Mathematical Methods in Physics-I
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Lecture - 11
Transformation of Basis (Contd.)

So, welcome back student to this mathematical methods 1 course where we are dealing with linear vector space. So, in last class if you remember we stopped one place, in the last class we studied about the basis transformation, basis transformation.

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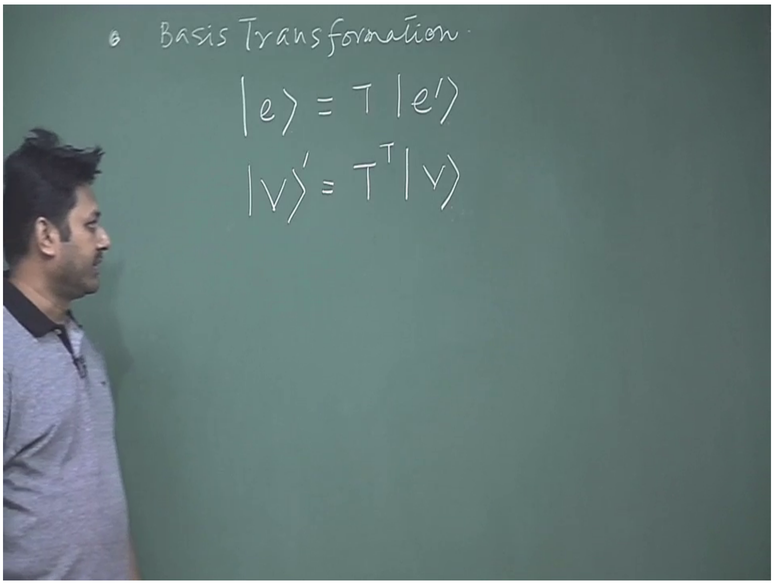

And we show that a basis e I can write in terms of another basis e prime with a matrix T this is the transformation matrix through which I can transform any one of the basis from one to another. If I do so then I can also find in prime basis some vector component this is the, this prime is a vector in prime basis some vector component and it will be something like this.

So, the rule is if I know the T matrix which is transforming the basis then I can also figure out what is the transformation of a given vector, this is a non prime frame I want to find out the vector component in prime frame. So, this is the corresponding transformation matrix I need to transfer that and T trans this T is related to the transpose of that, that we also shown in the last class and also we stopped one place by putting one problem and the problem was my e is given as e 1 is given as 1 0.

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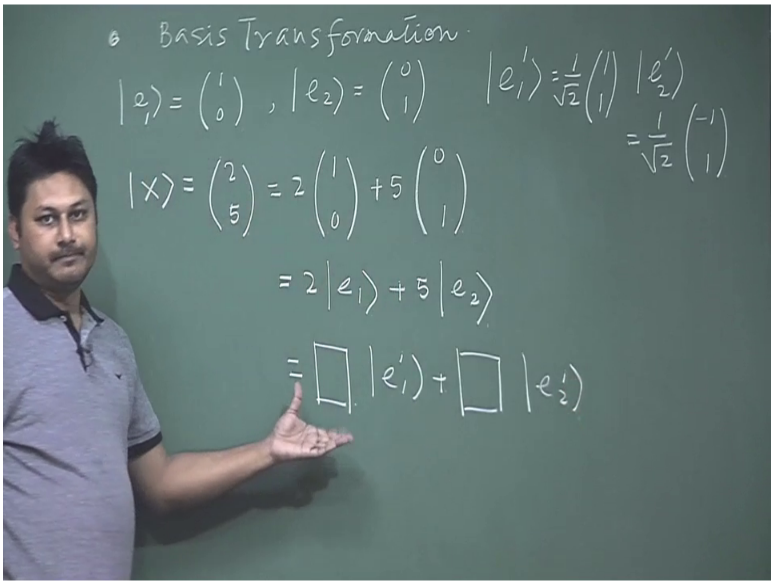

My e 2 was given as 0 1, my e1 prime was given as this and my e 2 prime was given as this. So, now, the question was a vector, that time I wrote x. So, let me with write it say 2 5 this component 2 and 5 is in non prime basis or natural basis. So, I can write it as 2, 1 0 plus 5, 0 1 like this, in terms of e1 and e2 it can be represented like this, the question was if I want to represent the same vector in prime basis then what should be the value.

