Astrophysics & Cosmology
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Lecture - 05
Binary Systems

Good morning and welcome to today's lecture. Today, we shall discuss binary systems.

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Let me remind you that we have already discussed the motion of a test particle in the gravitational field of another object whose mass is much more than that of the test particle.

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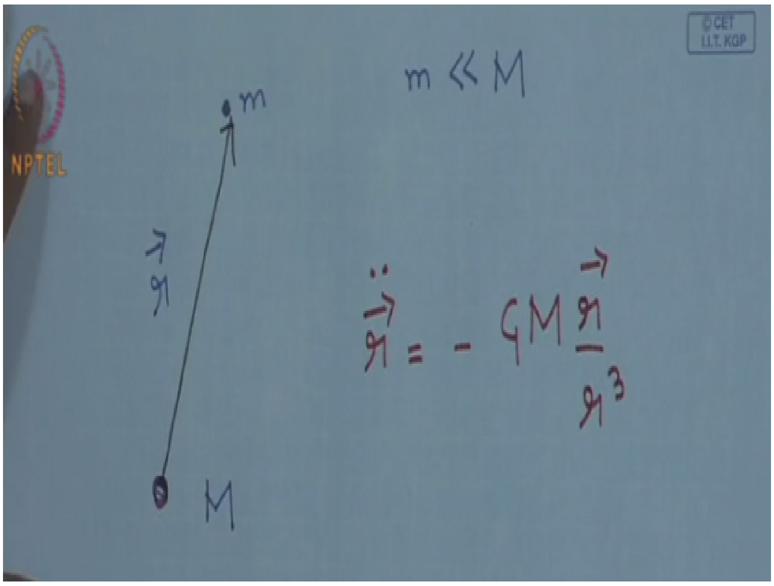

So this is the problem that we have discussed and the test particle m is assumed to be much smaller than the, mass that is much smaller than the mass whose gravitational filed we are interested in and essentially we can ignore the gravitational effect of the test particle on this object. Under this assumption the equation of motion of the test particle which we have already discussed.

Let me remind you again that the equation of motion is r.. the acceleration of the test particle = -GM, where M is the mass of the object that is exerting the gravitational force –GM/r square and I write it as the vector r/r cube. So this is the 1/r square law. So this is the motion of a test particle around a very massive gravitating object. Now today, we shall be discussing different system. Here we have 2 masses M1 and M2.

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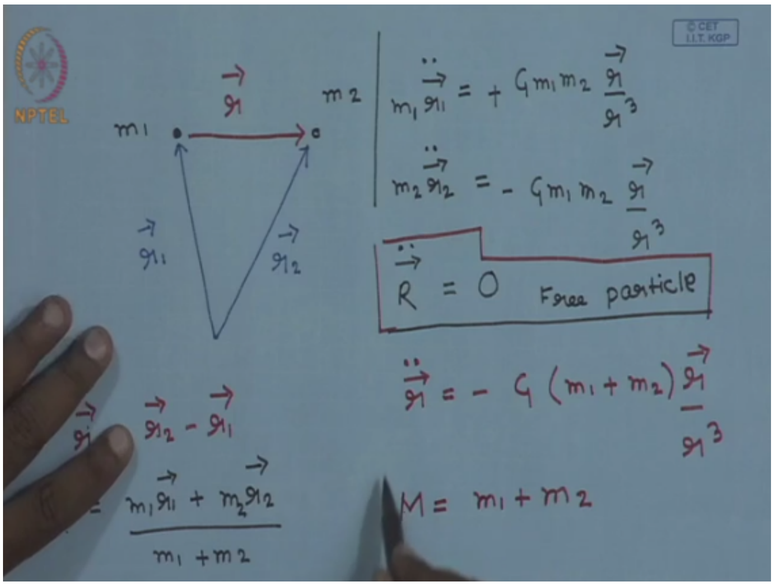

And these masses may be comparable. So this is the vector R1 with reference to some origin. This is the vector r2 corresponding to the 2 different masses and these masses may be comparable. So this is the system that we referred to as a binary. For example, we may have 2 stars in a bound orbit, okay. So this is the system that we are interested in and let me write down the equation of motion for this system.

So, this is the vector r, it points from r1 to r2. So r = r2-r1. And r is the relative displacement between the 2 particles. Now, we all know how to write down the equation of motion of these 2 particles. So let me do that, so for the particle m1, we have m1*r1 the acceleration of the first particle. So let me put a line here to differentiate between these 2. This is, so the rate of change of momentum of the first particle = - Gm1 m2.

So the gravitational force is proportional to the product of the 2 masses and the inversely proportional to the square of this distance and it is pointed towards the second mass. So there will be a + sign here and it will be r/r cube. Similarly, I can write down the equation of motion of the second particle. So this will be m1 and equation of motion of the second particle will be m2 r2 .. = now this force will be opposed to r.

So I have to put a – sign Gm1 m2 and the same thing r/r cube. Now the question is how do we solve the equation of motion of these 2 particles? You can see that they are coupled. So the motion of this particle depends on this, the coordinate of this particle and the motion of this particle depends on the coordinates of this particle. We have to simultaneously solve both of these equations.

So to do that it is convenient to change the coordinates instead of using r1 and r2 which referred to the 2 individual particles. It is convenient to use different coordinates and these different coordinates are the relative displacement between the 2 particles which we have already introduced and the position of the center of Mass r. Centre of Mass is defined as m1, r1 plus the weighted average of the 2 vectors m1 r1+m2 r2 divided by m1+m2. That is the center of mass.

So instead of using r1 and r2 we shall now be using the relative displacement and the center of mass to describe the system. So we had 2 vectors, we again have 2 vectors and we can equivalently use these 2 instead of these 2. So in terms of these 2 vectors what are the equations of motion. So lets us first work out the equation of motion for the center of mass this can be worked out by just adding these 2 equations if I add these 2 equations what I get is R = 0.

So what we see is that the center of mass of the 2 particles moves like a free particle. There is no external force acting on the center of mass. So the combined system the system made of these 2 particles combined can be thought of as the motion of the center of mass and that moves like a free particle. So if it is addressed to start with it will remain addressed and if it is in uniform motion it will continue to move with whatever velocity it has.

So this is like a free particle and for the subsequent discussion we shall assume that we are in the frame of reference where the center of mass is at rest. So this is the motion of the center of mass. The next thing that we have to work out is the equation of motion for the relative displacement r and that can be worked out by dividing this equation by m1 and dividing this equation by m2 and then subtracting this equation from this equation.

So let me remind you again what we have to do. You have to divide this equation by m1, divide this equation by m2 and then you just look at the difference of these 2 equations that will give you the equation of motion for r2–r1 and what you get? What you get is that r .. is = -G(m1+m2) r/r cube that is what you get. So what you see is that the difference vector, the relative displacement between the 2 particles.

That is governed by this equation which is exactly the same equation of a test particle in motion around a very massive object of mass M.

