Astrophysics & Cosmology
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Lecture - 09
Hydrostatics and the Solar Wind

Fluid mechanics and these are the equations that govern the material in different astrophysical situations. Now there are various other aspects of fluid mechanics, which we shall discuss as and when required.

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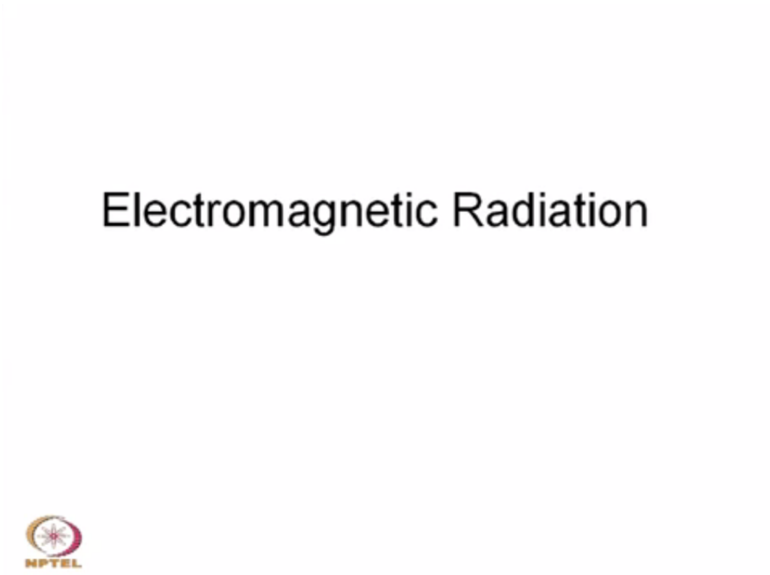

In today's lecture, we shall move on to a different topic much of our information about astronomical sources comes through the study of electromagnetic radiation, practically most of it comes through the study of electromagnetic radiation. Some information does come through high energy cosmic ray particles and some through neutrinos, but the bulk of it is through electromagnetic radiation.

There is currently a great effort underway to study gravitational waves and if that is fruitful then that shall open up another window in gravitational wave astronomy, but right now much of astronomy is deals with electromagnetic radiation.

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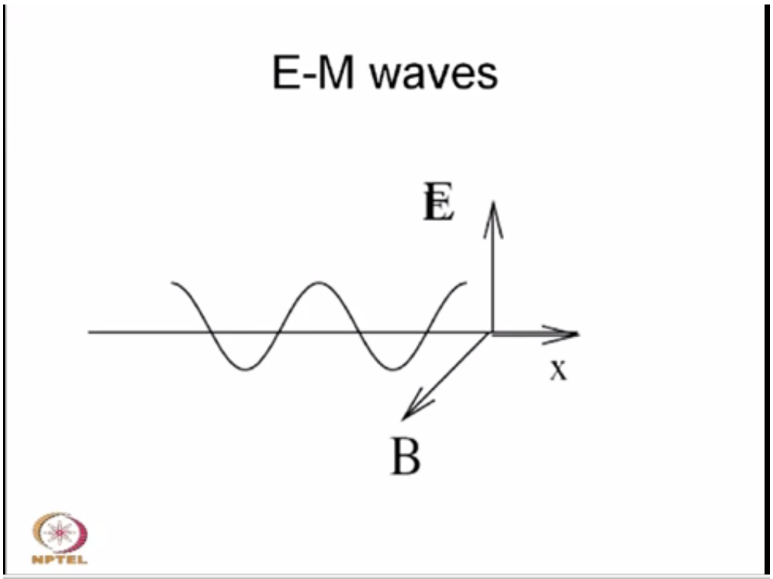

So what is this radiation? We all know that electromagnetic radiation is essentially an electromagnetic wave and I will briefly summarize the salient property of this wave. So if the wave is propagating along the x axis then we have an oscillating electric field pattern in the direction perpendicular to the x. So if I sit at one place and see the electric field, it is going to oscillate up and down possibly and the magnetic field is always orthogonal to both the direction of propagation and the direction of the electric field.

So this is in general what we mean by an electromagnetic wave, there is a direction of propagation and we have an oscillating electric field pattern that moves along the direction of propagation. Electric field is always orthogonal to the direction of propagation. There is also a magnetic field, which all 3 are mutually perpendicular and the electric and magnetic fields both oscillate with exactly in the same phase okay.

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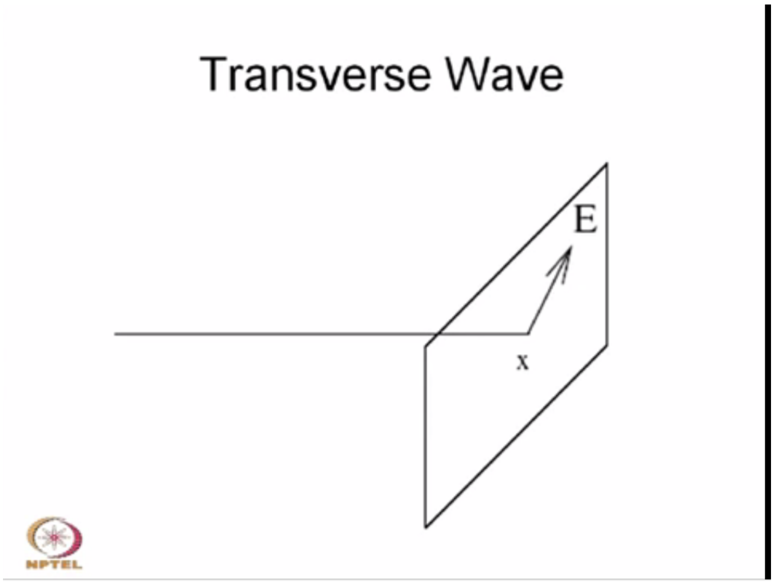

And this is a transverse wave. The electric field can oscillate in any direction in the plane orthogonal to the direction in which the wave is propagating okay and the magnetic field is always perpendicular to E. So this brings about another aspect that is polarization, but right now we shall not touch upon the polarization of the electromagnetic wave, having told you this much that it is a transverse wave with electric field oscillating perpendicular to the direction of propagation and this oscillating electric field pattern propagates.

