Mathematical Methods in Physics-I
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Lecture – 29
Complex Numbers

So, welcome back students, today we will going to start a new topic as I mentioned in my last class, which is interesting and important complex analysis. So, the name suggests that complex analysis is deal with complex number; I believe all of you have aware of what is complex number, but today's class or in the next class, I will just go through the old thing that you have already aware of. So, it is some sort of review kind of class, but it is interesting, because you need to know few things which may not be that trivial which that appears to be.

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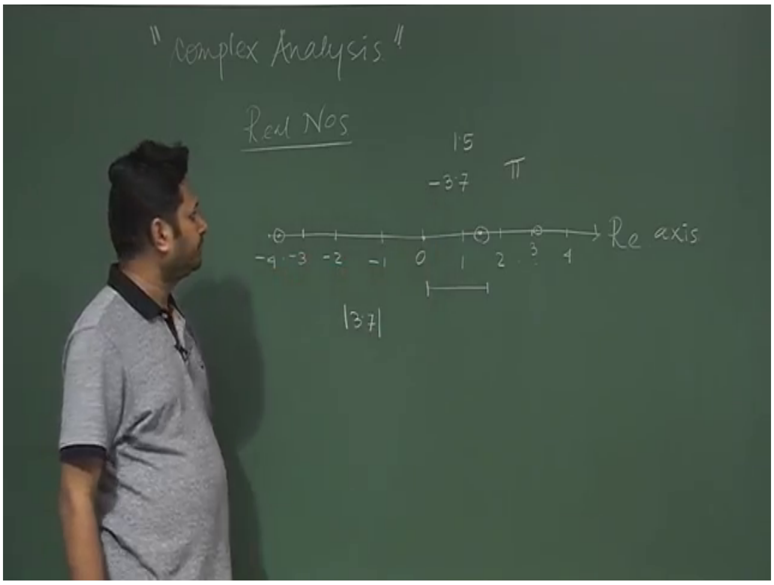

So, so far we are dealing with mainly the real numbers and there are different kind of real numbers are there rational, irrational, integers and so on. Whatever may be the real numbers this is the real line and in this real line you can put the number for example, if I say this point is 0, this is 1, this is 2 and so on, this is minus 1, this is minus 2 and so on. So, if I choose one particular number whatever may be say 1.5, it will be placed at here; minus 3.7 another number it will be placed somewhere here, pi which is approximately 3.14, it will be placed somewhere very close to 3, but greater than 3, somewhere here.

So, the point is whatever be the number if it is real, rational or irrational, whatever you can put this number over this real line, this real axis, it is called the real axis. And the magnitude of that thing is defined by the distance from here to here, if I say it is 1.5, so this length basically gives you the idea that what is the magnitude of that number. Since, this is a real number we do not need to mention anything about the magnitude the number itself is sufficient to define their magnitude. So, there is no issue with the magnitude and we do not need to separate out the magnitude and the value.

For example 3.7 the magnitude will be mod of 3.7 mod value of 3.7 or only the 3.7 whatever negative if I say mod of these things so that means, I am just talking about this length not the direction, which is given by this negative sign. After having the knowledge of this real numbers, then something is there which is not real and we are not able to find a suitable place to put this number.

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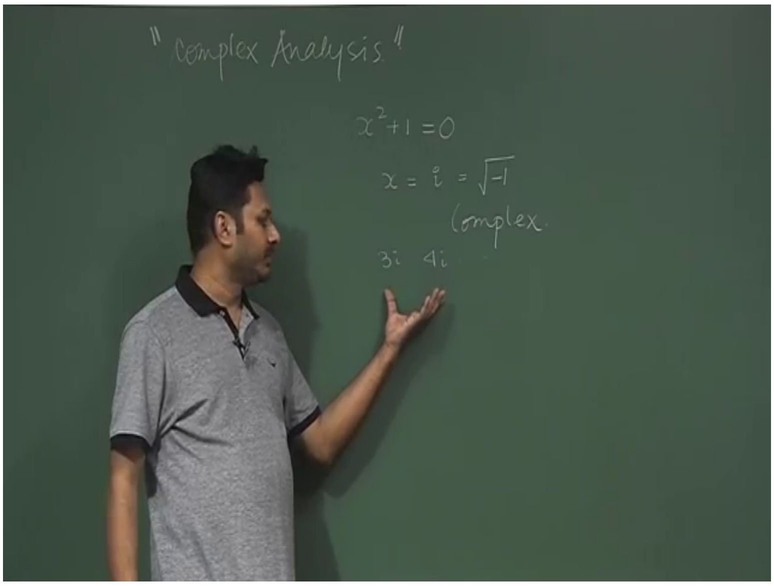