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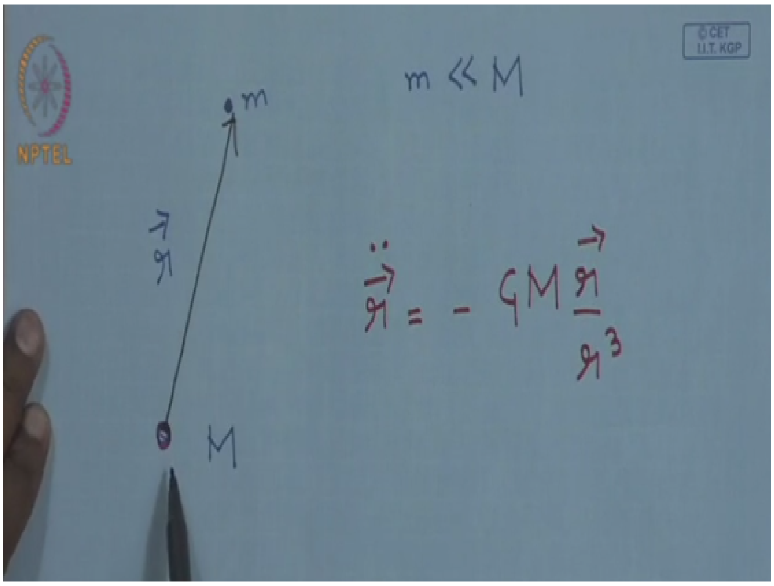

So the relative, the separation vector between the 2 particles behaves just like the motion of a test particle in the gravitational field of an object of mass m1+m2. And we have already discussed the motion of such an object. We have already discussed the motion of such an object and we have seen that the orbits are ellipses and we have, we can interpret in terms of the 3 Kepler's law as ellipses.

So we have already discussed this and we shall not repeat the discussion once more. So what we learn from this is that the 2 body problem where you have 2 masses and comparable masses in gravitational motion in each other's gravitational field can be reduced to a single mass moving around, single object moving around in a central force of a very massive object where we can ignore this particular mass.

So the problems are equivalent and we know that the solutions are ellipses and all of the Kepler's law's will hold. So once you know how the relative displacement between these particles evolves as a function of time you can easily work out what r1 and r2 will now do. And let me remind you again that we are going to work in the center of mass frame of reference where we are addressed at the center of mass of these 2 systems.

So the center of mass is at the origin so this is 0.

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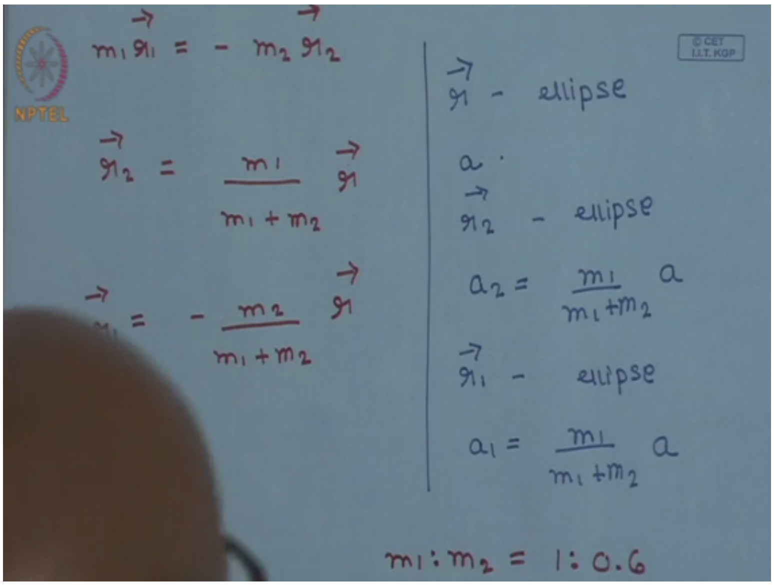

So we have m1 r1 = -m2 r2 and we can put this into this equation and what we will get is that r2 = m1/m1+m2*r. r1 = - m2/m1+m2*r. So we see that r2/r1 the ratio of the magnitude of the vector r2/r1 is = m1/m2, so this is all consistent with this, okay. And we have, let me remind you that we have seen that the vector r sweeps out an ellipse with semi major access.

Let us say a then r1 r2 will also sweep out an ellipse whose semi major axis is a2 which will me m1/m1+m2 a and r the vector r will also sweep out an ellipse and this will have a1 = m1/m1+m2*a. So let me now show you an example of what the orbit looks like.

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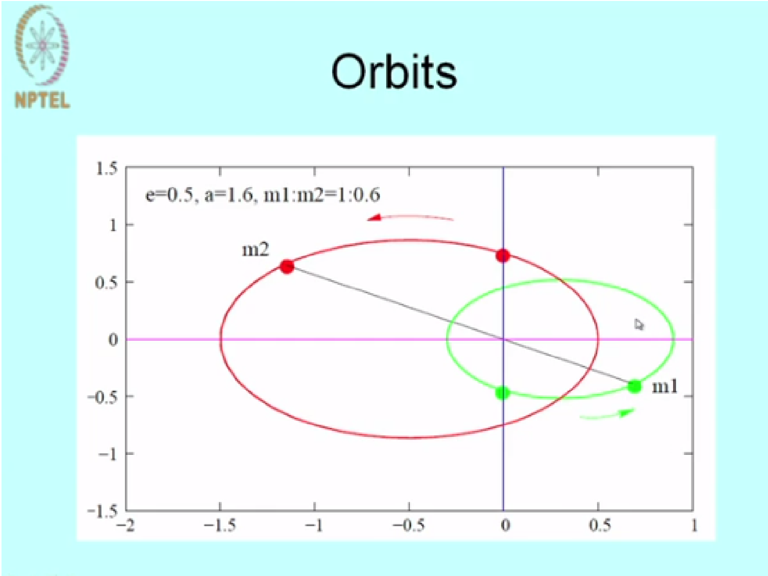

So this picture on the computer screen shows you the motion of 2 objects which are moving in each other's gravitational field and the object, the ratio of the masses m1:m2. So we are considering a particular situation where m1:m2 = 1:0.6. So you would expect the semi major axis of the second particle of the less massive particle to be larger the semi major axis of the more massive particle to be smaller.

So the more massive particle is going to have a smaller orbit the less massive object is going to have a larger orbit such that the center of mass remains fixed. So let us look at the picture again. Here the center of mass is located at this point the 2 objects are going to be located at 2 different sides of the center of mass. So the relative displacement between these 2 objects is always going to pass through the center of mass.

This sweeps out an ellipse, this also sweeps out an ellipse. This is more massive than this, so the radius of this the semi major axis of this ellipse is smaller, the semi major axis of this ellipse is larger. So when this object is here when the second object is here the first object is here then the first object moves in this direction the second of m2 moves in this direction and they have exactly the same time period and the line joining them always passes through the center of mass.

These distances are in the ratio 1:0.6. The semi major axes are also in the ratio 1:0.6. So this is in general what the orbits look like for binary systems.

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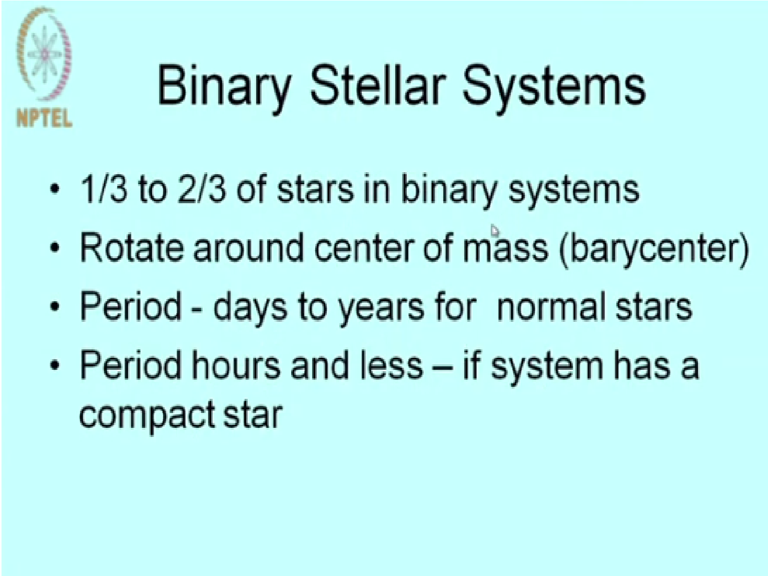

Now, let me now tell you what we are going to discuss in today's lecture. Today we are going to consider a system where we have 2 stars in binary orbit. So, 2 stars which are moving in each other's gravitational field. So we have learned about our solar system and in our solar system there is only a single star, the sun and we have all the planets going around revolving around the sun.

