

Astrophysics & Cosmology
Prof. Somnath Bharadwaj
Department of Physics and Meteorology
Indian Institute of Technology- Kharagpur

Lecture - 38
Thermal History: Neutrino Mass, Nucleosynthesis

Good morning and welcome to today's lecture. We have been discussing the thermal history of the universe and at the end of the last class, we were discussing the possibility of the neutrino having a mass. So let us resume that discussion and let me remind you of what we were discussing.

(Refer Slide Time: 00:50)

Massive Neutrino

$$n_\gamma = 420 (1+z)^{-3} \text{ cm}^{-3}$$

$$n_\nu = \frac{3}{4} n_\gamma \quad \text{if } T_\nu = T_\gamma$$

$$n_\nu = \frac{3}{4} \left(\frac{4}{11}\right) n_\gamma$$

$$n_\nu = \frac{3}{11} n_\gamma$$

$$n_\nu = 113 \times 10^6 \text{ m}^{-3} (1+z)^{-3}$$

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma$$

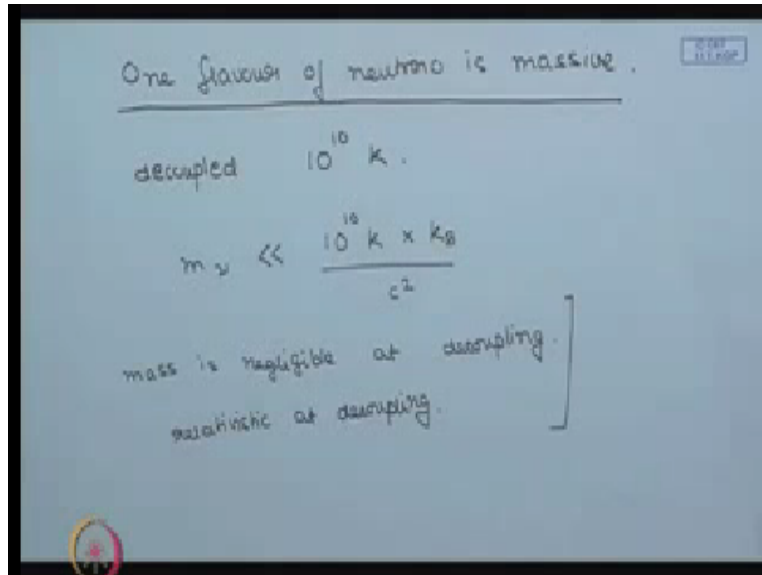
$$n_\gamma \propto T^3$$

So we started off by noting the fact that the photon number density in the universe is 420 photons per centimeter cube at present and the number density scales as $1+z$ to the power 3. Then we remember that the neutrino number density is lower than the photon number density and since the neutrinos are fermions, if the temperature were the same, then the number density of neutrinos would be $3/4$ for every species, every flavor of the neutrino, the number density would be $3/4$ times the number density of photons if the temperature was same.

But the temperature also is lower. So finally, the number density of neutrinos is $3/11$ of the number density of photons. So in addition to the photon background, we also have a neutrino background and each flavor of neutrino has a number density of approximately 113×10^6 to the

power 6 per meter particles per meter cube and the whole thing scales as $1+z$ to the power 3. So given this number density of neutrinos which is present in the universe, we were considering the possibility that one of the neutrino flavors, one of the neutrinos has a mass.