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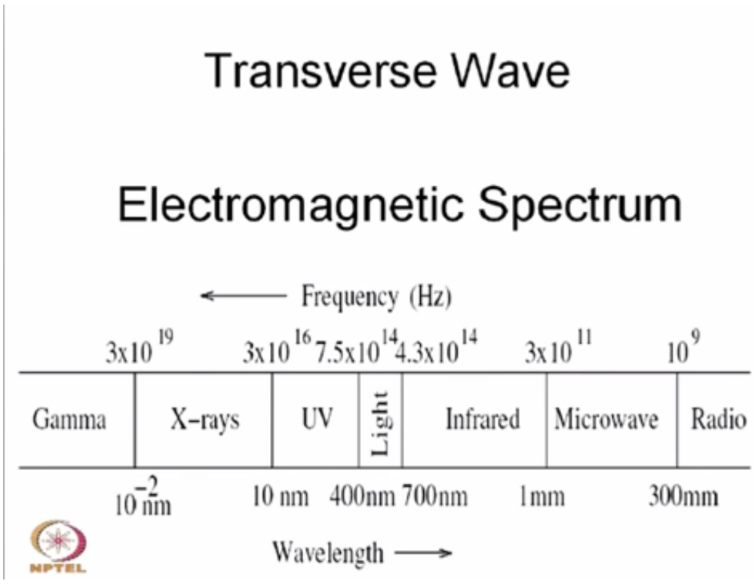

And we can have electromagnetic waves of a variety of wavelengths or frequencies and all of them are important in astronomy. So we gather information about sources about the universe in the entire electromagnetic spectrum and the different parts of the spectrum have been given different names and they have different properties also. Let me just briefly go through this, the lowest frequency electromagnetic waves okay.

So if you look at this diagram the frequency reduces as you go along this direction or the wavelength increases. We know that we are inversely related and they are related through the constant the speed of light okay. So as you move along this axis, the frequency goes down or the wavelength increases and the largest wavelength or the lowest frequency electromagnetic waves below 1-gigahertz.

These are referred to as radio waves and between 1 gigahertz and 3*10 to the power 11 or say between up to 1 mm roughly we have microwave okay. So the lower part of the spectrum is essentially radio and micro waves and the earth's atmosphere is transparent to a large range of wavelengths in this band okay. The lower part is cut-off by the ionosphere, the atmosphere we know the upper layer of the atmosphere there is an ionized layer where there are free electrons and ions.

And any electromagnetic wave of a low frequency coming through that is reflected or there are transmission effects and the intensity is attenuated considerably okay. Provided you are working at very low frequencies, but this cut off becomes important at frequencies say below 25 megahertz or 30 megahertz, 20 megahertz or below that okay. So if you look above 20 or 30 megahertz you can do astronomy, you can look outside the atmosphere.

And you have a transparent window and if you keep on increasing the frequency or decreasing the wave length again over here at high frequency end of the microwave or the low wave lengths say around 1 millimeter over there you start getting molecular transitions in the atmosphere. There are water molecules; various are the carbon dioxide and carbon monoxide, all these molecules in the earth's atmosphere.

So they have rotational and vibrational levels and the energies of these levels correspond to frequencies in this range at the higher end of the microwave and in the infrared. So the earth's atmosphere is opaque to much of this okay. There are bands where you may observe, but largely there are water vapour lines and various of the lines here. Again we have a transparent window in the visual region where our eye is sensitive.

And there is you can do terrestrial astronomy over here. Some parts of infrared also you might be able to do from the earth or if you go to very high altitudes you might be able to access them okay. Ultraviolet is blocked off fortunately because the sun emits copious amounts of ultraviolet and this is harmful to life on earth. X-ray and gamma rays are also blocked by earth's atmosphere okay.

So if you want to study these other bands you then have to send satellites outside the earth's atmosphere okay. They are heavily attenuated by the earth's atmosphere. You could study gamma rays inside the earth from secondary showers okay. And one thing that we should bear in mind so much of our information about astronomical objects outside the earth is gathered by the study of the electromagnetic waves coming at different frequencies from that source okay.

And the same source may look in quite different when you look at it in 2 different frequencies okay, so it is a frequency dependent fact water source looks like. For example, in the last class, we looked at the sun and we saw that in the visual, the radiation comes mainly from the photosphere the surface of the sun and that is the temperature of around 5800 kelvin. If you look at the sun in x-ray over here in x-ray.

Then, we saw that it is not the photosphere, the surface of the sun actually is relatively dark compared to the corona, which is outside the sun which is much hotter and where the bulk of the x-ray radiation originates okay. So you will have an entirely different picture of the same region of the sun okay so if you look at any astronomical source in different parts of the spectrum you may get totally different images and totally different information.

And possibly you may need all of the information to really get a picture of what is going on there okay and you have telescopes now which operate nearly across the entire electromagnetic spectrum and which gathers information about the techniques used are maybe different in different parts of the electromagnetic spectrum. And using these telescopes one can gather information about sources and different parts of the spectrum okay.

So now having given you this background let us now go on to the study of this electromagnetic radiation. So we have just heard that it is a wave okay. Now if you are dealing with length scales, which are comparably larger.

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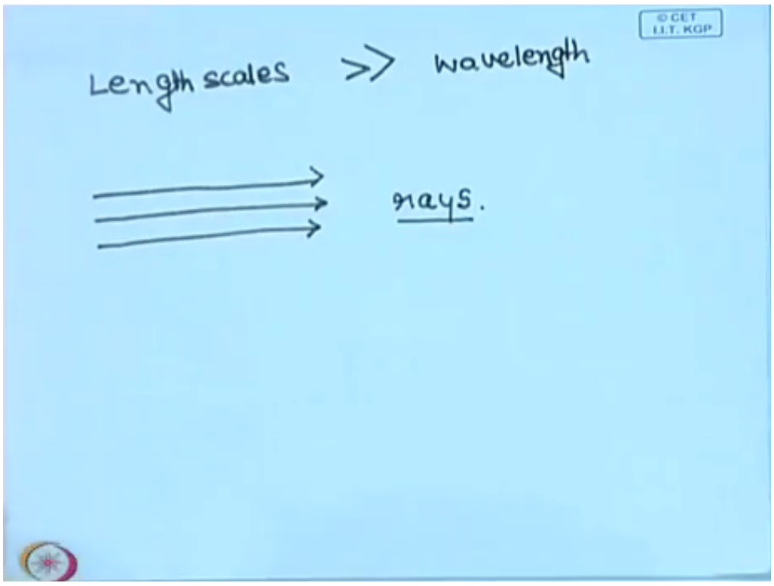

So if you are dealing with length scales that are much larger than the wavelength, which we do typically in astronomy and astrophysics. The length scales are very normal and the wavelength is quite small as we have just seen could be millimeter, centimeter, meter and could be Armstrong right. So in most of these situations, it is adequate to think of light as forget about the wave nature of light and think of light as rays.

