Astrophysics & Cosmology
Prof. Somnath Bharadwaj
Department of Physics and Meteorology
Indian Institute of Technology – Kharagpur

Lecture - 06
Binary Systems (Contd.)

Welcome, let me remind you that we have been studying binary orbits and in the last class we have seen that the motion of 2 objects in their mutual gravitational field can be reduced to the motion of a test particle in the gravitational field of a very massive particle. So the 2 body problem can be reduced to a single body problem and then I told you that binary systems where there are 2 stars in orbit around each other can be roughly classified into 3 types.

You have the visual binary, the eclipsing binary and the spectroscopic binary. In today's discussion we are going to continue binary systems and let me start of by revisiting the spectroscopic binary.

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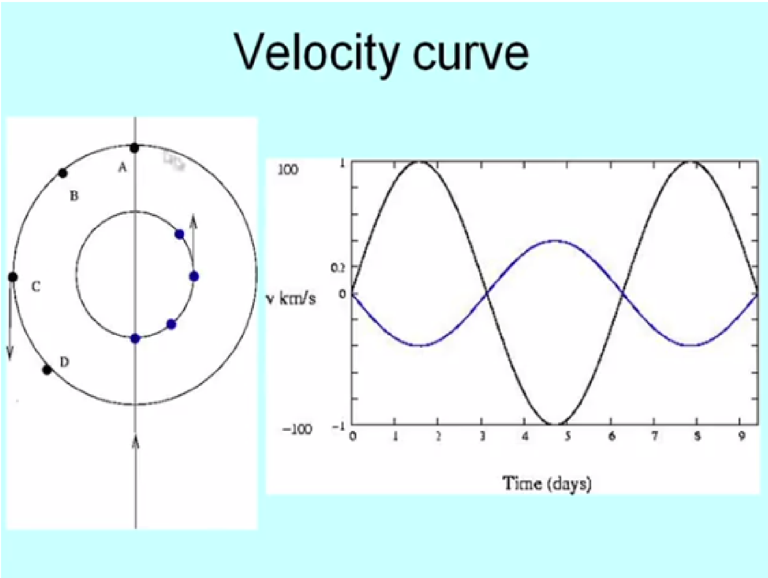

So this picture shows you a circular, 2 stars in a circular orbit and the one shown in blue is more massive the one shown in black is less massive and they are both moving around in circular orbits of the same time period. The line, the relative displacement vector joining the 2 objects at any given instant of time also traces out the circle and it passes through the center of mass.

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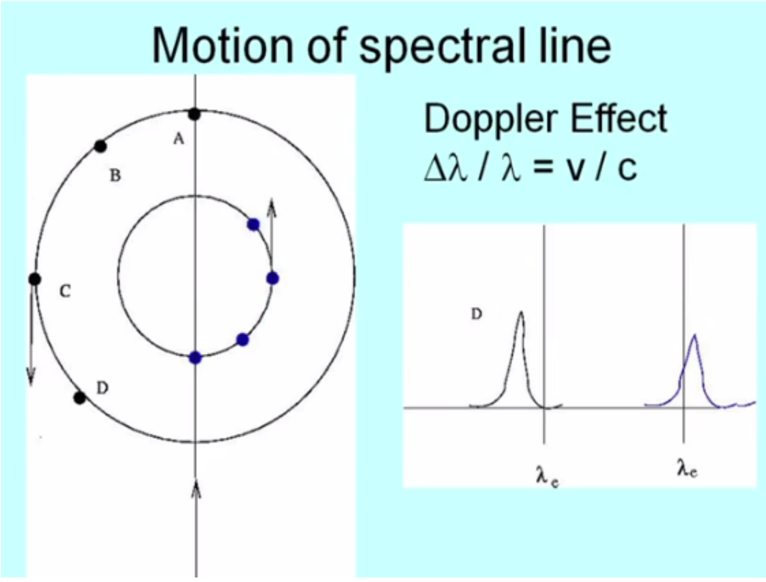

We have seen all of this and in this situation you can make out that even if you cannot make out the 2 stars which is usually the situation in a spectroscopic binary you see them as a single bright object on the sky.

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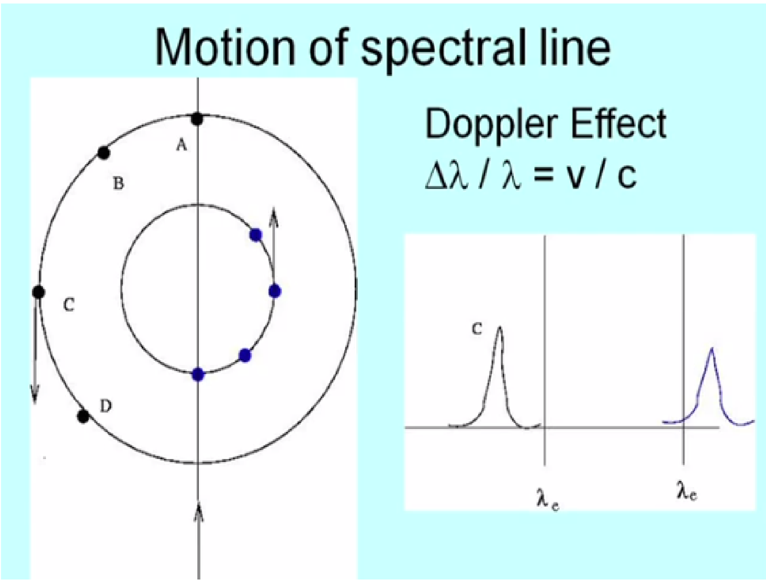

You will notice that the spectral lines shift and if you can see spectral line from both the objects you can determine the rotation curve, the velocity curve of the 2 objects. And here we have considered an object an observer who is in the plane of the orbit. So the inclination angle is 90 degrees. The inclination angle is 90 degrees, so the observer is located over here.

Distant observer located in this direction is observing both the spectral lines from both of these objects and what we will have is that the velocity that the observer see in first from Doppler shift will have a behavior V1 (t) for one of the objects it will go as V1 Sin omega t.

