Astrophysics & Cosmology
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Lecture - 39
Big Bang Nucleosynthesis

Good morning and welcome to today's lecture.

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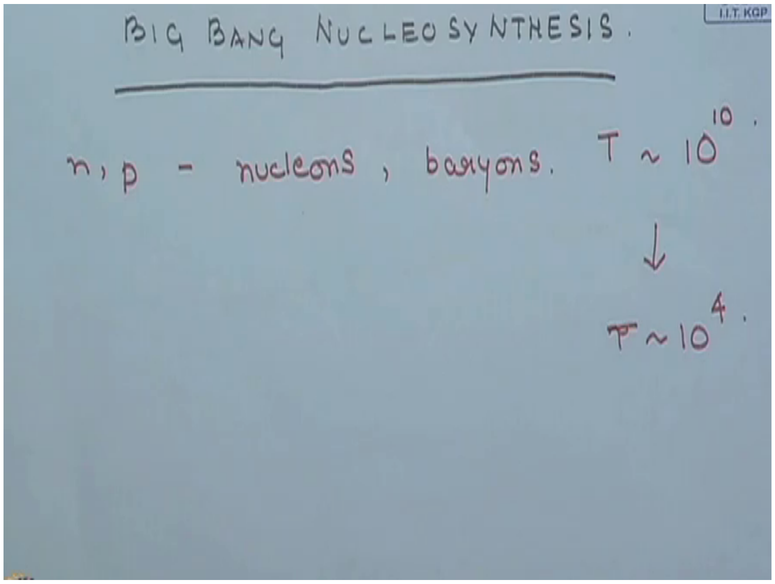

BIG BANG NUCLEOSYNTHESIS.

n, p - nucleons, baryons. $T \sim 10^{10}$.

$\downarrow$

$T \sim 10^{4}$.

We have been discussing Big Bang nucleosynthesis and before embarking upon this discussion let me just gave you a little prelude, it was recognized in the beginning of last century.

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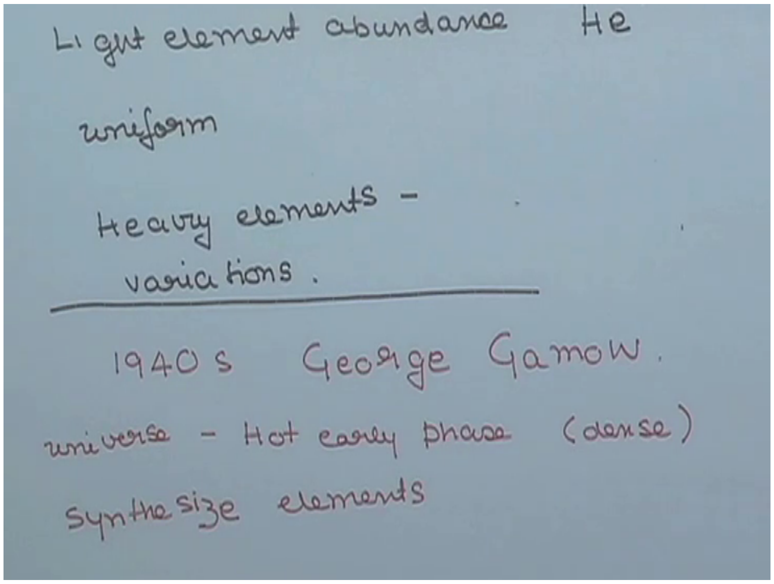

Light element abundance He

uniform

Heavy elements -

variations.

1940 s George Gamow.

universe - Hot early phase (dense)

synthe size elements

That the light element abundances particularly helium is more or less uniform throughout the universe, whereas heavy elements by heavy elements we mean elements heavier than hydrogen and helium their abundances shows considerable fluctuations, variations across the universe, so this is an observational fact and question is how to explain this? The remarkable theoretical insight was provided in the 1940s by the Russian physicist George Gamow.

Gamow made this remarkable proposition that if the universe had a hot early phase, why early phase because hot and dense is what you require for nuclei to form, a hot and dense medium, if the universe had a hot early phase hot and dense early phase then you could synthesize elements in the early universe and this prediction this proposal was made by Gamow much before the cosmic microwave background radiation.

And Gamow used the known abundances of light elements particularly helium to predict that there should be a radiation background the relic of this hot dense phase which was subsequently discovered as the cosmic microwave background radiation, so this is a brief history, historical note on this topic that we are discussing, so what are we discussing, we are discussing how the light elements were possibly synthesized during the hot early phase of the universe.

And this would be a natural explanation why the light element abundances or more or less uniform throughout the universe, we have studied how the heavier elements are synthesized they are synthesized inside stars which is why we expect there to be variations different parts of the universe because the star formation history has been different in different parts of the universe, so with this brief background let me remind you of what we had discussed in the last lecture.

So in the last lecture I had told you that we are essentially interested in following the evolution of the neutrons and protons which are the nucleons from which elements are built, so the elements that are similar to us are made up of nucleons neutrons and the nuclei of these elements are made up of neutrons and protons which are also referred to as baryons.

And we would like to follow the evolution of these particles during the temperature range 10 to the power 10 and lower which is where the universe was not enough for nucleosynthesis to have taken place and dense enough for nucleosynthesis to have taken place and I also told you that these elements constitute a very negligible fraction of total density during these epochs 10 to the power 10 to roughly 10 to the power 4, the universe becomes is radiation dominated in these era.

And the universe just becomes matter dominated around 10 to the power, the matter starts playing a significant role in the dynamics of the universe around a temperature of 10 to the power 4 Kelvin and lower.