For example the solution of this equation x square plus 1 is equal to 0, there is no real solution of this equation; there is no real solution. However, we have some other solution which are complex. So, basically there is a solution x is equal to i, what is i, this i is root over of minus 1, you can put plus minus sign here also, so that it became a solution, but I am taking just i to make this simple. So, root over of minus 1 is some number which is complex. So, first thing this is a complex number, this is a complex number. So, now we have something which is under root is defined as complex number and normally defined at i. So, now, we know that there is a different entire different set of number for example, I have 3 i, 4 i, 5 i, 6 i, 7 i anything multiplied by I will be a number not necessarily it should be integer it should be any number.

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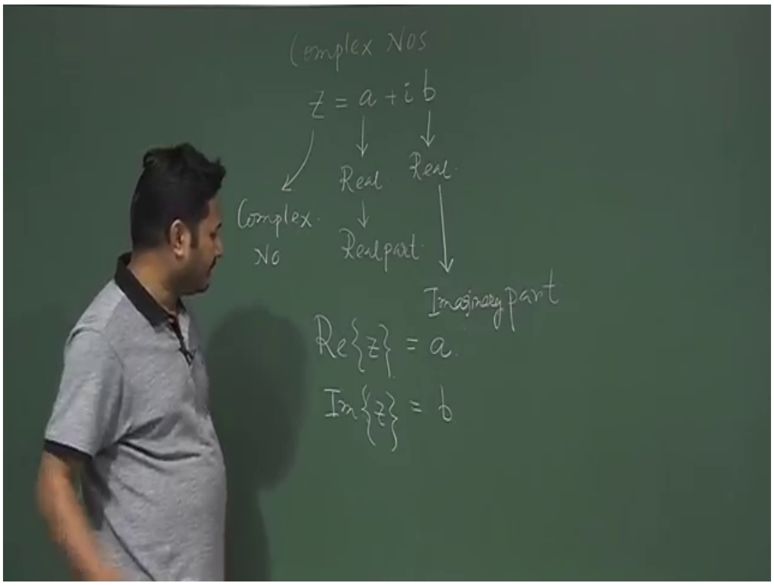

So, now we are dealing with something special and the special thing is called the complex number. Normally, we define this as Z, z is equal to a plus ib; this i, I have already defined what about a and b and how these things are constructed. So, this is a real number, a has to be real number; b is also a real numbers, but combination of these two with a multiplication of i, make z a complex number, z is complex number. So, the point is the complex number consists of two part one is real part and another part is also associated with some real number, but multiplied with i. So, this part is absolutely real part, but this part is not real, this is complex part, this is real part, this is complex part.

If I now write there is a way to write real of z what I try to say with this that means, real object is nothing but I am talking about this value a. In the similar way, if I write imaginary part of Z, it is better to write not complex part, it is better to write imaginary part, because this part is not associated with any real part, but i multiplied by something is entirely imaginary thing, so I call it imaginary. Now, real part and imaginary part is associated with each other when we add these two things, we will have something which is complex. So, when I say real of these complex things, it will be real; imaginary of complex things it will be some things like b. So, this is the imaginary part, this is the complex part that this is the real part real and imaginary part if I add these two things I will have something which is complex in nature. This I believe all of you are aware of.