(Refer Slide Time: 02:30)



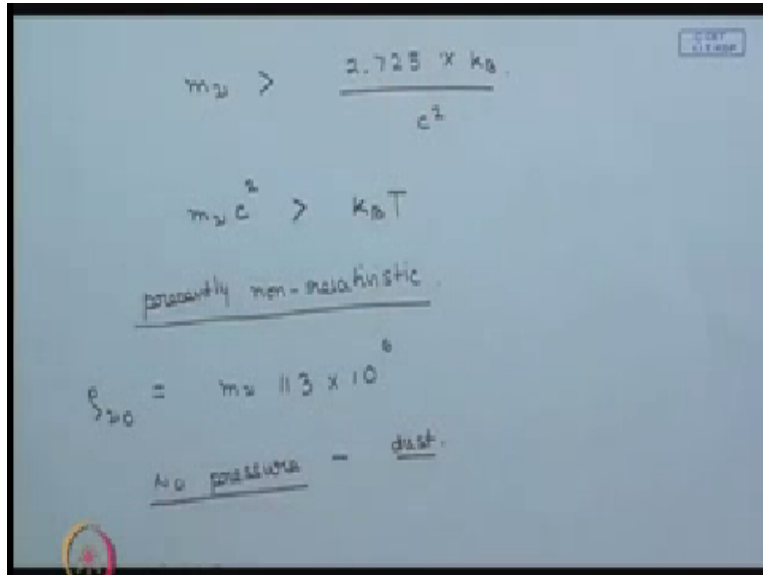
So one flavor of neutrino has a mass. This is the assumption that we had made and we had also assumed that the mass is much smaller than the temperature scale where the neutrinos decouple. So the temperature scale where the neutrinos decouple, we had seen is around 10 to the power 10 Kelvin. So if you convert this to mass scale multiplied by the Boltzmann constant, this will give you energy divided by C square, it will give you a mass scale.

So the mass of the neutrino is assumed to be much smaller than this number. If this holds then the fact that the neutrino has a mass, may be completely ignored at the time where the neutrinos decouple from the rest of the components of the universe. So at the time of decoupling, the number density of neutrinos would be exactly the same as this except that the redshift would be different.

And with the expansion of the universe, the number density, the phase space distribution of neutrino would be frozen and all that would happen is that each momentum would scale, would go down inversely with the expansion of the universe, we have studied this. Once the neutrinos decouple, all that happens is that the neutrinos free stream and the density in phase space is frozen

and each momentum essential scales as $1/a$.

(Refer Slide Time: 04:17)



Handwritten notes on a blue background:

$$m_\nu > \frac{2.725 \times k_B}{c^2}$$
$$m_\nu c^2 > k_B T$$

presently non-relativistic

$$\rho_{\nu 20} = m_\nu 113 \times 10^6$$

No pressure - dust.

So that is what would happen. Now the mass that we have considered, although it is much smaller than the energy scale 10 to the power 10 Kelvin, we have assumed that it is more than the energy scale at present. So it is more than the present energy scale which should actually be the neutrino temperature which is somewhat lower but it really does not matter. So we are assuming that the mass is more than the present energy scale.

So the mass of the neutrino manifests itself sometimes as the universe expands. And at present, we assume that the neutrino is, the rest mass energy of the neutrino is much larger than the kinetic energy. So at present, the neutrino is a non-relativistic particle, its energy is dominated by the rest mass and since the energy is dominated by the rest mass, we can calculate the mass density of the neutrino also. It was just the mass of each particle into the number density of particles.

So this is the mass density of that massive neutrino species at present which is predicted for such a neutrino and these particles, their random motion is also going to be negligible. Essentially the energy is dominated by the rest mass. So they do not contribute significantly to the pressure. You can think of them as pressure less dust which we have encountered earlier. So this is the consequence of there being 1 neutrino flavor which has got a mass.

So you will have a neutrino density like this, it predicts that the universe will now have a neutrino density, a matter component which has density given by this. Now let us ask the question, are there any observational constraints on this. Can the universe have been at present, can there be such a matter density in the universe coming from neutrinos.

Well, there are observations some of which we have discussed and some which we have not, but these observations tell us that the matter content of the universe, Ω_m is of the order of unity or less basically.

