Astrophysics & Cosmology
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Lecture - 40
Big Bang Nucleosynthesis (Contd.)

Good morning and welcome to today's lecture we have been discussing the Big Bang nucleosynthesis.

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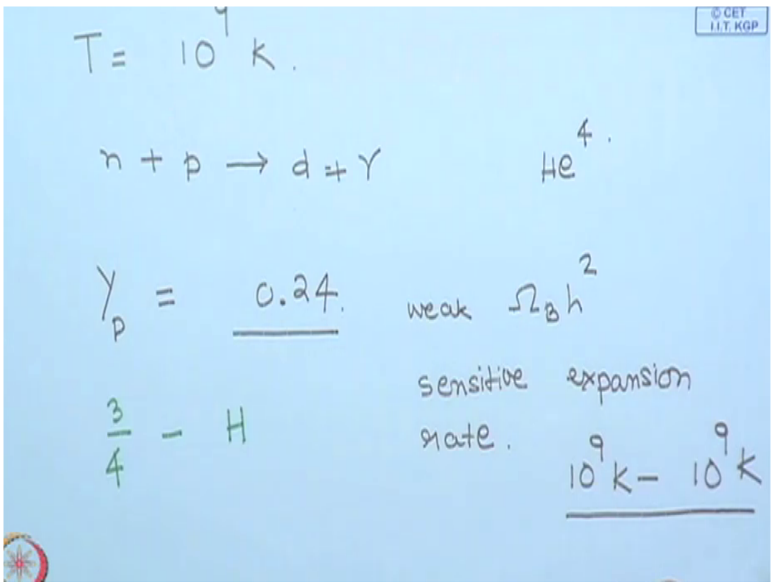

And we learnt in the last lecture that the Big Bang nucleosynthesis starts at a temperature of 10 to the power 9 Kelvin where the for the first time this reaction neutron + proton they combine to form deuterium, this reaction starts off in this reaction is able to produce significant amounts of deuterium, so this is where the Big Bang nucleosynthesis started and rapidly at the available the neutrons that where available at this temperature, they rapidly got bound into helium 4.

So the end product was helium 4 which is the light element which has the deepest binding energy per nucleon, so once the nucleosynthesis starts it proceeds very quickly and all the available neutrons are bound in to helium 4 and we saw that these are detailed calculations which we did not discuss but I told you that detailed calculations and the arguments which I presented in the last lecture all of these they indicate primordial helium abundance of the order of 0.25 and more precise number would be 0.24.

We also learnt that this is only weakly dependent on omega baryon h square but sensitive to expansion rate of the universe and it is sensitive to the expansion rate during the epochs prior to 10 to the power 9 Kelvin, so it is sensitive to the expansion rate between 10 to the power 10 and 10 to the power 9 Kelvin in this range, where the neutrons they decay can get converted into protons, so this is what you learnt in the last class that a brief recap of what we learnt in the last class.

Now today we are going to continue the first part of today's lecture we are going to continue our discussion of nucleosynthesis and so let us come back to this, so what I told us that the neutrons are quickly bound into the helium 4 nuclei, to form helium 4 nuclei but the point that we shall take up today is that helium 4 is not the only product of this nucleosynthesis and there are other nuclei other light elements nuclei of other light elements which are also produced in much less quantities.

So one of the first examples that we can talk about is deuterium, so there will be some deuterium which does not get converted into helium 4 left at the end of the nucleosynthesis. Similarly, so let me list all the elements all the nuclei that are left at the end of nucleus of the Big Bang nucleosynthesis.

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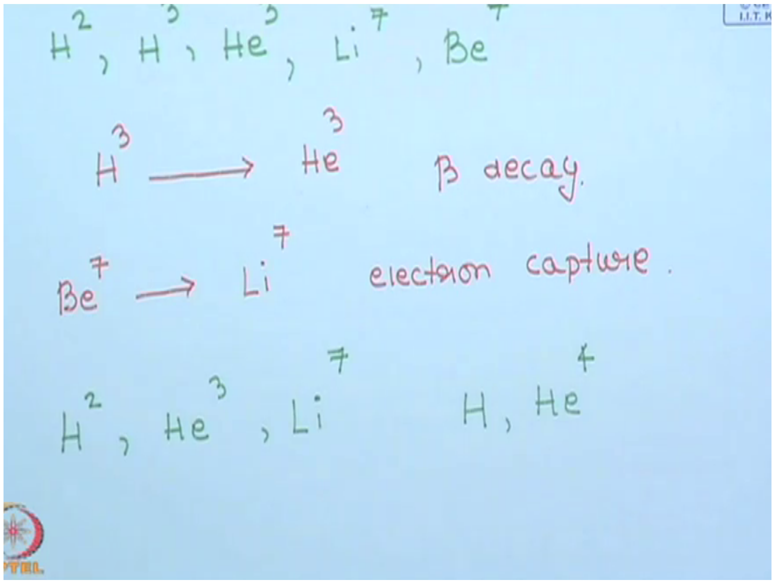

So at the end of Big Bang nucleosynthesis we have deuterium then we have tritium, we also have helium 3 all of these we have seen were intermediate stages that led to the formation of helium 4 through 2 body interaction processes reactions, so all of some amounts of all of these nuclei will also persist after Big Bang nucleosynthesis is over, in addition to this we also have some elements which are heavier than helium 4, we have lithium 7 and also beryllium 7.

Now the tritium gets converted to helium 3 through beta decay and we do not expect to see it around the tritium that was produced during the Big Bang nucleosynthesis. Similarly, the beryllium 7 gets converted to lithium 7 through electron capture and we expect to find traces of these elements which were produced during the hot phase early hot phase of the universe still around in the universe, so these deuterium, helium 3 and lithium 7.

So in addition to the hydrogen that is the proton and helium 4 which are the 2 most abundant nuclei roughly one fourth is in by weight is in helium 4 and the rest is in hydrogen, there are also very small traces of deuterium helium 3 and lithium 7. Now let us take up this deuterium for discussion next.