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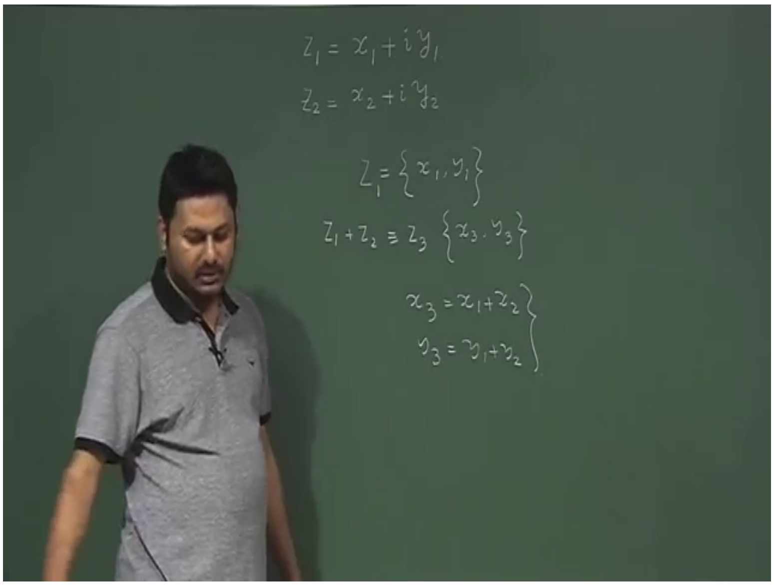

So, let me do view few simple things. So, how they behave say two numbers. If I take two complex numbers z 1 is say x 1 plus i y 1 when x and y are real numbers as I mentioned earlier and z 2 x 2 plus i y 2. So, addition is straightforward, if I write z 1 plus z 2 eventually I am doing that x 1 plus i y 1 plus x 2 plus i y 2, which is x 1 plus x 2 plus i of y 1 plus y 2, this is z 1 plus z 2. If I put z plus minus, so this quantity if instead of adding I will have a minus, so this sign will be minus and this sign will be minus. So, this minus this is this minus this plus this minus this, so sorry this plus minus not should be here it should be here. So, I will have minus sign here and minus sign here. So, this is the addition and subtraction of the complex number.

Now, if I multiply these two quantity, what happened? Normally, if I say this z equal to x and y essentially it is forming a pair. So, z is sometimes it is represented by this, so this is a pair, they form a pair, complex number which associated with real part and imaginary part. So, this real and imaginary part consists of a pair. And if I add two things, again z 1 plus z 2 which is another complex number say z 3, which is also having a pair x 3 and y 3, where x 3 is associated with x 1 and x 2, and y 3 is associated with y 1 and related to y 1 and y 2 with this. So, if I add two complex number, it will be a complex number.

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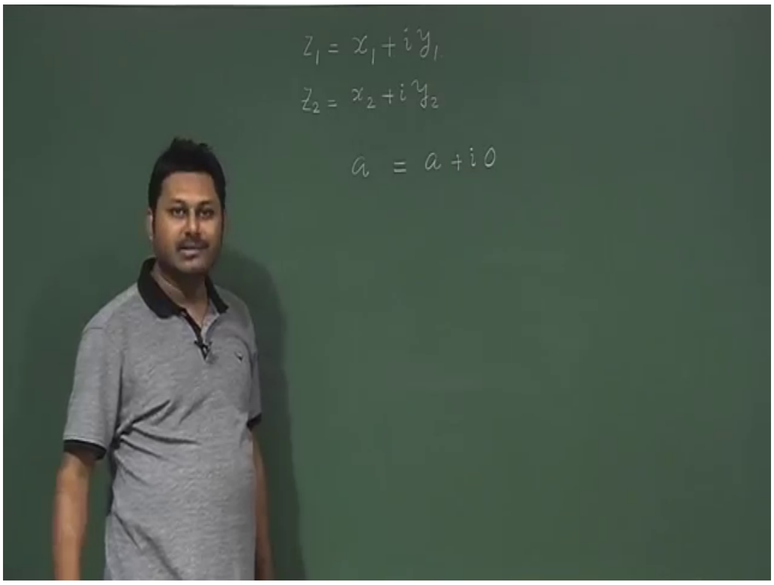

However, in general a real number say if I write a real number a, it can be also represented as a plus i into 0. So, the point is a real number can also be represented as a complex number, where the imaginary part where this part is 0. So, i will be there; i will multiply with 0, so that will give. So, in general a real quantity can also be represented as a general complex number. So, z 1 and z 2 is here.