Light propagates in a straight line along rays okay. So it is adequate to think of light as rays along straight lines along which light propagates. This is geometrical optics. So now in discussion today we are going to adopt this picture that light propagates in straight lines along rays. The wave nature is not important; it becomes important only when we look at length scale, when we have length scales, which are comparable to the wavelength of light okay.

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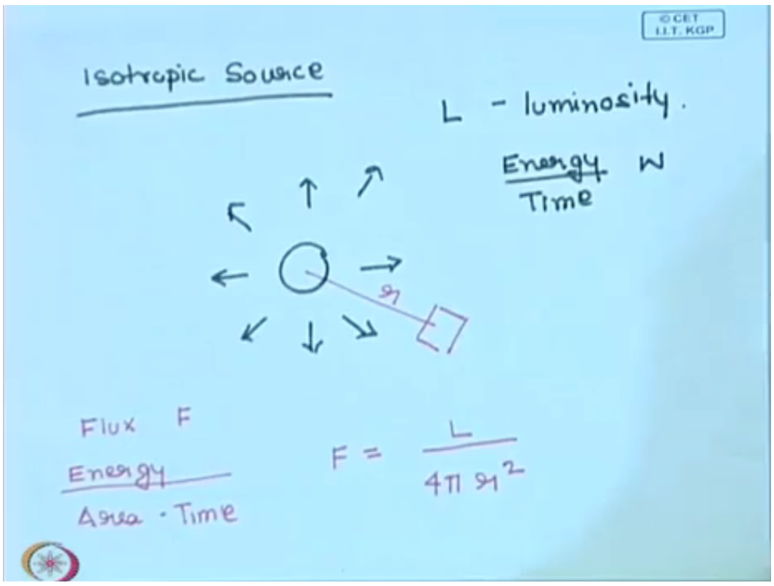

So let us for our discussion consider the simplest possible situation where we have an astronomical source here that emits radiation isotropically. So this is an isotropic source and it has a luminosity L. So this is the luminosity of the source. By luminosity, we mean the energy that it emits per unit time so you can measure it in Watts power okay. So it emits an energy L per unit time.

Now if there is an observer at some distance R from the source so let us consider an observer over here and this observer asks the question how much energy crosses a unit area placed normal to this direction in which the light is coming so the rays are coming from this direction. The observer places an unit area over here and he asks the question how much energy? What is the flux?

This is the energy coming per unit area per unit time. This is what a distant observer can measure. He cannot obviously go and directly measure the luminosity. He or she can measure the flux over here. You can put a detector and measure the flux and the radiation being emitted from here will be distributed since it is isotropic it will be uniformly distributed over a sphere of area 4 pi r square.

So we know that the flux is the luminosity/4 pi r square okay and further away the observer is the fainter the less the flux is going to be okay. Now this is the simplest description of the electromagnetic radiation, but it is rather simple because it considers all the rays coming out right. Now we interested in a slightly more detailed picture of the electromagnetic radiation.

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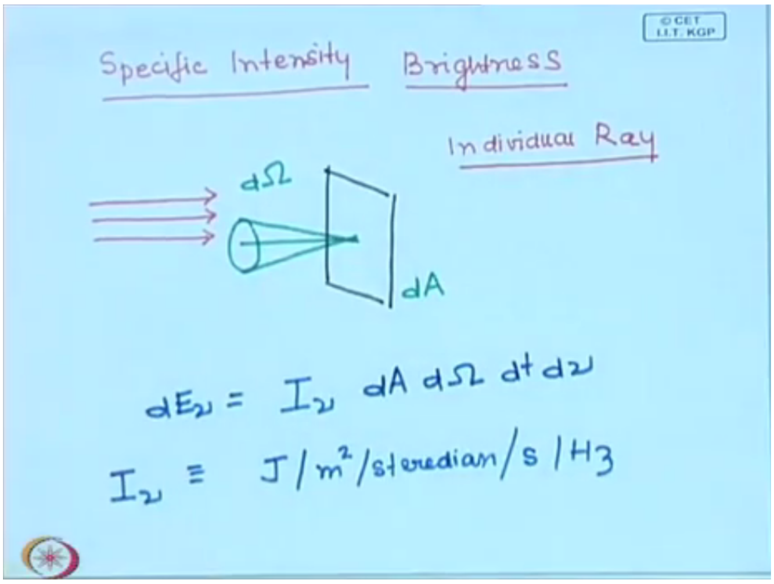

So we shall consider what is defined as the specific intensity or brightness. This is also referred to as sky brightness or brightness. So now in the earlier context, we looked at all the rays coming out from that source okay that is the rather crude picture. Now we are here interested in the energy that is carried by a ray, a particular ray, an individual ray okay. So this specific intensity is defined for an individual ray of light.

Now it is not meaningful to talk about ray propagating in a single direction okay. So it is more meaningful to talk of a ray propagating with the spread in directions right. So the question that we are going to address is as follows. Suppose we place a unit area detector of unit area over here normal to the ray and we ask the question, how much energy is incident? So we are interested in a particular ray so the question asked is how much energy is incident from this direction.

But I have told you that it is not meaningful to ask about a ray in a particular direction. So the question has to be rephrased what is the amount of energy that is incident in a solid angle d omega on the area dA in the time interval dt. So the question is as follows, what is the energy incident on the area dA from the solid angle d omega that refers tells you which ray you are looking at a particular ray, in the time interval dt, in the frequency interval d nu okay that is the question being asked.