So the important point is that the deuterium abundance after the Big Bang nucleosynthesis is essentially the deuterium abundance when the Big Bang nucleosynthesis starts off. So for the Big Bang nucleosynthesis to startup we require the deuterium to combine 2 deuterium nuclei to combine to form helium 3 or tritium and there is a critical density for that and the density the abundance that we have now is just basically just corresponds to that.

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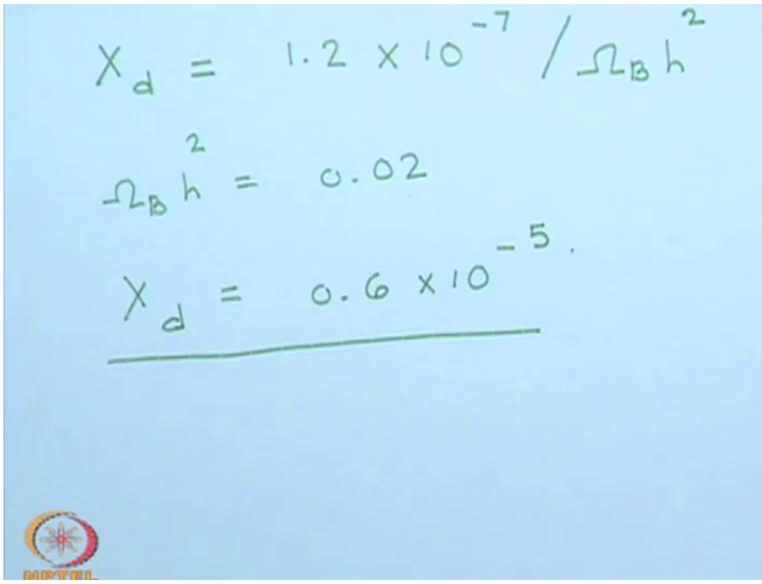

So the predicted deuterium abundance at the end of Big Bang nucleosynthesis is 1.2*10 to the power -7/omega baryon h square. So what are the points to note here, the first is that the predicted deuterium abundance is much smaller compared to the helium abundance which is roughly 1/4 of the total nucleons are in helium, roughly 3/4 are in hydrogen, the remaining 3/4 by weight of the nucleons are in hydrogen.

And we have this small amount in deuterium which is the third most abundant approximately the third most abundant species produced in Big Bang nucleosynthesis, so you see that the fraction of nucleons in deuterium is extremely small it has fallen significantly that is the first thing it is extremely small, the second thing is that the deuterium abundance is inversely proportional to omega baryon into this combination omega baryon h square.

And so this is very important because we have seen that the helium abundance is insensitive to the baryon density, so it does not really tell us much about the present value of the baryon density, it put some constrains on the expansion rate at that epoch when helium was produced before helium was produced, but in contrast the deuterium abundance is sensitive to the omega baryon.

So if you can measure the fraction of nucleons in deuterium one can constrains the observationally constrain the value of omega baryon*h square, and for the fiducial value which we have been adopting which is omega baryon h square =0.02 this is the fiducial value that we have been adopting the deuterium abundance comes out to be 0.6*10 to the power-5, so measurements of the deuterium abundance can essentially constrain omega value and h square.

And we also see that the deuterium abundance is extremely small it is of the order of 10 to the power predicted to be the order of 10 to the power-5, -6, so its observationally also a very a difficult and challenging task to constrain this, now let us take a brief look at what the observation tell us, so traditionally, historically there have mainly been 3 different kind of observations which have given input as to the deuterium abundance.

And the observations actually tell us the ratio of deuterium to hydrogen this is the quantity which can so if you can measure the deuterium line and the hydrogen line from the same place you can then determine the ratio of deuterium to hydrogen and observationally this is what has been traditionally determined by 3 different kinds of observations, let me just mention these kinds of observations.

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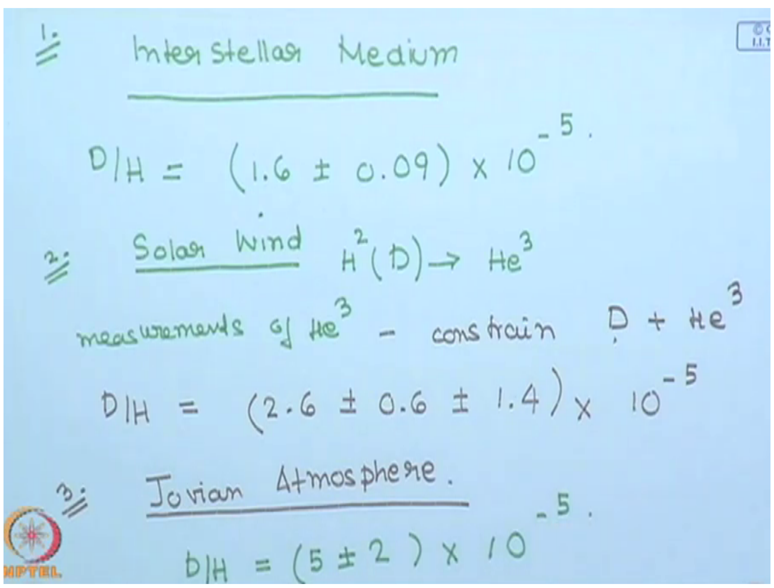

The first one is spectroscopic observation of the Interstellar medium, so this is the gas between the stars in our own galaxy, so spectroscopic observations of this gas they tell us that the deuterium to hydrogen ratio this = (1.6+- 0.09)*10 to the power -5, so this is one kind of observation then there is another kind of observation which also provides inputs on this. The second kind of observation is observation of the Solar wind.

And we have right in the beginning of towards the beginning of this course we have learnt about the solar wind this is a stream of energetic particles that are emitted from the sun, so the first conjected by Parker and then subsequently measured, so in the sun the deuterium gets converted into helium 3, the deuterium H2 in the sun which I also denoted by D gets converted to helium 3.