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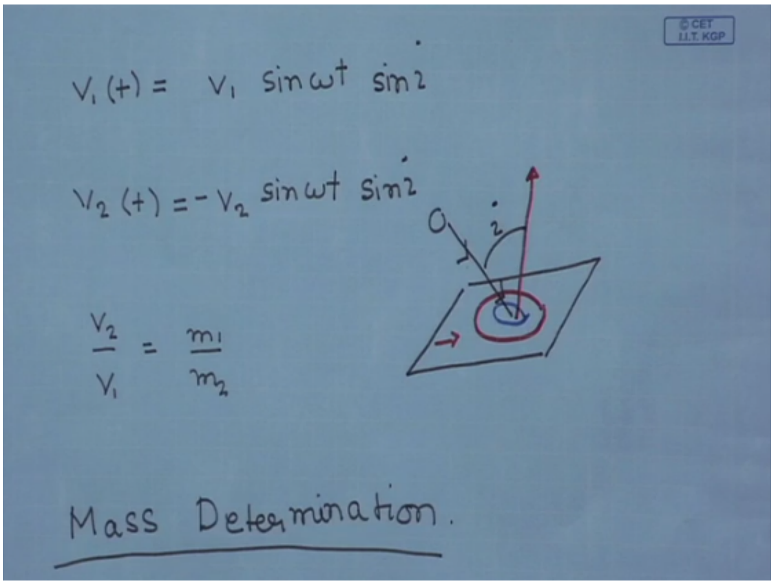

And for the other object V2 will be V2 the amplitude into the – sign sin omega t. So the 2 velocities are exactly pi out of phase and they have different amplitudes everything else is the same. And the ratio of the amplitudes we have seen V2/V1 = m1 inversely proportional to the masses/m2. Okay, so this is just a recapitulation of what we have discussed in the last class.

Now, today we are going to discuss how such an observation of a spectroscopic binary can be used to determine the masses of the 2 individual stars. So what we are going to discuss today is mass determination. This is something that I mention in the last class that binary systems can be used to determine the masses of the individual objects and today I am going to give you some idea about how this is done.

And we shall take up for discussion the simple situation where we have circular orbits. It is more complicated for elliptical orbits. Now the first thing that we need to consider is the possibility that the observer is not located in the plane of the orbit. That the inclination angle need not be 90. And if the inclination angle is i, so let me draw the picture. This is the plane of the orbit and these are the circular orbits that we have been talking about.

This is one of them and this is the other circular orbit that we have been talking about and till now we have been considering an observer who is located in the plane of the orbit. So the inclination angle which is the angle between the normal to the plane of the orbit and the direction of the observer is 90 degrees. We shall now consider a more general situation where the observer could be at an arbitrary inclination angle.

So this is the inclination angle i this is the direction of the observer. Now in such a situation the observer will only see the component of the velocity in this direction. And it is clear that the observer is located exactly perpendicular to the orbit he will not see any velocity. So there will be a factor of sin i that comes in, sin of the inclination angle. And if the inclination angle is 90 we recover back the thing that I had written down right at the start.

So this is the general situation that we are going to consider. Now the first point that we have to realize is that from the observation of the velocity curve. So the observer gets the Doppler shift of the lines, spectral lines and from there the observer determines the velocity, the line of site component of the velocity and from the peak value of this velocity curve the observer can actually determine the combination V1 sin of the inclination angle for the first object.

And V2 sin of the inclination angle for the second object.

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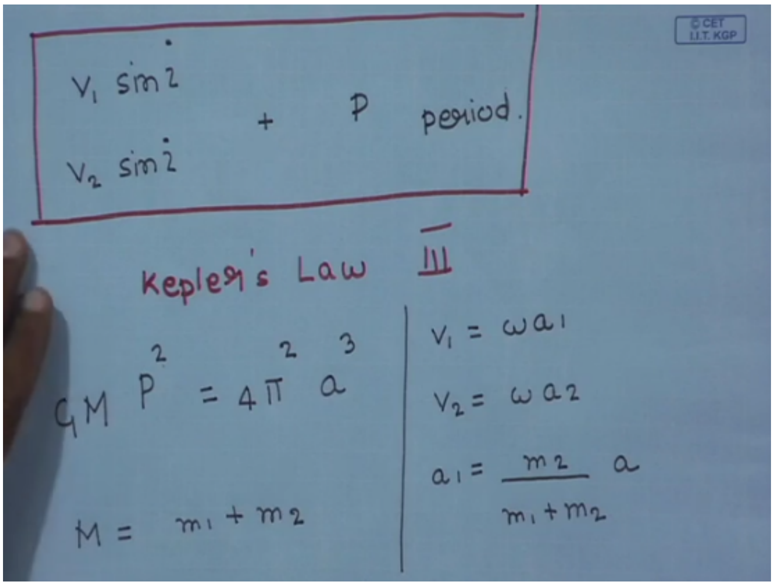

So what the observer can determine from the observations is actually V1 sin i and v2 sin i. These are the 2 things at the observer can determine from the observations in addition to this the observer can also determine the period of the orbit. So the period can be determined from this the velocity will show a sinusoidal variation and the period of that gives the period of the orbit. So this the period.

So these are the things that the observer can directly determine from the observations. And the question is that how is the observer going to interpret these observational quantities? That is the basic question. And how can the observer use these to infer the mass of the individual objects in the binary system. So to do this let us start off with Kepler's 3rd law. Kepler's law and we consider the 3rd law which says that the period of the orbit P square is proportional to the cube of the semi major axis a cube.

And the constant of proportionality is GM, I can write it in this way GM here and 4 pi square here. So the Kepler's laws give us this relation between the period and the semi major axis which in this case is the radius of the circular orbits. And I should remind you that the a over here is the radius of the circles swept out by the separation vector by the separation vector joining these 2. So a1, so the velocity, the peak velocity of the first object can be written as omega a1.

Where a1 is the radius of the first object and a2 is the radius of the second object with respect to the center of mass. And this will and V2 can be written as omega a2 where a1 = it is m2/m1+m2*a. So what we would like to do, similarly, a2 is m1 divided by m1+m2*a. and inversely proportional to the masses. So what we would like to do is that we would like to write this law in terms of the observe quantities.

Another point which I should remind you that the combined system of these 2 objects which have got masses m1 and m2, the respective masses are m1 and m2. It behaves like a system which has got a total mass a single body system which has the total mass m1+m2. This is again something that we had seen in the last class. So let me now write the Kepler's laws in terms of all of these quantities finally our aim is to write it in terms of these observe quantities, okay.

