Mathematical Methods in Physics-I
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Lecture – 01
Set, Group, Field, Ring

So, I like to welcome all the students, who have taken this course mathematical methods, one which is very important in physics because it deals with few mathematical techniques we always use in physics. In this particular course; however, we only restricted ourselves to 2 very basic topics. First one is linear vector space and second one is complex analysis. So, linear vector space is very important in understanding the quantum mechanics. Because in the higher class when quantum mechanics is deal with this Dirac notation and all these things, the fundamental aspects of linear vector space is very important.

On the other hand, the complex analysis is equally important. Because in many branches of physics quantum mechanics or in other branches of physics, it is invariably the linear the complex analysis is used. So, complex analysis complex numbers is a very important concept that we should learn also. So, let us start with the linear vector space. So, as I mentioned it is very important for the learning to complex learning to this quantum mechanics. So, let us start with a very basic concept which is set. So, set by definition it is something like a collection of well defined objects or elements, which are distinct.

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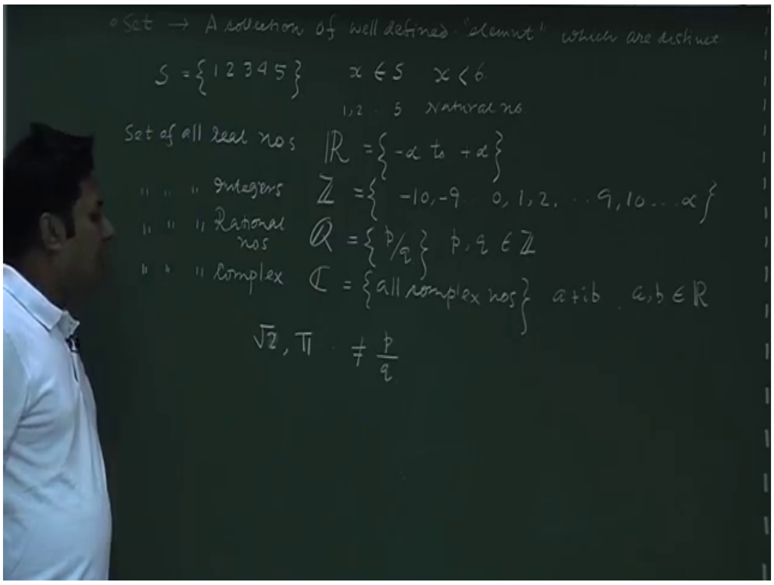

A simple example like if S is a set containing the numbers 1 2 3 4 5. So, the restriction of this numbers are up to 5.

So, I can say any general element x which is belong to the set S is restricted by this thing. 1 2 3 4 5 is; however, the natural numbers. So, 1 2 3 4 5 and so on is a set of natural number we call it is natural number. Similarly, we can form different sets for example, the set of all real numbers. Normally we defined as R which contain the numbers minus infinity to plus infinity all the numbers are the set of R inside. Next, I can say in the similar way that set of all integers, normally we write at Z all integers, I can write inside this set Z as say minus 10 minus 9 so on 0 1 2 so on 9 10 so on up to infinity only the integers with positive and negative values.

In the similar way a set of all say rational numbers, we write it as Q where the elements can be represented in the form of p by q where p and q belongs to Z. We know that that if a number is represented p by q, and if p and q both are belongs to these integers then this is the rational number. Similarly, the set of all complex numbers, this is a important set we going to use later. So, all complex numbers in the form a plus i b, where a and b belongs to the real numbers, a b are real, but as a whole a plus i b is a quantity which is inside this complex number. So, this is a different kind of number; however, similarly you can form different kind of sets like for example, rational number, I can also have a set which are irrational for example, root over of 2 root over of 2 pi etcetera.

These are the quantity which are irrational; that means, it cannot be represented in the form of p by q, but this is a real number. So, this can also form a set. So, with this example we try to find out that there are many kind of set that is present. And it is defined in this way after having a knowledge of sets next important thing, we need to know is called the binary operation. Binary operation is some kind of operation that we need to define over a set. So, set I already define there are few collection of numbers, there will defined and distinct now the next thing is that we need to understand that some kind of operation should be on this particular set and as a result we will get something.