So, this is box which I need to find figure out and it is e1 prime plus the box, this is e2 prime that was the problem so let us to the problem right now. So, first we need to find out. So, rule I need to write the rule somewhere here so that you can remember in the right hand side of the board say if e is related to e prime like this v prime is related to like this.

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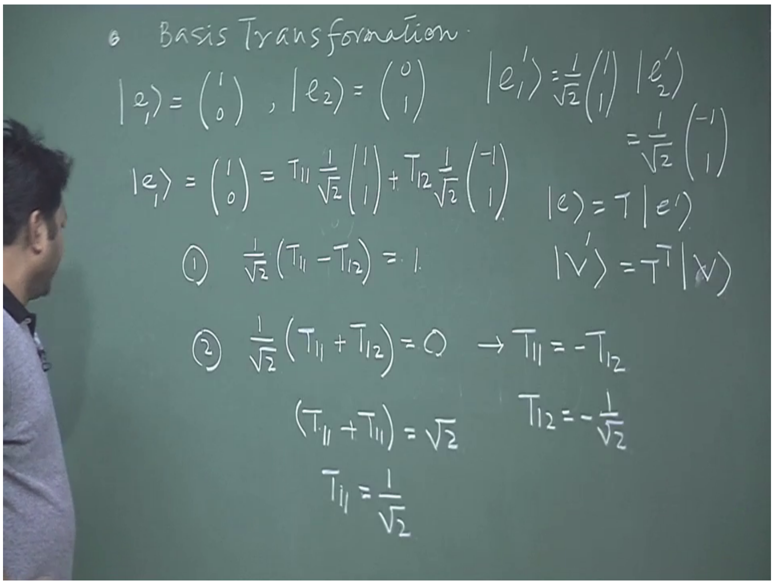

So, I need to find out the T matrix first. So, let me write e matrix in terms of, the first I need to write e matrix is in terms of prime. So, if I do then my e matrix e1 which is 1 0, I can write it as T 1 1, 1 by root over of 1 1 plus T 1 2, 1 by root over minus 1 1, T 1 1, T 1 2 I do not know right now I need to find out because this equation is there so I can figure it out what is the value.

So, if I write the equation from here I can have 2 equations, 1 by root over if I take common then T 1 1 minus T 1 2 is equal to 1 that is my equation 1, my equation 2 will be something like 1 by root over of 2 again I can take it common T 1 1 plus T 1 2 is equal to 0. From this T equation I can solve easily and I can figure what is my T 1 and T 2 2, from this equation I can readily have the information that T 1 1 is equal to minus of T 1 2. If I put it here then my T 1 1 I put this information here so minus T 2 would be replaced by T 1 1, plus T 1 1 will be root over of 2 and this gives me T 1 1 is equal to 1 by root over of 2. So, T 1 1 is 1 by root over 2, T 1 1 is minus t. So, T 1 2 is minus 1 by root over of 2. So, I can figure out 2 component T 1 1 and T 2 2.

So, I should write somewhere here because I need to use that. So, let me write it here, let me erase this T 1 1 I find as 1 by root over of 2 and T 1 2 I find minus of 1 by root over of 2.