And measurements of helium 3 in the solar wind tell us about the combined abundance of helium 3 and deuterium and this puts limits this tell us about the deuterium to helium to hydrogen abundance and these observations let me write it down they constrain deuterium + helium 3, so you send out a satellite and measure the abundance of helium 3 in the Solar went outside the earth's atmosphere and these constrains the combination of deuterium and helium 3.

And they tell us that the D/H ratio deuterium to hydrogen ratio is 2.6+-0.6+-1.4, so here we have 2 kinds of uncertainties one is the breaking up into deuterium and helium and other is the statistical uncertainty in the measurement anyway, these observation tell us that the deuterium to the hydrogen ratio is in this range.

And finally we have a third kind of observation these are observations of the Jovian atmosphere again right at the beginning we learned about the different planets and I told you that Jupiter is mainly made up of gas gaseous material and hydrogen ammonia etc. and so we can use the observations of the atmosphere of Jupiter to determine the abundance of whatever spectral lines you can identify and these observations they indicate that the D/H=5+-2*10 to the power-5.

Well this is these are the traditional, traditionally these are the kind of observations that people have used to constrain the deuterium to hydrogen ratio and off late I mean maybe past decade more than a decade possibly 15 years or so maybe going onto 20 years now little less than that maybe 15 years in the 90s and last decade there have been.

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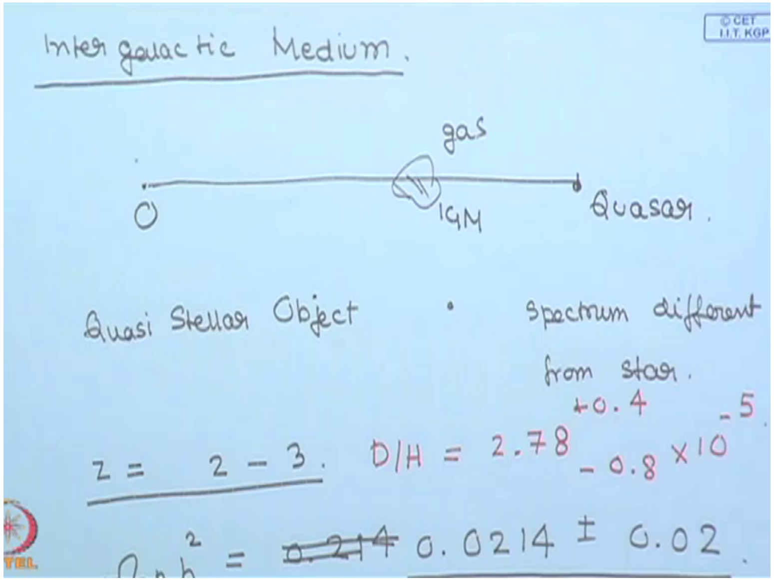

It has been possible to measure deuterium in the Intergalactic medium, so we have learnt that the universe is filled with galaxies and the but it is not only that the universe is filled with the galaxies the intervening space between the galaxies is also there is a gas gaseous material that occupies the intervening space between the different galaxies, it is largely ionized but there are hydrogen and there are helium, deuterium all these elements are present there also.

And so question is how do you detect this intergalactic medium? How do you observe it? Well the basic idea is that I am the observer over here and we look at a distant quasar, well let me briefly touch up on the, what is a quasar? Quasar is a very bright object which looks very much like a star, so it is a Quasi Stellar Object that is how the name quasar comes.

So it is extremely small it does not occupy any it occupies a very small angle in the sky so which is what we mean by quasi-stellar object, but the spectrum of a quasar is quite different from a star we have seen this is the light from a star is well described by a black body spectrum with absorption lines, on the contrary a quasar has a power law kind of it has a flat it is a power law spectrum with the emission lines and absorption lines.

So this is a very bright object over here and if you can now look at the light coming from this very bright object and there is some gas over here IGM intergalactic medium, then the gas over here will produce absorption lines in this quasar spectrum and you will be then able to do then find out what is there in the intergalactic medium and you can use this, now people have been carrying out observations of the IGM in the redshift range 2 to 3.

So this has got a very big advantage that a redshift 2 to 3 we have learnt means quite a distant object, so the gas that we are looking at is pretty far away not only that it is pretty back in the past, so it would not have been affected by stars, the ratio of deuterium to hydrogen this is something that I should have mentioned earlier.

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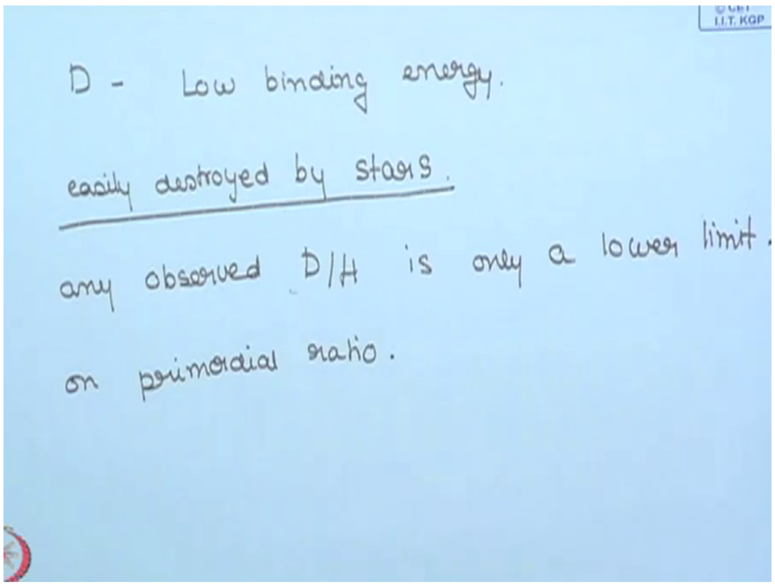

Deuterium we have seen as a low binding energy of the light elements that we have discussed deuterium has the lowest binding energy, so it has a low binding energy and it is easily destroyed by stars, so the fact that we have to take into account is any observation of deuterium is actually just a lower limit, there could have been more deuterium which has been destroyed by stars okay, so any observed D/H is only a lower limit on the primordial ratio.