This tells you the energy being carried by this ray okay. So we are interested in the energy being carried by this ray and this is given by I nu, which is called the specific intensity into these numbers. So that is how the specific intensity is defined. So let us go through the definition again. The specific intensity I nu tells us the energy coming on this area dA from the solid angle d omega that is a specific ray in the time interval dt, in the frequency interval d nu.

So that is the energy coming here okay. That is the definition of the specific intensity or brightness right and we are now going to study some properties of the specific intensity. So in astronomy or astrophysics, we characterize radiation using this specific intensity I nu or it is also referred to as the brightness. So this is the function of direction okay also right it is in a particular direction in a solid angle d omega in this particular direction.

Here it is a normal direction. Now what are the units of specific intensity let us first look at that. So the left hand side is going to be energy units say joules right that is the amount of energy coming on this. So the specific intensity should have units, energy so let us say joules per first area per meter square then we have per steradian. Steradian is a measure of solid angle then we have per time as second per hertz right.

So that is the unit of specific intensity joules per steradian. You can order them differently per second per hertz per steradian okay. So that is how the energy carried by a ray is quantified good. So with this definition let us now calculate a few things so that we get a feel for what we are talking about. Let us first calculate the energy flux on a surface okay.

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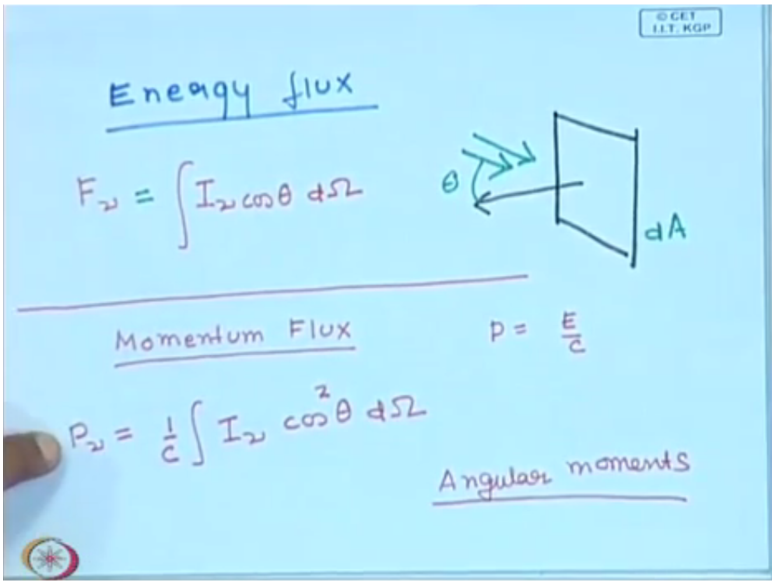

So we are going to first calculate the energy flux or just the flux let briefly just calculate it for an isotropic source. So we will first calculate the flux F and here it is the specific flux right because we are talking about a specific frequency. So it is a specific flux F nu, incident on an area like this whose normal is over here okay. Now a ray coming at an angle so this area is dA, the ray coming at an angle like this will see a smaller area, it will see the project area projected normal to this right.

So the energy coming flux from a ray coming at an angle is going to be reduced by a factor of cos theta where theta is the angle that the ray makes with the normal. So if you want to ask the question what is the energy coming in this area dA in the unit time dt, in a frequency interval d nu then you have to integrate over all the different solid angles. So energy coming from the normal direction is I nu d omega and if it comes at an angle it is going to be reduced by a factor of cos theta so this gives me the energy flux.

I nu cos theta*the solid angle and I nu is the function of the direction in which the light is coming. I nu could be a function of the direction, it could also vary from place to place okay. You should bear this in mind so this is the energy flux. Let us next consider the momentum flux. Now for electromagnetic waves, if you think in terms of photons the idea is simplest. We know that it has an energy and a momentum both.

And the momentum p is E/C okay. So if you can calculate the energy and divide by the C, you will get the value of the momentum. Now we are interested here in the momentum normal to this surface okay. So radiation incident on the surface is going to impart momentum to this surface and the momentum imparted, we are interested in the component normal to the surface.

So if you want to calculate the energy is coming like this and if you want to calculate the momentum imparted in this normal direction you have to then put a factor of cos theta okay. There will be 2 factors of cos theta, one because you are looking at the particular component of the momentum, one because the area gets projected right. So the momentum flux P nu = 1/C I nu cos square theta d omega okay.

And these are all different angular moments of the specific intensity that these are what are called different angular movements. So multiply with cos theta and integrate over solid angle. This is the first movement, this is the second movement okay. So the first movement is the energy flux, momentum flux is the second movement, 0th movement will have no cos theta at all okay.

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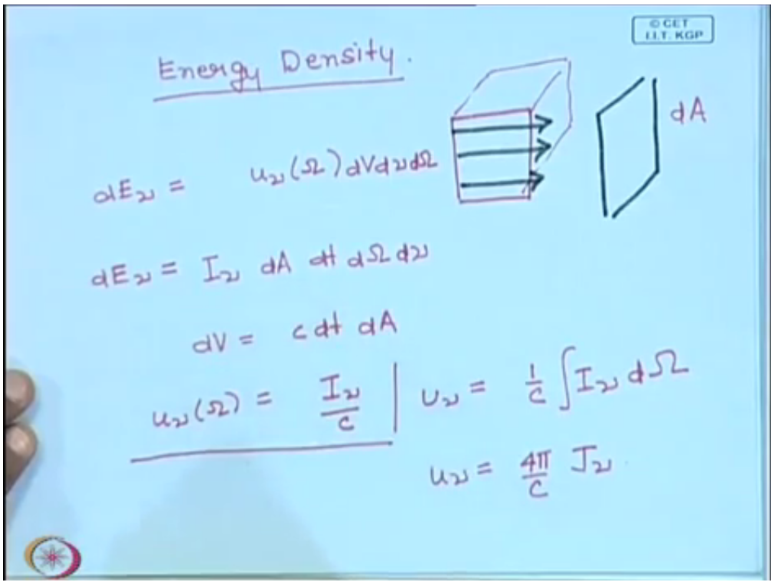

Let us next look at the energy density so the energy density along the ray in some region. Before we look at the energy density in some region, let us first consider the energy density along a ray, which we shall define as U nu omega. So it is the energy density along so let us draw a ray here first and it is the density of energy carried by this ray okay that is given by U nu omega.