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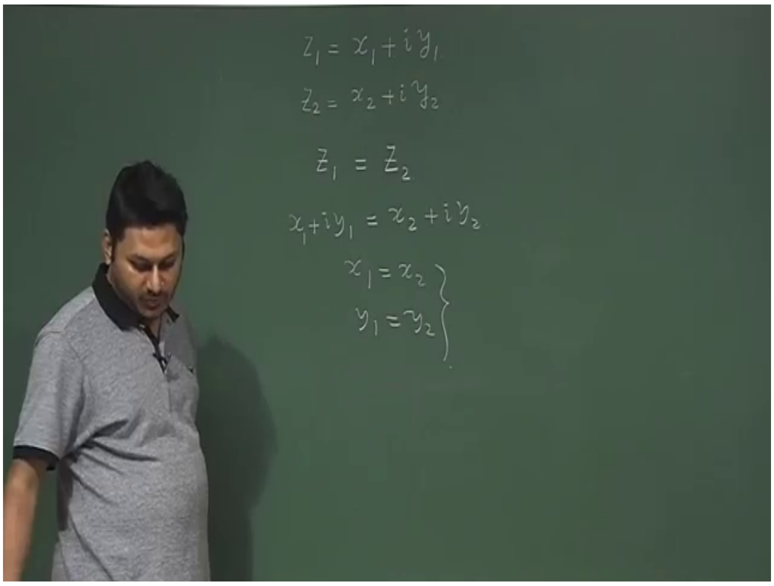

So, now interesting thing the first fundamental thing that I should mentioned earlier, but anyway if I say z 1 is equal to z 2, I add these two things, I subtract two things, which is important because this is the basic operation that you always do. But before that we need to know that if two quantity z 1 and z 2 are equal to each other what is the meaning of that; that means, x 1 plus i y 1 is equal to x 2 plus y i 2. It essentially means that x 1 is equal to x 2, and y 1 is equal to y 2. That means, if two complex number are equal to each other, this is a complex number, this is a different complex number, they are equal to each other, if they are equal to each other then the real part associated with this complex number of this complex number are equal; not only that the imaginary part is also equal. So, this is a very fundamental identity of the complex number.

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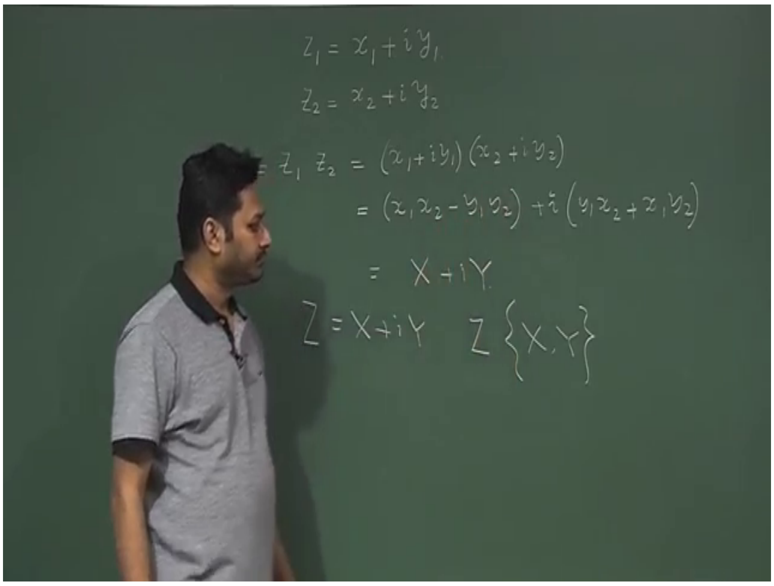

So, go back to the fundamental operation. So, so far we deal with the addition and subtraction. So, multiplication is another important operation. So, if I multiply z 1, z 2, essentially it will also give you a complex number x 1 plus i y 1 multiplied by x 2 plus i y 2. It is interesting, because let me now do one thing x 1, x 2 and minus y 1, y 2, if I multiply i into i will have minus 1 and because of that I will have this quantity plus i y 1 x 2 plus x 1 y 2. So, this is also forming a complex number. Uf I now write this as say big X plus i big Y, then it is also forming and z 1, z 2, if I write this is my big Z. So, big Z is equal to X plus i Y, which is also forming a pair. So, Z is forming like a pair two quantity, but these two pair are related to x and y like this, this and this.