So our system is obviously not a binary system. So the question arises why are we so interested in binary stellar systems where we have 2 stars. Well, let me give you some motivation for this and observe fact that half to 2/3 of all the stars are in binary systems. So our solar system which has only a single star is more of an exception than a rule. Roughly half to 2/3 of all the stars are found in binary systems.

Now, both of the stars as we have seen rotate around the common center of mass that is what we just saw. So in all of these systems where you have 2 stars they both rotate around common center of mass. In orbits that looks some like this. And the center of mass is also referred to as the barycenter. The periods could be from days to years for normal stars. Normal stars are stars that look like our sun.

May be somewhat larger, somewhat smaller they all burn hydrogen. This is something that we shall discuss in much more detail later. Now the periods could be much less, could be hours and less if one of the stars is a compact star like white dwarfs or neutron star. We have seen that the period depends on the orbital of the semi major axes of the orbit and the smaller to semi major axis the smaller the period.

So if you want a small period the semi major axis of the orbit also has to be small and the stars also have to be small if they have to orbit in a very close in a small elliptical orbit. So if you have compact stars if one of the stars in the system is a compact star or if both them are compact stars like neutron stars or white dwarfs or one of them could be a black hole then you have periods which could be hours or even less.

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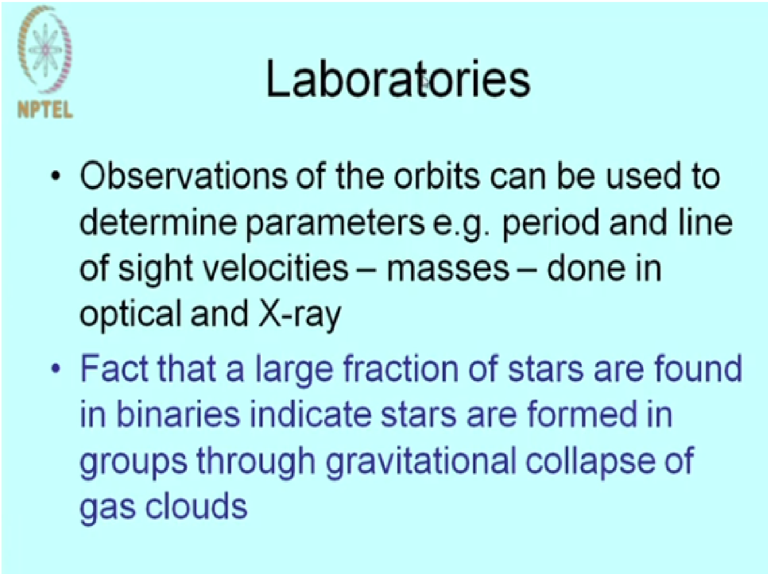

Now binary stellar systems are some of the most interesting things that one could study. They are actually laboratories, astrophysical laboratories, why? Well observations of the orbits can be used to determine parameters of the stars. If you have an isolated star for example it is extremely difficult if not impossible to determine the mass of the star. For example, the mass of the sun can be determined from the period of the earth's orbit using Kepler's law.

We know that the period and the orbit the semi major axis is related through the mass. So if you can measure the period in the semi major axis you can determine the mass. So individual stars it is very difficult to determine the mass of the star but if you have a binary system you can determine various parameters about the stars, like the mass and this is done in optical and in x-ray.

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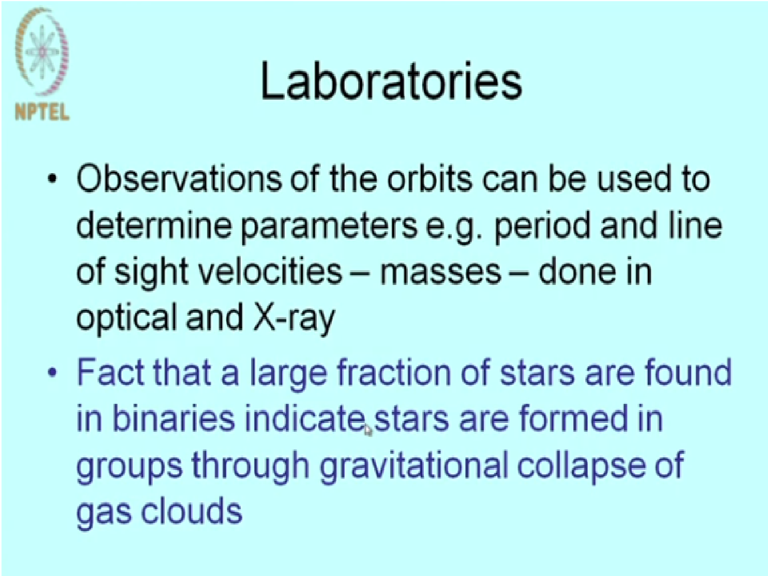

So they are very useful laboratories where you can determine different parameters from the period and the line of sight velocities you can determine the mass of the stars. The second thing that you can learn from binary system is the fact that you have a large fraction of stars in binaries and individual stars are rather rare. This is an indication that stars are formed in groups through the gravitational collapse of gas cloud.

So, when in the interstellar medium you have these gas clouds this and when a such cloud collapses you do not form a single star but you form several stars simultaneously and the fact that you have binary systems in excess of individual stars is gives us the handle on the star formation process. You have different kinds of binary systems where both of the stars are normal stars you could also have binary systems.

Where one of the star is a white dwarf or may be a neutron star and the fact that you have these varieties, this variety of binary systems provides tests of models of stellar evolution.

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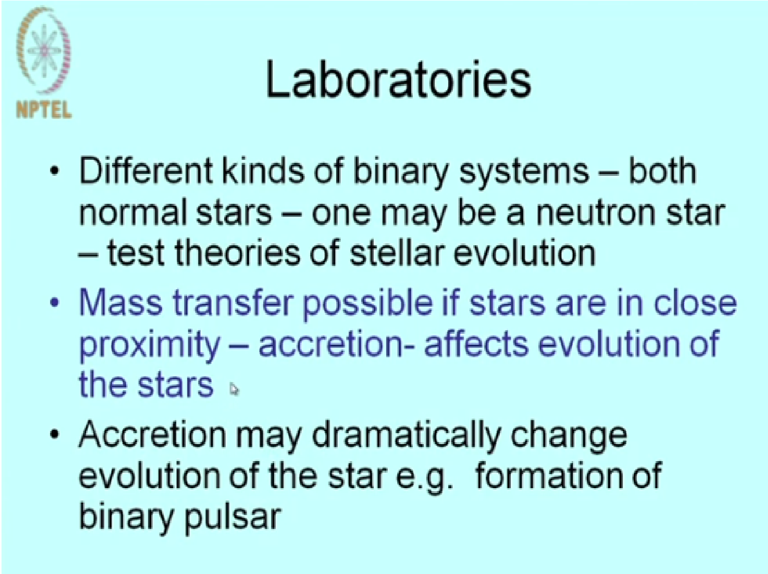

And very interestingly when you have 2 stars in close proximity or meeting each other one another it is possible that there could be mass transfer from one star to another. This is what is referred to as accretion and an isolated star would evolve in a particular way if you lead its life in a particular way. Whereas if it were to, if a star were to receive material from outside it would completely change its evolution.

