinate which is r e to the power i theta, we show that how to write from here to here. So, will not going to discuss this more, but importing things that Z star was something which is called the complex conjugate and when I say conjugate; that means, I need to replace this i with a minus i that is why this minus sign will be here.

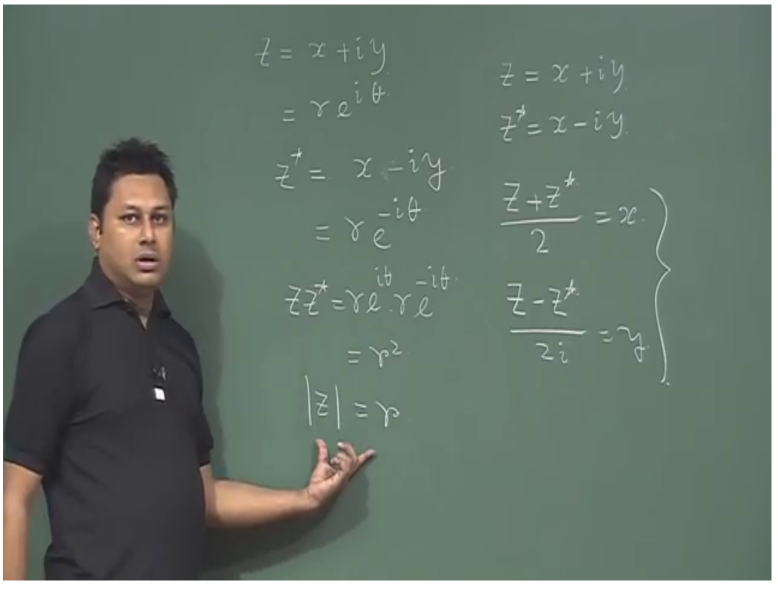
In the similar way if I write these things in terms of polar coordinate, then this polar coordinate this i will be replaced by minus i. So, the equation from this and this there is this change this changes from this i. Now if I try to find out geometrically; what is happening for this Z star. So, this is say my complex plane where real and imaginary axis is represented by this i have a point, I have a point Z which is x plus iy which is here somewhere xy.

So, Z star is nothing but the replica of this point or the image of this point against in real line. So, if this point is x plus iy, I will have another point here which is x minus y and the point will be defined as x minus iy which is just the image of this point against the real axis as simple as that also if I say these things in terms of theta this is the value of r by the way; there is a relationship between r and theta that we also derived last class with x and y r will be this quantity and theta will be this quantity.

So, now if I say this is r both the cases; there is no change whether it is Z or Z star both the cases, the r will not going to change because this is a real part when I make a Z star these real part should not change; only thing is the angle and if we measure the angle theta for this case from this direction, I need to measure theta in this direction which is opposite and that is why the negative sign will appear here so; that means, this point in polar and Cartesian coordinate if each it is here.

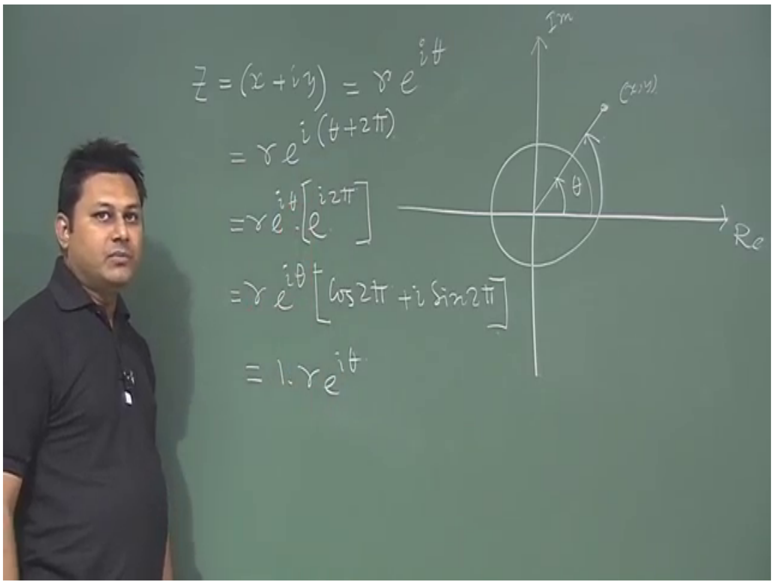
Then I will have a point here which is the mirror image of that point and that is nothing but Z star this is the few small small thing that I want to quickly mention regarding this complex analysis course which is well known, but still I believe it is important for you to make a review that is why this is the first 2 or 3 class will be a review class.