So to do this let me proceed, so what we will do first is that we will write this as

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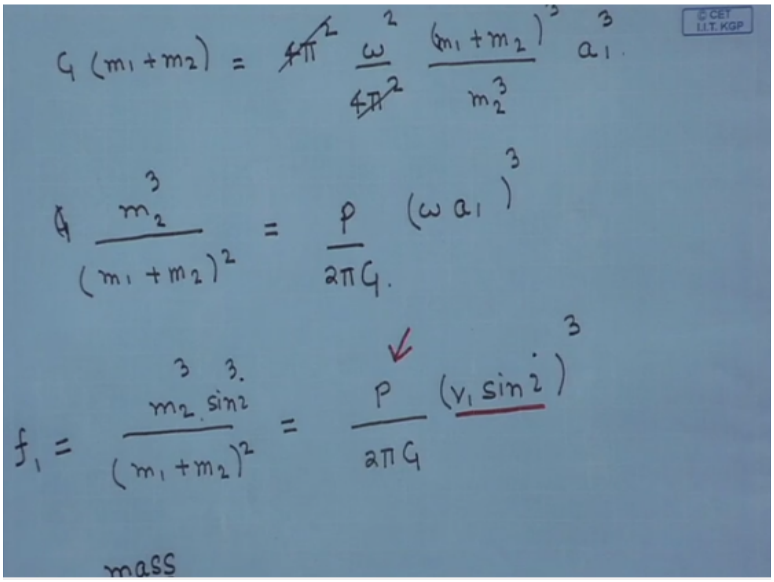

G (m1+m2), G m1+m2*the periods square = 4 pi square. So let me keep this here. This = 4 pi square and the period, the period is 2 pi, period is 2 pi/omega. So I can write this as omega square/4 pi square and I have a cube over here and a cube can be written as m1+m2 divided by m2*a1. So I want to write it in terms of quantities I can observe from the first object which is here, right.

So what I can write this as m1+m2 cube a1 cube. And a1 cube*omega cube we know is the maximum speed of the first object provided to multiplied with sin i. So what we have to do is we have to write this as we will write this, so let us try to simplify this first. So we will take all the factors of m onto that side on to the left hand side. So what we have is m cube m2 cube/m1+m2 square = these 2 factors cancel out. So we have omega square a1 cube and I would like to write this as omega a1 cube.

So I have to divide by factor of omega and dividing by omega means I have to multiply by the time period into divided by so this can be written as this divided by 2 pi and I have the factor of G over here. And in order to write this in terms of observational quantities, I have to introduce another factor which is sin i. So I have to introduce a factor of sin i cube. If I introduce a factor sin i cube, then I have the observe quantity V1 sin i cube over here.

So what I get is that m2 cube/m1+m2 square = the sin i cube = P the period divided by 2 pi G*V1 sin inclination angle cube. So this is the quantity which i, so these are 2 quantities this is something I can determine from observations. This also can be determined from observations 2 pi G we know so the right hand side is something that we can determine from observations and this gives me what is called the mass function for the object 1.

This is called the mass function of the object 1 and this is something that can be determined observational. So the mass function can be determined observationally and it is from the theory of orbits it = m2 cube/m1+m2 whole square*the sin of the inclination angle cube. It has the dimensions of mass. But it is not directly the mass of any of the objects.

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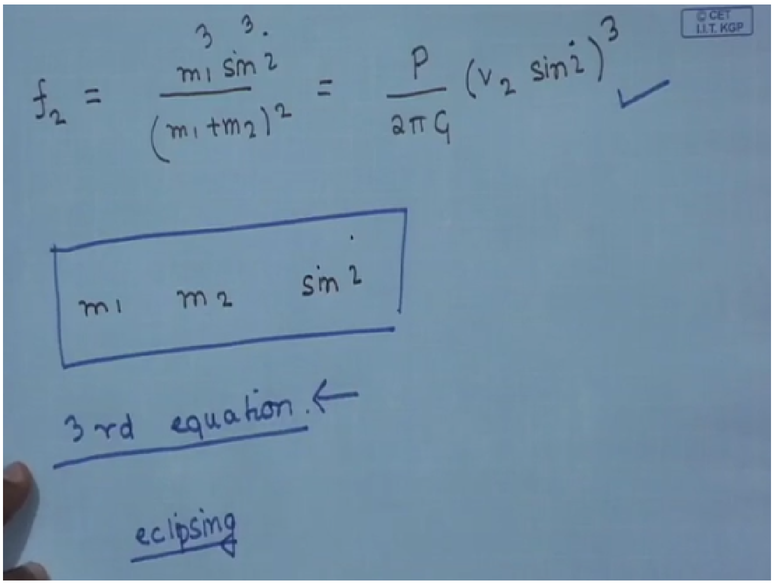

Similarly, if I had the observations of the second object they will give me V2*sin of the inclination angle and what I will get from this is the mass function of the second object which = m1 cube sin i cube/m1+m2 and observationally this is P/2 pi G V2 sin i cube. So from observations we can determine 2 quantities the 2 mass functions. The mass function of the first object and the mass function of the second object.

The entire problem let me remind you has 3 unknowns. The 3 unknowns here are the masses of the 2 objects m1 m2. Further so we have 2 objects of unknown mass m1 and m2 in circular orbit. Further we also have the inclination angle sin i. So these 3 quantities are unknown and we only have 2 relations which can be determined from these observations one is the mass function of the first object other is the mass function of the second object.

These are the 2 things that can be determined observationally and these 2 quantities together are not adequate to uniquely determine the masses. So that is the first point that we have to understand. So if you have a spectroscopic binary from observations of the 2 spectral lines if you can measure the Doppler shifts of the 2 spectral lines you cannot determine the masses uniquely.

What you can determine are the mass functions which have the dimension of the mass but unfortunately did not give us the individual masses. In principle you require a 3rd observation, a 3rd equation. And such an equation becomes available such an observation 3rd constrained it becomes available if you have an eclipsing spectroscopic binary for example. So if you have an eclipsing spectroscopic binary then from the light curve from the duration of the eclipses.

And the dip in the light curve during the eclipses you can get constrained from where you can determine the masses and the inclination angle. But with from just a spectroscopic binary you cannot determine these 2 things uniquely, okay. Now there are one can make progress, so let me try to give you some insight as to what you can do from just a spectroscopic binary next. So for example if I have a situation where I have, where I know that one of the masses is much larger than the other. For example, let me just take up an example.