So the primordial abundance could have been higher than what is observed, so the observations of the intergalactic medium allows us to probe high redshift, so if you go back in the past less stars would have been formed and there is less chance of having removed some of the deuterium and these observations they indicate they are interpreted.

So observations along the line of sight to high redshift quasars they now tell us that D/H has a value 2.78 let me also give the error bars so you have +0.4 and -0.8*10 to the power-5 and I have told you that observations of this D/H ratio are sensitive to the value of omega baryon h square, so this value of the ratio of D/H the deuterium to hydrogen ratio they tell us that omega baryon h square has a value 0.0 sorry 0.0214+-0.02, so this is the value which is inferred from observations of the deuterium abundances.

So there are all these other elements helium 3, lithium and lithium which are also produced during the Big Bang nucleosynthesis, but the abundances are smaller and I shall not go into for the details of the observations and the predictions for these elements. Let us now change the topic of our discussion and turn our attention again on the cosmic microwave background radiation.

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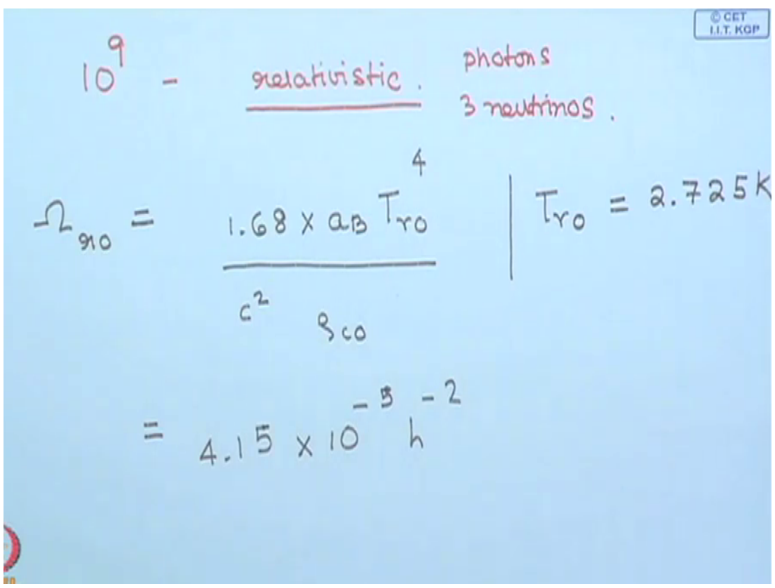

So the discussion till now has been centered at a redshift of around 10 to the power 9 and we have seen that when the universe is dominated by relativistic particle, so it is relativistic and we have 1 photon the CMBR photons and 3 neutrinos and if you calculate the density parameter corresponding to this constituents that is the present value of the density parameter omega r0.

Then we have seen that this combination of photons and 3 neutrinos massless neutrinos whose temperature is somewhat lower than the photons, they finally in the final contribution gives us a density 1.68 into energy density into the Stefan Boltzmann constant into the present temperature of the CMBR we want to calculate the present value of the density parameters, so this is the present energy density corresponding to this mixture.

And to calculate the density parameter what we have to do is divide this by c square to convert it into mass density and divided by the critical density now and this will give us the density parameter and just to remind you the present value of the photons of the CMBR is at temperature is 2.725 Kelvin and this has a value which is a 4.15*10 to the power-5 h to the power-2, the age dependence comes from here.

So we see that at present this density of the photon and the 3 neutrinos CMBR and the 3 neutrinos is extremely small compared to the critical density and of course compared to the matter, present in the universe also compared to the dark energy which is more than matter, but at an epoch of 10 to the power 9 the universe was dominated by these relativistic particles, let us just look back and determine how this ratio.

So let us look at how as the universe evolves? How does the ratio of the matter density and density of the relativistic particles how does this evolve? So this is something we have discussed earlier also but let us look at it again.

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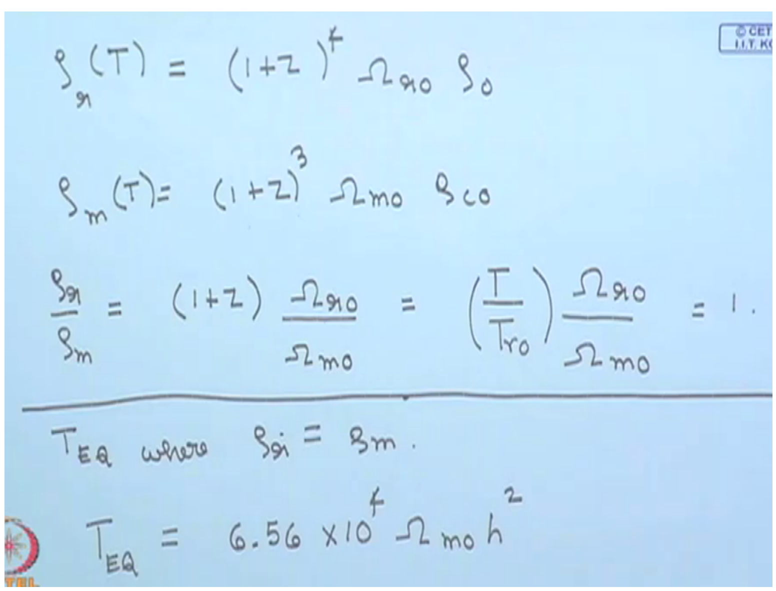

So the relativistic particles rho r as a function of temperature or redshift is essentially 1+z to the power 4, the scale factor to the power-4 into the present density which is omega r0*rho critical 0, whereas the matter density as a function of scales as a to the power-3 so 1+z to the power 3 omega matter 0 rho critical 0.

So if you calculate the ratio rho r/rho matter the ratio of the relativistic particles density in the relativistic particle to the density in the non-relativistic matter this will have a 1 + z dependence and then we will have the ratio omega relativistic 0 which we just calculated/omega matter 0 and 1+z is basically one by the scale factor which I can write as the temperature at that redshift/the present value of the temperature/the ratio of the density parameters.