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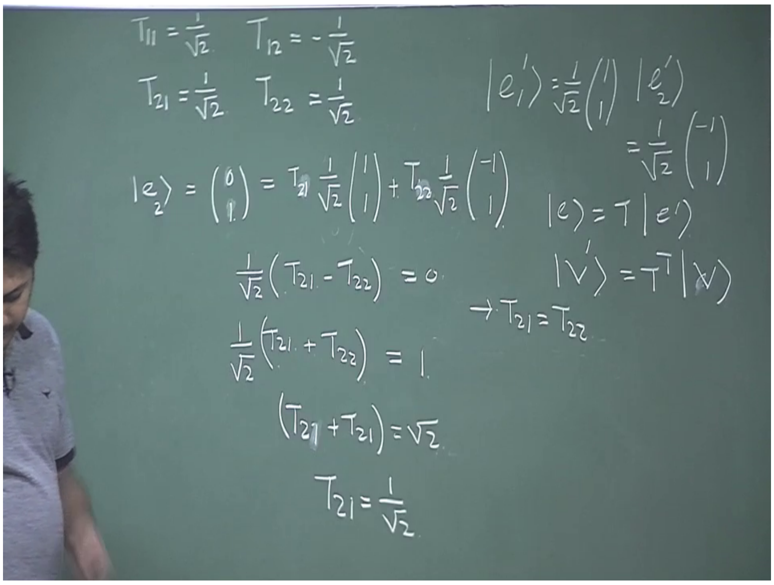

Similarly, I will do the other equation other, other ma other vector I can do the similar thing. So, I can write e2 which is 0 1, now this coefficient will going to change I should write it at T 2 1 one 2 1 and it is T 2 2, I need to find out this also to complete the matrix is it will be a 2 by 2 matrix T will be. So, 2 component I figured out another 2 component T 2 1 and T 2 2 I need to find.

So, now with this equation also I can write 1 the similar kind of equation that will find here T 2 1 minus T 2 2 is 0 first equation. So, when the first equation is here I can readily write from this equation that T 2 1 should be equal to T 2 2. Second equation I can write 1 root over of 2, T 2 1 plus T 2 2 this is equal to 1, T 2 1 1 by root over 2 I take common T 2 1 plus T 2 2 that is equal to 1, T 2 2 and T 1 1 are same. So, I can write T 2 1 plus T 2 1 is equal to root over of 2 or sorry it will be T 2 1. So, T 2 1 is 1 by root over of 2 if my T 2 1 is 1 by root over of 2 this T 2 is also 1 by root over of 2. So, another 2 value I will get which is 1 by root 2 and 1 by root 2. So, all the 4 components of T I figure out ok.

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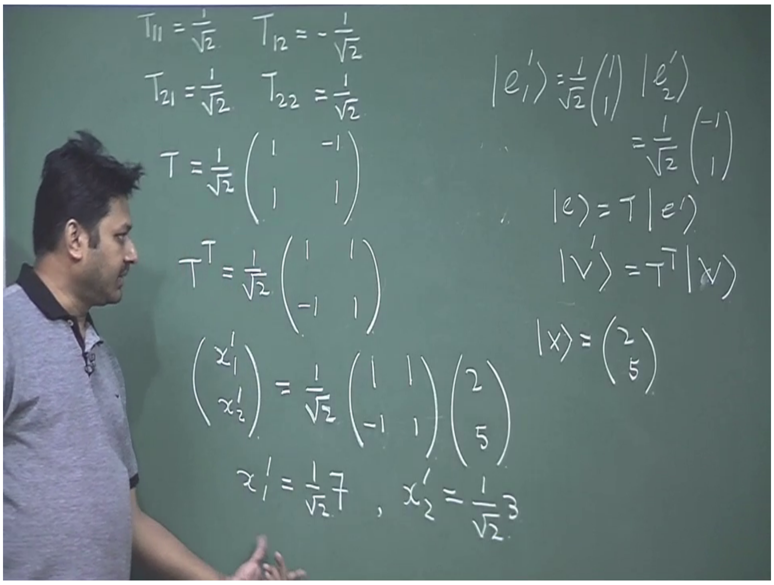

So, my T matrix how that T matrix will look like, 1 by root 2 I can take 1 by root 2 common then it will be just 1 minus 1, 1 1 like this, this will be my T matrix now according to the rule if I want to find out what is the vector component in prime frame I need to multiply T transfer T t and this I need to make a transpose. So, let me transpose that to T transpose will be 1 minus 1, 1 1 this is my T transpose. So, I can just need to transpose that and then I need to multiply with my original vector that is given in so original vector according to the problem it was 2 5. So, I should write it here x1, x2 at prime frame this component I want to know is equal to this matrix multiplied by the original matrix that is given fine. So, I almost done the part so I just need to figure out what is x 1 and what is x 2 by solving this. So, let us solve and try to find out what I am getting is a correct thing or not.