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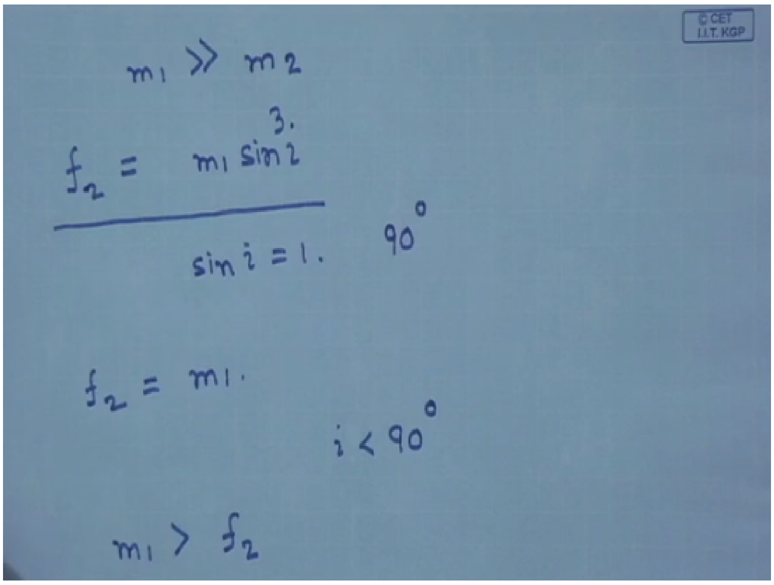

So suppose I know that m1 is much larger than m2. So if I know that one of the masses is much larger than the other then m2 can be dropped from this equation and what this gives me is that f2 = m1 sin i cube. So in this case I have only this constraint. So I have a constraint only on the more massive object. If I know that one of the objects is much lighter than the other.

For example, for the earth sun system we may ignore the mass of the earth and interpret the sun and we will have such a thing, okay. So if I have reason to believe that one of the objects is much lighter than the other then I can get a direct constraint on the mass of one of the objects from so if I can measure the Doppler shift the mass function of the second object which the lighter object I can get a constraint on the mass of the first of the heavier object.

And this does not tell us the mass of the heavier object directly but it gives us an upper limit, a lower limit because sin i so if sin i is 1 which is the largest value it can have then f2 is directly = to m1. Whereas if the inclination angle, so this is the inclination angle is 90 degrees. If the inclination angle i is < 90 then this factor is going to, this factor is observationally determined this factor is going to be < 1.

So the mass then m1 is going to be more than f2. So this is an lower limit on the mass. In general, the mass functions set lower limits on the masses of the 2 objects which is quite clear from here. Because these factors are all going to be < 1. So if I can measure f2 then it sets an lower limit on the mass of the first object. Similarly, if I can measure f1 it sets a lower limit on the mass of the second object.

Let me now take up an example of where of such a situation where I know f1 and f2.

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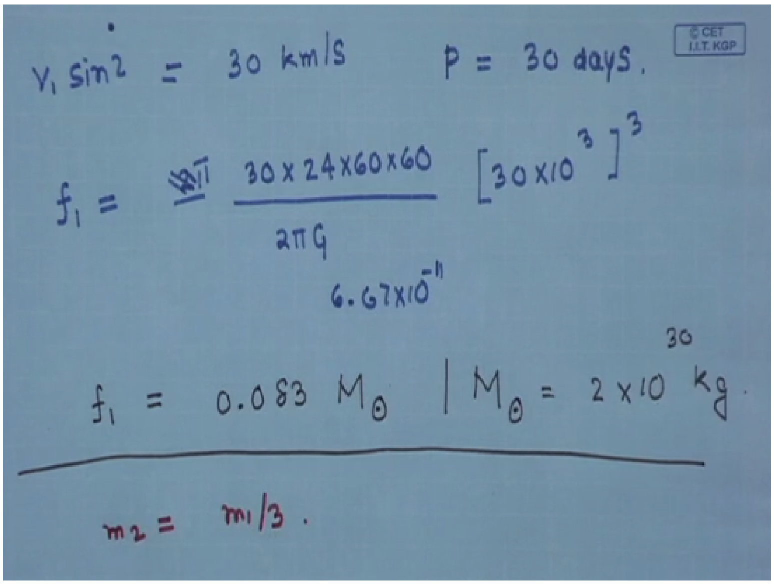

So the example that I will take up, we have measurements of V1 sin i that is the only thing that you can measure, you did not know the inclination angle. So what you can measure is V1 sin i and we know that the situation the example which I will discuss this has a value V1 sin i

= 30 kilometers per second. And the period of the object, of the system is 1 month 30 days. So in this case f1 can be observationally determined from these observations.

And we have seen that f1 is 2 pi by sorry 1 pi P1 is period. So the period is 30 days and we have to convert it into hours into minutes into seconds so this is the period in seconds and this divided by 2 pi G and we know the value of G is 6.67 10 to the power – 11 in SI units. And then we have the cube of the velocity which is 30*10 to the power 3 cube. And what this gives us is that the mass function of the first object f1.

If you work through this f1 turns out to be 0.083*the mass of the sun. Let me remind you that the mass of the sun we are writing it in units of mass of the sun is roughly 2*10 to the power 30 kg. So this is the mass function that you would infer from velocity measurement of 30 kilometers per second and a period of 30 days, okay. Now let us see what you can, how you can use this to infer the masses.

Now let me just shortcut the entire discussion, it is equivalent to what we have been talking about. The ratio of the peak velocities we have seen that the ratio of the peak velocities is inversely proportional to the masses. So suppose we have observations of both the velocity curves and we find that the ratio turns out to be such that m2 = m1/3.

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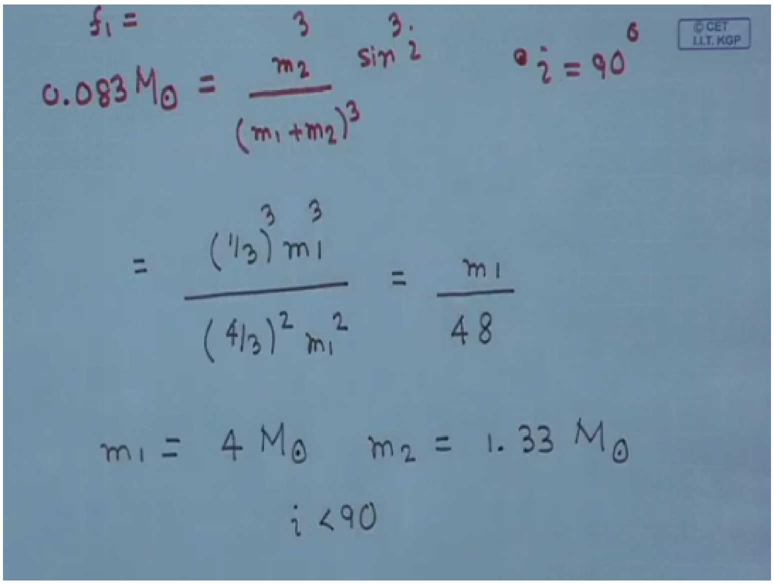

So, what we then have is that f1 which is 0.083 M sun solar masses = the, so f1 this is f1 this = m2 cube/m1+m2 square*sin cube i. Let us for the time being assume that the inclination angle is 90. Then what we have is that this = so this is 1/3 of cube of m1 and here I have

1+1/3 which is 4/3 square of m1 cube and here I have m1 square. So this gives me m1 divided by 48 because here I have, so we have 16*3 essentially.