So very interesting physical phenomena takes place in binary systems and which affects the evolution of the stars and accretion may also dramatically change the nature of the stars. For example, there are binary stars called binary pulsars which are formed, some of these are formed during accretion in binary systems, very high millisecond binary pulsars. So variety of phenomena take place in binary systems and one can study.

So interaction between different kinds of stars, a variety of such phenomena can be studied using binary stellar systems.

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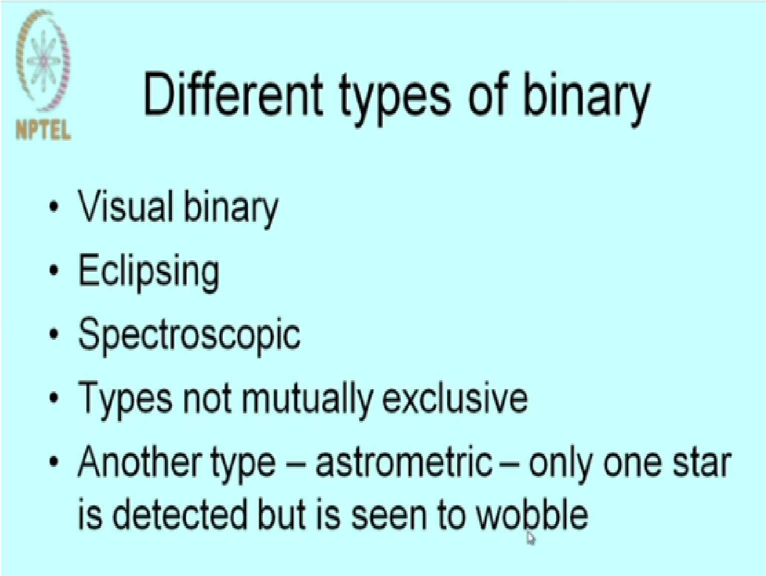

Now, there are typically 3 different types of binary stellar systems. So we have the visual binary, the eclipsing binary and the spectroscopic binary. These are the 3 main types of binary stellar systems. And we should note that these types are not mutually exclusive, so there could be a stellar system, binary stellar system which is both a visual as well as an eclipsing binary or a visual and a spectroscopic binary.

There is another type called an astrometric binary where you only have, where you only see one of the stars on the sky. And you know that it is a binary you inferred the fact that it is a binary, it is a part of a binary system from the fact that the stars seem to wobble because of its motion around the common center of mass. You do not see the partner, such stars where you only see one of them and it wobble this called in astrometric binary.

So let me now go in some detail not very much in some detail and tell you about these 3 different types of binary systems the visual binary, the eclipsing binary and the spectroscopic binary.

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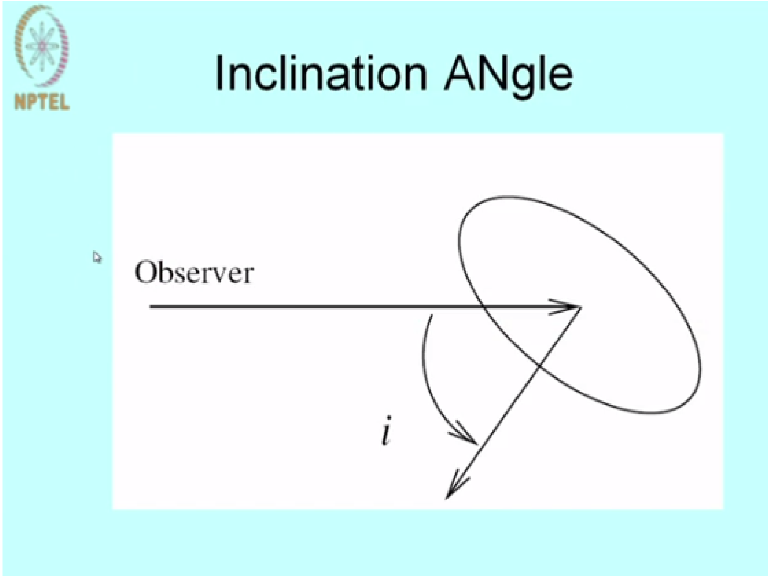

Before that we need to introduce certain aspects. So we have the inclination of the binary system, the inclination angle. So this picture shows you a binary orbit and this is the plane of the orbit and the observer is located in this direction. So the observer is looking at the orbit at an angle called the inclination angle which is the angle between the direction at which the observer is looking at this the direction to the observer and the normal to the plane of the orbit.

So this is the inclination angle. If the orbit is, if you have an inclination angle of 0, it essentially means that the orbit is face on. You are seeing the orbit face on. So it is like this, the binary system is, you are seeing it like this, the star is going around like this and you are seeing it face on.

Whereas, if the inclination angle is 90 you are seeing the orbit edge on like this, the plane of the orbit is like this, the star is going around like this in this plane and you are seeing it edge on that corresponds to 90 degree. So, let me repeat inclination angle of 0 the orbit is being viewed face on. Inclination angle of 90 the orbit is being viewed edge on and every other and other angles lie in between these 2 extreme cases.

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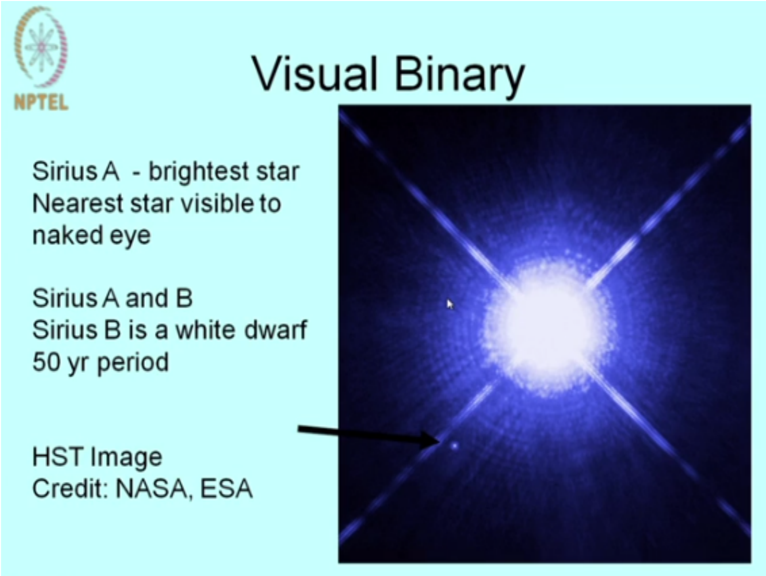
binary.



So the first type is a visual binary and this picture shows you a particular case of a visual binary. So this picture shows you the star Sirius A and this star is the brightest star in the sky and it is also the nearest star that is visible to the naked eye. So this is Sirius A, the brightest star in the sky what is interesting from our point of view is that there is another star which you would not notice.

If you were to look at this Sirius A and this star is called Sirius B and Sirius B is a white dwarf, so very small star and the arrow here shows you the location of the white dwarf. It is over here. So the white dwarf and the Sirius A, so Sirius A and Sirius B form a binary system and this binary system has a period of around 50 years. And visual binary is a system where you can see both the stars in a photographic plate or a picture of the sky.