We are finally interested in the total energy so this is the volume that we are interested in. So the energy contained in this volume dE nu is dV and there will be d nu and d omega right. So this is because this we are looking a particular ray so there will be a factor of d omega, that is the ray in a small solid angle and this is in a frequency interval nu and the energy density * the volume will give me the energy okay.

So that is how the energy in this particular ray in this volume is defined okay. Now we also know that dE nu = let us put an area over here unit area and we know that the energy that will cross this is I nu is dA dt d omega d nu that is the energy that will cross the area dA okay. Now let us say that this dA has the same area as the area of the volume we are interested in okay.

Then what is the length over which this energy is distributed. The energy that will cross in a time dt, that length is Cdt. So this volume dV we can say = Cdt okay into dA. So if you identify these 2, it is quite straight forward that the energy density of a ray = I nu/C okay. This is the energy density in this ray. If you are interested in the total energy density, you have to integrate over all possible rays, you have to integrate over all possible solid angles.

So this is = 1/C the integral of I nu d omega, which we can also write as U nu = 4 pi/C J nu where J nu is the average specific intensity. What do you mean by the average specific intensity? You integrate it over all solid angle and then you divide by the total solid angle, which is 1/4 pi that is the angular average okay. So what we see is that the energy density is related to the 0th moment of the specific intensity okay.

And you can now also integrate all of these over frequency to get the total energy flux, the total momentum flux, the total energy density.

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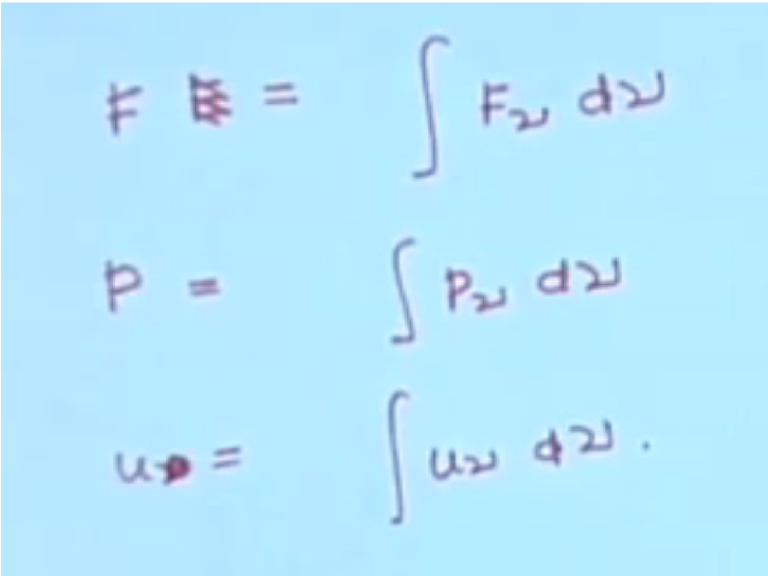

That these were the specific ones so if you for example you want to calculate the total flux? What you have to do is you have to integrate F nu d nu. Total momentum is going to be P nu d nu this is the specific momentum and the total energy density U is U nu d nu okay. Those were certain properties of the specific intensity.

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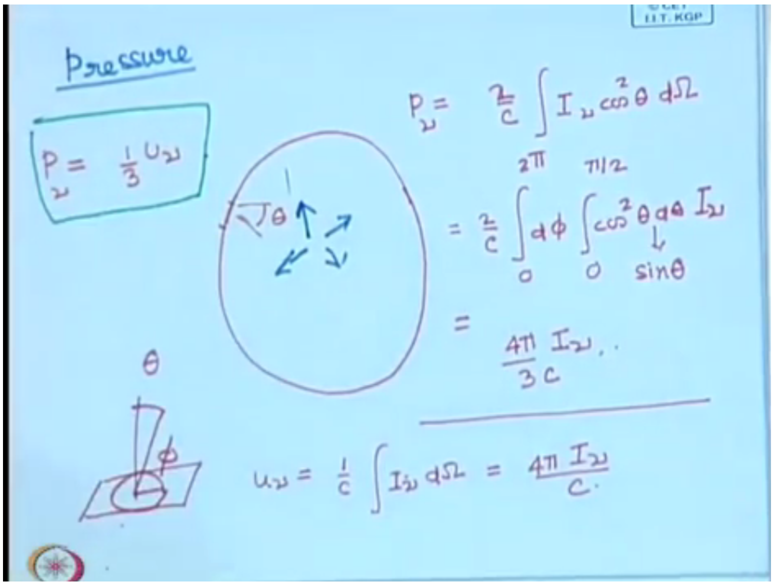

Let us now consider another simple example. Let us consider a reflecting cavity. I have drawn it circle, it need not be circle okay. There is a reflecting cavity, which is filled with isotropic radiation okay. So it is equal in all directions. The radiation inside is isotropic. So radiation goes and gets reflected and then goes again and gets reflected and it is isotropic inside.

This is an example, which is going to be considerably important so we want to calculate the pressure inside this. So pressure we know is the force per unit area and we can write it as the momentum flux, the rate of change of momentum, momentum flux is also the pressure. So let us calculate the momentum flux in some element over here. So the momentum flux we have seen is 1/C*the integral of the specific intensity, this is P nu let us say in a particular I nu cos square theta d omega okay.

That is the momentum that is incident, but this will get reflected so there will be a factor of 2 because it is reflecting right and the integral here is now going to be only over one direction, only on one incident, only on this half, it will not be on that half okay. Now this integral over solid angle we can write in spherical polar coordinates I hope you are familiar with this so we can write it as 2/C.

So cos theta remember is this angle theta. We also have the azimuthal angle right just to get things clear. This angle is theta; this angle can also be rotated like this; this angle is phi okay. So we can write this as d phi. I nu is isotropic, it does not depend on direction so it can just be taken outside so we have cos square theta d theta and I have I nu completely outside okay and this integral is from 0 to 2 pi and the theta integral only goes from 0 to pi/2 there is nothing coming from the other side okay.