(Refer Slide Time: 06:46)

$\Omega_m \lesssim 1$ or less expansion of universe
 $\rho_m \leq \rho_c \Omega_m$ | $\rho_c = \frac{3 H_0^2}{8 G}$
 $m_\nu \cdot 113 \times 10^6 \text{ m}^{-3} \leq 1.88 \times 10^{-26} h^2 \text{ kg/m}^3 \Omega_m$
 $m_\nu \leq 1.7 \times 10^{-384} \text{ kg } \Omega_m h^2$
 convert to eV $\times (c^2/e)$
 $m_\nu \leq 94 \text{ eV } \Omega_m h^2$

So it would be more appropriate to put a, so the matter content of the universe at present is predicted to be $< \Omega_m$, at present is predicted to be < 1 , we have seen that there is dark energy, etc. So there are observations from the expansion of the universe and various other observations, which indicate that the density parameter of matter in the universe at present is ≤ 1 .

Which essentially tells us that if one species, one flavor of neutrino is massive, then the density of that neutrino should be \leq the critical density of the universe or we can say that this should be $= \rho_c \Omega_m$. So, we have worked out what this would be in terms of the neutrino mass. This essentially tells us that the neutrino mass $M_\nu \cdot 113 \cdot 10^6 \text{ m}^{-3}$

should be \leq the critical density of the universe.

The present value of the critical density of the universe is a 1.8. We know that the critical value is 3 of the density is $3H_0^2/8G$ and if you right H_0 in terms of hundred small h kilometres per second per megaparsec, then what this gives you is the critical density at present is $1.88 \times 10^{-26} \text{ KG H square}$. This H square comes from here KG per metre cube, that is the critical density.

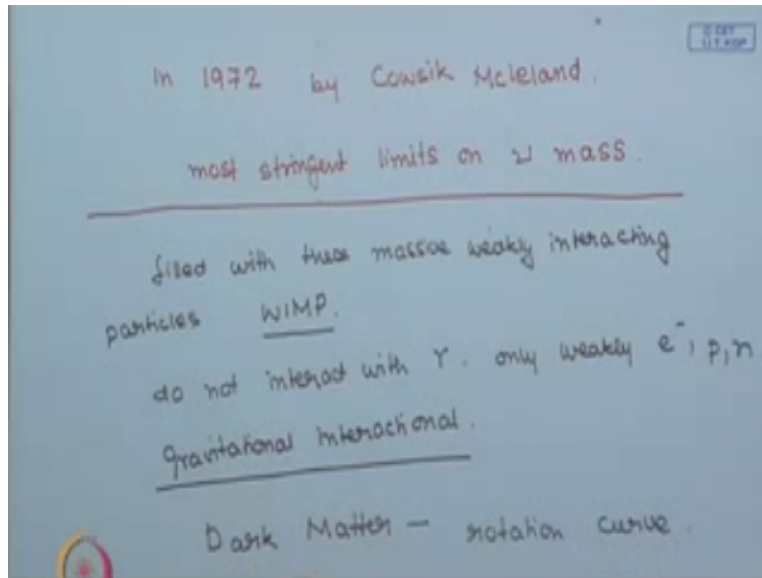
So, the density of neutrinos coming from the neutrinos should be $<$ the matter density in universe. So, the matter density is the critical density into $\Omega_{\text{matter } 0}$. So, what it tells us is that m_{ν} , the mass of the neutrino should be \leq . So, we can divide this by 113×10^6 to get a limit on the mass of the neutrino and what it tells us is that this should be $< 1.7 \times 10^{-34} \text{ KG} * \Omega_{\text{matter } 0} H^2$.

So, this is the constraint that we get on the mass of the neutrino. It is more meaningful to express this in terms of electron volts. So, what we have to do is we have to multiply this by convert this into energy multiplied by C square. So, we have to multiply to convert to EV (electron volts). We have to multiply $\times C^2$ by E, that is what you have to do. So, the limit that you get if you do this is that the mass of the neutrino should be $\leq 94 \text{ electron volts } \Omega_{\text{matter } 0} H^2$.

So, this is the limit that you get on the neutrino mass as a limit from the fact that the density parameter of the mass of the universe should be < 1 . So, from the fact that you know, you can determine the mass parameter of the universe. So, the mass of any neutrino if one of the neutrino species flavour is massive, the mass should be $< 94 \text{ electron volts}$ of the order of 94 electron volts.