(Refer Slide Time: 04:37)



So, now Z and Z star can also be written in terms of x and y sometimes you also require that. So, Z is equal to x plus iy and Z bar or Z star is equal to x minus iy if these 2 are given, then ready you can find that Z plus Z star divided by 2 is your x and Z minus Z star divided by 2 i is your y just a relationship between Z and Z star with x and y that is all nothing special, here also one thing if from this also, you can readily understand that Z star is equal to r e to the power i theta multiplied by r e to the power of minus i theta which is nothing but r square. So, mod of Z is equal to r same thing in a different way.

(Refer Slide Time: 05:49)

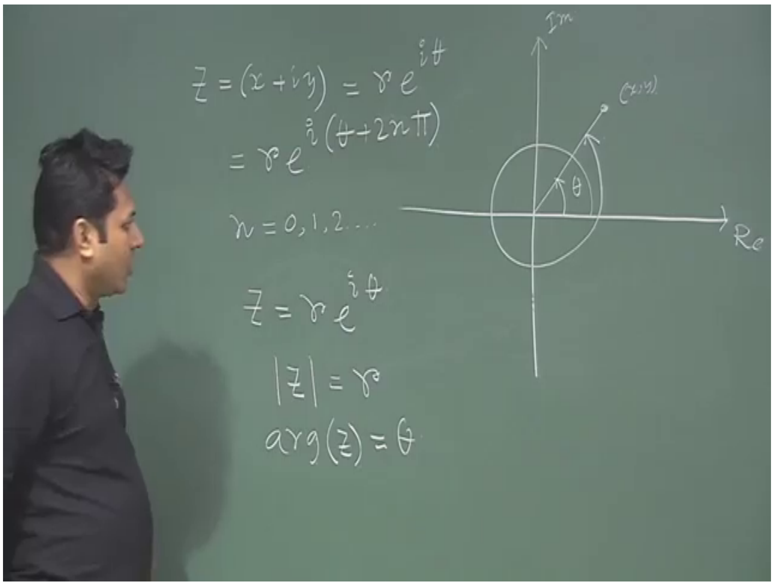


Now in the next thing which is important if I represent a quantity complex quantity Z which is say x plus iy and which is r e to the power i theta, I represent this as usual this is my complex plane real and imaginary axis and I say the point is somewhere here; for example, this is my x and y, fine.

Now, what is the value of theta? Theta is this quantity; right still there is no problem, but is this the value of theta is unique here because if I go with this, then again I will have a theta which is the same point which locate the same point in polar coordinate; that means, this I can write in this way also; if I multiply; if I add 2 pi within my initial theta; if I add 2 pi, then I will have a angle which is 2 pi plus theta, but this will represent the same point here the angle of the theta is just added to 2 pi and it will represent this point why it is representing the same point; mathematically, it is easy because e to the power i theta and e to the power 2 i pi, you can haven these things additional things and what is the value of this quantity r e to the power i theta multiplied by cos 2 pi plus i sine of 2 pi sine of 2 pi is 0 cos of 2 pi is 1.

So, eventually I am having one multiplied by e to the power i theta which is the original value and for that I am getting the same that is why I am getting the same point r, but theta is now change from theta plus 2 pi; this will true by the way for another round also this is one round if I took another round then still it is true and so on.