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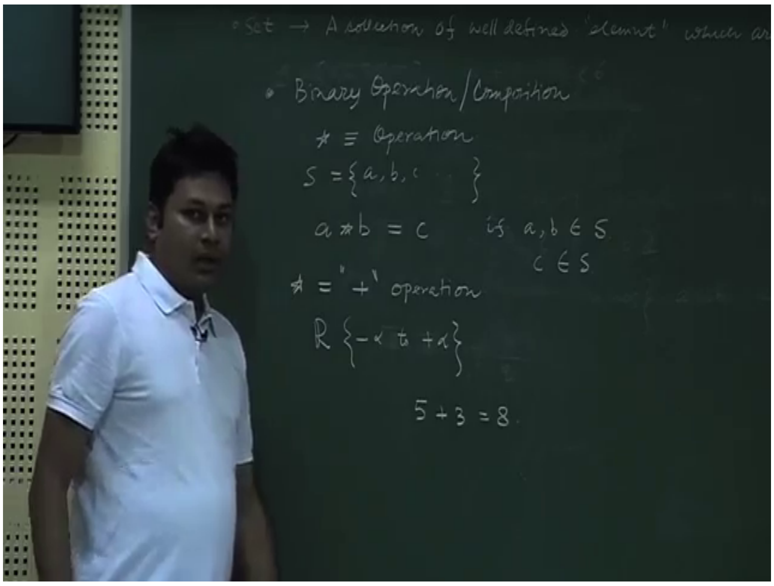

So, let us put some kind of definition in the form say I called it binary operation. Sometimes it is called binary composition also the composition. And binary operation are same thing which by definition says that if this correspond to some operation, and I operate this star over 2 elements of a set say a set has some element a b c and so on and I want to operate the star over this element like that a star b, then I will going to get some other element c. Now the binary operation by definition is that c if a and b belongs to the set f then c has to be in S also so; that means, if I operate the binary operation over 2 elements a and b whatever the result I find here. For example, I find c this has to be inside the set you will not going to with something that is outside the set what will be the example. For example, I can write this as plus operation. Now I define this general binary operation to a particular operation say plus which is addition.

Now, after having a binary composition, we need to know over which I need to apply this what is the set over which I will going to on which I going to apply that. So, I going to take a set real number. So, real number as I mentioned earlier it contains all the numbers ranging from minus infinity to plus infinity. If now I operate this plus operator over any 2 elements, for example, 5 plus 3 it will give me something 8, if you do this exercise for whatever the element you like you always find that the end, result that I am getting after applying the binary composition is a element of r. So, this is a valid binary operation and when the set is defined as my real number set.

So, now take another example, I will going to change these things say minus I can always do that instead of taking the addition.

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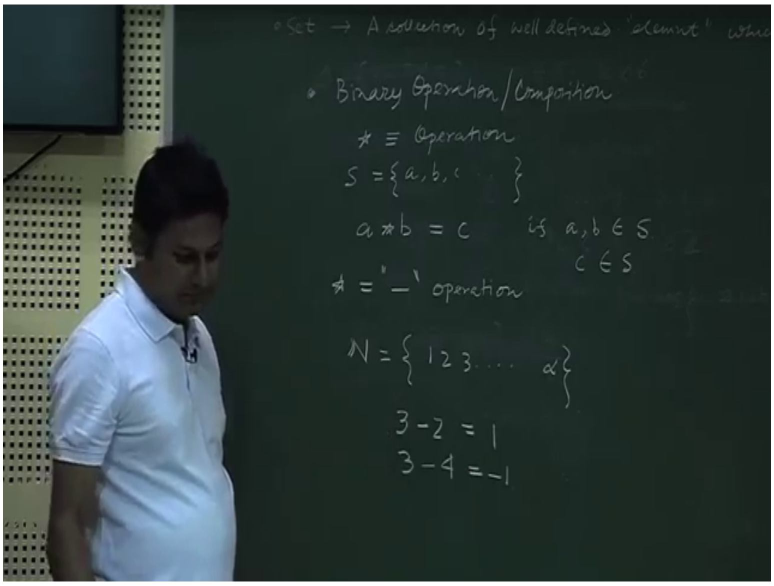

Now, I am taking a operation say minus if I take a operation minus then I will going to apply that on over the set of natural numbers. So, natural numbers I just mentioned earlier it is a set which contain 1 2 3 4 5 6 up to infinity, these are the set of natural numbers mind it there is a difference between the set of natural numbers and integers also contain the negative values of 1 2 3 and so on, but for natural numbers there is no negative and also the 0 is absent. Now if I operate this minus operator over this for example, I take 3 minus 2 where 3 or 2 are the elements of this set in and I am operating this binary operation minus over 2 elements 3 and 2 as a result I will give I will get 1. You find that in this case at least for these 2 elements after putting the binary operation.