So this goes from 0 to pi/2 okay. So this gives me a factor of 2 pi and there will be a cos theta, sin theta, d theta right that is the solid angle. The solid angle is d phi sin theta d theta okay. So if you do this integral then you will get a factor just 1 okay it is a straight forward integral so what you get from this is you can just write this as d of cos theta and do the integral okay.

What you will get is 4 pi/3 C*I nu. Now if you also calculate the specific energy density inside this is 1/C the integral of I nu d omega since I nu is isotropic it is the same in all directions, this is equal to how much is it? 4 pi, integral of a solid angle is 4 pi, so 4 pi I nu/C this is = 4 pi I nu/C okay. So what we get by comparing this and this is that the pressure = 1/3 of the energy density okay.

This is very important so for isotropic radiation field, the pressure is 1/3 of the energy density. So I request you to please go through this yourself once or twice to make sure that you have really understood it okay. So we have been calculating various properties like pressure, energy flux, momentum flux etc using the specific intensity now let us ask the question how does the specific intensity change as a light propagates?

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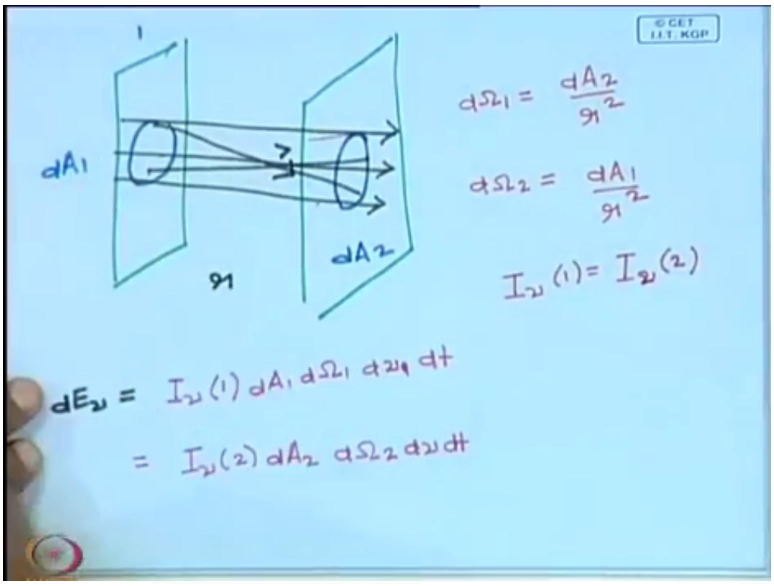

Okay how does the specific intensity change as the light propagates? So to do that let us consider first a particular ray so okay let us look at 2 surfaces located at 2 different position so this is one surface and this is another surface and let us look at 2 different areas on these 2 surfaces so this is the area dA1, this is the surface 1 and this is the area dA2. Now I have told you that a particular ray does not refer to a single direction, there is a spread in direction.

So let me also draw the spread in direction so you could also have something like this right, small spread in direction, the whole thing is propagating in this direction. Now we have drawn these dA1 and dA2 such that all the rays that go through this also cut through this okay nothing more comes in here between here and here okay. So this entirely everything that goes through this the entire ray also passes through this okay.

That is how we have defined these 2 areas, dA1 and dA2 and the quantity that we are now interested in seeing is how does the specific intensity change as you go from the first surface to the second surface okay that is the question that we are interested in addressing right and these 2 are located at distance r apart. So that is the problem that we are interested in solving okay.

So let us calculate the energy that passes through this in a time interval dt, in the frequency interval d nu okay. So let us see. So the energy that passes through this is I nu 1 that is the specific intensity value over here*dA1 d omega 1 d nu 1 d nu 1 and dt are equal okay in the same frequency interval, in the same time interval this is going to be = I nu2 dA2 d omega because the entire ray passes through this area and this area.

So we are looking at the area cut by this ray over here. Now let us ask the question how are the 2 solid angles related? How much solid angle does this subtend over here? Solid angle we know is the area over here divided by the distance square right. So we know that d omega1 = what is d omega 1? It is dA2/r square right, that is the solid angles subtended by this area over here? What is d omega 2?

It is dA1/r square. So you can see that if I put these 2 over here what I have is dA1 dA2/r square. So all of these factors are essentially exactly the same. So we are led to the conclusion that the specific intensity at 1 is equal to the specific intensity at the place 2. So specific intensity is something that does not change when the light propagates in free space that is a very useful and interesting property okay.

So the specific intensity that comes off the radiation from the sun for example is the same whether you look at it on the surface just outside the sun or if you look at it here, it does not change okay. Let us try to understand now how does this connect with the picture that if you go closer to the sun obviously you will receive more radiation right. The specific intensity does not change.

But we know that if you go further away and further away from the sun, the sun will appear faint right.

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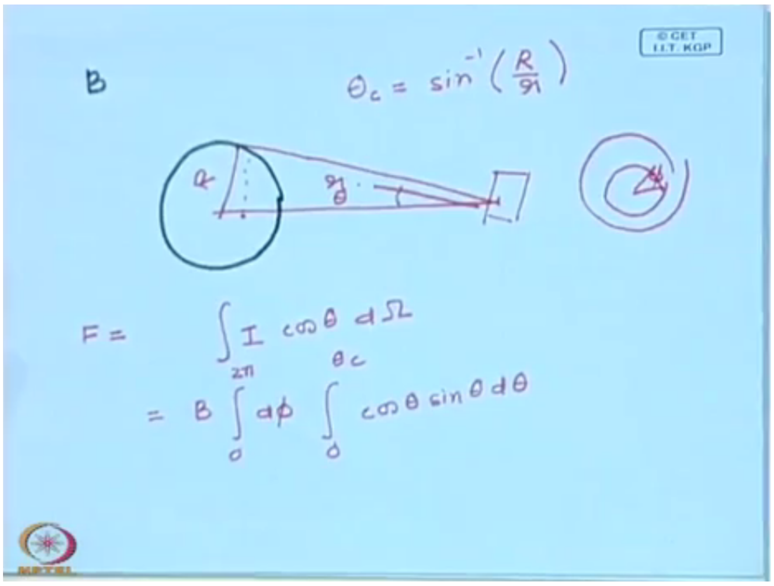

So let us try to see that this is actually consistent with that okay. So for that again let us consider a spherical source like this and this source is an isotropic source with brightness B so what we mean by that is surface brightness is B, all the rays that leave this have the same specific intensity B, it is an isotropic source emitting with the same specific intensity. So all the rays that leave this have the same specific intensity B okay.