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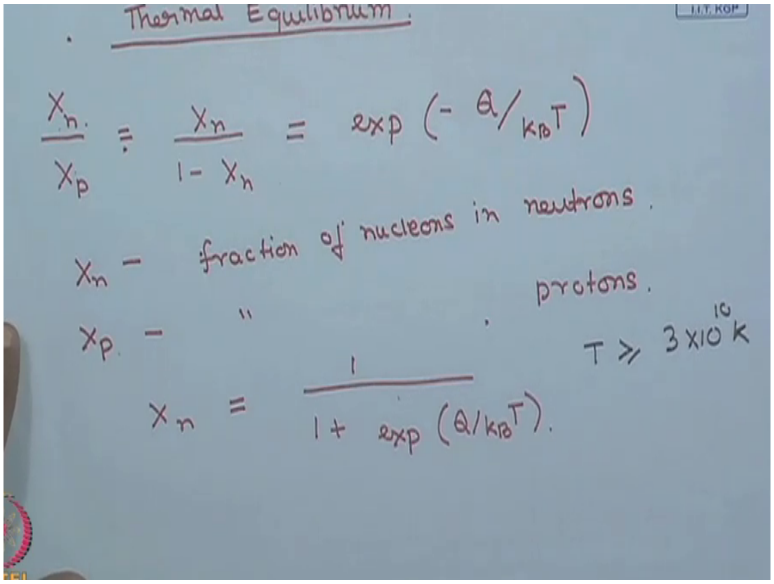

Thermal Equilibrium.

$$\frac{X_n}{X_p} = \frac{X_n}{1 - X_n} = \exp\left(-Q/k_B T\right)$$

X_n – fraction of nucleons in neutrons.

X_p – " protons.

$T \gg 3 \times 10^{10}$ K

$$X_n = \frac{1}{1 + \exp\left(Q/k_B T\right)}.$$

Then I had told you that the neutrons and protons are in thermal equilibrium at temperatures >3*10 to the power 10 Kelvin and we had also shown you the reactions through which they are in thermal equilibrium, so the fraction of nucleons which are in the form of neutron that is Xn remember Xn is the fraction of nucleons in the form of neutrons, Xp is the fraction of nucleus in the form of protons.

So Xn we had I had shown you is given by 1/1 + exponential (the mass difference between the neutron and proton/KB T) and this mass difference roughly corresponds to the temperature of around 10 to the power 10 Kelvin, so this thermal equilibrium formula holds at temperature above 3*10 to the power 10, subsequent to this these 2 elements that these 2 nuclei particles go out of thermal equilibrium.

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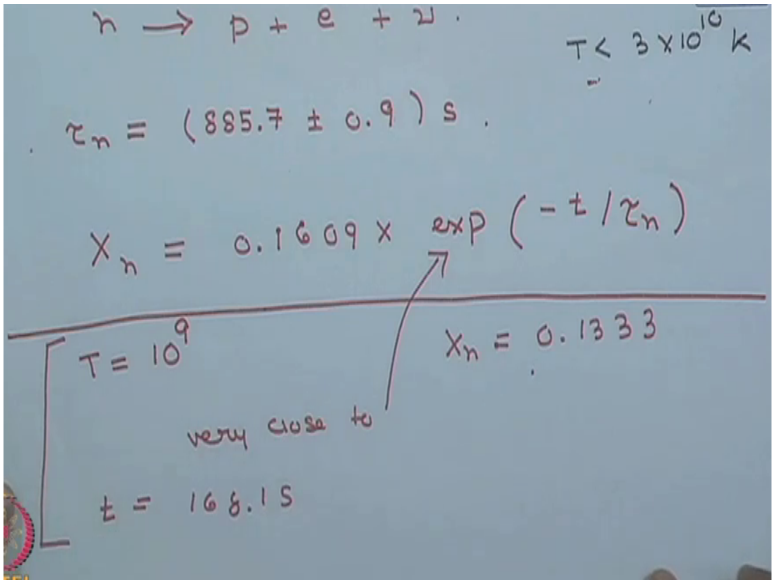

$n \rightarrow p + e + \nu$.

$T < 3\times10^{10}$ K

$\tau_n = (885.7 \pm 0.9)$ s.

$X_n = 0.1609 \times \exp(-t/\tau_n)$

$T = 10^9$

$X_n = 0.1333$

very close to

$t = 168.1$ S

And you then have only the reaction where the neutrons all decay to protons also producing a positron and an antineutrino and this decay has a mean lifetime of 885.7 seconds and this decay starts off at a temperature <3*10 to the power it starts off at 3*10 to the power 10 and it proceeds and during these temperatures <3*10 to the power 10.

The fraction of nucleons in the neutrons is well described by this formula 0.1609*exponential - the age of the universe / the lifetime of this neutron and this gives you a good approximation as to the fraction of neutrons nuclei in the form of neutrons and I had also shown you the number this number Xn at temperature T=10 to the power 9 Kelvin, so this is what we had done in the last lecture.

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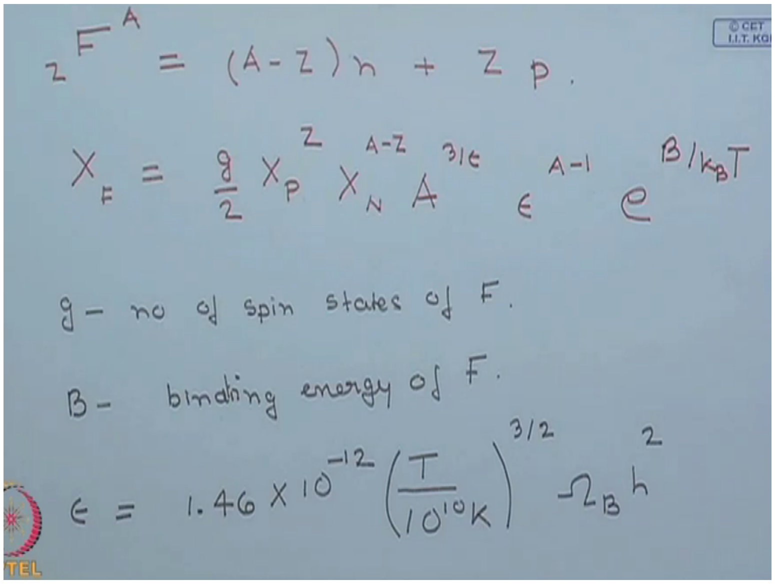

$${}_Z F^A = (A-Z)n + Zp.$$

$$X_F = \frac{g}{2} X_p^Z X_N^{A-Z} A^{3/2} \epsilon^{A-1} e^{B/k_B T}$$

g – no of spin states of F.