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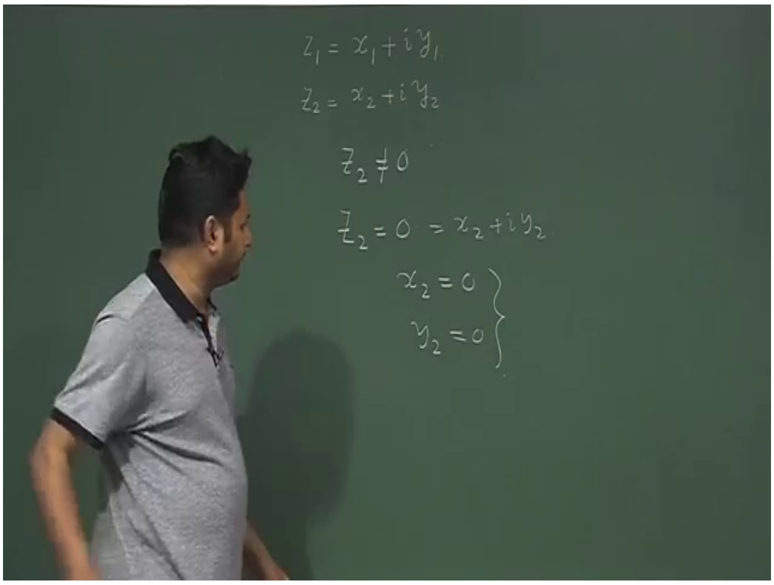

Next multiplication we understood. Now, subtraction, subtraction is important. So, z 1 by z 2, this subtraction this subtraction is valid only when z 2 is equal to not equal to 0, because if it is 0 then we have some problem, we do not have any kind of solution for that or we cannot do these things. But the question is let me go back to our thing, if I say z 2 is equal to 0, where it is x 2 plus i y 2, if a complex number is 0 that again essentially means the real part is 0 and the complex part is also 0. If it is zero then the real and complex part both are 0 separately. So, it is nonzero that means, that any of these terms is not equal to 0; if it is not equal to 0, then still it is ok; it is not equal to 0, still it is ok, but if it is nonzero; that means, both are zero it is not possible. So, if it is zero then both should be zero if anyone is not equal to 0 then I can say z 2 is not equal to 0.

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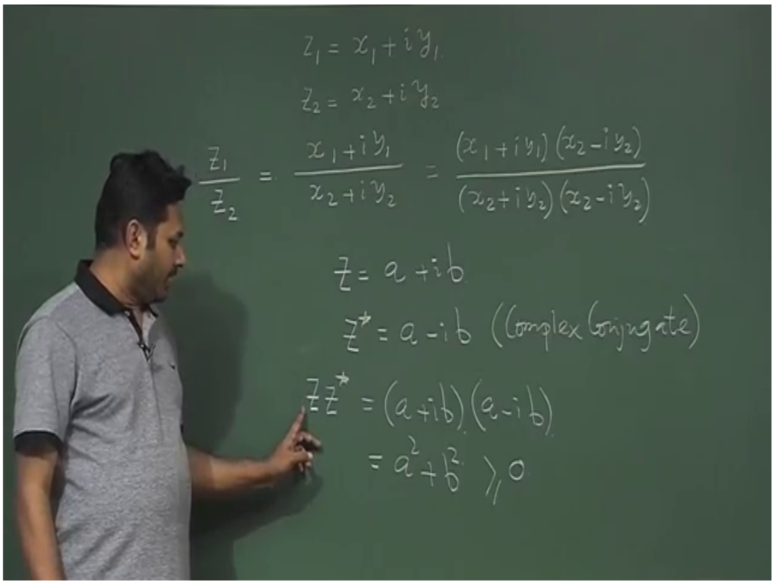

So, this quantity also, if z is not equal to zero then this quantity is x 1 plus i y 1 divided by x 2 plus i y 2, but normally we do not put this complex number in this form. So, the denominator has to be in a reading in a proper way, so that no complex component will be here that is the standard procedure. So, that I can write this z whatever maybe I can call it as z 3, which is z 1 plus z 2, so z 3 comes out to be another complex number. Now, we need to find out what is the relationship, what is the pair thing for this. So, it is x 1 plus i y 1 multiplied by x 2 minus i y 2 divided by x 2 plus i and x 2 minus i y 2.