So a visual binary refers to a binary system where both the stars are visible in photographic plate or an image of the sky. And if you wait for 50 years and if the so you can then see the orbit you can then see the 2 stars moving around on the sky and from where you can infer certain properties of the orbit depending on the inclination angle which in general is not known.

But you can make out certain properties of the orbit for a visual binary. You will see the 2 stars moving around over the course of time. So this is the first type of binary, the visual binary.

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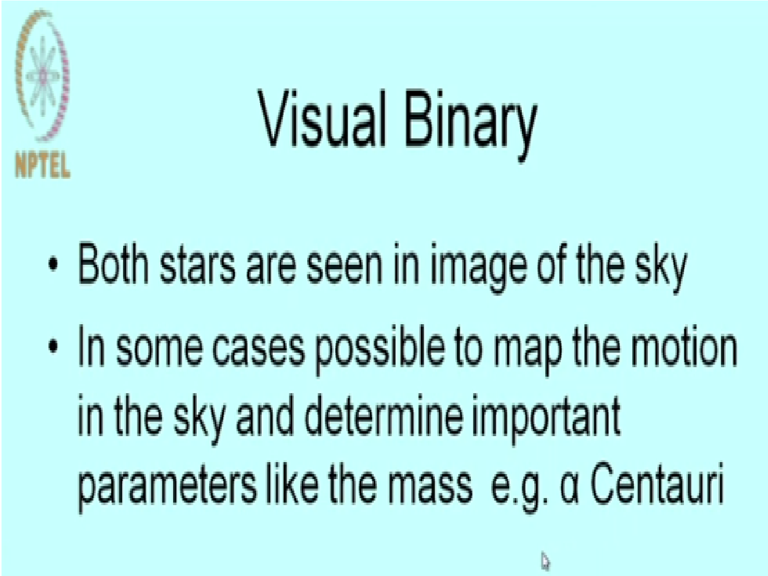

And select be repeat again, both stars are seen in images of the sky and in some cases it is possible to map the motion in the sky and determine important parameters like the mass. Example is alpha Centauri. Let me now move on to the next kind of binary system which is an eclipsing binary.

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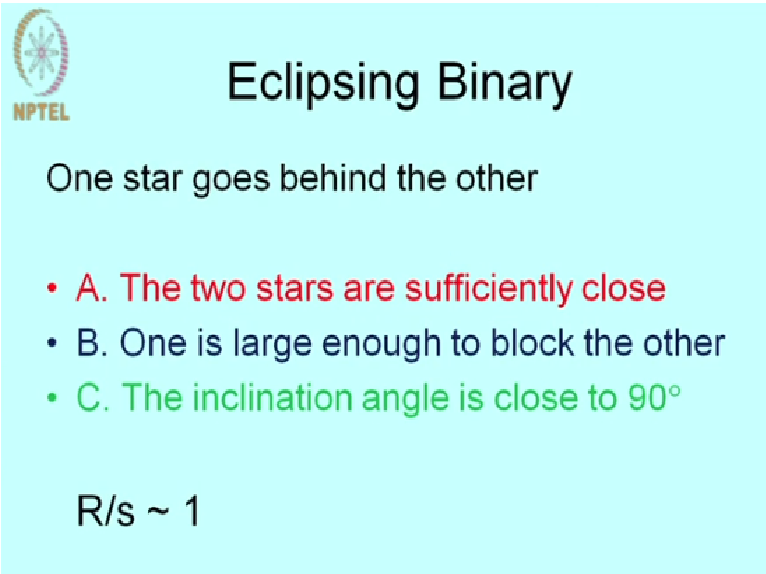

So in an eclipsing binary you typically do not see the 2 stars as distinctly cannot make out the 2 stars in an image of the sky. So the question is how do you know that you have a binary system? Well you know that you have a binary system because of the fact that one star goes behind the other creating an eclipse. So this is the situation where one-star eclipse is the other. And question is what you require for an eclipsing binary?

Well for an eclipsing binary in general the 2 stars should be sufficiently close. So if you have 2 very distance stars it is highly unlikely that one of them will block the other. Further one of them should be large enough to block the other star and the inclination angle should be close to 90 or else it is unlikely that one of these star is going to block the other. For a phase on orbit it is not going to happen that one of the stars is going to block the other.

So you require the inclination angle to be close to 90. Now the first 2 conditions are satisfied if the ratio of the radius of the star and the separation between the 2 stars is approximately 1. If this condition is satisfied then the inclination angle also need not be very close to 90 and you will have an eclipse for a large variety, for a large range relatively large range of inclination angels.

So the situation is that the size of the star should be comparable to the separation between the 2 stars. This is the situation.

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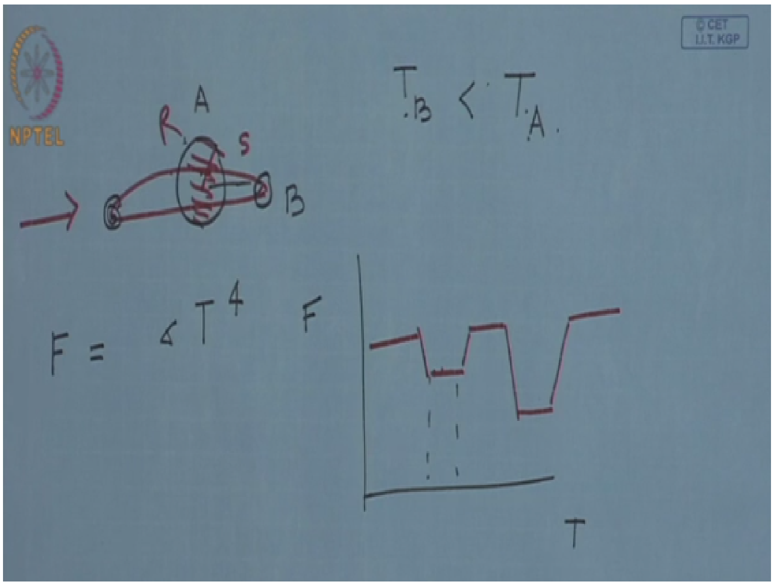

If you have such a situation then for a large range of viewing angles one of these stars you are going to have an eclipse, okay. So such a situation is very favorable and if this R and this is S the separation, such a situation is extremely favorable for an eclipsing orbit. So, when one of these stars and goes behind the other, okay? So we are considering a situation where we cannot distinguish the fact that there are 2 stars in an image of the sky we cannot make out the 2 stars.

But there will be a reduction of light when one of the stars goes behind the other and this is how we can make out that there is an eclipsing binary. So let us briefly now discuss what kind of a reduction light you expect and its visible consequences. So let us consider a situation like this a hypothetical situation like this where you have 2 stars, one of them is rather big and the other one is smaller and they are going around in a circular orbit and the observer is located over here.

So when this star is behind then the small star, let us call this A and this B. When B is behind A it is completely eclipsed and then it comes around and you can see both the stars and then B comes in front and it obscures out a part of A and then again it moves out. So question is what will the total light that comes from the system look like? That is the question. This is what is called the light curve.

So let us try to make an estimate of what the light curve from such a system will look like and we will assume that the smaller star is cooler than the larger star. So we will assume that the larger star is hotter and the smaller star is cooler. So and we know that the radiation that you receive from any star per unit area of a star is proportional to sigma T to the power 4. The flux of radiation from per unit area of a star is proportional to sigma T to the power 4.

So the radiation per unit area from this star from the bigger star is more than the radiation per unit area from the smaller star. So let us try, let us now try to schematically draw the light curve for such a star. This is the total radiation flux of radiation coming from the star. So let us start with the situation where both the stars are visible and in such a situation the light that you will receive is the combination of both A and B.