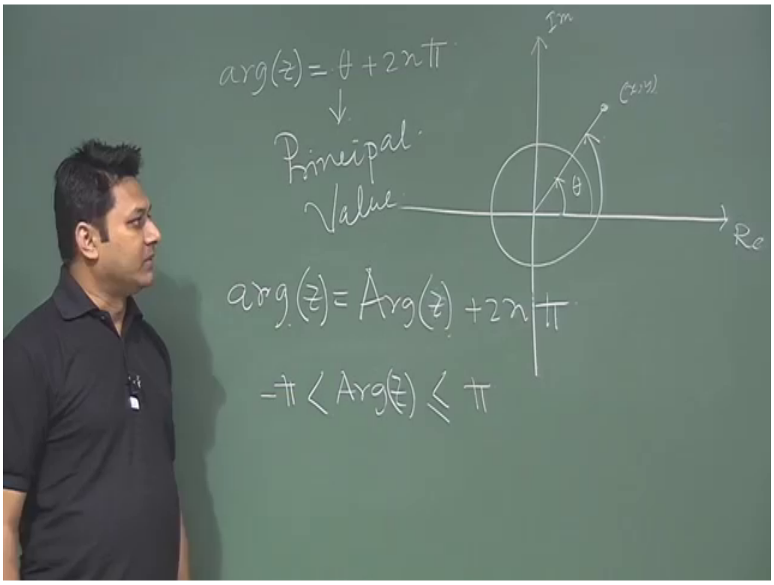
(Refer Slide Time: 08:31)



So, in general this equation should be represented as r e to the power of i theta plus 2 n pi where n is 0 1 2. So, on so; that means, this theta is not unique this theta is not unique if I add something, then I will get the same value of this when I was representing a number Z is equal to r e to the power i theta when I try to write the r and theta in terms of Z.

Then normally we wrote mod of Z is equal to r just few minutes ago I show that not only that this theta we wrote argument of Z is theta, right small argument of Z I write it is as theta, but here I find that this theta is not unique theta rather to n pi is added over that and this 2 n pi again give me the same point that is why this uniqueness of theta is gone. So, that is why argument of Z is represented as theta plus 2 n pi.

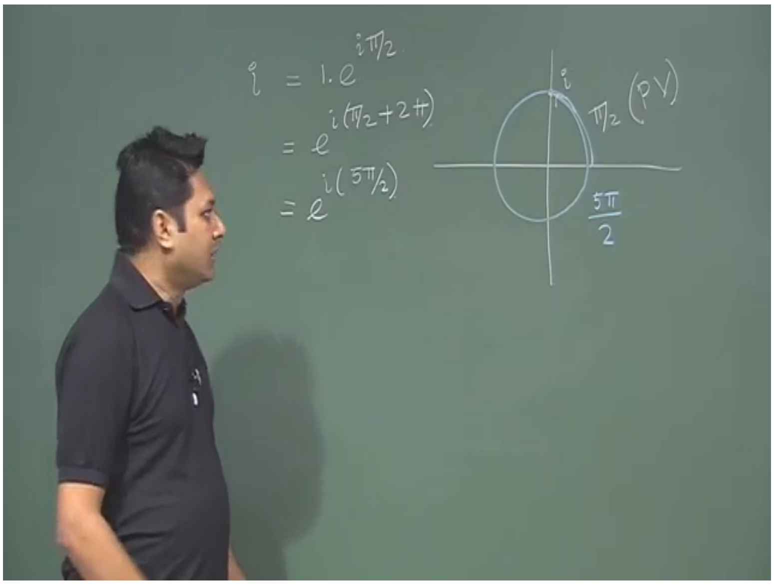
(Refer Slide Time: 10:04)



Now, this theta which is the value here from here to here is called the principal value principal value; this principal value of theta should have some restriction; obviously, because 2 n pi, if I add with that I will get argument or argument of Z. So, this theta if I represent at a principal value there is a way to represent it normally it is represented as argument of Z is argument of Z plus 2 m of pi this argument is the argument of Z in general where to n pi is added, but this argument of Z is associated with this theta which we call the principal value. But there is a limit what is the limit. So, argument of Z big argument of Z which is a principal value should lie in between pi to minus pi this is the limit. So, theta if this theta is in between these, then we call this as the principal value.