So, x 1 prime is simply 1 by root over of 2, 2 into 1 and 5 into 1 so 7 it is coming like this and x 2 prime is coming as 1 by root 2 minus 2 plus 5 so 3. So, 2 components are having like this and this by this treatment. So, now, how these things it look like. So, let me write T matrix here somewhere because I may be require to use that later.

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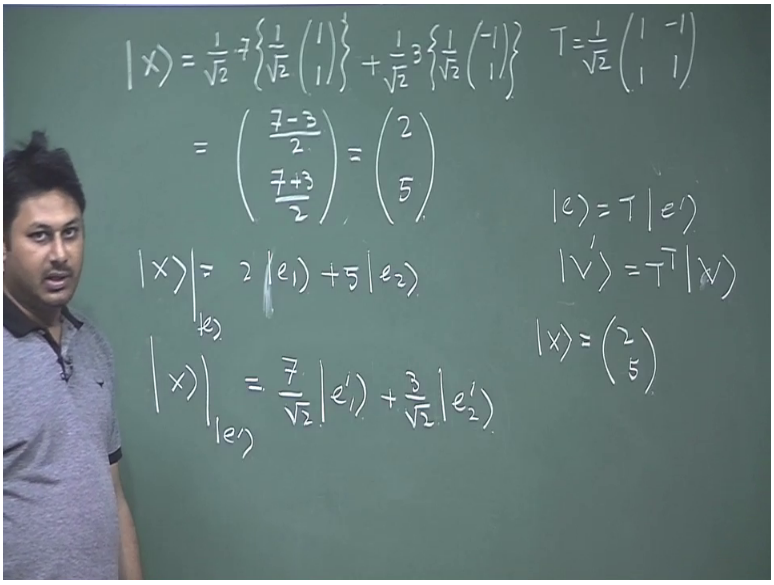

So, my T matrix was 1 by root 2 1 minus 1 1 1 this is a transpose of that matrix. So, that is why the minus sign is coming here anyway. So, I got the component so; that means, in non prime frame, in non prime frame if I write x this will be the first component 1 by root over of 2 multiplied by 7 into the basis which is this plus next component which is this, into the basis. Now, if I calculate that then ill written back my old matrix 2 5 if it is correct if these things are correct so let us check also weather this is correct or not.

So, what will be this matrix 1 by root over of 2, 1 by root over of 2 it will be multiplied and it will give half here also half so 7 and minus 3. So, first component will be 7 minus 3 divided by 2 and the second component will be 7 plus 3 divided by 2 these are the 2 components in this form and this is essentially my old matrix 2 and 5 so. That means, the vector always can be represented in form of his natural basis which is 2 5, when I write any vector in this form that essentially means that I am writing this vector in non prime or natural basis here it is e, but I can also write the same thing in non prime basis also, in non prime basis it will be. So, let me write a total thing.

So, x in say original natural basis is 2 e 1 plus 5 e 2 x in prime basis is this component that I figure out, 7 by 2 e 1 prime plus 3 2, e 2 prime. So, I can if you think I can write any vector in terms of whatever the basis I like for example, this is the most natural way to write the entire vector that is why the basis is natural. But the point is I can also write the same vector in another basis only thing is that the component will going to change and if I want to find out the component this is the rule, I need to use this rule to find the component this component in any basis which is not natural, but one thing I need to know that is this T matrix. I need to know the T matrix which transform this component to the component which is in prime basis.