The factor of 3 square and 3 square from here cancels out. So we have m1/48 which tells us that the more massive object = 4 solar masses and the less massive object then can easily worked out it is 1.33 solar masses. Okay, so this is an example where we have, where we start off with measurement of the velocity and the period which you can observe from the velocity curve and we determine the mass function.

And then we use the mass function assuming that the inclination angle is 90 degrees to determine the masses of the 2 objects. Now the point to note is we have assumed that the inclination angle is 90, if the inclination angle is < 90 then masses will get increased and these are just then lower limits, the masses will get increased if the inclination angle is < 90.

And you easily workout how the masses are going to get increased because there is the factor of sin i cube over here. So basically it will get increased by a factor of 1/sin i cube, okay. So let me end this discussion over here. So what we have been discussing till now is how you can use observations of binary systems to determine the masses of the individual objects and this is a very unique thing that you can do with binary systems there is practically no other way even which you can determine the masses of stars.

And what we have seen is that if you have a spectroscopic binary you cannot determine the masses uniquely. You can put lower limits on the masses the masses have to be more than certain value. The inclination angle still remains unknown. If you can independently determine the inclination angle then you can completely solve for the masses and workout everything.

Let me now move on to something else that is related to binary something that is very exciting and in which a very active area of research at present and this is the topic of Exoplanets.

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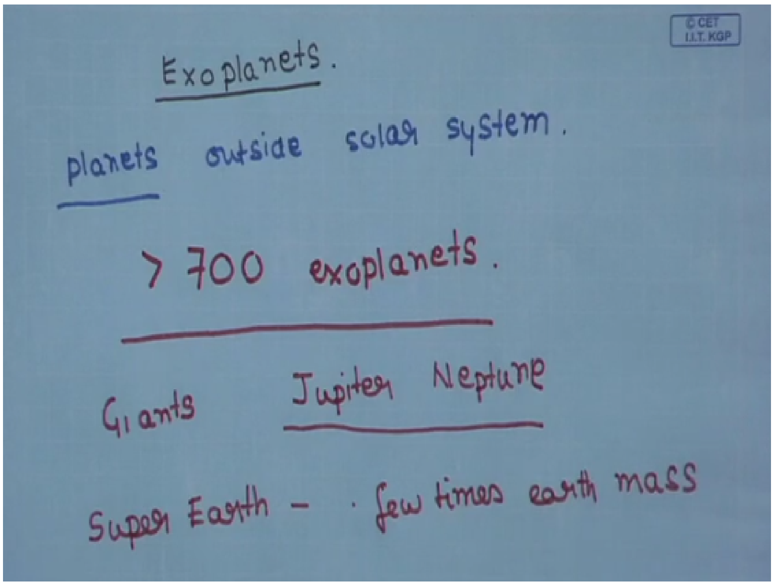

So what do we mean by an exoplanet. We have discussed planets in our solar system and we have seen that there are 8 planets and now there are the minor planets also, so 8 full-fledged planets. Exoplanet refers to a planet which is outside the solar system. So these are planets outside solar system. So they are an orbit around other stars not the sun. And it is a topic of great interest.

Because it holds in principle it holds the potential of allowing us to discover life extra-terrestrial life. So if we can, the first thing that you will have to do is to find a planet which is hospitable for life and this holds tremendous opportunity if you can determine planets in around the orbit, around stars besides our sun. So these are referred to as Exoplanets. And at present there are more than 700 Exoplanets than have been discovered.

And this is an ongoing process. So the number keeps on increasing. Now most of these planets that have been discovered are actually giants like Jupiter and Neptune. So these are all mainly giants. But there are a few planets which have been discovered which have few times whose masses are a few times more than the mass of the earth. These are called super earth. So some few such planets have also been discovered.

So these are few times larger earth mass. They are all still larger than the mass of the earth. Now the question, the interesting question that I am sure must be in your mind at present is how does one go about discovering a planet around a star outside the solar system? Well the planets are typically very faint objects they did not emit light of their own.

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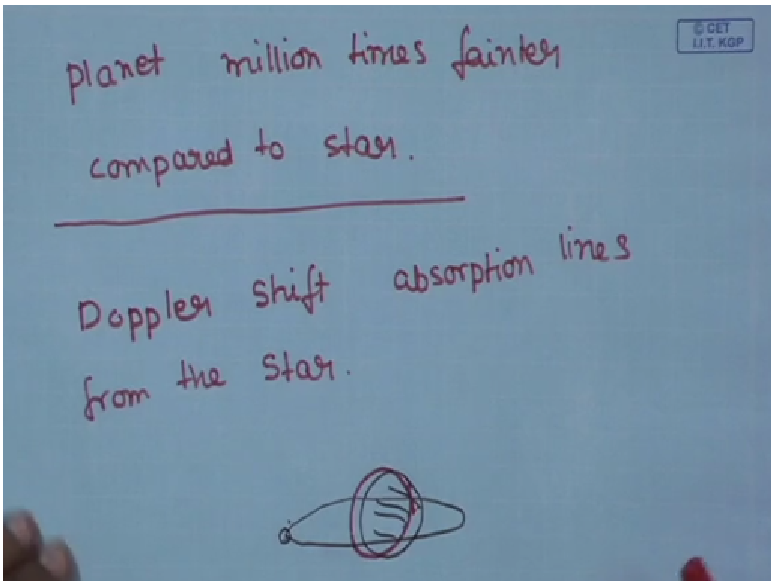

And a planet around a star is roughly a million times so a planet is a million times fainted. At least around a million times fainter than the star which it is orbiting. So the question is how does one discover an object which is next to another object which is a million times brighter. After all every image has limited dynamical range and it is extremely difficult to make out one object which is next to another object which is a million times brighter.