Now let us ask the question when are the density of matter and the relativistic particles the same this is what is called the epoch of matter radiation equality, so let us calculate a temperature then these 2 have the same density, so we all that you have to do is you have to set this =1 and this gives us the temperature, so we have worked out the numerical value for this and we know that T gamma 0 is 2.725.

So putting in these numbers we find that the temperature at which matter radiation equality occurs has a value 6.56*10 to the power 4 omega matters 0 the present value of the matter density into h square and we have seen that present values.

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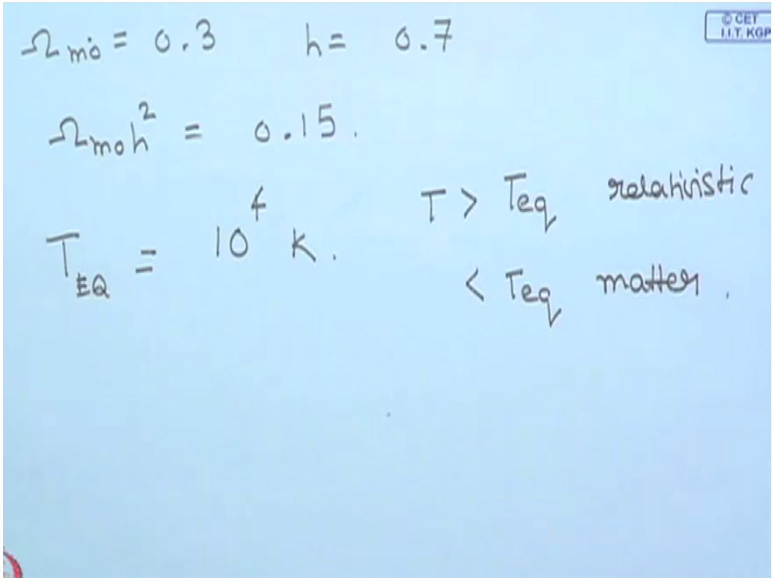

So the present observations seem to indicate that omega matter 0 has a value 0.3, h has a value 0.7 which is more or less indicative of omega matter 0 h square having a value 0.15 for these values the equality occurred at 10 to the power 4 Kelvin which is what I had mentioned earlier, so above 10 to the power 4 so in the range 10 to the power 4 to 10 to the power 9 Kelvin the universe is actually dominated by these relativistic particles.

And below 10 to the power 9 so T>T equality the universe is dominated by relativistic particles and <T equality it is dominated by the non-relativistic matter okay, so this is a brief background about the expansion which it is a recapitulation we had discussed this several times earlier. Now let us look at the CMBR at a temperature of around 10 to the power 9 Kelvin the CMBR photons have adequate energy to keep the nuclei ionized we have seen that.

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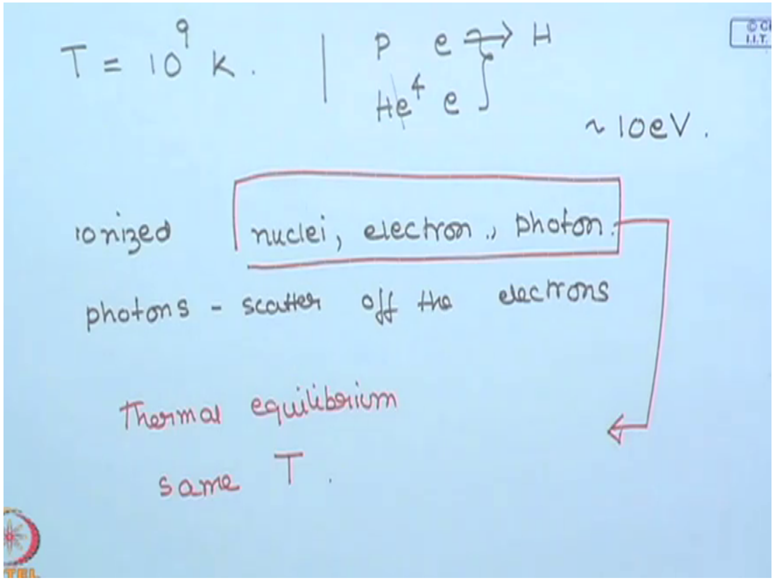

So at temperature of 10 to the power 9 Kelvin that is the temperature we are starting off at and the typical so what do we have we have the protons and electrons and the helium mainly the helium nuclei and the electrons and we are assuming that the universe is neutral, so the number density of the positive particles and negative particles are more or less are the same.

The electronic binding energy for to form a hydrogen atom so suppose you look at this possibility of the proton and electron combining to form a hydrogen atom or helium 4 and electrons 2 electrons combined to form the helium 4 atom, these have binding energy of the order of 10 electron volts, whereas the temperature here corresponds to a energy scale which is considerably higher than this.

So at these temperature the nuclei and electrons are separate, so they are all ionized so you have the nuclei and you have the electrons separate and the photons the scatter of the electrons and the electrons also scatter with the nuclei and the whole thing is so they are all very tightly bound, so this and the photon they are all very tightly coupled and in thermal equilibrium through scattering and have the same temperature.

So at a temperature of around 10 to the power 9 Kelvin this is the kind of situation that you have, now let us ask let us take a look at this scattering and the equilibrium and scattering, let us take a closer look at this, now this CMBR photons are essentially being scattered of the electrons that is the main thing.

