Mathematical Methods in Physics-I
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Lecture – 03
Span, Linear Combination Of Vectors

Welcome back student. So, in our last class we started the concept of vector space. And we know from where this vector space is coming there is a field and over that a vector space is defined they form ring and all these things there is a group the concept of all these things you know. Today I like to start more detail about how to define a space what is the basis what is called span and all these things important thing.

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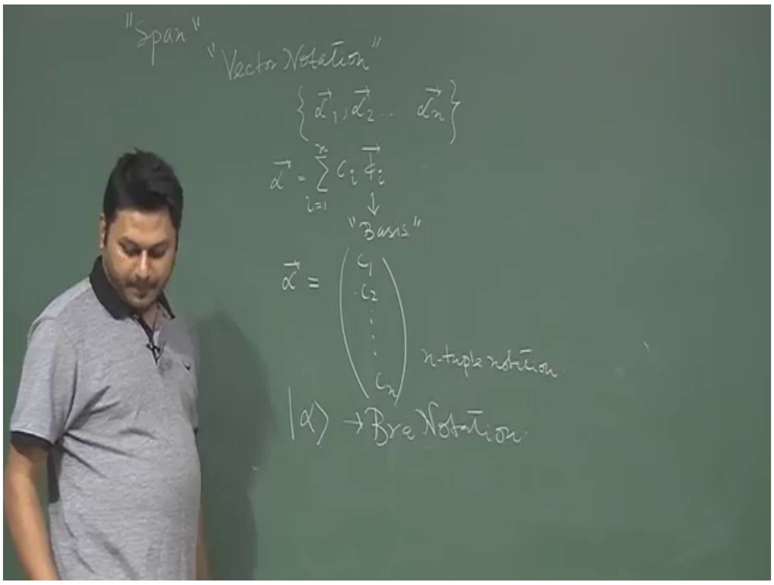

So, let us start with span, a very important term. Before dealing with this span let me once again write the different vector notation. So, vector notation. So, there are different way to note the vector. For example, normally I write alpha 1 alpha 2 alpha n say this is a set of vector now these are different vectors.

So, one vector alpha 1 2 3 are different vector, but now I am writing one vector alpha with different components say this vector alpha, can be written in terms of c i phi i, this ci are the coefficient of this vector where phi i is a basis vector, I will come to this point just to for the time being just you bear with me, with the fact that I am define something basis and a vector can be represented in this form. So, this is a summation form of a vector. Now the same vector alpha can be written in terms of row or column matrix where this c 1 c 2 this coefficient are there, i to 1 to n for example, this is called n tuple notation maybe we will going to use that in our next class also, but this for here this understanding span.

This is important another important notation also have this is called the bra notation, this is a very important notation in quantum mechanics, but here whatever I am writing is basically the vector form, but in quantum mechanics this vector form is will be within, but as a they will use as a state vector they call a state of a function, but essentially the mathematical structure is same. Now after having the knowledge of how to write a vector in this form; however, the basis and all these things is still there.

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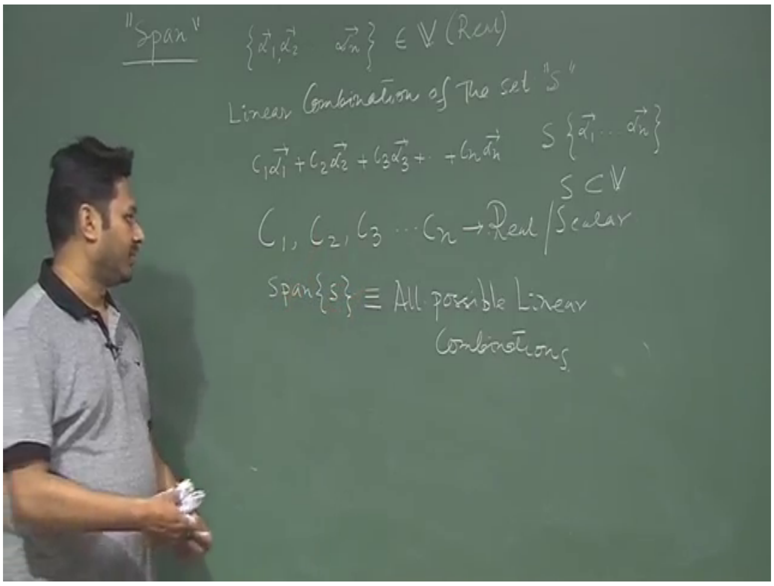

Now, go to our old topic span what is the meaning of span. So, let us take few vector of alpha 1 alpha 2, alpha n which belongs to some vector space v. So, some vector space is there and these are the few elements n elements there are maybe more the n element.