Well one of the most common techniques for discovering Exoplanets is very similar to the thing that we have just been discussing it is through the Doppler shift of the spectral lines. And here it is Doppler shift of absorption lines from the star. So what is the situation? The situation is that you have a star and you have a very small mass which is the planet in orbit around the star and normally you would think that the star is fixed and the planet just goes around it.

But as we have seen both of them actually move around the center of mass. So the planet also wobbles the sun also, the star also wobbles back and forth. So if you look closely at the star it also goes around its center of mass and it wobbles back and forth. And this motion of the star around its center of mass gives rise to a Doppler shift in absorption lines from the star.

So the basic question is that we have to measure this shift Doppler shift of absorption lines emitted from the star. So if the star had no companion no other mass around it, it would remain fixed or it would move with uniform velocity but the presence of planets around it. So the effect is very similar to that which you have you for binary stellar system except that the effect is extremely small.

So the orbit of the sun, of the star is going to have a very small radius and the corresponding Doppler shift is also going to be extremely small. So let us to make progress, let us first make an estimate of the kind of numbers that we expect in such a situation. So let us take up the Jupiter Sun system, the familiar. So, Jupiter is the largest planet in our solar system.

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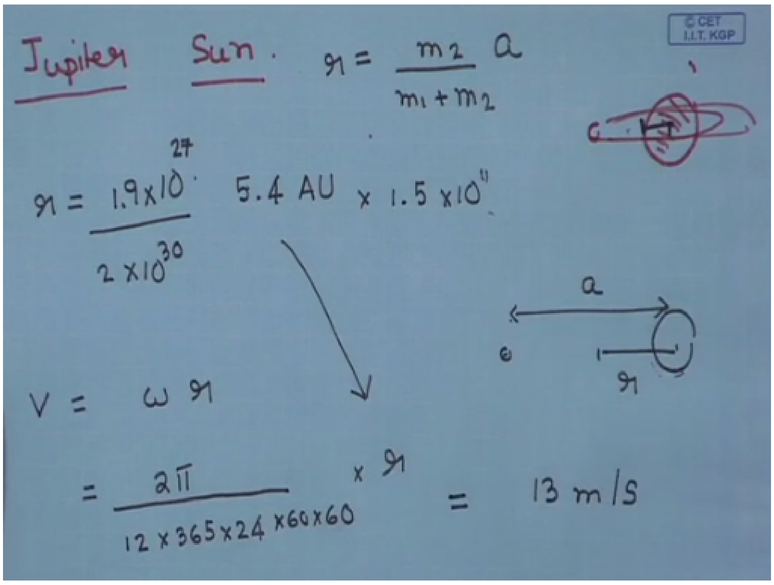

So let us take up the Jupiter let us ignore all the other planets and just consider Jupiter in Orbit around the sun. Let us take up this system. And see what will be the Doppler shift of spectral lines from the sun. So the question that we are addressing is that we have Jupiter going around the sun. So just imagine that we have the sun over here and Jupiter is going around the sun and considered the situation where we have an external observer somewhere far away looking at the solar system.

The observer cannot see Jupiter, observer only sees the sun can the external observer who sees the sun make out that there is a planet called planet Jupiter orbiting around the sun from the Doppler shift of spectral lines on the sun. So question is what is the velocity of the sun due to Jupiter? That is the basic question. So that is the thing that we would like to estimate.

So to do this let us first what we have to do is we have to first estimate the distance from the sun to the center of mass of the Jupiter Sun system. So question is how are we going to estimate this? Now the distance from the sun to the center of mass of the Jupiter Sun system is going to be the Sun Jupiter distance. So let us say that it is the Sun Jupiter distance let us call it r. r is going to be the Sun Jupiter distance.

And the Sun Jupiter distance we know that Sun Jupiter distance, it is 5.4 astronomical units. So the distance between the Sun and Jupiter is 5.4 astronomical units. This is the Sun Jupiter distance and the Sun Jupiter distance will get scale down by the mass of Jupiter. So the mass of Jupiter is 1.9*10 to the power 27 kg. Let me remind you that r or a1, r = m2 divided by m1+m2 a. That is what we are calculating.

So, this is the Sun, this is Jupiter this is r, the earth the distance from the center of mass and this is a that is the notation that we are using. So the distance between the sun and the center of mass is going to be the mass of Jupiter divided by the total mass into the Sun Jupiter distance. And the sun Jupiter distance we have just seen is 5.4 astronomical units. The mass of Jupiter is 1.9*10 to the power 27 kg.

And we will divide this by the total of the 2 masses and when calculating the total we can ignore the mass of Jupiter. This is much larger. So the mass of the sun is 2*10 to the power 30 kg 1000 times larger. And so what we see is that this is approximately a factor of 1000 smaller and we can convert this into meters. So I have to convert it into meters, what we have to do is multiplied by 1.5*10 to the power 11. This will be give me the value in meters.

So that is the value of the distance from the sun to the center of mass. We have to calculate this. It is 1000's of the distance from Jupiter to Sun. What we need is the velocity of the sun which is going to be omega*r and omega is the 2 pi. We know that the period of Jupiter, Jupiter has a time period of 11 of 12 years let us say, so it is going to be 12*365*24*60*60.

So this is omega 2 pi/the period of Jupiter*r from here. And if you work through this put in all the values, so I have given the values here, you have to just calculate them. You will find that the speed of the sun the velocity of the sun comes out to be 13 meters per second. So the sun is going to do a circular orbit around the center of mass with the speed which is 13 meters per second.

The question is can we detect, can an external observer, suppose we had an external observer outside the solar system whose capabilities were like those of ours would such an observer be able to make out the motion of the sun because of the presence of Jupiter. Well present day, high resolution high dispersion spectroscopy allows us to measure Doppler shifts of the order of 3 meters per second. This is a tremendous thing.