That is the source we are looking at and let us consider an observer at some distance r from this source okay and the source let us say has radius R and let us ask the question so what is the flux of radiation that will be measured by an observer over here energy flux. What is the total amount of energy that an observer over here will receive the flux that is the flux right, energy per unit area that is what we finally measure?

So the flux F is the integral of the specific intensity into cos theta*d omega right where theta is this angle so this is the maximum value of theta, theta is the angle that it makes with the normal, this is theta okay. So the radiation is going to be incident for a range of theta starting from 0 over here till a value what is the maximum value? The maximum value let us call it theta C is going to be sin inverse of this divided by the radius that is where this line of site is going to be tangential to this curve okay.

So this is going to be sin inverse R/r okay. So this is the limit so you have to integrate over possible all solid angles over this disc. So this will appear like a disc, each point on the disc will have this brightness, will be arriving with this brightness and I have to integrate the specific intensity over the entire disc to calculate the flux over here okay. So there are 2 angles involved, 1 is theta, which changes like this and another one is phi, which if you look at the disc this is the disc I am looking like this so that will be in this direction phi okay.

So we can write this integral this specific intensity is a constant that comes outside and we have 0 to 2 pi d phi that is the phi integral, phi is the angle with respect to some fixed direction and then I have 0 to theta C, theta C is this angle of cos theta sin theta d theta. Let me remind you that the infinite decimal solid angle d omega is d phi.

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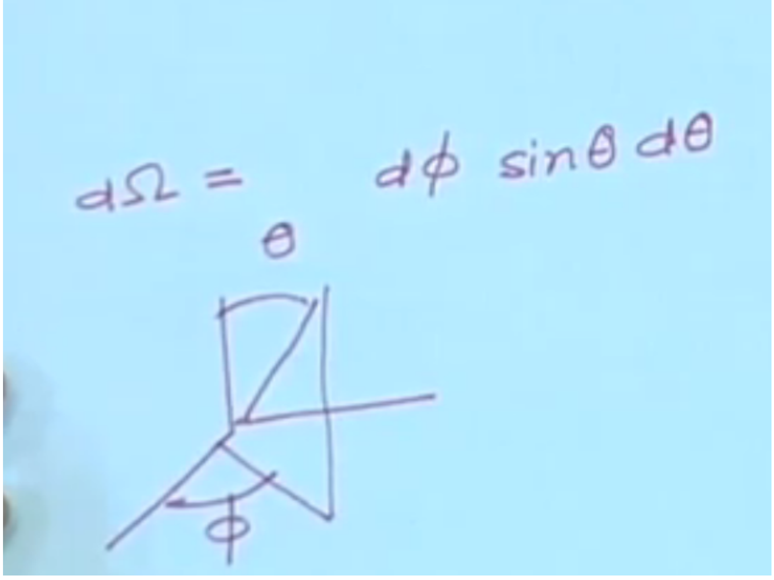

Let me write it somewhere. The infinite decimal solid angle d phi sin theta d theta okay. This is how you in spherical polar coordinates. This angle is theta, you project this on to the x, y plane, take the angle here this is phi okay and you want to integrate over solid angle d omega then you have to do the integral d phi d theta cos theta and the range of theta is 0 to pi, the range of phi is 0 to 2 pi.

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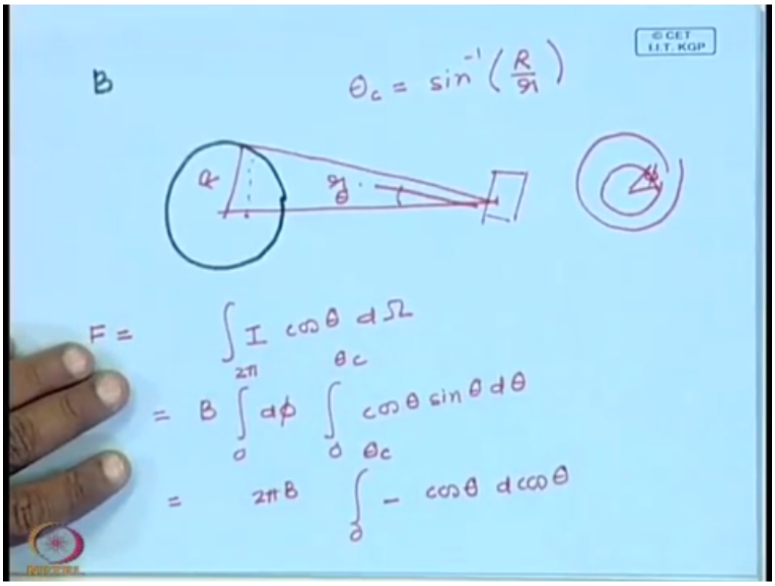

In this case, the range of theta is just 0 to theta C okay and if you do this integral so this integral can be written as -cos theta d cos theta right. So we can now do this integral, this integral just gives us cos square theta/2 right.

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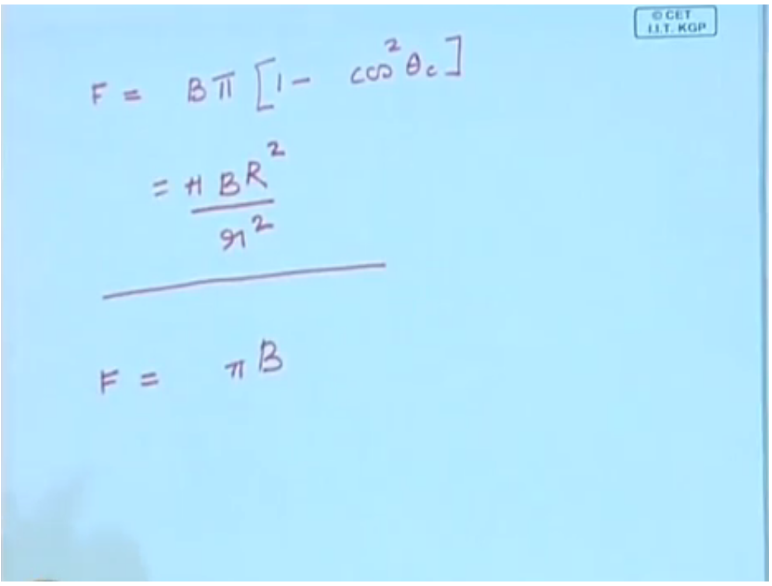

So we have the flux = B pi and if you integrate this you will get cos square theta with the minus sign. Now the 2 limits are theta C and 0. So since there is a minus sign I can take the limit 0 first so that 0 cos square theta is 1 so this becomes 1-cos square theta C, which is sin square theta C so we get B pi R square/r square okay where we have just used the value that sin is the theta C is R/r okay.