But I am taking only the in elements of that if I make a linear combination of that linear combination of the set I put this set as s; that means, s is a subset of the given vector space v which contain that the vector alpha 1 to alpha n. So, the s contain basically alpha 1 to alpha n these vector. I put a set this set is basically the subset of the entire vector space v, and if I write s as a subset of inter vector space of v. There are few elements that is inside the subset s which is also belongs to the vector space v and linear combination of s means I will take all the vectors and write it as this way all the vector which belongs to s, I take and write c 1 c 2 c 3 c n in this form. This is c 1 c 2 c 3 c n are real quantity real scalar it may be imaginary also it is not necessarily it is a real.

If the vector space is real for the timing let us take a real vector space. So, this quantity may be real for the time being I am taking real vector space. So; that means, the field over which the vector space is formed is a real field. So, this quantity is taking from the scalar quantity and this scalar is a real quantity, but essentially this is a scalar this is not a vector this linear combination, if I make a linear combination. If I say it, I am taking the value and make a linear combination it will look something like this. I will take all the vectors multiplied with this constant and then make a summation over that. So, this is linear combination now this linear combination is called the span.

So, span of s; that means, I am taking few vectors from the given vector space and make a linear combination over that then span of that set when I make a linear call linear combination then I called a span of this thing. So, span of these things is not equivalent to all possible linear combinations span of s, means all possible linear combination; that means, whatever the value c 1 c 2 c n. I like this is basically forming a span. So, let us take some example this is a rough this is something which is this is something which is just the definition. Now if I write if it is a definition, now if I write in a mathematical form then say alpha 1.

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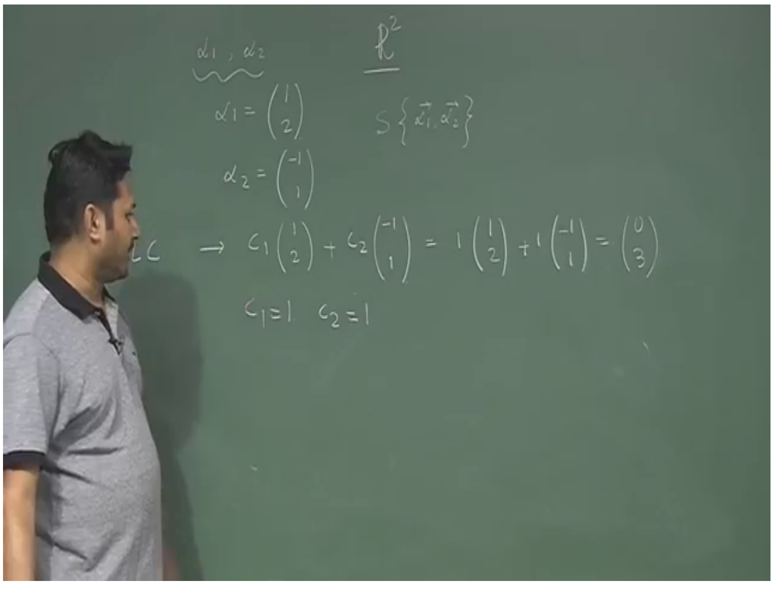

Let us take R 2 R 2 means all the vector, that I am taking will be in 2 d. So; that means, alpha 1 and alpha 2 these are 2 vectors, that I can take from the entire vector space which belongs to R 2.

So, if I write say alpha 1 1 vector 1 2 and alpha 2, another vector say minus 1 1, then this 2 vectors I am taking from R 2 and form a set where alpha 1 and alpha 2 are there. Now I am saying that is it possible to write a vector what will be the linear combination. So, let me find out the linear combination. So, one linear the most general form of the linear combination is this one, this is the linear combination l c of the vector. This is the linear combination of the vector these 2 vectors are giving to me, I have a linear combination and then c 1 c 2, I multiply now it is my choice what c 1 c 2 value I will take. So, let us take c 1 as 1 c 2 also one if I take c 1 equal to 1 c 2 is equal to 1, then if I put here then what value I will get 1 1 2 plus 1 minus 1 1 as a result.