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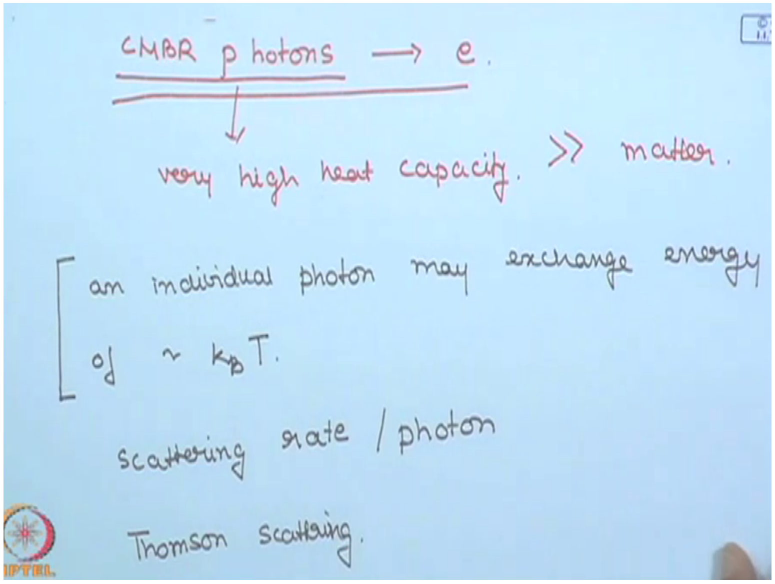

And this is so the CMBR photons and the electrons this is the scattering, now let us so we have seen that the photons have a very high heat capacity compared to the matter rest of the matter, so this scattering is actually we know for sure that this scattering is not going to change the energy content of the total CMBR that is not going to be possible, because the photons, the CMBR photons have a very high heat capacity compared to matter>>the matter.

We have evaluated these numbers earlier on and we know that this has a very high heat capacity compared to the electrons and nuclei, so the scattering between the interaction between the electrons and matter and the CMBR is not going to change the energy content of the total CMBR because it has a much higher heat capacity, but it is not possible that an individual photon may exchange energy of the order of KB T, so that is the possibility that we are looking at.

So it is possible that an individual photon energy of order KB T which is the typical energy of the photon at any temperature T, so let us see what is the rate at which an individual photon may exchange energy of the order of KB T through scattering with the electrons, so to do this what we have to do is we have to look at the scattering rate per photon, so that is the rate at which a single photon will scatter of electrons.

And the process here is for us to calculating the scattering rate we have to essentially just look at the Thomson scattering cross-section and we can calculate the scattering rate.

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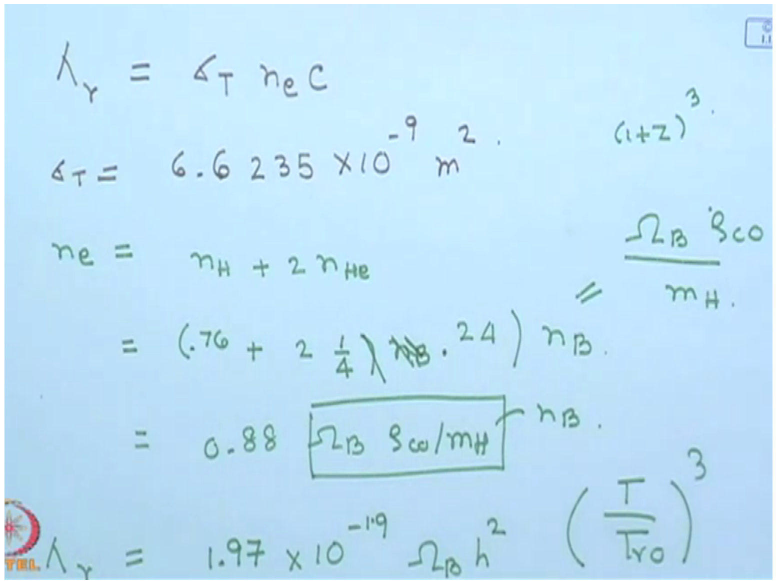

So this is quite simple to calculate let us denoted by lambda gamma, so we have the Thomson scattering cross-section sigma T which we have considered come across earlier this cross-section into the number density of electrons into the speed of light this will give us the rate at which the scattering takes place, so sigma T has a value which we have encountered earlier 6.6235*10 to the power-9 meter square.

So that is the Thomson scattering cross-section which you have encountered earlier and we have here the speed of light which we know, the only unknown that we have to calculate is the number density of electrons, the number density of electrons we can assume is that the universe is charged neutral, so the number density of electrons will be the it will be basically the number density of hydrogen atom nuclei + twice the number density of.

So it will be number density of hydrogen nuclei + twice the number density of helium 4, because every helium 4 has 2 protons and doubly charged, so this and we have seen that the total baryons are divided between these in the ratio, this is 0.24 and this is 0.76 roughly, so this will be 0.76+

by weight 0.24 is in the form of helium, so the number density of helium will be half of that rather 1/4 of that, so I have because they have 4 so *1/4 * the number of baryons.

So by weight 1/4 of the number of baryons by weight has gone into helium and there will be a factor of 2 here because there are 2 protons in each of them +0.7 this into sorry this into the fraction of into 0.24 this will give us the electron density and number of baryons is basically omega baryon/into critical density/the mass of hydrogen mass of the photon or mass of the hydrogen atom.

So this 0.88*omega baryon*rho c0/mH and this is basically the number of baryons, so we know that the number density of baryons scales as a, this into a factor of (1+ z)cube this is the present density, so (1+ z)cube and if you put in all the numbers here it turns out that this the scattering Thomson scattering rate per photon has a value 1.97*10 to the power-19 omega baryon h square (T/T gamma 0) to the power 3 which comes because of this scale factor (1+ z)cube, T gamma 0 is the present value of the CMBR temperature.

So this gives us the scattering rate per photon, so this is the rate at which a single photon gets scattered, now what we would like to calculate is the rate at which a single photon loses or gains energy exchanges energy this is not the rate at which the single photon loses or gains energy, we are interested in the energy transfer.