So, one important thing we know here that 1 vector can be transformed to any of the basis that we know, but most importantly how I can transform this to this is given by this rule. So, I just need to know how this unit vectors or the basis transformed to each other that is why it is called a transformation of basis, I erase T matrix, but I may require this T matrix anyway I can repeat that. So, next thing, important thing is the condition of orthogonal transformation, orthogonal transformation of basis.

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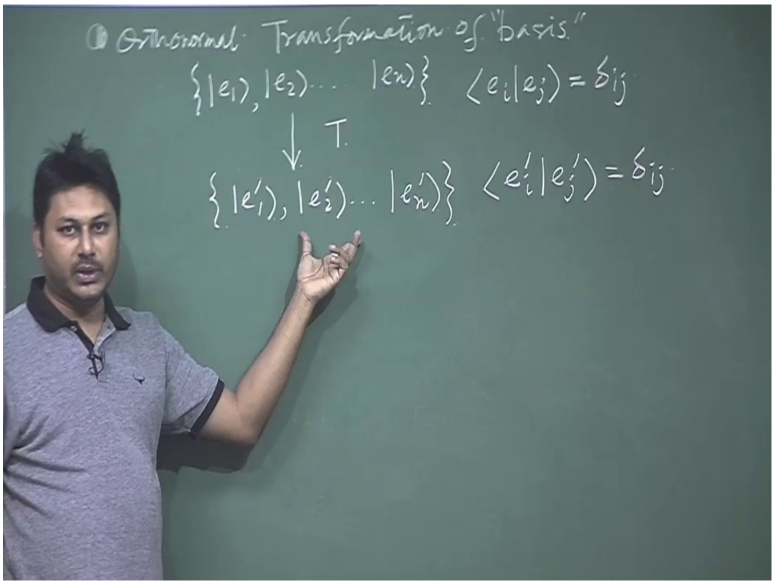

So, or rather orthonormal transformation of basis so make it more general. So, orthonormal transformation of basis, what is the meaning of that what is the meaning of that, say e1, e2, en are a set of vectors which are forming a basis where this condition is satisfied, this is the condition of earth normality I several time I mentioned. So, this is that, now I want to transform and I want to make a transformation and I find another set of basis e1 prime e2 prime en prime, I can transform that through T matrix that I already shown.

If I do that then I will demand that my new set of basis will also follow this formula; that means, my new set of basis will be also orthonormal what is the condition for that so; obviously, the transformation matrix T should be a special matrix that can transform this to this in such a way that the new basis will also be orthonormal. So, I know the problem, I know the problem the definition of the problem is known. Now, let us try to find out the how this T matrix is play the important role, what will be the special condition, what will be the condition about T matrix?

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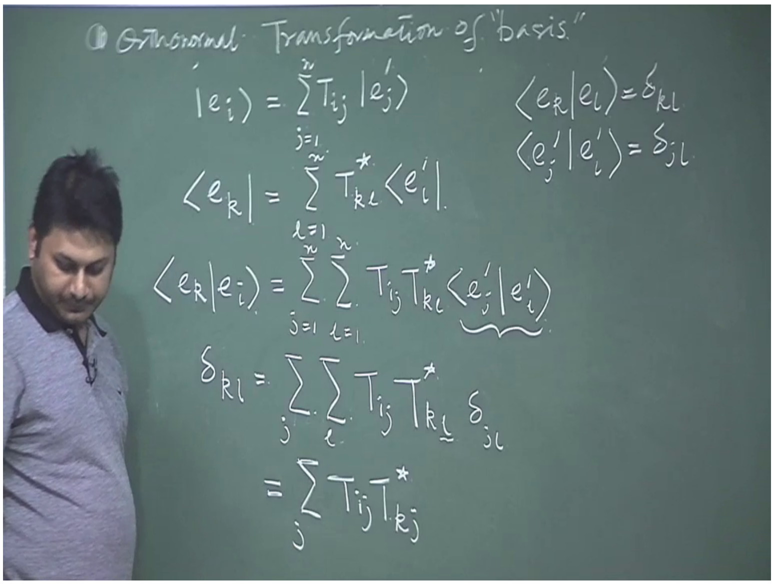

So, I should write ei like T i j, ej prime because this is the transformation rule that I am using, I am just writing in form of matrix component, where is j is running from 1 to n suppose. I make a dual conjugate and I want to find out ek which is Tkl el prime, here l runs to 1 to n where tkl is have to be star because if you remember the dual conjugation thing when I make when I transform a Kate vector to a broad vector, then which is multiplied with that here it is a number should be make in such a way that I can put a star here.