I am having something like 1 minus 1, it is 0 2 plus 1 3 0 3 is a new vector. And it depends on what value of c 1 c 2, I choose I can start generating the new vectors all time if I take for example, if I take c 1 is equal to 2 c 1 equal to c 1 equal to 2 c c 2 is equal to say minus 5. I will get a different vector if I take this is 0, this is 1, I will get a different vector whatever the possible value you imagine if you put this value you will start getting every time new and new set of vectors this is merely an example.

So, now the next problem is the next thing is this is the linear combination of these 2.

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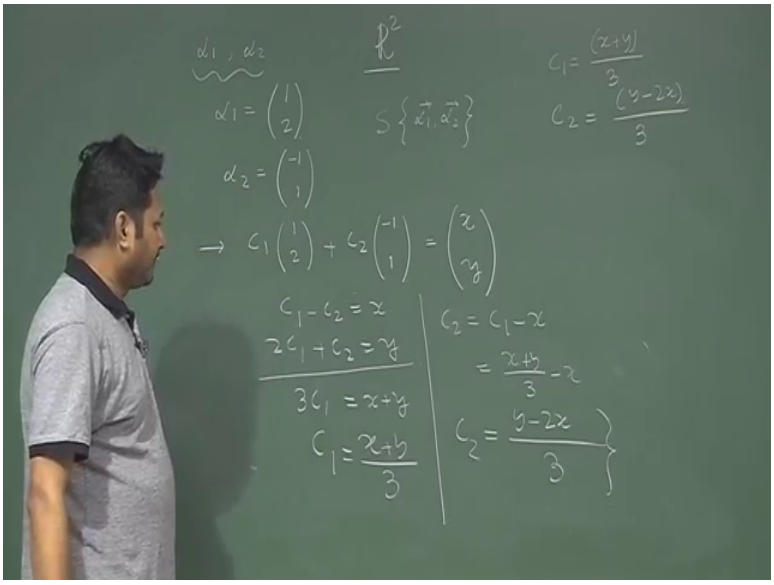

Now, I am saying that I want to find a specific vector x and y out of the linear combination a specific vector x and y, y are I will choose what will be my x y previously what we are doing we are just choosing c 1 and c 2 and I am getting a new vector now I am saying the other way that I will going to choose c x and y. And try to find out what is the value of c 1 and c 2. So, let me do that. So, quite simple. So, c 1 minus c 2 will be x 2 c 1 plus c 2 will be y this is my equation the first equation. So, now, if I try to solve that mind it I need to find out what is my c 1 what is my c 2 when x and y is given. So, it is not that x and y I try to find out I try to find out c 1 and c 2.

So, I need to solve in such a way that I will have c 1 in terms of x y and c 2 in terms of x y. So, if I add this I will have 3 c 1 is equal to x plus y; that means, c 1 is equal to x plus y divided by 3 this quantity. And now if I put if I multiply these things with 2. So, this is one solution and if I multiply this with 2, you can do in any way you can do it another another way also. So, c 2 is c 1 minus x. So, c 1 is this x plus y divided by 3 minus x it is 3 y minus 2 x c 2. So, I can have a value of c 1 I can have a value of c 2 both in x and y. So, if my x and y is given to me then I can readily find out what is my c 1 and c 2. So, let us do this thing. So, let me write it somewhere. So, what is a value of c 1.

So, c 1 is your x plus y divided by 3 and c 2 is y minus 2 x by 3. So, up to this it is fine. Now I need to figure out the value of c 1 c 2 for a given x 1, now let us take.

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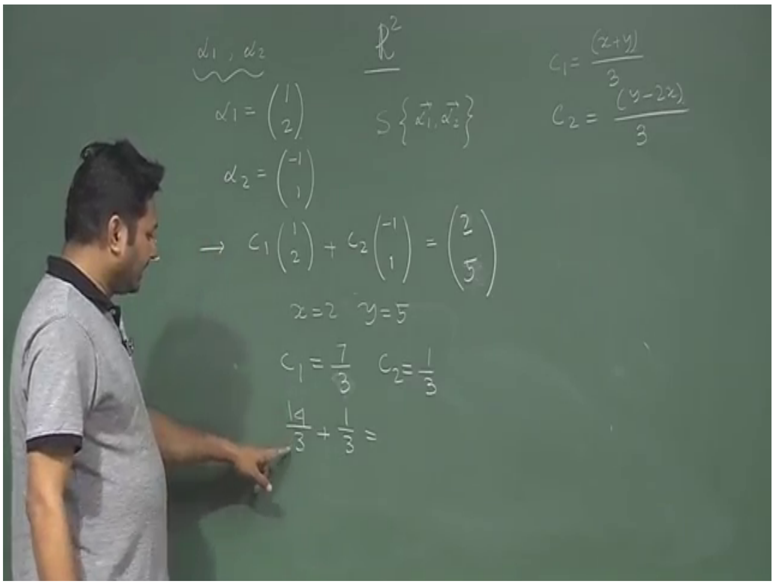