B – binding energy of F.

$$\epsilon = 1.46 \times 10^{-12} \left(\frac{T}{10^{10} K}\right)^{3/2} \Omega_B h^2$$

So essentially at the neutrons are decaying and we would like to now study the formation of a south of neutrons and protons so we would like to study their formation of a nucleus of an atom F, let us say the final product which has atomic number Z and mass number A, so this is formed out of (A–Z) neutrons and Z protons, so this is the thing that we would like to consider that the neutrons and protons they form a nucleus F of an element F which is atomic number Z and the mass number A.

And if all of these things that is the final product the neutrons and protons are in thermal equilibrium, then it is possible to work out from the Thermodynamics from statistical mechanics consideration, the fraction of the nucleons which are in the final product, the fraction of nucleons that go into this element F and this turns out to be g/2 the proton fraction to the power Z the neutron fraction to the power A-Z.

Then the mass number to the power 3/2 epsilon to the power A-1 and we have this Boltzmann factor e to the power Beta/KB T where T is the Boltzmann factor, so let me explain the various terms that occur in this expression which give us the fraction of nucleons in this element in this nucleus F, so here g refers to the number of spin states of F, we all have already seen what Xp is, it is the fraction of nucleons and protons, Xn is the fraction of nucleons and the neutrons, A is the mass number, epsilon okay.

B before the d is the binding energy, so it is the binding energy of the nucleus F, so B is the binding energy of the nucleus F and epsilon is 1.46*10 to the power-12 (T/10 to the power 10 Kelvin) to the power 3/2 omega baryon h square, so let me remind you that omega baryon refers to the present value of the baryon density parameter, so let me also write this down because they are going to keep on referring to omega baryon.

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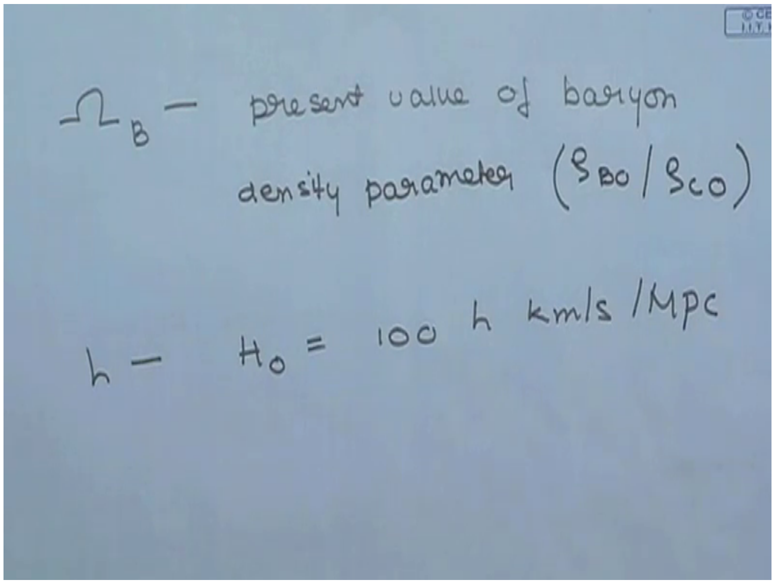

Ω_B – present value of baryon density parameter (ρ_{B0}/ρ_{c0})

h – $H_0 = 100\ h$ km/s /Mpc

So omega baryon here is present value of the baryon density parameter that is the ratio of the baryon density, the ratio of the baryon on density at present to the critical density at present this is omega baryon and h so we were expressing the Hubble parameter at present as 100 h kilometer per second per megapersec, I have already told you what this is and h is believed to have a value around 0.7 at present.

So in thermal equilibrium what we see is that the fraction of nucleons in the form of this nucleus F is given by this formula and this number epsilon which is dependent on omega baryon is extremely small it is of the order of 10 to the power-12 and the fraction of nucleons in any nucleus F is also going to be consecutively extremely small until the epoch until the temperature falls sufficiently low.

So that e to the power the binding energy/KVT this factor over here compensates for the fact that epsilon is extremely small.

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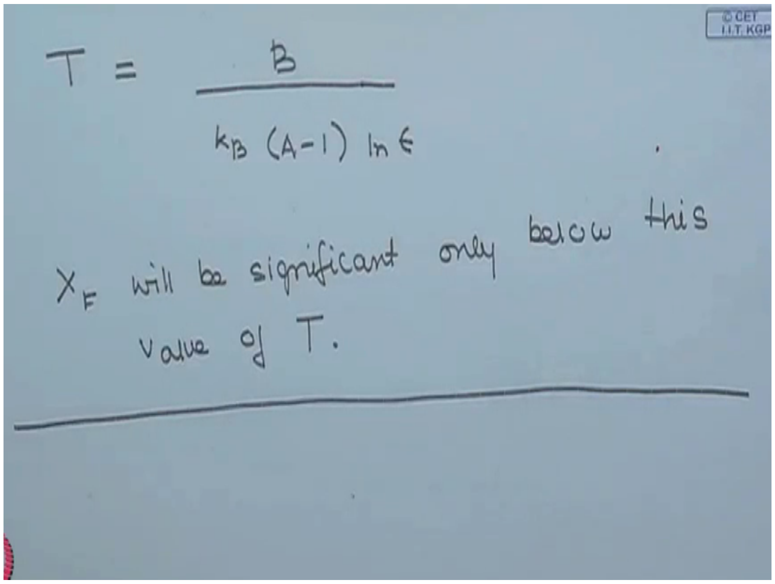

$$T = \frac{B}{k_B (A-1) \ln \epsilon}$$

X_F will be significant only below this value of T.