I am getting something which is inside the set, but it not necessarily be true for all the elements. For example, what happened if I do 3 minus 4 again 3 is a elements here 4 is element here minus is my operation I will do 3 minus 4, I will get something which is minus 1 which is outside of the set because minus 1 1 is not inside the set. So, even though my binary operation is well defined my set is well defined, if I operate this binary operation, I will find that the result that I am getting is outside the set. So, this is not a at least this operation is not a valid binary operation for this particular set. So, now, I know what is the binary operation and what is the set, now a more important concept we need to know we know what is binary operation, we know what is set the definition is clear to us.

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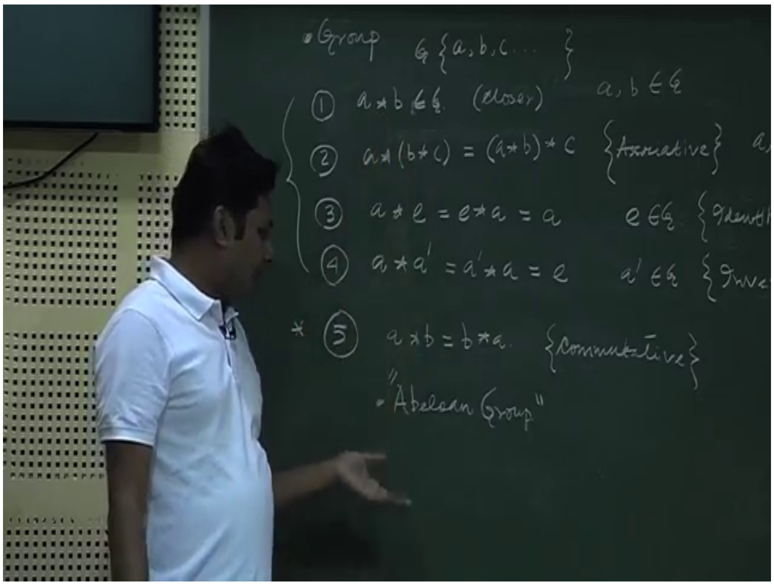

Now, the next thing is that group the concept of group. It is a very important concept in understanding many things in physics and also in mathematics the concept of group is very important. So, let us try to find out what is the meaning of group say G is a set containing the elements a b c and so on. Now there are few properties that the set G has to follow if this say j G form a group what are this property, first property a star b is equal to is belongs to G by the way, when I say group when I define a set then I also define a binary operation here consider star is a binary operation, I do not know for the time being what is the star it is plus minus multiplication, I do not know, but this is an operation that I operate over 2 elements and I am getting something.

So, the first thing is that a star b, I operate 2 elements with a binary operation and then I am getting something that has to be inside G. So, this is by definition this is true because binary the definition of the binary operation is that only it is called closer property closer; that means, I operate a operation over 2 elements whatever I am getting is inside the set. So, it is a closer to a star, b star c is equal to a star b star c. So, I first operate b c and then after that I operate that quantity with a this is the same thing, that I operate a b first and then operate with c so this is called associative. The first one is closer second one is associative these 2 rules these 2 rules has to be followed by G.

If it forms group anyway there are other also other rules also. The third rule is a star e is equal to e star a is equal to a, a is element of the group e is element of the group here I should write a b are element of this group here a b c are the element of this group. Every time you should we should note that all the elements that I am taking is from this group and that is trivial. So, here also e is some element that is inside this set G. So, e belongs to G. Now this suggest that if I operate a with e, I will get back to the same element. So, this is called identity it is like it is this particular element behave like a identity. So, existence of identity is important this is called identity. So, inside the group I should have it essentially means inside the group I should have some kind of element here.

For example e, which is which is which behave like in such a way that if I operate any of the operate any of the element a it will return back the same element here. 4 a star where a star is also an element of G, this a prime also is element of the set G. And I operate this a with this a prime and as a result I am getting back my original identity element. So, this is called the existence of inverse. So, inverse should exist. Now if this 4 properties satisfied I now defined a binary operation I defined a group and now I operate this binary operation and check whether these 4 properties are satisfied or not. If it is satisfied then I must say that this particular set G forms a group; however, apart from this 4 properties there are another property.