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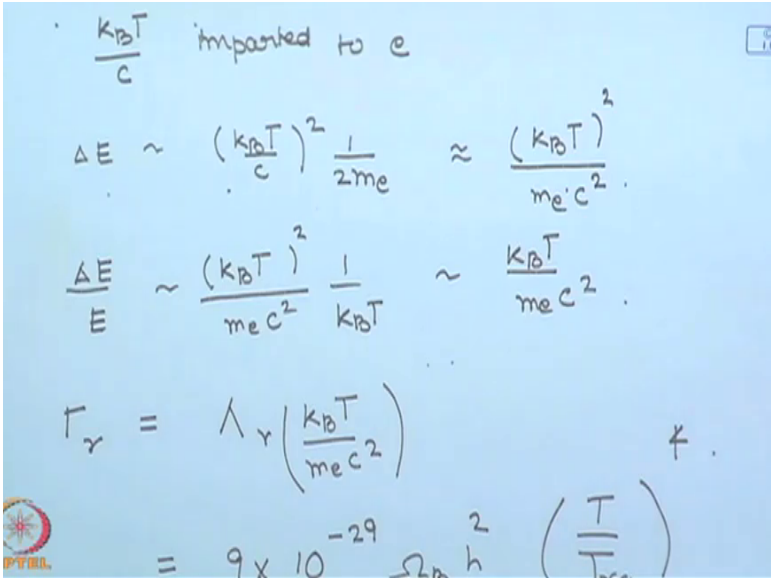

So to calculate that let us consider the scattering in the scattering process a photon has momentum KB typically has momentum KB T/c and this is the momentum that it also imparts to the, this is imparted to an electron in a scattering, so this is the momentum imparted, so the energy imparted to the electron will be the momentum square which is delta energy in a scattering will be of the order of KB T/c square that is the momentum square/the mass of the electron p square by 2m.

And so this is of the order of (KB T) square/m electron c square, so if you look at the fractional energy fraction of the energy that is transferred in one scattering then this will be of the order of (KB T)square/m electron c square/KB T this is the energy of photon, so this is of the order of KBT/m electron c square.

So we can calculate the rate at which energy is being exchanged by a photon by multiplying the scattering rate into the fractional energy that is exchanged in the every, fractions of its energy that is exchange in every scattering which is KB T/the rest mass energy of the electron, so what you have to do is you have to use this value that we have just calculated over here and this turns out to be 9*10 to the power-29 omega baryon h square T/T gamma 0 to the power 4.

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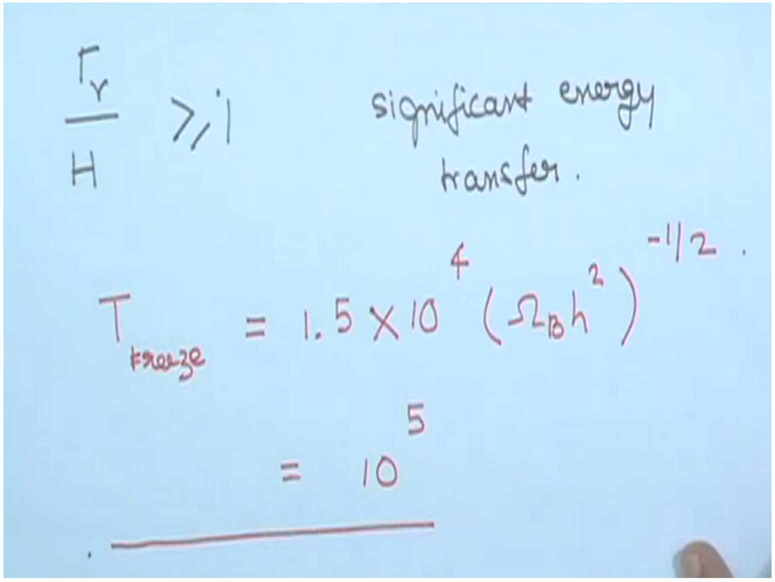

So what is the consequence of this? The consequence of this is we have to now compare, so for there to be significant energy transfer we require this ratio to be >=1 then there will be significant energy transfer and we have calculated the Hubble parameter during this relativistic epoch this is all we are assuming this is all occurring in the relativistic epoch okay, so we have calculated the Hubble parameters.

So if you apply this condition where the rate that the rate of the energy transfer to the Hubble parameter this ratio should be of the order unity it turns out that this occurs at temperature which we can called the temperature freezing where the energy distribution of the photon gets frozen this has a value 1.5*10 to the power 4 omega baryon h square to the power of-(1/2) and it has a value of 10 to the power 5 for the fiducial value of the omega baryon h square that we have been using.

So what do we learn from this exercise what we learn is that the photon exchanges energy with the electrons till the universe is hotter as long as the universe is hotter than this, once the universe is cooler than this, this ratio falls below 1 and the energy distribution of the photons whatever it is, is frozen the energy transfer becomes insignificant and all of this occurs in the relativistic era because the universe becomes matter dominated at a temperature of 10 to the power 4.

So the energy transfer stops at very at this temperature 10 to the power 5 but the collisions between the photons and electrons still continue subsequent to that for looking whether the collisions are effective or not you have to consider the ratio of the collision rate to the Hubble parameter and this.

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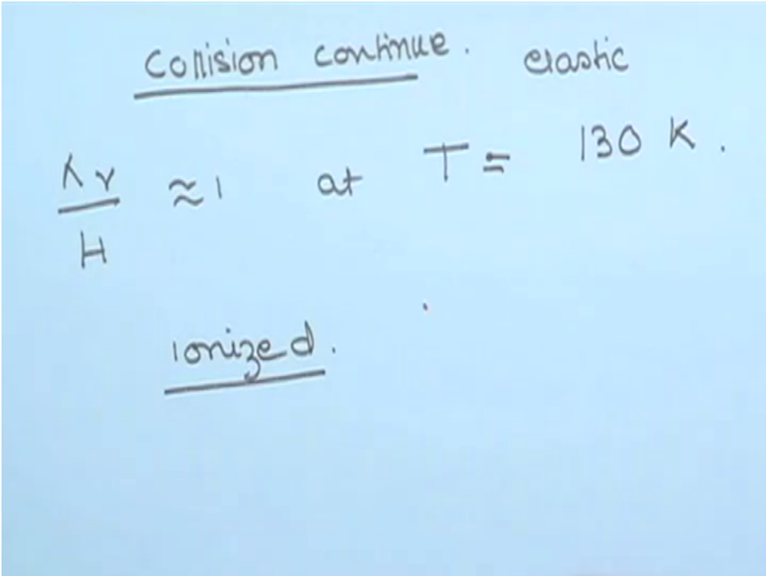

So the collision if you were to push this calculation all the way to the present and then it follows that the collision rate is of order unity at when the universe has a temperature of 130 Kelvin, but so the collision but these are all elastic collisions after a temperature of 10 to the power 5 the dominant process are elastic collisions and there is no exchange of energy they continue to be coupled but there is no exchange of energy in these collisions.