So this gives us a temperature so corresponding to this element F we have a temperature which is T which is the binding energy/KB the Boltzmann factor*(A–1)*ln epsilon, so the at temperatures if the temperature of the universe is more than this value which I just showed you here is more than this value then the fraction of nucleons in the nucleus F is going to be extremely small and we are going to have a significant fraction of this nucleus forming only at temperatures.

So XF will be significant only below this temperature and we see that essentially this is related to the binding energy, so you will form a particular nucleus only if the temperature falls of the universe falls below the binding energy with these extra factors over here and you will not form the that nucleus if the temperature is more than the binding energy divided by these factors over here, binding energy per nucleon B/(A-1) is the binding energy per nucleon.

So it is essentially determined by the binding energy per nucleon, so you have if the temperature only if the temperature is adequately low compared to the binding energy per nucleon then you will form that nucleus above that temperature the temperature is adequately high to disrupt that nucleus okay, so I hope this point is clear.

And so this is the fact this is the basic consideration when we have to if you have to consider the formation of different nuclei in thermal equilibrium, if you have thermal equilibrium and this kind of a process then this is the guiding consideration and the value of this temperature for the different nuclei of our interest here, the value of this temperature is weakly dependent on omega baryon h square through epsilon right.

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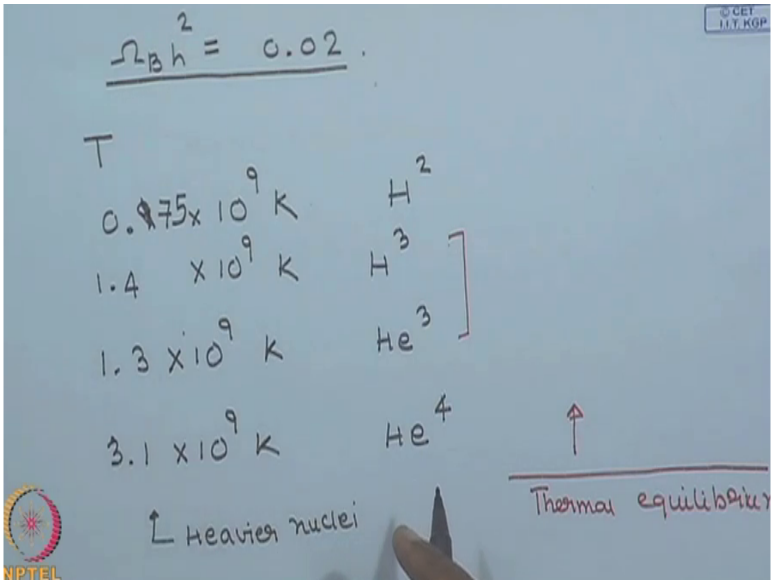

$\Omega_B h^2 = 0.02$.

T

0.975×10^9 K $\quad H^2$

1.4×10^9 K $\quad H^3$

1.3×10^9 K $\quad He^3$

3.1×10^9 K $\quad He^4$

Heavier nuclei

Thermal equilibriu

So the value of this temperature is weakly dependent on omega baryon h square only logarithmically for omega baryon h square =0.02 which is the fiducial value that we shall be considering throughout, for this value this temperature T has the following values, so it has the values 0.97*10 to the power 9 Kelvin for deuterium, so you will not form significant fraction amounts of deuterium at temperatures above this.

It is only when the universe cools to temperatures below this that you will form any significant amount of deuterium that is the basic information carried in this. Similarly, you have the value 1.4*10 to the power 9 Kelvin for tritium and 1.3*10 to the power 9 Kelvin for helium 3 and you have 3.1*10 to the power 9 Kelvin for helium 4, the binding energy per nucleon is more or less same for remains more or less constant for heavy nucleons heavier nuclei.

So this heavier nuclei also you may take a value which is somewhere around here, so if you had nuclear these heavy this nucleosynthesis taking place in thermal equilibrium you would first form helium 4, so if nucleosynthesis were taking place in thermal equilibrium let me just put it here, if nucleosynthesis were taking place in thermal equilibrium you would first form helium 4, the whole process would go this way.

Because as the universe cooled you would first form helium 4 which has the highest binding energy per nucleon and after that when the universe cooled further you would form helium 3 and tritium you would form these 2 and the tritium would then decay to helium 3 and you would then finally form deuterium, so it would proceed in this way in thermal equilibrium if the universe were in thermal equilibrium.

Let me state this again, you would first form helium 4 as the universe cooled we would first form helium 4, then you would form helium 3 and tritium, the tritium would decay to helium 3 subsequently and finally would form deuterium, now what does the nucleosynthesis actually takes place this way, well it does not why? Well the problem with this picture is that the density of the universe is not sufficiently high.

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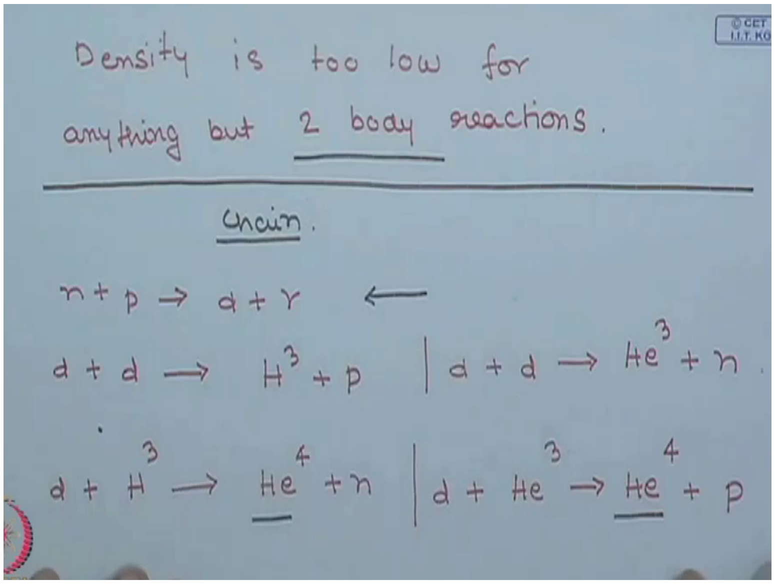

Density is too low for anything but 2 body reactions.