And then let us consider the situation where B is completely obscured by A. So if B is completely obscured by A the light from B contribution from B is locked out and for a certain period and this is what you get. Next again you can see both of them and then B comes in front. Question is what happens when B comes in front. When B comes in front it blocks out a certain part of the hotter star and that gets replaced by an equivalent part which is cooler.

So the radiation that you receive in this situation is actually going to be lower than the radiation that you receive when B is completely blocked out. So the light curve from the 2 stars will look like this. And this corresponds to the situation where B is obscured by the brighter star hotter star and this is the situation where the cooler star comes in front of the hotter star which is larger.

So this is what the typical light curve is going to look like and if you can measure the different intervals, time intervals, the different time intervals you can infer very useful information about the different stars in the orbit.

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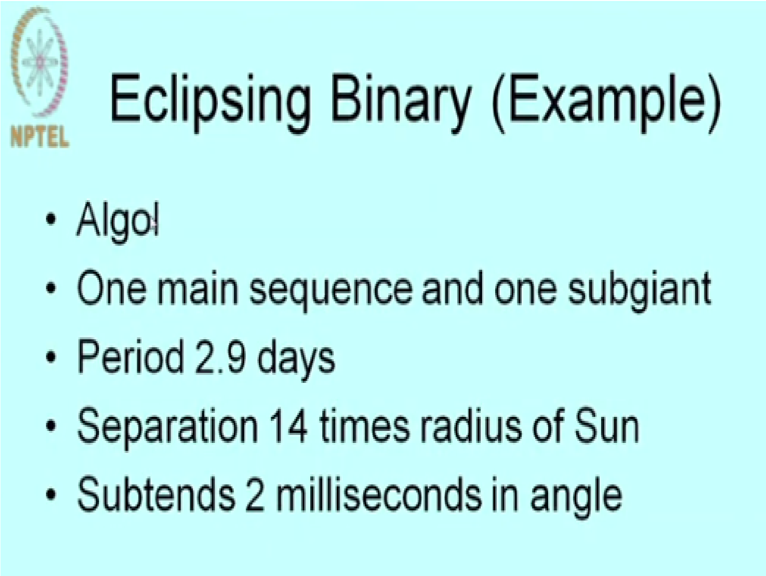

So this is the eclipsing binary and an example is Algol and one of the stars in this eclipsing binary is a main sequence star. Main sequence star is a normal star like ours which burns hydrogen and another star is a sub giant. It has a period of 2.9 days, well the period has to be small will typically be small in an eclipsing binary because we have seen that the distance between the 2 stars should be comparable to the radius of the star if you want the probability for an eclipsing binary goes up if you have the situation.

So typically the eclipsing binaries will have a very small orbit. Orbit of the semi major axis of the orbit will be small. So the period is also going to be small and in this case we have a period of 2.9 days and the orbit is the separation between the 2 stars is just 14 times the radius of the sun. So it is an extremely tight orbit. Just 14 times the radius of our sun and the 2 stars in this binary system subtend 2 millisecond of arc in angle.

So in the usual traditional imaging you will not be able to make out that there are 2 stars in the sky. But if you use high resolution imaging like interferometry or adaptive optics you can make out this separation 2 milliarc second separation in the sky and you can distinguish and make out the fact that there are 2 stars.

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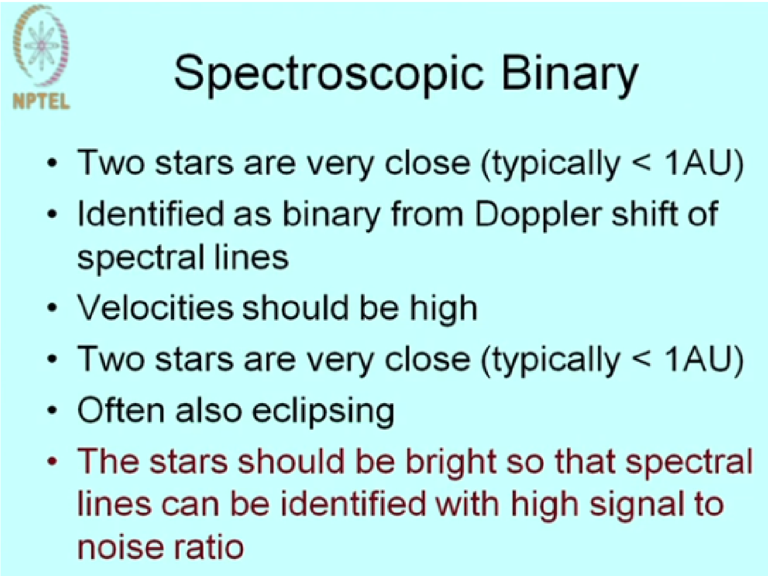

So finally let me come to the third type of binary system which is common. So these are spectroscopic binaries. Again these are systems where you need not make out that there are 2 stars. So looking at the image you would think that there is just a single star in the sky. Then the question is how do you make out that you have a binary system? Well, here we shall answer that question as we go long.

So what is the spectroscopic binary here again we have 2 stars which are very close typically < one astronomical unit and we make out that we have a spectroscopic binary from the Doppler shift of spectral lines. So the basic idea is that you have to be able to identify spectral lines coming from the 2 distinct stars. And as the stars move around you will have Doppler shift.

And since the 2 stars have different motions you will have different Doppler shifts for the 2 different lines and then from that you can make out that you have a binary system. For appreciable Doppler shifts you need the velocities to be high. And for the velocity is to be high the orbits have to be the semi major axis of the orbit has to be small, which is why you require them to be in very, the 2 stars to be very close.

So for a circular orbit we know that V square is proportional to 1/r, right. So you would like the velocity to be large so the radius, the separation between the orbit, between the 2 stars has to be small. So the Doppler shifts of the 2 stars should be discernable further the star should be bright. So that the spectral lines can be measured with the high signal to noise ratio. After all you have to determine the Doppler shift of the spectral lines.

And for this you require very high signal to noise ratio, spectral lines from the stars. So the stars have to be bright. Now very often spectroscopic binary is are also eclipsing because the 2 stars are very close together. So very often they are also eclipsing. So this is what is spectroscopic binary, they are typically stars which are very close together very bright, so that you can make out spectral lines.

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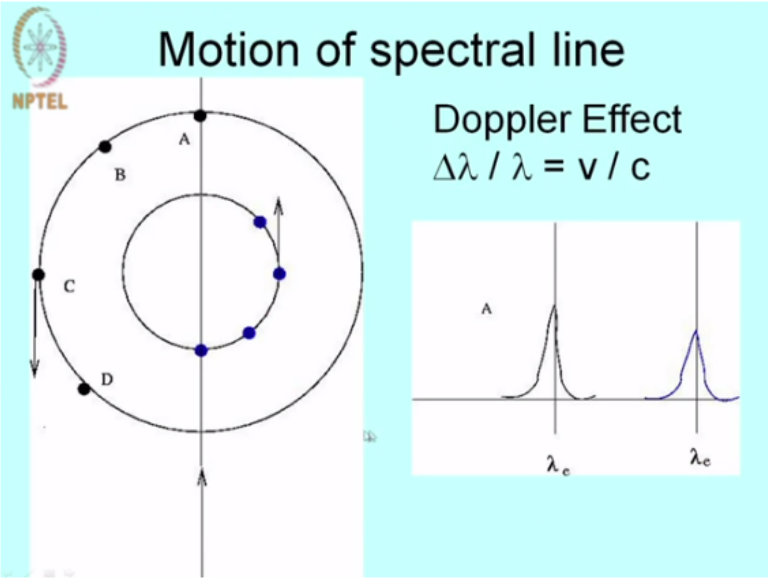

So let me try to explain to you what will happen if you have spectroscopic binary. So this picture shows you a binary system of 2 stars in circular orbit. Circular orbit has been particularly chosen to keep the analysis simple. So you have these 2 stars going around in circular orbit around the common center of mass over here and the black object is less massive, the blue object more massive and they are both doing circular orbits.