And this is the collision would continue till 130 Kelvin after which the collisions also would become insignificant, well this calculation is incorrect why is it incorrect? Because it assumes that the universe is completely ionized all the way till present.

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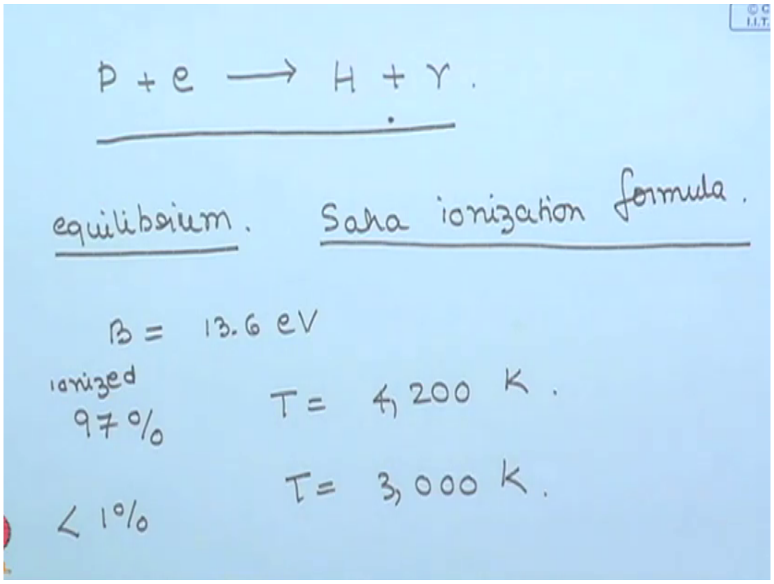

In reality what happens is that as the universe cools, the protons and the electrons when the universe becomes sufficiently cooled the protons and electrons they combined and to form the hydrogen atom giving out a photon, and one can work out this ratio, so one can work out the ratio of neutral hydrogen and ionized hydrogen assuming that it is in equilibrium and using the Saha ionization formula which we have encountered earlier.

And this calculation you have to put in the binding energy 13.6 ev for the hydrogen atom and workout the Saha ionization formula and this calculation based on the assumption that these are all in equilibrium, they give results that 97% of the hydrogen is ionized at a temperature of 4200 Kelvin and <1% is ionized, this is ionized at a temperature of 3000 Kelvin, so what we see is that the universe that it became neutral at around 3000 Kelvin, this is if we assume equilibrium.

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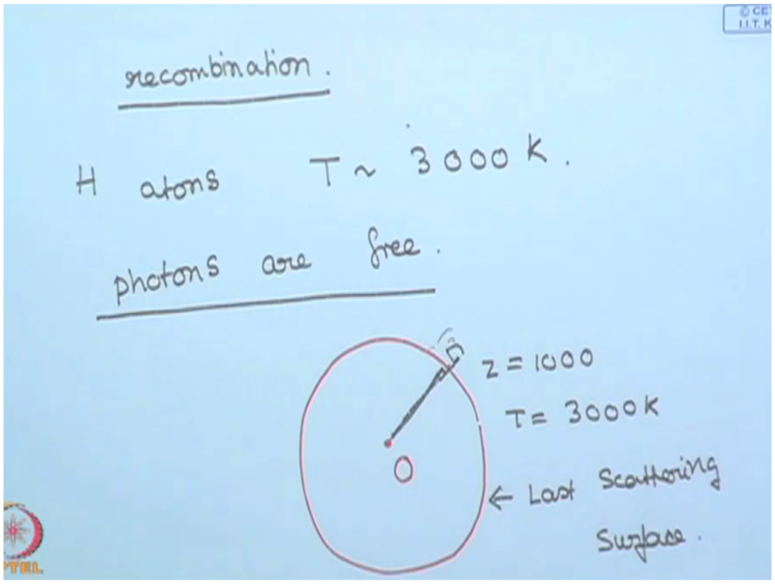

In reality this assumption of equilibrium does not hold and one has to actually solve the rate equations because when the density of protons and electrons falls when the proton and electron combines to form the hydrogen atom this process is called recombination, the identity falls and once the density falls this reaction goes out of equilibrium and one has to look at the detailed calculation, well the bottom line is that the universe is neutral.

So bottom line is that the universe is no longer filled with ions it is filled with hydrogen atoms by a temperature of around 3000 Kelvin and once the universe becomes neutral, the scattering process scattering becomes insignificant and the photons are free particles effectively photons are free which brings us to the picture that we have for the CMBR, so let me just draw this picture for you.

So I am the observer we are the observer here and we are sitting in a part of the universe that is neutral and if I look at a photon trace back of CMBR photon which is arriving at me now and I trace it backwards, tracing it backward means also going back into the past and the universe is hotter at when the universe is at of temperature 3000 Kelvin the universe was ionized.

So here there are no ionized particles so the photon propagates freely, once it is at a redshift of around 1000 temperature of around 3000 Kelvin the universe is ionized and the photon no longer propagates freely it gets so this is propagating freely till here the photon gets scattered and we cannot see anything beyond this, because once the photon gets scattered the optical depth is >1.

So this is also referred to as the last scattering surface. So let me now bring we running out of time, so let me now bring today's lecture to a close. Let me just briefly recapitulate what we have learnt in this course, we started off by discussing planets and then stars, our galaxy and finally we moved on to discussing the entire universe.