So, now, here we have doing something quite interesting though it is quite simple, but for example, let me before doing before calculating this let me do these things. So, I have z is equal to a plus i b. If I say z star which is called complex conjugate, I should write a minus i b, complex conjugates is something where i should be replaced by minus i. So, this is called the complex conjugate this is called the complex conjugate.

So, every complex number should have a complex conjugate because every complex number can be represented in this form a plus ib. And if a plus ib is there, then we should have another quantity a minus ib. What is the geometrical significance of these two numbers I will say, but let us first understand, what is the meaning of these things. So, by definition when i make a complex conjugate my i will be minus i the interesting thing is that if I multiply z into z star then I will have a plus ib multiplied a minus ib which is equal to a square plus b square.

Now, a square plus b square, a is a real number, b is a real number, we are having a square and add these two things. So, this quantity is always greater than equal to 0 and real. When I say greater than equal to 0 that means, this quantity has to be real. So, this is something which is important. If I have a complex number, if I want to make this complex number a real number, I just operate with the complex conjugate here we are doing the same thing. We have a complex number, I do not want a complex number here in the denominator. What I will do I just multiply this by the complex conjugate which is x 2 minus iy 2. When I multiply that I need to multiply something here also, so that these two things are canceled out and the original thing remains same.

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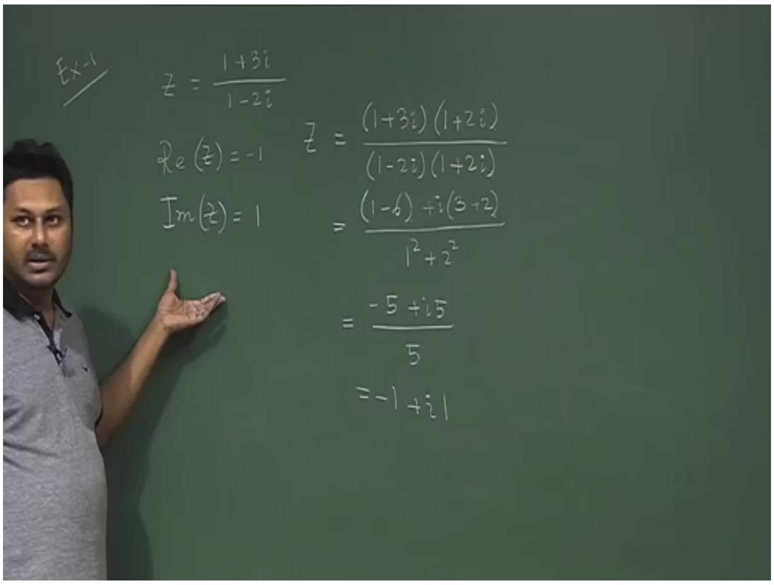

If I do that then I will have something x 1 x 2 minus y 1 y 2 plus i y 1 x 2 minus x 1 y 2 whole divided by this quantity which is nothing but x 2 square plus y 2 square. Again we can separate it out in two part x 1 x 2 minus y 1 y 2 divided by x 2 square plus y 2 square plus i y 1 x 2 minus x 1 y 2 divided by x square plus y square, this is the general form. And I can write these things in form of say big X again plus i big Y where X and Y again is a is a complex number where X is related to x 1 x 2 and y 1 y 2 with this relation, and Y is related to with that relation.

So, small kind of problems will be there. So, for example, a complex number is given to you and you are asked to find out what is the imaginary part of that. So, for example, say z is equal to some number 1 plus 3 i divided by 1 minus 2 i. And you are asked to find out the real part of z and the imaginary part of z. From this, it is not quite trivial because I am making a division of that. So, real and imaginary part you need to figure out exactly the way I do, I did just before.