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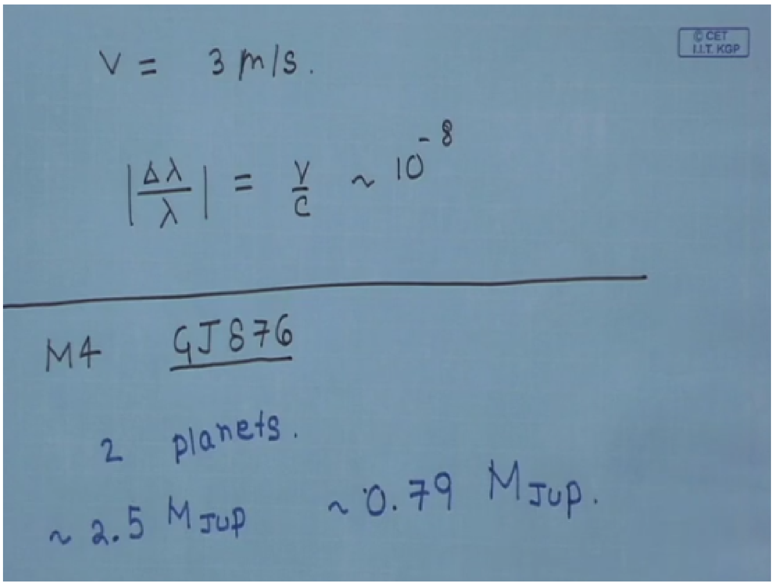

So it is at present possible to measure Doppler velocities through Doppler shift it is possible to measure velocities of the order of 3 meters per second. So delta lambda/lambda which = v/c is of the order of 10 to the power – 8, that is the accuracy which is possible through high dispersion spectroscopy. So it is possible, it is possible to make out the speed of the sun due to the presence of Jupiter.

So, similar technique has been used now to look at stars outside the solar system. So what the basic idea is that you look at stars and see if there are Doppler shifts in the spectrum of the star you will only see one of the Doppler shifts, you will not see any Doppler shift. You will not see any radiation from the planet. So you will just see the Doppler shift from the, of the light from the star and if the Doppler shift shows a sinusoidal variation.

Then you can interpret that in terms of an Exoplanet and this is what is done. Let me give you an, let me just mention an example of an Exoplanet. So there is an Exoplanet there is a star. Stars are classified into different types which we shall discuss later on in this course. So there is a star of type M4 the star has the name GJ 876 this is the number given to the star. So this is the name of the star.

And it is found from the Doppler shift of absorption lines from that star which is found that the Doppler shift shows sinusoidal variation not exactly sinusoidal it is more complicated.

And that variation has been interpreted and it is now believed that this particular star has 2 Exoplanets not just one, 2 planets of masses. So one of the planets have mass approximately 2.5 times the mass of Jupiter.

Jupiter remember is 1000 times less massive than the sun and the other mass, other star has a mass of approximately 0.79 M Jupiter mass of Jupiter. So this is just an example of a system, of a star which has got not 1 Exoplanet but 2 planets around it. So obviously the velocity curve is going to be not just a single sinusoidal because the 2 planets will have their own periods.

So it is going to be more complicated and it is a difficult task interpreting it but people have interpreted it, scientists have interpreted it and they infer that there are 2 planets going around this star one of them of mass 2.5 and other of mass 0.79. So let me just summarize what I have told you. So I have very briefly just told you that the Doppler shift from produced by a binary in a binary system can be used to determine the masses of the 2 stars if I have stellar binary system.

Not only that if I have planet going around the star then if I can detect, if you can detect absorptions lines from the star the Doppler shift of the absorption lines allows us to infer the presence of planets around the star. So let me bring our discussion of binary stars to a close over here and let me take up one problem. Problem is on Keplerian orbits. So let me state the problem first. The problem is as follows;

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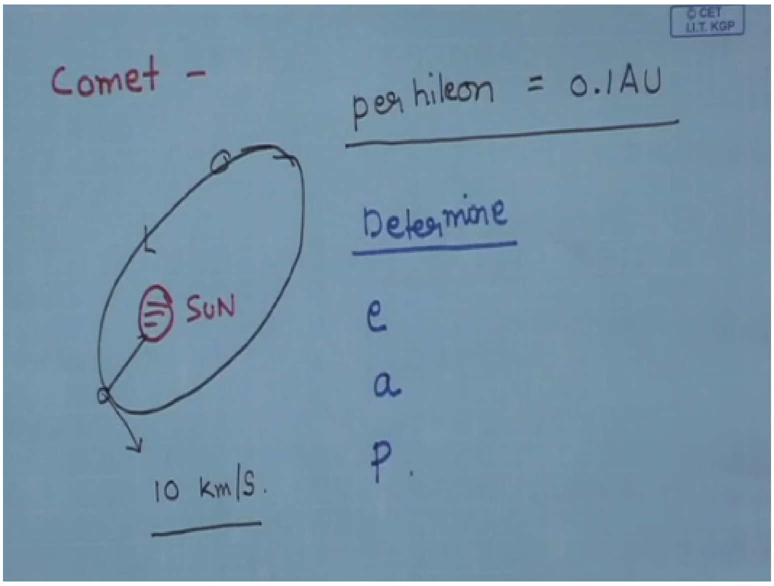

There is a comet which is observed in motion around the sun. So this is the sun and there is a comet observed in motion around the sun. It goes in an elliptic orbit sun as one of the focuses and it is found that the per hileon the orbit has a per hileon is the distance of closest approach to the sun. So this is the comet the per hileon of this comet is 0.1 astronomical unit and further it is possible to measure the speed of the comet at the per hileon.

And it is found that the comet has a speed of 10 kilometers per second at per hileon. So this is observed, so there is a comet going around the sun and what is observed is the comet is observed when it is closest to the sun that is when it will be very bright and what is observed is the distance the per hileon distance it is 0.1 astronomical unit and the speed is 10 kilometers per second. These are the 2 observed quantities for that comet.

The problem that you have to solve is to determine first the eccentricity. Calculate the problem is to determine eccentricity then determine the semi major axis and finally determine the period of this comet from these 2 observations, okay. So this is the problem that we are going to discuss, okay. So the problem that we are going to discuss is that we have a comet and we have measured it is per hileon distance and its speed at per hileon.

We have to determine the eccentricity, the semi major axis on the period of the comet. So let us first determine the eccentricity.

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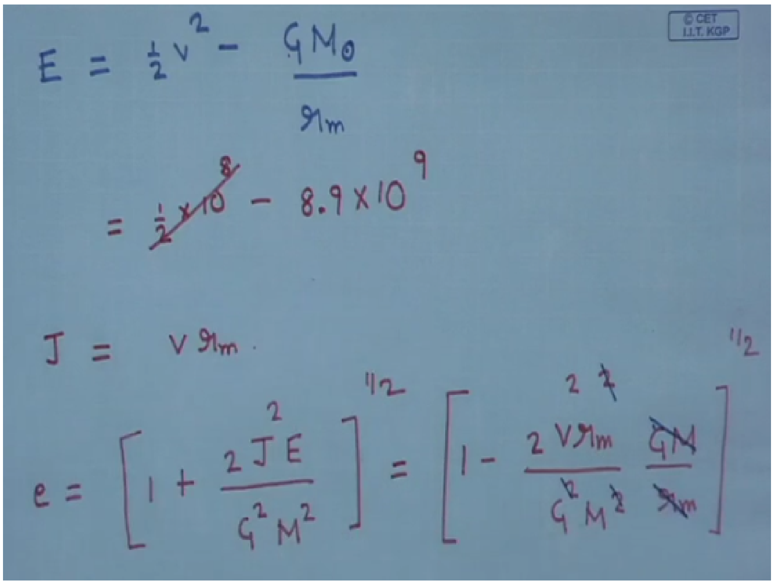

The eccentricity can be determined if we know the energy per unit mass. And the energy per unit mass of the comet is half v square – GM sun/the minimum distance r min. And we see put in the numbers half v square the velocity the speed is given, the minimum distance is given. So you have to just put in the numbers and it turns out if you put in the numbers that this will give me so this is going to be half.