Let us check to few cases how these things are. So, for example, here the first theta is this one which is in between pi to minus pi. So, for this theta it is; obviously, the principal value of the theta, but if I make one round and go to this point; that means, theta plus 2 pi which is not in this range, but still we this gives you the same point. So, that theta is not a principal value, but it is a value with added 2 pi with the principal value. Now, we will come to this point once again, but before that let me write what is the point i.

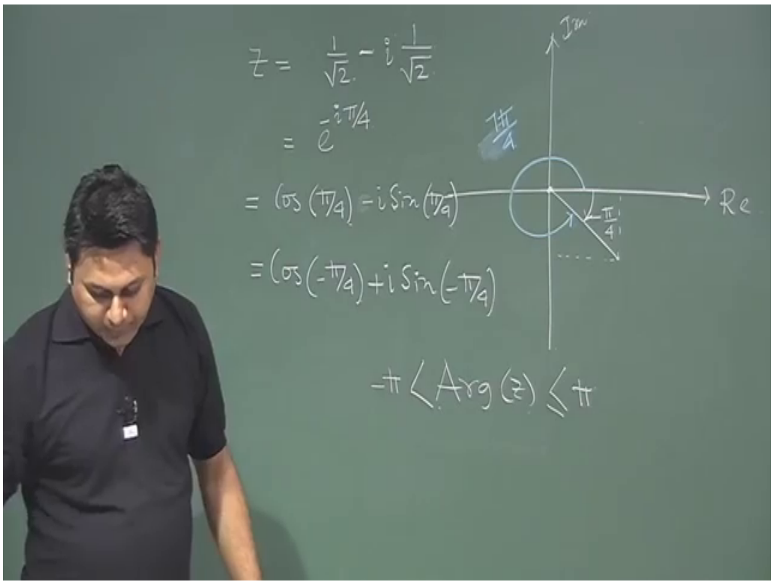
(Refer Slide Time: 12:26)



How to write i in terms of r theta form? So, it is one; obviously, the magnitude is one. So, where the i locate; i is locate here. So, it would it will be e to the power of i pi by 2, but as I mentioned if I add 2 pi with that e to the power i pi by 2 plus 2 pi is also the same value, but here the value is e to the power of i phi pi by 2 seems to be.

So, e to the power i 2 pi by 2 1 into the power 5 i pi by 2 both are giving the same i. So, in one case the angle is measured from here to here which is pi by 2 and by the way this is the principal value pv another thing is adding these 2 i am giving this just one example so; that means, other value is adding these 2 things. So, I will go here, here, here, here and then go to this point. So, what is the value of this 5 pi by 2 which is not the principal value which is something added with that also one thing for example.

(Refer Slide Time: 14:07)



I have one point say Z equal to minus 1 by root 2 plus i 1 by root 2 this you can widely figure out that the point let me first figure out the point minus.