Chain.

$n + p \rightarrow d + \gamma \quad \longleftarrow$

$d + d \longrightarrow H^3 + p \quad \Big| \quad d + d \longrightarrow He^3 + n$

$d + H^3 \longrightarrow \underline{He}^4 + n \quad \Big| \quad d + He^3 \longrightarrow \underline{He}^4 + p$

So the density of the universe during these epochs, the density of the universe is too low for anything but 2 body reactions to be in thermal equilibrium, so you have to build up the entire nucleosynthesis from only from 2 body reactions not more than that and obviously something like this that we started off with or anything else that you can imagine is not a 2 body reaction it is a many body reaction.

So you have to build up whatever heavier element starting from protons and neutrons whatever elements heavier elements you can from a chain of 2 body reaction, so you have a chain of 2 body reactions which takes place, so there is a chain, the chain starts off with the neutron and proton combining to form a deuterium + giving out photons, so this is the first step in the chain.

The next step in the chain is that deuterium + deuterium so they can give rise to a tritium +a proton this is one possibility, the other possibility is that deuterium + deuterium can also give rise to helium 3+a neutron, so this is the next step in the reaction and the final step is you have a deuterium + a tritium giving you helium 4+a neutron, so the extra neutron when you form helium 4 comes out over here.

And the other possibility is that the helium 3 produced here reacts with fuses with deuterium and gives you a helium 4 and in this case you have an extra proton which comes out, so this is the chain of reactions and this so this is the chain of reaction then finally you are left with helium 4, now this is all fine.

So you would expect that these reactions to start off but the only problem is that neutron the first step in this reaction is this, this reaction is in thermal equilibrium, so let me write down the reaction again.

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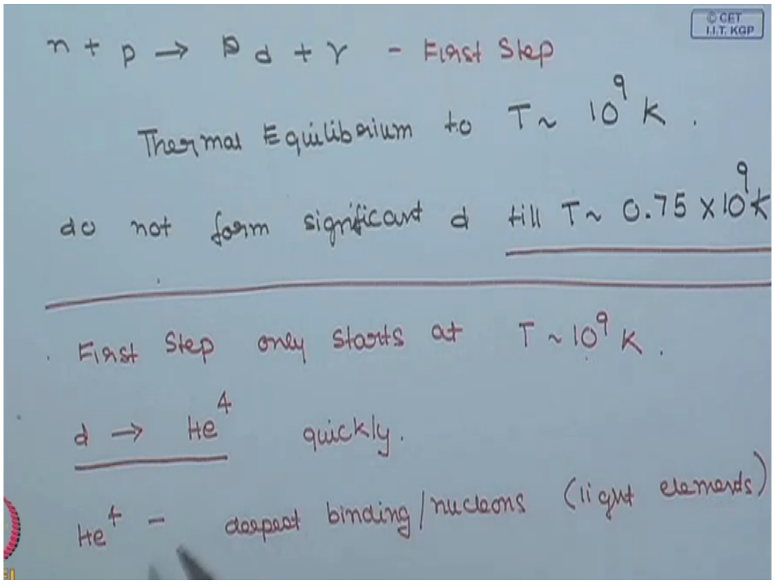
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$n + p \rightarrow \cancel{p}\, d + \gamma$ - First Step

Thermal Equilibrium to $T \sim 10^9$ K.

do not form significant d till $T \sim 0.75 \times 10^9$ K

First Step only starts at $T \sim 10^9$ K.

$d \rightarrow He^4$ quickly.

He^4 - deepest binding / nucleons (light elements)

The first step is neutron + proton going to deuterium +a photon this reaction is in thermal equilibrium all the way to temperatures of the order of 10 to the power 9 Kelvin and we have just seen that you will not form significant amounts of deuterium till the temperature drops to 0. sorry we should be 0.75 you will not form significant amounts of deuterium till the temperature drops to 0.75 Kelvin which is the problem.

So you do not form deuterium till temperature of the order of 0.75*10 to the power 9 kelvin, so the basic point is you do not form significant amounts of deuterium till the temperature falls below this because that is decided by the binding energy per nucleon and deuterium has a rather low binding energy per nucleon which is why you have to wait till the universe cools to this value till you can form significant amounts of deuterium.

So you have to the universe has to wait, so the basic thing is that the neutrons and protons are there initially at thermal equilibrium and then they go out of the equilibrium at 3*10 to the power 10 Kelvin, subsequent to that they have they cannot form the first step in this chain cannot take place till the temperature reaches a value of around so that the first step only starts at temperature of 10 to the power 9 kelvin.

Where you form significant amounts of deuterium through this significant amounts of deuterium cannot form much before this that is the basic problem and once you form deuterium the other steps go through very quickly, so from deuterium to helium 4, so the whole of nucleosynthesis only starts at the temperature of 10 to the power 9 Kelvin and deuterium to helium 4 goes through quickly once this starts.

And helium 4 has the deepest binding energy per nucleon for the light elements, so the nucleons whatever are available the neutrons and protons whatever neutrons are available at this temperature they quickly get bound.