So, z can be represented as 1 plus 3 i multiplied by 1 minus 2 i 1 plus 2 i rather the complex conjugate; and the denominator is 1 minus 2 i 1 plus 2 i. So, it will be one minus six something like this plus i 3 plus 2 and the denominator I have a plus b into a minus b a square minus b square this is a standard formula. So, basically we have a square plus this. So, I will have minus 5 plus i 5 divided by 5 1 square plus 2 square which is 4 plus 1 - 5. So, I will have minus 1 plus i 1. So, real part of this z this comes out to be minus 1, imaginary part is 1. So, the solution of these things are this. So, now this kind of different kind of form you can expect. And for different kind of form you have to calculate how the real and imaginary part are there and you need to calculate. So, if these things are multiplicative then also you can figure out.

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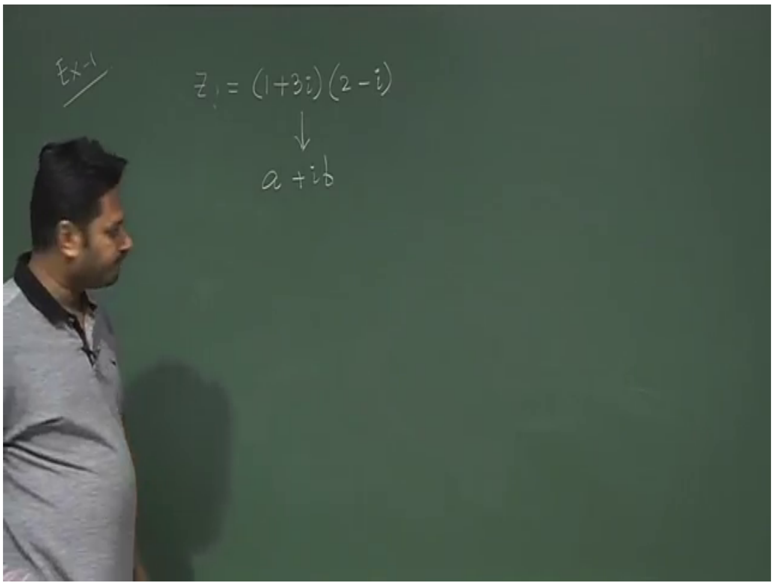

For example, a similar kind of problem maybe giving to you but instead of having a plus sign instead of having a for example, z is 1 plus 3 i 2 minus i again you need to find out what is the real because it is the multiplicative form since it is a multiplicative form you need to make these as a plus ib form. When you make this a plus ib form, then things will be easier because then you know what is your real and what is your imaginary quantity separately. And it is essential that you should know what is real and what is imaginary quantity for that. So, the procedure is same, you just multiply you take something which is you take something which is not real which is imaginary and we take something which is completely real and separate out with a plus ib, and calculate that.

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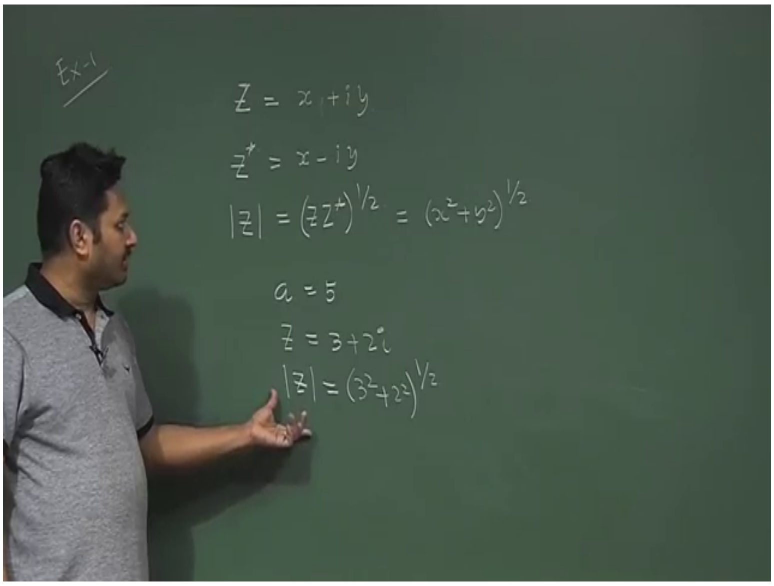