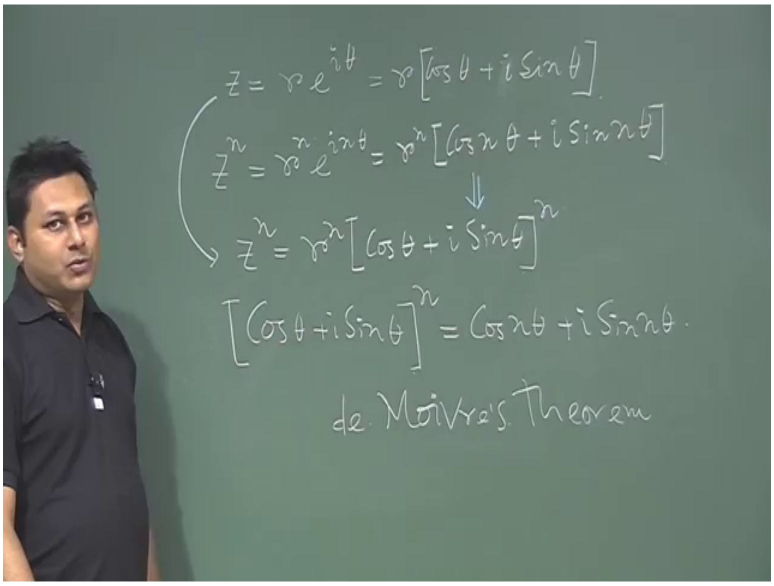
Let me change slightly, it will be not this one it good minus then it will be easier. So, this is my real axis this is my imaginary axis. So, I want to locate this point which is easy that I go real axis to one by root 2 and the same value I will go down. So, that somewhere here this is the point. So, what theta I will put here the question is what theta I will put here because if I write this in. So, it will be just e to the power I pi by 4 because pi by 4 is 45 degree. So, it is cos pi by 4 plus i sin pi by 4, I should put a negative sign here. So, that this became negative this.

This I can write this equation I can write as cos of minus pi by 4 cos 45 degree pi by 4 means what cos 45 degree cos 45 degree means one by root 2 and sin 45 degrees also 1 by root 2, but negative sign I am taking care of this way so that my theta become minus pi by 4; why I am taking writing this equation in this form. So, that I should understand that whatever the theta I am taking is the principal value because here 2 way, you can have this angle first is this angle you have this angle which is how much 2 pi minus pi by 4. So, it is 7 pi by 4 something like that 7 pi by 4 and you can also have this quantity which is minus of pi by 4 because you are measuring you know this is anti clockwise and you are measuring this as a clockwise.

So, clockwise is normally we put as negative. So, this is, but the question is which theta is a principal value. So, you should remember this thing that argument of Z which is the principal argument that is why this bj is associated for that it should be this quantity so; that means, it should be greater than less than pi by 2 less than pi and it should be greater than pi by 2. So, 7 pi by 4 is; obviously, greater than pi. So, that is why this argument of these things should not be this blue one rather it will be minus pi by 4 because minus pi by 4 is falling in this limit. So, the principal value of the angle should be minus pi by 4 that is the thing I wanted to mention here.

After that I will also like to mention one thing this is important this principle value principle argument these things we will going to use that soon, but before that let me write something.

(Refer Slide Time: 18:35)

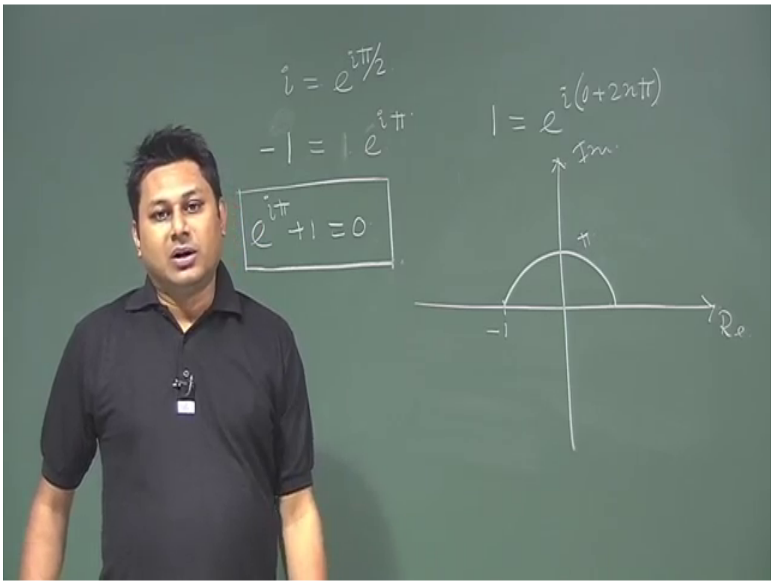


So, Z was represented by r e to the power of i theta in this form which is r cos theta plus r plus i sin theta this right. Now if I put Z to the power n some value which is; obviously, r to the power n e to the power I of n theta this quantity I can write r to the power n cos of n theta plus i sin n theta, but already Z is equal to this. So, when I make a to the power n from here if I take these things then it will be r to the power n cos theta plus i sin theta whole to the power of n.