So what we see is that the flux indeed falls off as 1/r square as you go further and further okay that is the first thing which is what you expect 1/r square so the flux is what falls as with distance. Why does the flux fall? The flux does not fall because of the reduction in the specific intensity, the flux from the sun falls off as you go further and further away not because the specific intensity has gone down, but because the solid angle subtended by the sun actually goes down as you go further and further away.

So for any source the specific intensity does not change as you further and further away, the solid angle changes and it is usually the flux that we measure so the flux falls as one/R squared okay. Another interesting fact is that on the surface the flux is exactly = pi*B okay right. Now this was all in vacuum so in vacuum we saw that the specific intensity does not change as light propagates.

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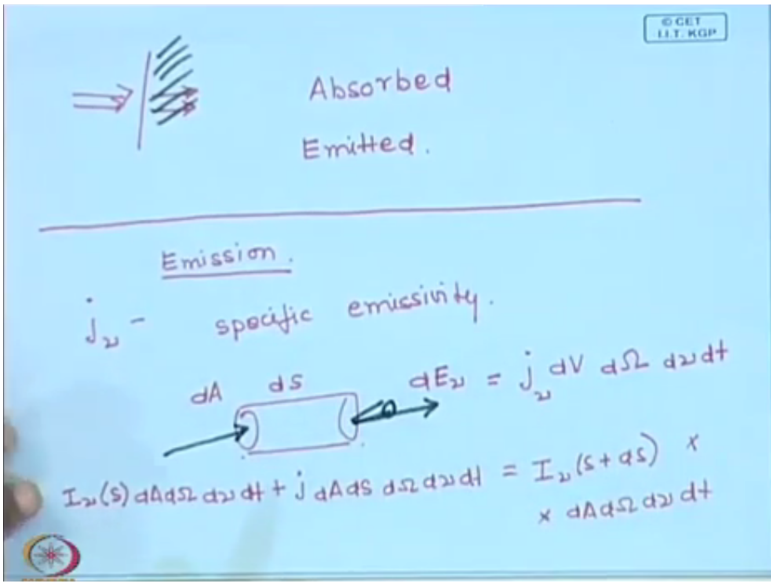

Now what happens inside a medium let me very quickly derive the equations for what happens inside a medium so let us now consider a ray like this and it enters some medium okay so this is my medium over here. Once it enters the medium you can have either absorption, the light can get absorbed or emitted, that you can have emission again at the same frequency.

So there are 2 processes, there is also scattering, which we can think of as both absorption and emission combination okay. So let us first consider emission. So we introduce J nu which is the emission coefficient, specific emissivity okay. So how is this J nu defined? Let us consider a volume of material dV okay. So there is a volume of material dV and the amount of radiation that this volume of material dV emits in this solid angle.

So we are interested in the production of a particular ray okay so in a particular solid angle d omega in the frequency interval d nu in the time dt, J nu into this is the energy that is produced by this material okay. So volume dV of this material produces energy dE nu in this solid angle d omega so the energy that is going out in the solid angle d omega in the time interval dt in the frequency interval d nu okay.

So let us now think of this as having cross sectional area dA and length dS let us say okay. So there is light incident over here. There is ray incident over here and the ray comes out from this side. Let us ask what is the incident energy? We know that the incident energy is I nu at the point S, this is S, this is S+dS*dA d omega d nu dt. Now the amount of energy that gets added over here when it travels through this the amount of energy that gets added is plus this thing over here.

So it is +j, now the dV here is dA dS*d omega d nu dt and this will be equal to what comes out, what comes out is the specific intensity at the point S+dS* all of these factors again so we have dA, d omega, d nu, dt. So we can see that all of these factors exactly cancels out and you can take this on to the left hand side so what you will get is the difference in specific intensity = J nu*dS.

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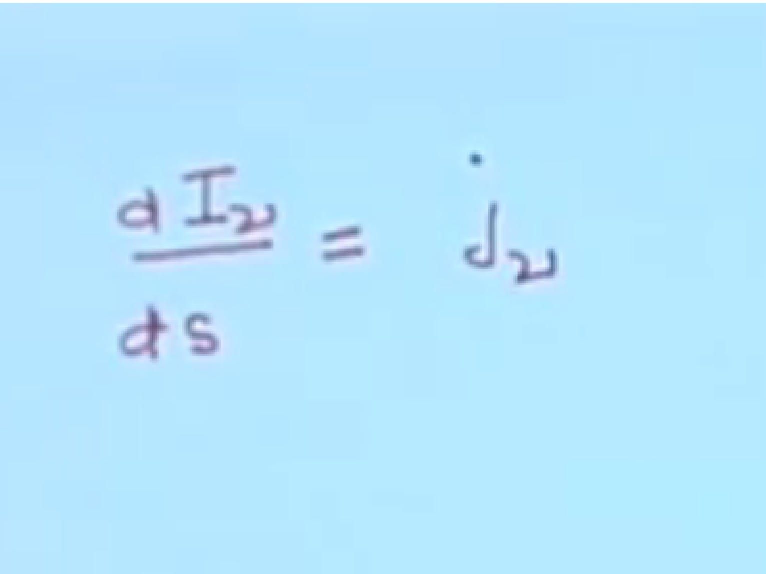

All the other factors cancel out okay or you can write it in this form d I nu dS = J nu okay that is the radiation that gets added due to emission okay. I think we are running out of time now so we shall postpone the discussion of the absorption to the next class okay so we have considered what happens due to emission.