Let us take this is 2 5. So, I will try to find out the value vector 2 and 5 in terms of linear combination of 2 vector 1 2 and minus 1 minus 1 1 my result will be 2 5. So; that means, my x is here 2 y is here 5. So, c 1 will be x plus y; that means, 5 plus 2 7 divided by 2 and my c 2 is y minus 2 x; that means, 5 minus 4; that means, 1 by 3 this is it very correct cross verify 7 by 2 minus 1 by 3. So, it should be 2 7 by no 7 by 3. This is I am making. So, this is x plus y divided by 3. So, x is 2 7 2 by 3 then it is coming 7 by 3.

So, 7 by 3 minus 1 by 3 it is 6 by 3 it is 2 it is 2, then y minus ah then the next one, next one is 7 by 3 multiplied with 14 by 3 plus c 2 is 1 by 3. So, it is 15 by 3 15 by 3 means 5. So, I have 5 so; that means, I can generate from these 2 vector I can generate whatever the vector I like in R 2, that is the most important thing that we figure out with this treatment what is these 2 vector these 2 vector. I took from R 2 this is a arbitrary vector, but these 2 vector has some power through which if I make a linear combination of that I can generate any vector I like x y can be anything. So, I can generate any vector in ah if a given vector is to be. So, I can find my c 1 and c 2.

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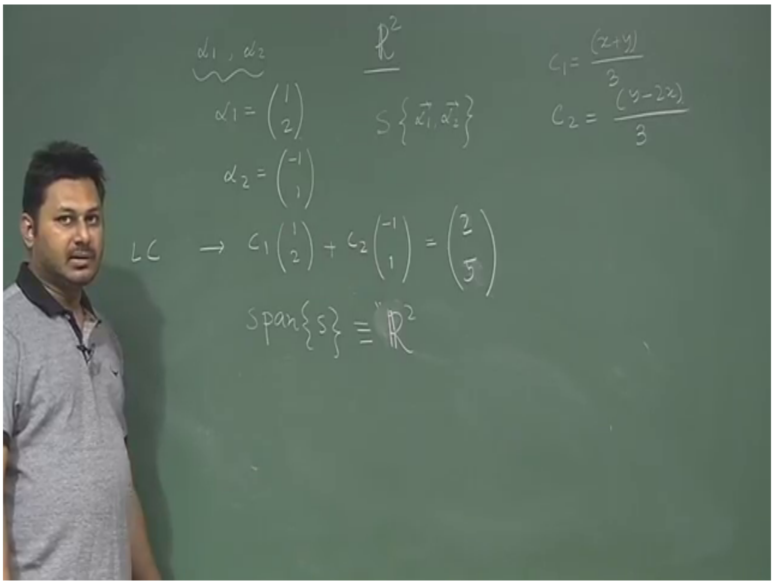

So, that I can generate any vector with this linear combination. So, for this alpha 1 alpha 2. So, I can write span of this s is equal to the entire vector space R 2. We called it this vector alpha 1 and alpha 2 span the entire vector space; that means, give me 2 vector I make a linear combination and I can generate any vector. I like, but is it always true that any arbitrary vector if you took and you will generate the span of these things it will going to generate a span these 2, vector span of these things is basically the entire vector space it will do, but it is really true for this or any other things other relation is required for that that is a very important thing.