Now, these things and these things are same these things and these things are same if they are same then we have something interesting and this suggests that cos theta plus i sin theta whole to the power n is equal to cos of n theta plus i sin of n theta and I believe all of you those who are taking this course are aware of this equation this identity which is called the de Moivre’s theorem this de Moivre’s theorem suggests that if we have cos theta plus i sin theta whole to the power n then this value will be nothing but cos n theta plus i sin n theta which comes quite obviously, but this is a very very interesting identity in my opinion.

. So, next thing is we are still. So, next thing is few equation. So, before that let me show you one thing that p to the i represent i as e to the power i pi by 2 this is the representation of i.

(Refer Slide Time: 21:30)



In terms of exponential term, but I can also represent one in terms of one is a pure real quantity, but still we can represent in terms of polar coordinate r theta where I is one obviously, but the phase term this is called the phase term is say e to the power one is represented sorry this is minus 1. So, minus 1 I want to represent. So, minus 1 is e to the power i 1 can also be represented as this anyway one will be e to the power of i into 0 as simple as step plus 2 n pi plus 2 n pi because 0 is a is a principal value and then this value is the addition, but if I put n equal to 1, then we have e to the power i 2 pi e to the power i 2 pi will give you the value one.

So, no problem with that, but I am interested in minus 1 which is represented in this form if you have this point here. So, always I am doing this because you people when you have a complex number, then try to find out what is his location in this complex plane and when you have the location of this point in the complex plane then you will have the idea of what is his amplitude and what is his value. So, this minus one is located somewhere here because this is a real point it will be in the real axis and angle if I measure it will be something like this.

So, this angle is pi, but interesting thing is this equation e to the power i pi plus 1 is equal to 0, this is a very famous and important equation because all the information I like to finish this class with the node and try to make you make you aware of these things that all the important terms are associated that is associated in the in the in the physics as well as in the math is here first of all you have e here second thing you have pi here e is the irrational number pi is irrational number I; the imaginary quantity e to the power i pi is something plus 1 gives you 0.

This is the more fascinating one of the most fascinating equation to be very honest in mathematics as well as in physics because this pi e are always used in the different-different part of the physics, but you should also appreciate with the fact that 0 is something which is very important in the math because these this is the identity element for addition operation one is identity or element or multiplication operation pi which is very important e is very important i is very important all these things are together. So, 1, 2, 3, 4, 5; 5 very important terms are together and they are forming some a very fascinating equation something like this.

So, this is a just a byproduct of what we are doing, but I thought that I should mention these things and those who are really concerned about this fact that e pi and these things are. So, important they have an equation in their hand which is associated with all the terms together and gives a very simple and beautiful form of this equation. So, with this note; o, let me conclude this class here; still in the next class, we will do few things like using this concept this principal value concept we will find out how you can measure the root of a complex number which is important.

So, in the next class, we will find how to how to calculate the root of a complex number and using this concept of the principle value and all these things. So, with this note let us conclude the class here in this class you learn few important things that how the complex number and the principal value and all these things are related to each other; this is nearly nothing but a point in the complex plane and you need to know the exact location of this point because in the future class everything will be associated in this plane which we call the complex plane and you should have a clear cut idea about the location of a complex point in this plane.

So, the theta and r is some parameter which is useful, but apart from that you should have a realization that in real case, everything in over the real axis to live becomes simpler, but since it is a complex thing. So, you will have entire plane where the points are located and not only one variable is associated with that you have 2 variable x plus iy. So, these two variable; we need to deal with these two variable and how this will gives you some fascinating thing, we will learn in the future classes.

So, with this note let us conclude this class.

Thank you.