So, it is just change in the dimension from row to column, if you remember this is changing also this has to be multiplied with the, this I need to put a complex conjugate of that. So, that is why it is there. So, now, if I multiply this ek ei, in the right hand side I should write it as j 1 to n l 1 to n Tij Tkl star e j prime el prime equation like this, now the initial basis the basis that I used or that is initially there is already orthonormal. So, this condition was there that is first, second thing is that whatever the new basis I am getting should also follow that rule; that means, I demand this has to be a delta jl also now I put this both these condition here.

So, that I can get a relationship here so delta kl is equal to sum over j, sum over l tij tkl star and this quantity I demand that this should be delta function or delta jl kind of thing, this means whenever I this is the dummy both that j and l both are dummy in this is. So, if I run a new one. So, when n is equal to j only this quantity will be 1 otherwise it will be 0. So, I can remove my this integration and I can write something like Tij Tk l is j then only this term will survive. So, I replace this l to j with a star. So, I will have this. So, like let me write this equation here in the top.

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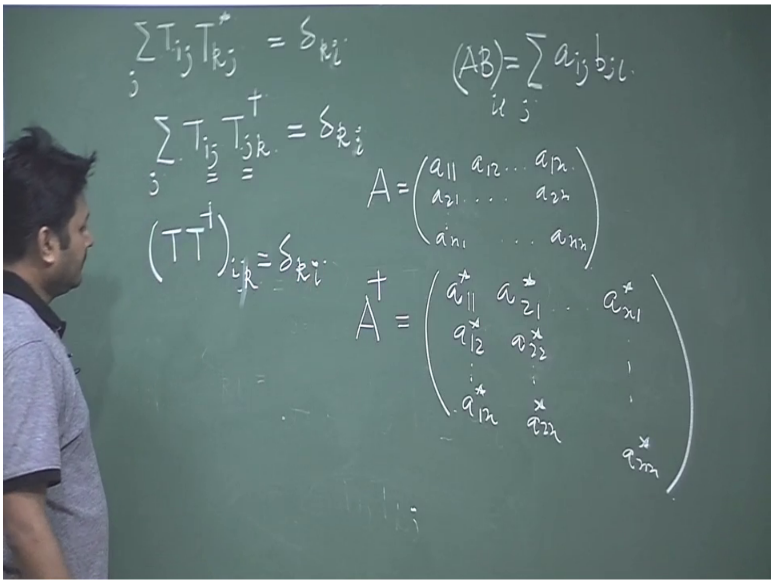

So, finally, I have j T ij, T star kj is equal to delta of kj, this equation I got with the with putting all the condition, now look carefully what is the meaning of that j is summed over here so the here is a j here is a j. So, few class back I already mentioned that when 2 matrix is are multiplying say a and b if you remember.

So, let me remind once again. So, a b 2 matrix are multiplied then the component should be represented in. So, it is the a b multiplication il component where j is this. So, the j should be put side by side so that the valid matrix operation or matrix multiplication can be done. So, this is the rule, but here I find j and j sitting here and here not here. So, I need to rotate these things, I need to rotate these things if I do that then; obviously, I need to make a transformation or I need to write the T matrix in different way say, if I do then it will be i j T jk dagger of that what is the meaning of dagger. Dagger, let me explain quickly here a if a matrix a is a 1 1, a 1 2, a 1 n, a 2 1, a 2 n, a n 1, a n n this is a matrix a where these are the matrix component if I make a dagger of that, a dragger; that means, first I need to make a transpose of this and then the complex conjugate. So, a dagger component will be something like a 1 1 star a 1 2 star a 1 n star I make a transpose of this and then make a star of that, a 2 1 star, a 2 2 star, a 2 n star, a n 1 star to a n n star.