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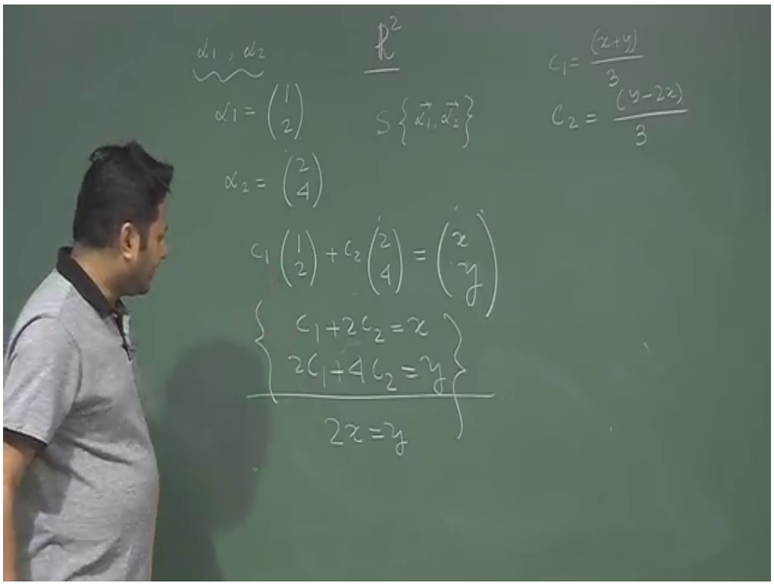

So, let me change these things let me took another vector alpha 2, which is 2 4. This is also another vector they will have this and these 2 vector, I can take from R 2. Now try to find out whether these 2 vector can span the inter vector space or not so; that means, c 1 1 2 plus c 2, 2 4 is equal to x y. I demand that I will solve this equation from here I will have one equation c 1 plus 2 c 2 is equal to x 2, c 1 plus 4 c 2 is equal to y. I demand that I will solve this. And I will find some unique c 1 and c 2. So, that any vector is given to me I can figure out. So, now, if I do try to solve that you will find let me solve it these 2 equation what I will do I will multiply here these things and these things from these things.

And this thing I do not need to do anything if you want to solve that you will not going to get any solution over that. Because these things if I multiply 2 over that then I will have a relation 2 x is equal to y this relation, I will end up if I want to multiply say if I multiply 2. And then if I when you multiply this equal 2 then if I try to solve you will never find a solution of these things why it is that why this is like that you merely have a relationship between these 2 so; that means, this relation only you can get, but not a unique solution; that means, you cannot have any arbitrary value of x and y for which c 1 and c 2 you can define specifically, why it is that it is because the reason is because these 2 vectors alpha.

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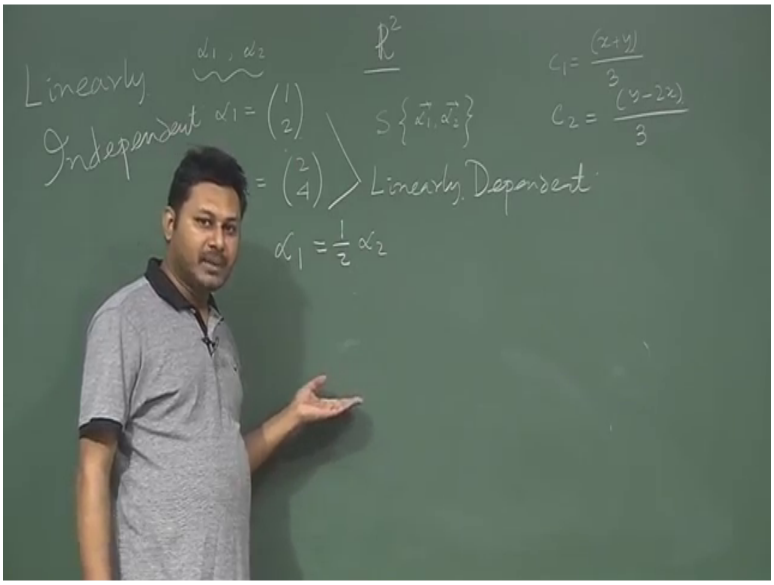

And alpha 2 is not linearly independent. A new term is here which is called linearly independent I will describe this term later, but for the timing I am just writing this what is the meaning of linearly independent and dependent very simply, I can understand that I can write alpha in terms of alpha 2 like this. If I take a half of that if I write one vector in terms of other vector, then these 2 vector is called linearly dependent vector. So, these 2 vector is linearly dependent these term linearly independent I wrote previously to show that whatever the vector you are taking to make a span they should have some relationship between them to span the inter space you find that for this 2 vector even though they are forming a linear combination.

But this linear combination cannot leads to any vector, I want in the vector space; that means, there is a restriction between these 2 vectors. And this restriction is they has to be linearly independent if they are linearly dependent then from that span I will not able to whatever the vector I will have if I make a span or make a linear combination of that I will not get a vector any vector in this thing. So, they will not going to span. So, these 2 vector not going to span the entire vector space the linear combination will be there, but they are not going to span the entire vector space.