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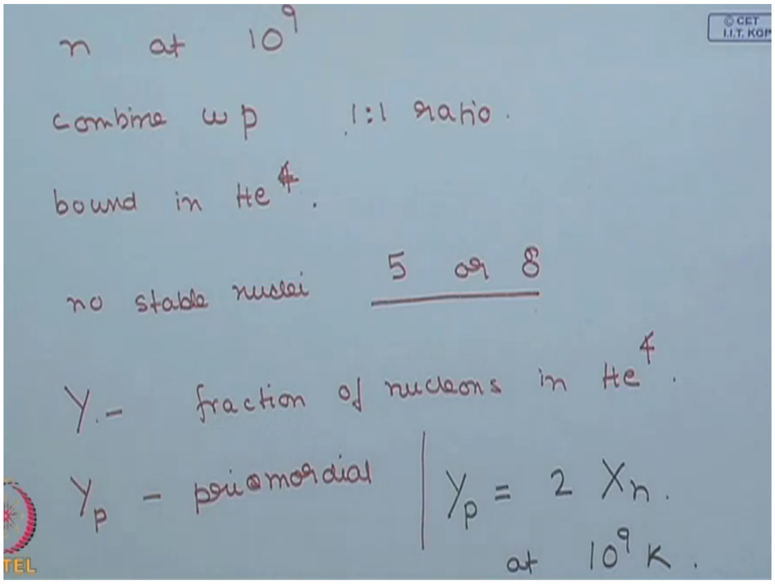

n at 10^9

combine w p 1:1 ratio.

bound in He^4.

no stable nuclei $\underline{5 \text{ or } 8}$

Y.- fraction of nucleons in He^4.

Y_p - primordial | $Y_p = 2 X_n$. at 10^9 K.

So the neutrons n at 10 to the power 9 combined with protons 1:1 ratio and they are all bound up in helium 4, they all get bound in helium 4 and the fusion the nucleosynthesis does not proceed beyond this because there are no stable nuclei of mass number 5 or 8, so it kind of this is where it goes up and so the fact that the neutrons and protons combine at a ratio 1:1, essentially means that the fraction of nucleons.

So we shall use Y to denote the fraction of nucleons in helium 4 and YP is the primordial, by primordial we mean after the Big Bang nucleosynthesis, helium 4 can also be produced in stars, so primordial refers to the fraction of nucleons in helium 4 before star formation takes place, so this is the primordial value before star formation takes place and the primordial value YP is essentially twice the fraction of nucleons in neutrons because every neutron combines with 1 proton.

So you have helium 4 has 2 protons and 2 neutrons and so the ratio is 1:1, so the fraction of helium 4 is essentially twice the fraction of neutrons at a temperature of 10 to the power 9 Kelvin, so this decides the primordial fraction of helium 4 and we can use this to calculate the primordial fraction of helium 4, let me do that here, so we have seen that the neutron fraction is given by this formula over here.

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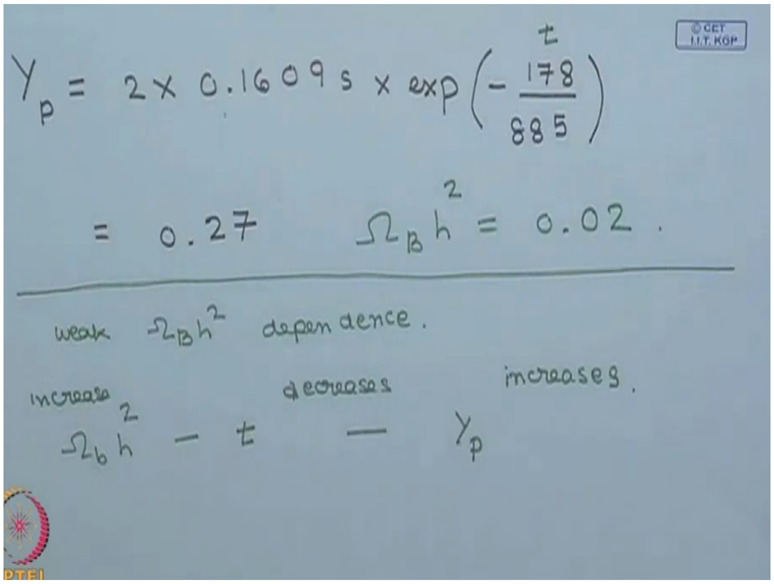

$$Y_p = 2 \times 0.1609\,\mathrm{s} \times \exp\left(-\frac{\overset{t}{178}}{885}\right)$$

$$= 0.27 \qquad \Omega_B h^2 = 0.02\,.$$

weak $\Omega_B h^2$ dependence.

$$\overset{\text{increase}}{\Omega_b h^2} - \overset{\text{decreases}}{t} \quad - \quad \overset{\text{increases.}}{Y_p}$$

So the primordial fraction of helium 4 is given by 2*0.1609 seconds*exponential we need to find the time the age of universe when the temperature is 10 to the power 9 Kelvin and this is something that we have worked out, we have worked out the age of universe and the relation between the age of universe and the temperature a few lectures ago, so let me remind you, so at a temperature of 10 to the power 9 kelvin this will be 178 seconds so -178/885.

So this gives us the primordial helium abundance and this turns out to be 0.27 for omega baryon h0 square, so this is the primordial helium abundant helium 4 abundances that we expect from these considerations of the Big Bang nucleosynthesis and these values are for omega baryon h square=0.02, now this number will change if we change omega baryon h square it has a very weak omega baryon h square dependence.

Omega baryon h square dependence essentially comes in through this epsilon which has a dependent on omega baryon and h square, so let me also show you, so epsilon over here has a dependence on omega baryon on h square and this is what introduces the omega baryon, so this is as a consequence of this the temperature where the deuterium fraction becomes significant depends on the omega baryon h square and consequently the time.

So the neutrino, the neutrons decay the time that the neutrons have to decay before the deuterium fraction builds up also depends on omega baryon h square and if you increase omega baryon h square then the time also decreases, so if you increases this t decreases and the primordial helium fraction increases, but this is a very weak dependents this is a point that I should mention and it is the modern value of the numerical computation.