And we start from the position A where one of the objects is here and the other object is over here. And the orbit is circular, so and the observer is located in this direction. So in the position A there is no velocity component of the motion along the line of sight and it is the line of sight motion that causes component of the velocity that causes the Doppler effect. What is the Doppler effect?

Let we remind you the Doppler effect is a shift in the wave length of a spectral line. So there is a spectral line which is emitted at a wave length lambda and if the source is moving with the velocity V then an observer will see a shift in wave length. And the shift in the wave length delta lambda is such that delta lambda/lambda = v/c. So it is the fractional shift in the wave length is proportional to the speed which the object is moving.

And it is only the line of component, line of site component of the speed. So it is only the component in this direction. And if the source is moving away from the observer then the velocity is positive and there is a positive shift in the wave length. The wave length increases this is called a redshift. This is called a redshift.

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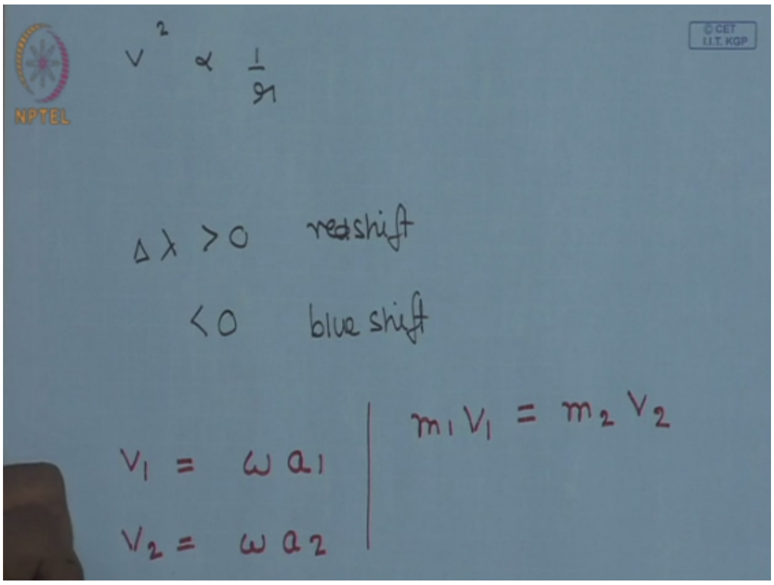

So, delta lambda > 0 is called a redshift. This happens if the source is moving away from the observer. If delta lambda is < 0 you have a blue shift. This happens when the source is moving towards the observer. So in this position A, the source has no velocity component relative to the observer its line of site. So the spectral line is observed where you would expect it at the wave length lambda. There is no shift in the wave length.

Now, after some time the star A, the star over here has moved to position B similarly this has move to this position, position B and we now have a line of site component of the velocity. So the velocity is tangential. We have a line of site component of the velocity and this object is moving towards the observer so we have a blue shift this object is moving away from the observer, so we have a redshift.

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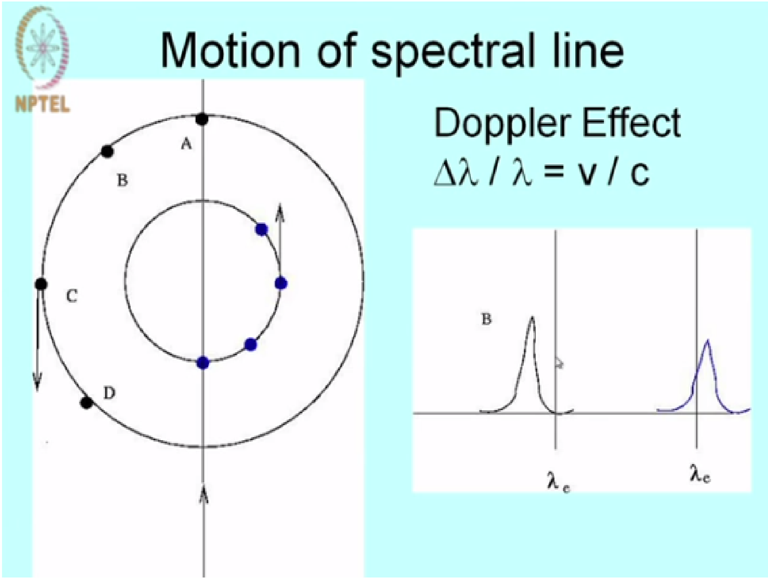

So the wave length of this object in position B has shifted it has become less. Whereas, the wave length emitted from here has the object here is moving away from the observer. So the wave length has increased. Now when it moves from position B to position C the line of sight component of the velocity has increased further. It is now completely along the line of site, so the shift in the wave length has also gone up.

There is a larger shift in the wave length and then in position D again the line of site component of velocity has gone down so it has decreased and then it will move to this position where there is going to be no shift at all and then it will repeat.

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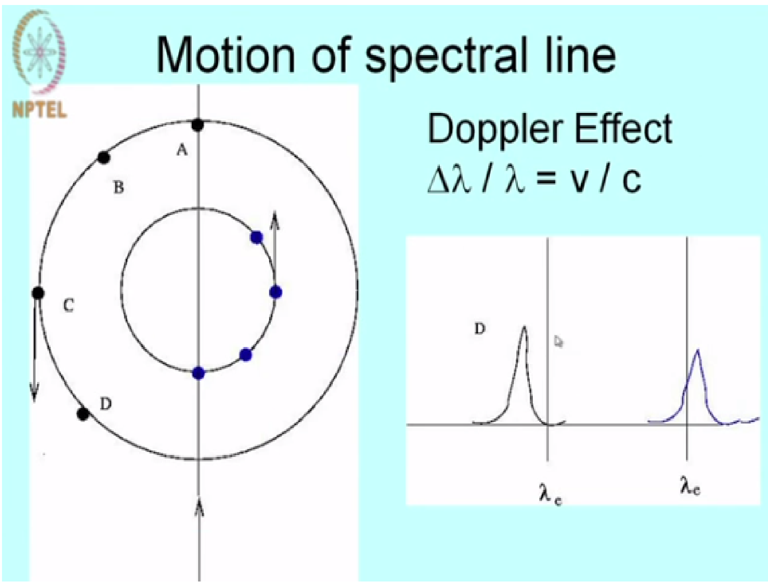

So this will now, this will so the wave length, the shift in the wave length is going. The wave length is going to go up and down like this. Similarly, this wave length is going to go up and down like this.