So, transpose plus complex conjugate. So, 1 matrix is given I make a transpose and then make a complex conjugate, whatever the matrix I am getting I can write this a dagger. So, here I am doing the same thing, when I replace rotate these things eventually I am making the transpose of that and then I remove this star. So, when I remove this star I need to put a dagger so; that means, this operation is nothing, but I am changing the matrix in such a way I can have my jj here sitting. So, this is kj. So, this now this entire matrix having the form of matrix multiplication T and T dragger is a 2 matrix which is multiplied. So, I can write this as T t dagger of, it is not kj it will it will k k I component. So, T T dagger ij ik is delta ki so; that means, T T dagger component I generalize I know now what is the components here.

So, this components if I look carefully then I will find a beautiful thing which suggest that T T dragger ik component which is equal to this quantity; that means, when I and k are not equal then this is 0 when I and k are equal then this is 1.

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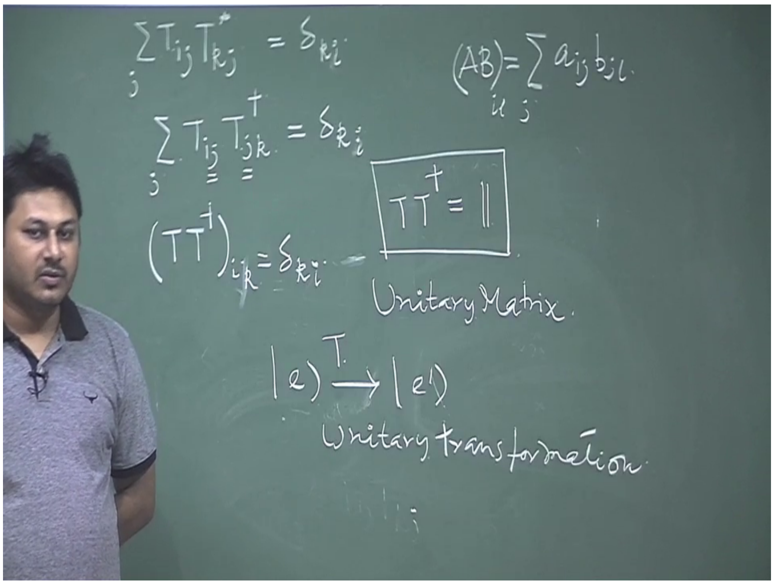

So; that means, it is nothing, but T T dagger matrix is nothing, but unity a matrix multiplied by that dagger of that matrix gives you 1 is called unitary matrix, this is called unitary matrix and this transformation has to be unitary transformation. So, this transformation e to e prime this transformation has to be unitary transformation. So, that both the, both the basis are forming a set of orthonormal basis. So, here e prime e and prime both the basis are forming orthonormal basis under which condition they are transforming by a matrix T, but this T has to follow a rule that is T T dagger is equal to 1 this is called the unitary transformation. So, the (Refer Time: 30:08) is for unitary transformation when the matrix is T is related to this equation which is a unitary matrix.

Under that condition if I transform 1 basis to another basis if my first basis is a orthonormal basis my final basis also be a orthonormal basis only the condition is that my T matrix should be unitary in nature.

So, with that today I will stop the class here. In the next class which again start with unitary matrix and show the example that I just show today that how the T matrix is forming a unitary matrix. So, with that let us conclude the class today. So, 2 very important concept we learned here what is the transformation of basis and the rule, how the vector components can be given and another thing is that the unitary transformation for which you can get a orthonormal set of basis from a orthonormal set of basis by using a suitable T matrix with that let us stop here. So, next class see you in the next class where we deal with this unitary matrix in more detail.

Thank you.