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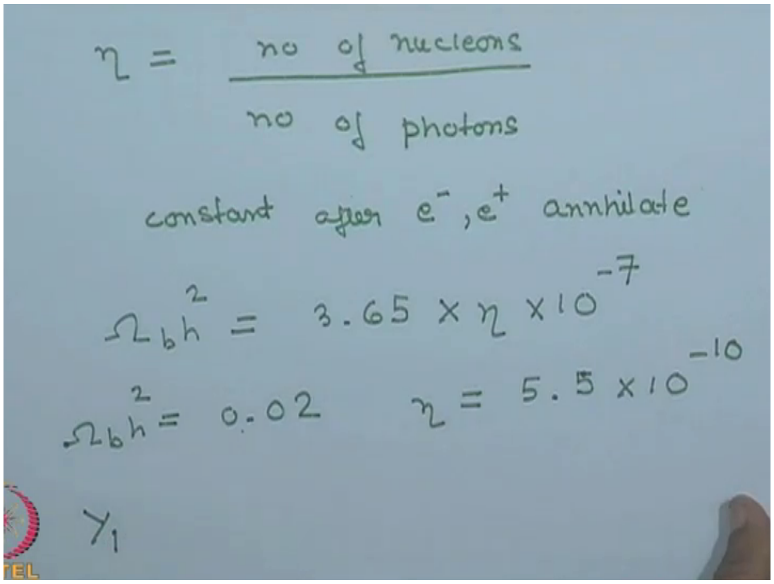

$$\eta = \frac{\text{no of nucleons}}{\text{no of photons}}$$

constant after e^-, e^+ annihilate

$$\Omega_b h^2 = 3.65 \times \eta \times 10^{-7}$$

$$\Omega_b h^2 = 0.02 \qquad \eta = 5.5 \times 10^{-10}$$

Y_1

So this entire exercise actually to get the precise answers you have to do it numerically putting in all reactions rates and the modern values are usually presented in terms of eta and eta is the ratio of the number of nucleons to the number of photons, we have calculated this ratio several lectures ago and this ratio let me remind you is constant with the expansion of the universe, after electron and positron annihilates.

So it is a very convenient way of parameterizing the baryon density and this is related to the baryon density omega baryon h square as this is =3.65*eta*10 to the power-7 and so that is the relation and it so the fiducial value that we have been considering corresponds to eta =5.5*10 to the power-10.

So remember that the ratio of the nucleons to photons is extremely small, so if you have omega baryon h square 0.02 it means in terms of eta 5.5*10 to the power -10. Similarly, value of eta so we have let me give you the values of the primordial helium abundance and you can see that eta and omega baryon h square are directly proportional.

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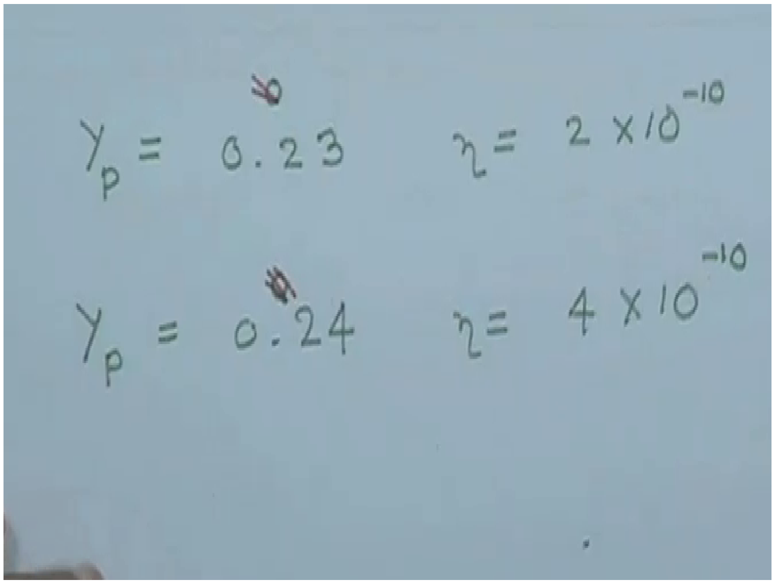

$Y_p = 0.23 \qquad \eta = 2 \times 10^{-10}$

$Y_p = 0.24 \qquad \eta = 4 \times 10^{-10}$

So the results of the numerical computation of the how the abundance of helium they tell us that YP in the primordial helium abundance has a value 0.23, for eta =2*10 to the power-10 and this =0.24, eta =4*10 to the power-10, so we see that the primordial helium abundance the nucleosynthesis the results of other nucleosynthesis calculations show that the primordial helium abundance is only a very weakly dependent on the baryon on fraction.

And you can see over here that it only changes from 0.23 to 0.24 sorry 0.023 to 0.024, if the value of eta is doubled which essentially also boils down to saying that if the value of omega baryon h square is doubled, now I have told you that observations indicate that the ratio of the primordial that the helium abundance is more or less uniform across the universe, these observations are essentially spectroscopic observations.

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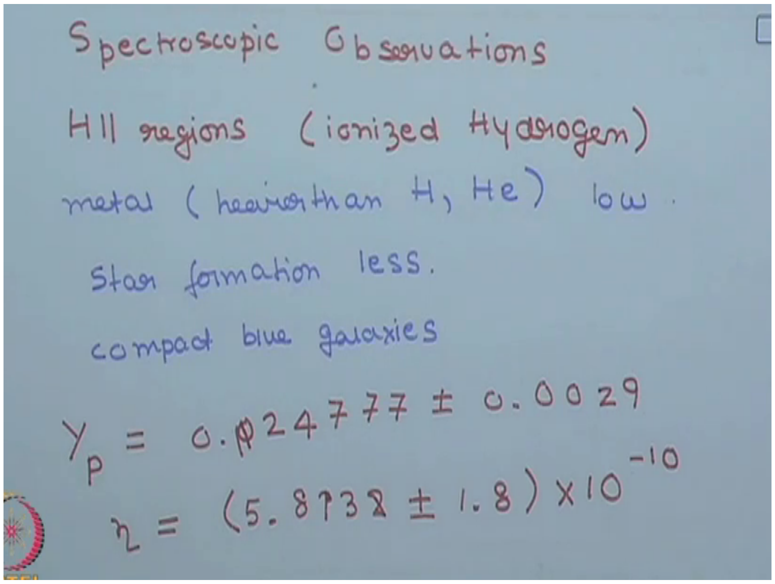

Spectroscopic Observations

HII regions (ionized Hydrogen)

metal (heavier than H, He) low.