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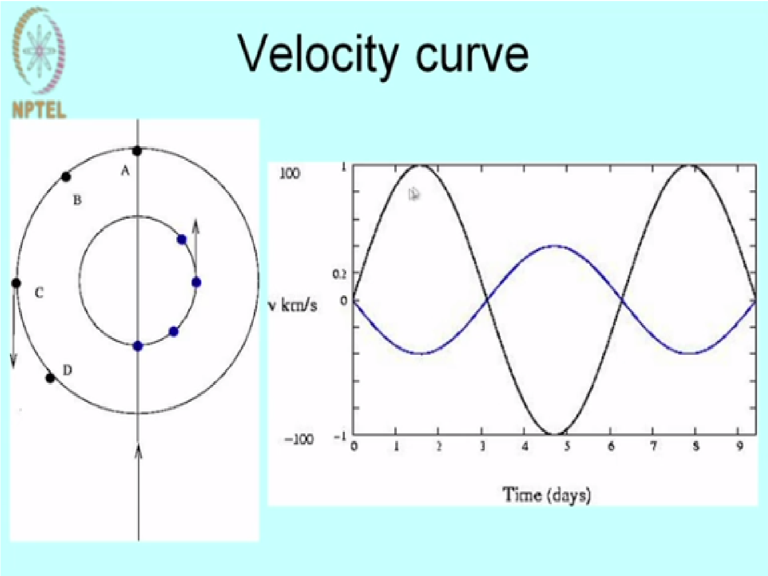

And if I plot the wave length, the wave length can be used to interpret to infer the line of site component of the velocity. The shift in the wave length can be used to infer the line of site component of the velocity and this is what the line of site component of the velocity looks like. So in position A there is no line of site component of the velocity in position B it has gone up, position C it has gone up further for this object.

And then it reduces and it becomes negative in this half and again positive. For the second object it starts off with 0 and then you have a negative line of site component it becomes negative and is moving away from the observer and then it again goes down like this. So you have, you will see that there are 2 spectral lines shift and from there you can infer for a spectroscopic binary you have, if you can determine spectroscopic lines from the 2 different objects these 2 spectroscopic lines will shift.

The shifts will be pi out of phase because the motions of these 2 objects are exactly pi out of phase and they will, in this particular case you will have exactly sinusoidal behavior with time. The 2 sinusoidal curves are exactly pi out of phase.

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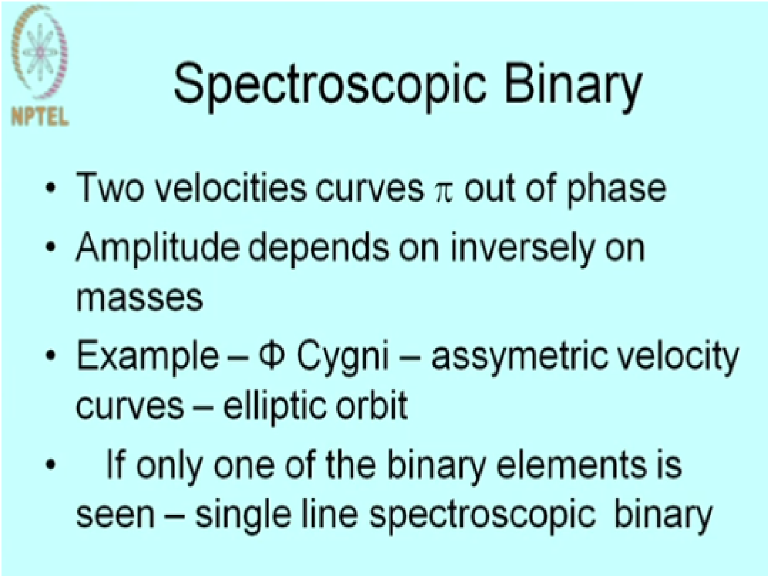

So these are characteristic features of spectroscopic binaries. In a spectroscopic binary you have 2 velocity curves. If you can identify 2 spectral lines from the 2 different objects then you will find that they have 2 different velocity curves which are exactly pi out of phase. The velocities of the 2 objects the amplitude of the 2 objects are inversely proportional to the masses and the velocity V1 is proportional to in a for a circular orbit omega*a1.

V2 = omega*a2. Omega is the angular frequency which is common for both of them and a1 and a2 are inversely proportional to the masses. So the velocities also are inversely proportional to the masses and this is true even for elliptic orbits. So the peak, the amplitude depends inversely on the masses. This is another characteristic feature.

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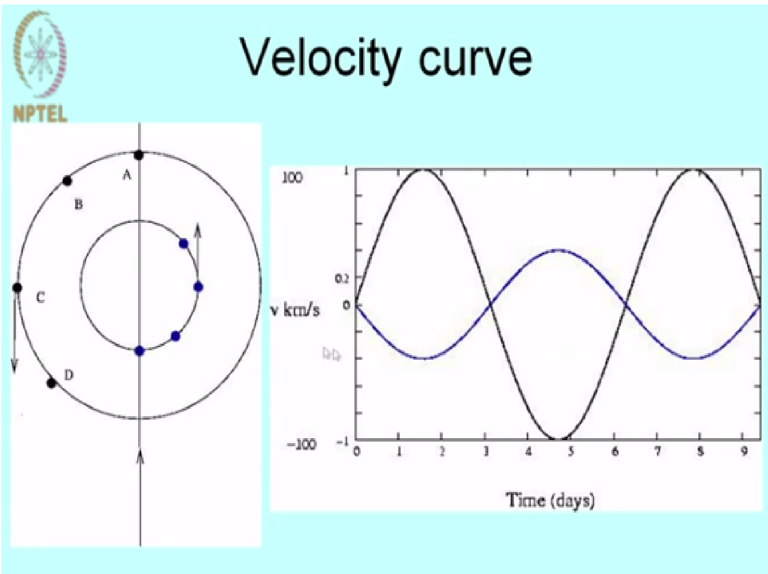

So here, the amplitude of the less massive object is more and the amplitude of the more massive object is less. So the ratio of the amplitudes of the velocities tells you the ratio of the masses. And an example is phi Cygni. For phi Cygni the velocity curve is asymmetric. So what we have considered over here is a circular orbit. In general, the circular orbit will not be even if you have circular orbit it will not be exactly in the plane, exactly adjourn.

It will not be viewed exactly adjourn it will there be an inclination angle, effect of an inclination angle and typically you will not have circular orbits you may even have ellipses, elliptic orbits and for elliptic orbits you will not have exact velocity curves like this. You will have asymmetric curves which is the situation for phi Cygni. And there are situations where only one of the binary elements is seen in the spectroscopic line and these are called single line spectroscopic binary.

So let me now briefly recapitulate what we have learned today. Today we have been discussing binary systems where you have 2 objects in a binary orbit, in an orbit in each other's gravitational field and we saw that this can be reduced to exactly the problem that we had discussed earlier where you have a test particle moving in the mass of a, in the gravitational field of a much larger mass M.

So the 2 body problem of a binary system can be reduced to a one body problem if you go to the center of mass frame of reference. And the one body and the displacement relative displacement between these 2 particles behaves in exactly the same way as a single particle, test particle in the gravitational field of a much more massive object, that is the first thing.

So each of the displacement vector between these 2 particles chases out ellipse and ellipse follows Kepler's law and each of the particle also traces out ellipses and follows Kepler's laws. The amplitude are inversely, the ratio of the amplitude is inversely proportional to the masses. Then having told you this, I give some motivation why we are interested in binary systems.

A large fraction of the stars are found in binary systems and they are also very useful laboratories for studying a variety of things including the, they are possible the only situation where you can determine the mass of stars and they also let us study the interaction, accretion between stars. And finally, I told you that there were 3 kinds of binary stellar systems. The visual binary, eclipsing binary and spectroscopic binary.

These 3 types are not mutually exclusive so you could have a binary system which is both the visual and the spectroscopic binary for example. And then I briefly discussed each different type of binary system. So let me bring our today's discussion of binary systems to an end over here. In the next lecture I shall resume our discussion of binary systems.