Now, after having that let me give you few information about. So, z I find that z is equal to x plus iy; and we called z star which we called complex conjugate sometime it is reading z bar which is x minus iy. And mod of z which is important mod of z which is z z star whole to the power half is equal to x square plus y square whole to the power half the same thing I am writing. But you need to understand one thing that for the real numbers, if say a is a real number, which is equal to 5, then the magnitude of this quantity is 5, we do not need to do anything with that. But if I write another number z which is 3 plus 2i, then the magnitude of these things is something different, it is not trivial for that. For real number it is trivial the magnitude of these things is already 5, but here the magnitude is what should I write 3 or 2, it is not like that. The magnitude is something which is 3 square plus 2 square whole to the power half. Now, why is that I will going to explain, but not today in the next class, but you should remember one thing let me conclude with this important stuff.

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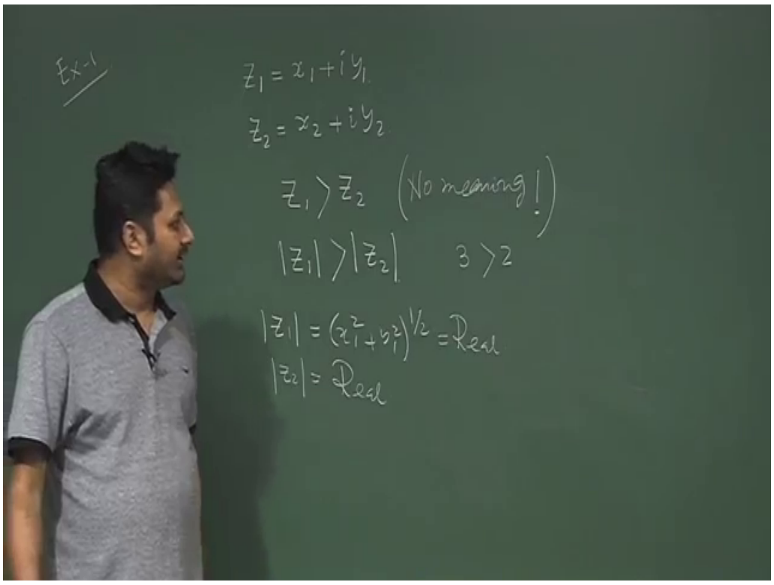

That if I say some z 1 which is x 1 plus i y 1 and some z 2 which is x 2 plus i y 2 if I say z 1 is greater than z 2 this argument, this is absolutely no meaning of this argument, no meaning. Why, because z 1 and z 2 are merely a points I will show that in the next class their points I should not compare the point to point, this is one point and this is another point, I will not able to say which point is greater than which point because both are defining a point. However, these may be this is much meaningful thing why because mod of z is not a point mod of z is some kind of length a real quantity as I mentioned in the last. So, what is z 1, z 1 is x 1 square plus y 1 square whole to the power half, this quantity is real because x 1, y 1 both are real square of that and making half of that. So, this is purely a real quantity, and z 2 in the similar way is also real.

If I compare the real quantity eventually what is the meaning of that I am comparing the magnitude of these two. So, whenever I am saying this and this, this is a real number, this is a real number. So, basically the magnitude of this real number may be greater than that. For example, if it is 3, it should be greater than 2, because this 3 is the magnitude of this, this 2 is the magnitude of this. So, I have some real numbers which can be measurable and which is basically gives you idea that what is the length of this quantity or what is the magnitude of this quantity then I can compare. But if I write this it is absolutely wrong because two complex number never be compared unless we can make the unless we can find out what is the amplitude of these things or what is the mod value of these things.

With that let us conclude that today's class. In the next class, we will start from here and show from where this kind of concept is coming. So, the geometric representation of a complex number is very important. So, in the next class, we will show the geometric representation of a complex number; it eventually gives you some sort of point in a complex plane. And when you put the some sort of point in the complex plane then you can have different kind of properties like a vector is following. So, with that let me conclude today's class. See you in the next class, we will start from this point and will show the geometric representation of a complex number.

Thank you and see you in the next class.