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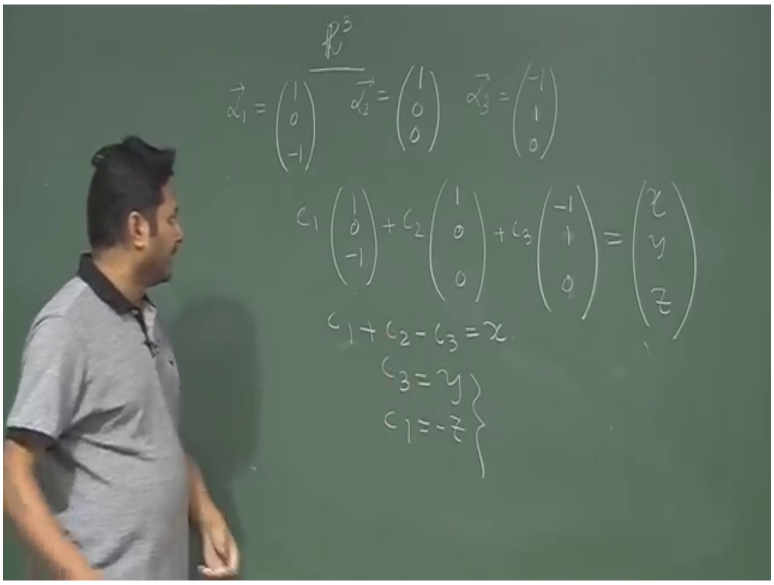

So, now I will go back to this you can do that for other dimension also. So, I should mention here for example, R 3 say I will have one vector alpha 1 say 1 0 minus 1 alpha 2 another vector say 1 0 0.

And alpha 3 I have minus 1 0 1. These 3 vectors can also potentially form a span because if I make a linear combination of that this is a homework please do that for R 2. I did for R 3 you should do that this is the linear combination of the vectors. These vectors I make a linear combination and my demand is that linear combination can leads to any vector in R 3.

Now if this is true then I should have a relationship between c 1 c 2 c 3 to x, and I can write my c 1 c 2 c 3 in form of x y z. If I do, then I have a recipe that what should be my value of c 1 c 2 c 3. So, that I can generate any vector in my hand. So, this is the same treatment that we have already done for 2 d case which was simpler, but here is also simple it is not very difficult thing just need to just need to find out this. So, let me write down the equation for that also, to make thing easy this is one equation you have the next equation c 2 is 0 c 2 is 0 c 3 is 0.

Now, there is a problem that I will have something 0 0 0. So, if this vector may not be let me have another equation. So, make it change because if we have all 0 in the middle then I should have shouldn't have this this, vector first I need to check whether they are linearly dependent or not. So, first equation is the I will show in maybe in the class second equation is c 3 is equal to y and third equation is c 1 is equal to minus of z. So, it is easy it is quite easy c 3 you can readily understand, c 1 you can readily understand and from that c 1 and c 2 you can readily find out what is your c 2 c 1 c 2 c 3. You can readily find out with x y z quite easily. In fact, So, next I will show few things.

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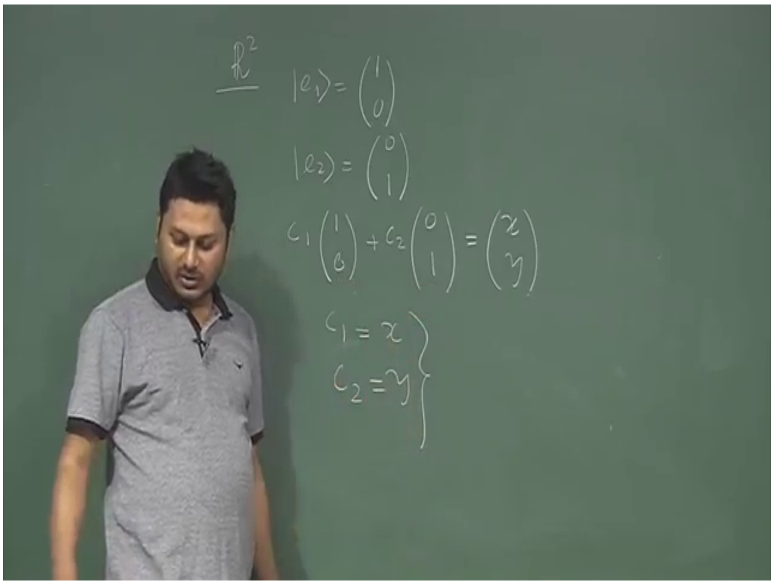

So, in R 2, in R 2 these are my natural choice. Why this is natural choice in R 2, I mentioned that span is 2 vectors any 2 vectors and if I make a linear combination of that 2 vector I will get I will generate a new vector.