And I have 10 kilometers per second so half *10 to the power 4 square. That is the 10 to the power 8 in SI units – G* the mass of the sun/the minimum distance and this factor G times the mass of the sun divided by the minimum distance. This factor turns out to be much larger than this and this you can see is 10 to the power 7 and this factor turns out to be 8.9*10 to the power 9.

The entire thing is in SI units so it is per kg. I will not write the units anyway. So I can essentially ignore this term. So, the energy is – GM/r and it is 8.9 10 to the power 9. Now how to, okay next we also need to calculate the angular momentum per unit mass and for this orbit you can straight away take the velocity and multiplied by the per hileon distance it will give us the angular movement per unit mass. So let me just write it as v*r minimum.

Now how to calculate the eccentricity? The eccentricity is 1+2 J square E/G square m2 to the power half. And E we have seen, so this can be written in our situation this can be written as 1 – the energy is –G. So this will be 2 V square rm square, I have written J and in the denominator I have G square M square and let me write the energy, energy is -GM/r minimum to the power half. That is the eccentricity.

So what you see here is that a factor of G cancels out with this, GM cancels out with this, power of V cancels out with, 1 factor of r cancels out with this.

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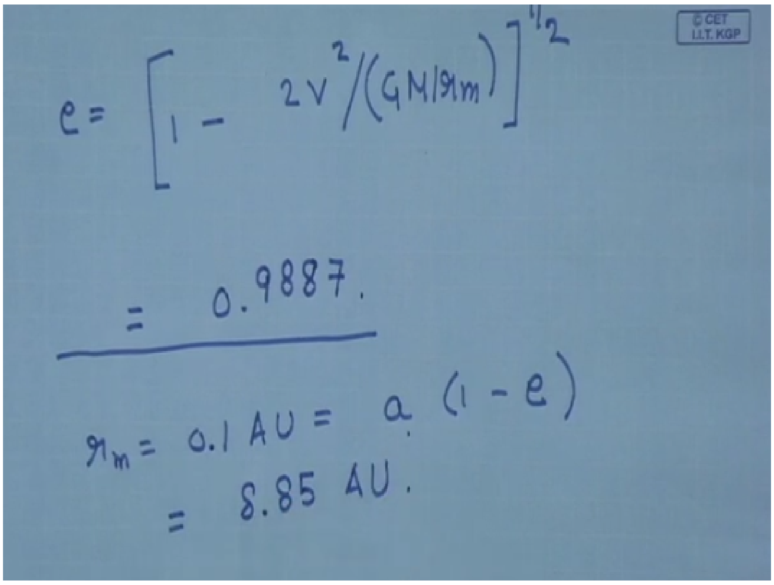

So what we are left with is that the eccentricity E = 1–2 v square divided by GM/rm. So 1 – 2 v square divided by GM/rm. And you have to just put in the value of the velocity here. We have calculated GM/r the per hileon distance and what you find is take the square root of that what you find is that the eccentricity comes out to be 0.9887. So that is the eccentricity of the orbit.

So we have determined the eccentricity of the orbit. Next to determine the semi major axis, the minimum distance rm which is 0.1 astronomical units. We know this is the semi major axis into 1 – the eccentricity. We know the eccentricity, so you can determine the semi major axis and the semi major axis comes out to be 8.85 AU.

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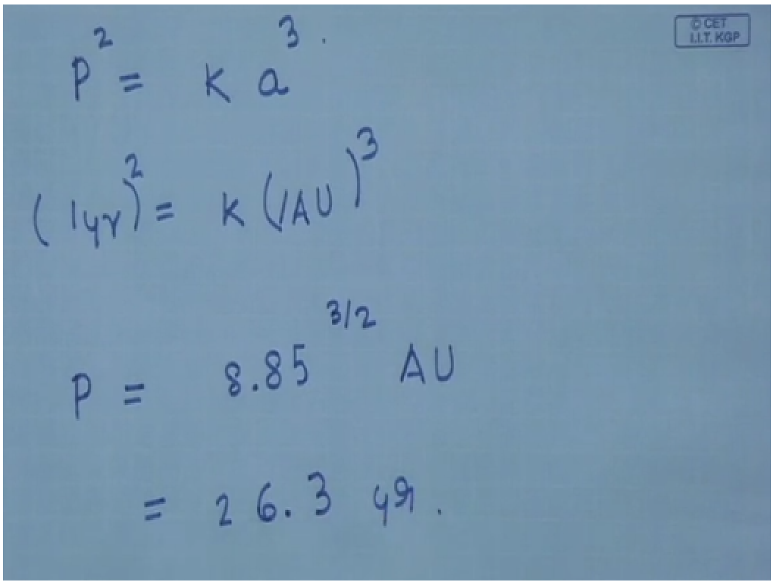

We have to next determine the period, so we know that the period square = some constant into the semi major axis cube. We also know that for the earth the period is 1 year square = the same constant*1 astronomical unit cube. So just taking the ratio of this gives us the period = 8.85 to the power of 3/2 in astronomical units and this comes out to be 26.3 years that is the period of this orbit.

So let me finish our entire discussion over here. We started off by discussing the motion of a test particle around a very massive object and we saw that it is governed by and the orbit is an ellipse and there are the 3 Kepler's laws and we use this to discuss the solar system. After that we moved on to binary systems where both the masses are comparable but we showed that such a situation can also be reduced to a test equivalent to a test particle moving around a very massive object.

And we learned how the binary systems can be used to determine the masses of stars. We got some idea about this and we also learned how this knowledge can be used to discover Exoplanets. So let me bring today's discussion to a close over here. We shall take up some new topic in the next class.