Which is not essential to be a group, but which is important to know that a star b is equal to b star a. If that is true; that means, I can reverse the operation, I first take a and then operate over b whatever the result I will get the same thing if I just exchange this element I take b, first and then a and I will get the same thing it is quite for it seems to be quite, but it is not true for all the all the groups and different groups this property does not hold always if it is hold anyway then we call it is the commutative property. So, this is an additional property, which is an additional property if a set with binary composition star if has this additional property then we call a special name to that group and this name is called Abelian this group is called Abelian group.

So, the important thing that to be remembered that the group form by the set S under the binary operation star, if this 4 property needs to satisfy if they satisfy then we will say that the set G are forming a group under the operation star. Apart from that if the extra property is satisfied by the group then we called it as Abelian group. So, we can have a very simple example of formation of the group. There are many examples. So, let us start with a very simple example. So, in order to put the example I need to first define a set and then I need to define a suitable binary operation over that.

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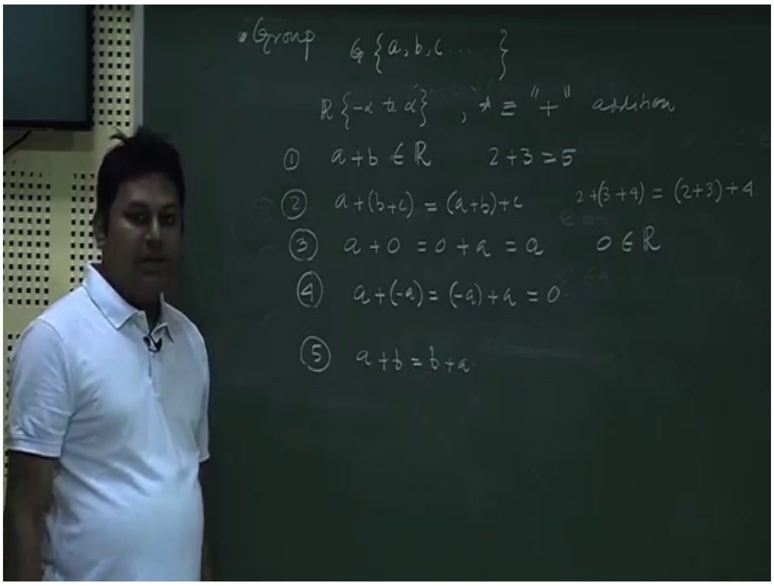

So, let us say R is my set minus infinity to infinity all the numbers.

So, I define set and what will be my binary operation star it is equivalent to the same thing plus addition, simple addition operator I consider to be my binary operation star. Now I apply this or the elements over that. So, you can readily understand that any element a plus b if I do it will be belongs to the real number R. This is quite trivial that if I add 2 things whatever the results 2 real numbers, whatever the result I will get that will fall in the real number example 2 plus 3 is equal to 5. You do you can take any number you want second thing a plus b plus c is equal to a plus b plus c, this is also true if I take another element 2 plus 3 plus 4 this is equal 2 plus 3 plus 4, true third thing it is important.

So, a plus 0 is equal to 0 plus a is equal to 0. So, 0 in this case behaves like a identity element so; that means, if I 0 belongs to R, I should write 0 belongs to R. So, this is not an element that is outside r, but if I add these 2 things I will get back sorry, here should I should write a. So, if I add these 2 things I will get back my original element a. So; that means, the identity is there this is my identity fourth thing. So, I can write this, as this a is any element I want minus a is again an element which is inside the set R. Now if I add these 2 things I will return back my identity element. So, minus a here is the element which I call which we should call is inverse in plus operation.

So, all the 4, 4 properties is satisfied when I take the real number real number say an addition. So, I can say; however, the fifth operation a plus b, b plus a fifth property rather it is also satisfied here. In this case it is also satisfied if I see we will find that this is satisfied. So, if it is satisfied then I can say that under addition operation this real set form a group it is not forming a group. We called since these commutative relations also valid here we call they are forming a an Abelian group. So, now, we have a very rough understanding of what is group and with a very basic example with very simple example like addition, and all these things, but there are other examples also. So, let me do that here only. So, it is not necessarily that always we take the operation addition that we know that add to real numbers it may happens that the addition is different.