So, what I did I take say some vector 1 1 and then 1, minus 1. And then make a linear combination and try to find out what is in the right hand side and. So, on, but if I take these victor with this form one 0 and 0 one life will be much simpler. So, let us find out. So, c 1 1 0 plus c 2 0 1 is my x and y. This is my equation this is a linear combination c 1 multiplied by vector plus c 2 multiplied by vector this is a linear combination and because of the linear combination, I will get a new vector I will have a new vector if I have a new vector. Then I can say this new vector have the value of c 1 and c 2 in terms of new vector coefficient is simply x and y. So, there is no need to make a solution. So, readily I can find my c 1 and c 2, if I define if I use these 2 vector as my spanning set spanning the vector of my spanning set.

So, any vector; that means, any vector say 2 5.

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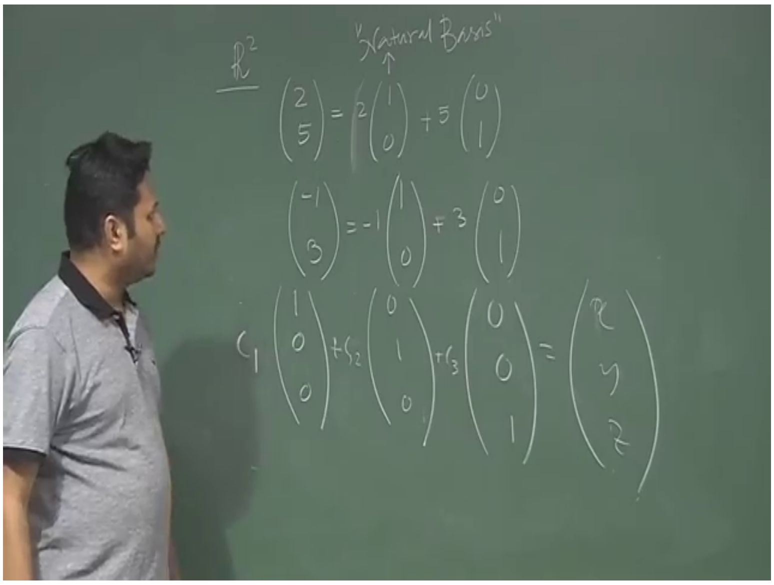

I can write this 2 5 as 2 multiply 1 0 plus 5 multiply 0 1 as simple as that minus 1 3. I can write this vector that 1 0 3 0 1. So, readily I can expand a vector in terms of in terms of this basis this spatial vectors this spatial vectors is called this, this spatial vector is called natural basis. The word basis is still unknown to us we will do that in the next class, but please remember this is called the natural basis you can have these things for 3 d also 3 dimension, it will be something like this if you make a linear combination c 1 plus c 2 plus c 3 you will simply have a vector x y z. Whatever the x y z you have the same value of c 1 c 2 c 3 is there as simple as that. So, with this note I like to conclude here.

So, one important thing we learn todays class, that is span is what the vector space is given to you choose certain vector and make a linear combination of that when you make a linear combination of that 2 vectors. Then you will find that you can generate any vector you want from that chosen set of vector. So, if it is possible to generate any vector from that chosen vector by making the linear combination of them, then you can call this vector as a spanning set. And now we find that there is some unique choice of the spanning set vector which we call the natural basis using which you can generate the vector very nicely.

So, that you do not need to know what is the relationship between c 1 c 2 c 3 with x y z and so on. So; that means, you have some kind of spatial vector which is a preferable thing. In the next class we will learn how these things were there and also we need to know one important thing that how this spanning thing will span 2 d to 3 d to 4 d and also on it is not necessarily that you will take 2 vectors and span 3 d. So, it is not possible I will show in the next class. So, with that let us stop the class here. In the next class we will start from here and trying to find out what is the meaning of basis which is much more important with this I will like to conclude the class here.

Thank you for your attention.