Star formation less.

compact blue galaxies

$$Y_p = 0.\cancel{0}24777 \pm 0.0029$$

$$\eta = (5.8132 \pm 1.8) \times 10^{-10}$$

So these are let me just tell you about what the observations tell us, so these are essentially spectroscopic observations of H2 regions, so H2 essentially H2 regions means regions of ionized hydrogen and one usually chooses regions where the metal the metallicity, so the metal these are elements heavier than hydrogen and helium they are all in astrophysics referred to as metals, so metals have low abundance.

So the helium abundance is measured by looking at H2 regions that is regions of ionized hydrogen where the metal abundance is low this indicates the fact that mental abundance is low indicates that star formation has not fully taken place, so star formation is less we would like to look at regions where the star formation has been less, because then it will give us an estimate of primordial helium abundance, stars also produces helium in the nucleosynthesis we have already learnt about this when we studied stars.

So typically preferably one looks at these compact blue galaxies, these are galaxies where star formation has not fully taken place as yet and these observations indicate that the primordial helium abundance has a value 0.02 sorry the helium abundance let me correct this the abundance is 0.23 correct there should be no 0 here, it is roughly one 4th so the helium abundance is 0.24777+-0.0029 which essentially tell that eta has a value 5.8132+-1.8*10 to the power-10.

So we see that the observations are more or less consistent with the predictions of the theory and they predict they indicate that eta has a value 5.81+-1.8 10 to the power-10, what we see from this is that the primordial helium abundance is not very sensitive to the baryon density present value of the baryon density it is rather weakly sensitive to the present value of the baryon density, however, so it cannot tell us much about the present value of the baryon density.

But it tells us that the overall picture that we have of how the light elements were synthesized is more or less is correct, so the picture which was proposed by Gamow and worked out by a variety of people later on is gives a good explanation for the observed light element abundances.

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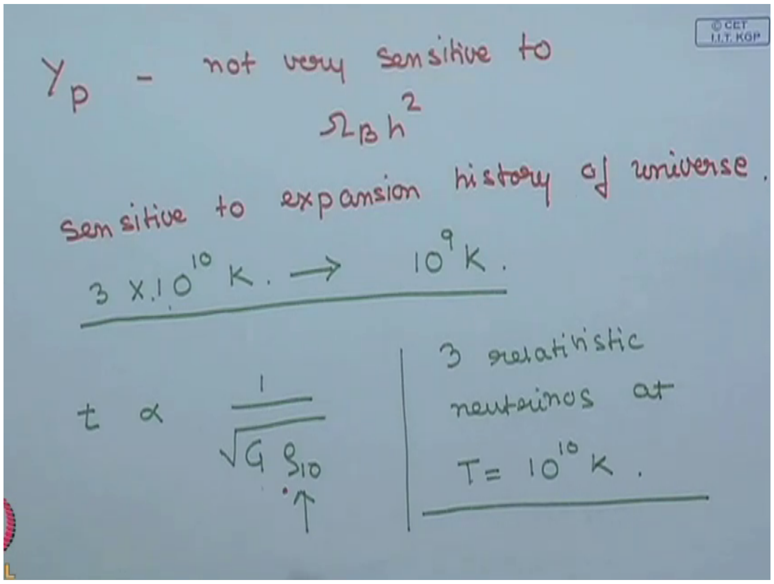

Y_p - not very sensitive to $\Omega_B h^2$

sensitive to expansion history of universe.

3×10^{10} K. → 10^9 K.

$t \propto \frac{1}{\sqrt{G \rho_{10}}}$ ↑

3 relativistic neutrinos at $T = 10^{10}$ K.

Now before moving on to something else I should also mentioned that the YP helium abundance is not very sensitive to omega baryon h square. However, it is sensitive to expansion history of universe between the a time when the neutrons start to decay, so the neutron start to decay at 3*10 to the power 10 Kelvin and nucleosynthesis starts at 10 to the power 9 Kelvin and the helium abandoned is essentially very crucial to the expansion history during these epoch why?

Well the helium abundance in a nutshell the helium abundance is decided by the following criteria, the neutrons start to decay at this epoch and they decay till this temperature is reached whatever neutrons are there at this temperature are all converted into helium which is what this formula is essentially all about, so the question is how much time did the neutrons have to decay between here and here.

What was the time available for the neutrons to decay? Because that is what will come here is actually the time elapses between these 2 epochs, we have put in the value over here but this value will change if you change the expansion history of the universe, so this is so the helium abundance is extremely sensitive to this and that time we know we have calculated this is proportional to 1/square root of G rho 10 where rho 10 let me remind you is the density of the universe at the temperature 10 to the power 10 Kelvin.

So the helium abundance puts quite stringent limits on the value of the density of the universe here and it tells us that there are at most 3 relativistic neutrinos at a temperature of 10 to the power 10 Kelvin. So let me summarize what we have learnt today, what we have learnt today is that nucleosynthesis essentially starts at a temperature of 10 to the power 9 Kelvin at temperatures hotter than this you cannot form significant amount of deuterium for nucleosynthesis to take place.

You can first form deuterium significant amounts of deuterium at a temperature of 10 to the power 9 Kelvin and the moment you formed deuterium the rest of the reaction chain of reactions proceeds very quickly, so it quickly takes place at 10 to the power 9, so whatever neutrons you have at 10 to the power 9 are all quickly bound into helium.

So the helium abundance is essentially determined by the neutron abundance at the time that when the universe is 10 to the power 9 Kelvin and so this is and it is twice the neutron fraction at that epoch this is essentially it and this the helium abundance does not depend on the baryon density very sensitively, but it depends on the expansion rate of the universe at the epoch between the epoch when the neutron and proton where out of thermal equilibrium to the epoch when deuterium formed a nucleosynthesis started.

So we now have a very good picture of how the light elements were synthesized, I have told you how helium was synthesized and what it tells us, so what observations that the observation are consistent with this, in the next lecture I sell briefly touch upon the other elements particularly deuterium and then finish over there, so let me bring today's lecture to a close over here.