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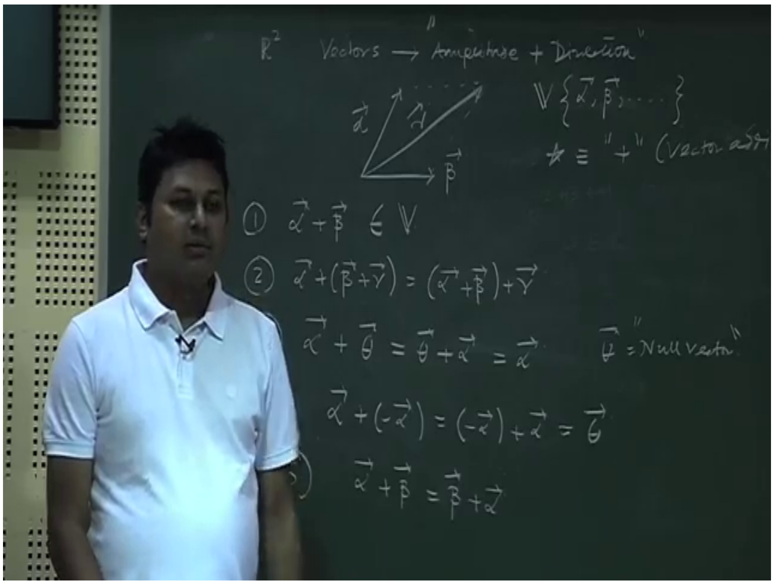

For example, let us consider the R 2 this symbol suggests that I am dealing with some space this is called the Euclidian, Euclidian this is plane Euclidian plane where the vectors are defined this vectors. So, let me first say this vectors. Vectors is something which has amplitude and direction. It has amplitude and direction if these 2 things are there we call some quantity vector, we know that I mean in 2 d if vector is defined like this say alpha is a vector another vector can be defined like this say beta in 2 d they have 2 components we will come back to this point in more general way later. So, let me first try to explain you how this group how this vector can form a group under this addition.

Now, you should remember, that this addition is not a simple addition. So, far we deal with the addition of these real numbers. So, we know that how to add, but if I now if I say that my group my set vector set is defined by these numbers and my star binary operation is addition I must say this is a vector addition this is a vector addition. So, now, under the vector addition we will quickly check whether this under vector addition this set b forms a group or not this is very important. So, let us start with this and this belongs to v, if I add 2 vectors. I will have another vector say here say gamma see if I add 2 vectors I will find another vector that will be inside this set. So, this set is all the vectors all the vectors you can imagine this inside this set.

So, first property is hold second property. If I add 2 vector beta and gamma and then add the another vector alpha, I will get the same result. If I add alpha and beta and then add gamma it is trivial third property very important alpha is a vector theta, if which what kind of vector it is it is basically a null vector. We know that there is something called null vector here theta is null vector. So, it is a special kind of vector which has direction, but 0 magnitude. So, if I add these 2 things whatever the thing I will get is just the original vector. So; that means, here null vector is my identity under this vector addition fourth alpha plus minus alpha. Now if I add 2 vectors one is alpha and another is the same vector.

But with a negative sign; that means, in opposite direction if I add to this to these 2 vectors then essentially I will get something is null. So, again I can have one this vector this vector here. So; that means, identity whatever I have I can return back this identity here with a 4, and this now be behave like a inverse. So, all the 4 properties according to the definition of the group is followed by this vector addition, which is not entirely the addition that we know this is a different kind of addition you can see there is a rule of this addition; however, here you should also note that alpha plus beta is beta plus alpha this fourth fifth things are also valid. If that is the case then we can also say the vectors can form a group under this vector addition.

So, for this class I would like to conclude this class here. So, very important few concept we learn here. First the concept of set and binary operation binary operation is something when we apply it over a 2 elements of set, then I will return back something which is inside the set that is one and. Secondly, the group the set can form a group when they follow few properties which is closer associative existence of identity existence of inverse and so on, if they follow this rules then I will say that this particular set can form a group. In our next class we will extend this idea we will learn more than group there is something called ring then field, and finally, we will try to find out applying this concept how the vectors the concept of vector space emerge. So, with that let me conclude this class. So, in the next class as I mentioned, I will start from here. I will start define something called ring.

Thank you for your attention. So, that is all. So, just wait for the next class.