So, basically what we find is that cosmological observations of the expansion rate of the universe and the large scale structures can put limits on the masses of fundamental particles. In this case, the mass of the neutrino.

(Refer Slide Time: 12:59)



This very interesting limit, this very interesting idea was first worked out in 1972 by Cowsik and McClelland and when it was initially worked out, this was one of the most sensitive, most stringent limits on neutrino mass. Now, let us ask the question, you see what happens if one of the neutrinos species is actually massive. So, if one of the neutrino species is actually massive, then we have the whole universe at present filled with these massive weakly interacting particles.

Which are also referred to as WIMP. So, these are particles which do not interact with photons and interact weakly with electrons and baryons. So, these such particles would essentially be only visible manifestation of such particle would be through the gravitational force, gravitational interaction. The gravitational field, the gravitational interaction produced by such particles would be just like any other particle, but these particles would not emit light.

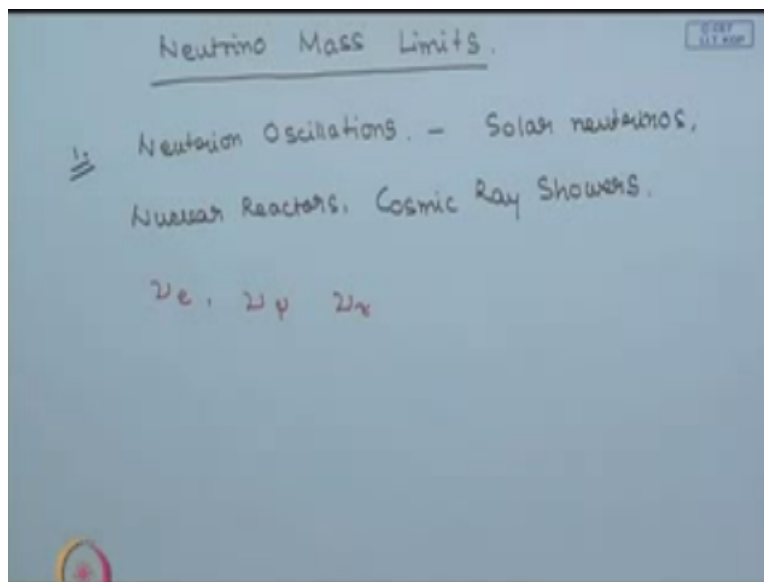
They would not interact with photons and they would only interact very weakly with electrons and protons and neutrons. So, if we had a sea such particles floating around which would happen if one of the neutrinos were massive, this would be what a very good candidate for what is called dark matter, and we have already discussed what evidence for dark matter that is the rotation curve of galaxies.

There are a variety of other observational evidences which indicate that the universe at present has a large component of the matter in the universe is at present made up of this dark kind of

matter which dark matter. So, the question arises are neutrinos the dark matter that we have in the universe. I have told you that there is evidence that around 25% of the matter of the constituents of the universe is dark matter.

So, the natural question that arises is that are the massive neutrinos dark matter candidates. Well let us see. This is the limit that we have using the kind of arguments which were given in 1972. Let us now very briefly just look at the current situation. So, let us ask the question what are the current limits on the neutrino mass.

(Refer Slide Time: 17:30)



Well, the first point that I should tell you is that now are very stringent limits on the neutrino mass, very stringent limits which are imposed by various other kinds of experiments and observations.

So, let me briefly just tell you about some of these. So, the first kind of limit comes from observations of neutrino oscillations. So, observations of neutrino oscillations and neutrino oscillations are observed in neutrinos coming from the sun, so solar neutrinos then nuclear reactors and from cosmic ray showers. So, what are these neutrino oscillations. Well I have told you that there are 3 different flavours of neutrinos.

So these are the electron neutrino and the mu neutrino and tau neutrino. So, what happens in

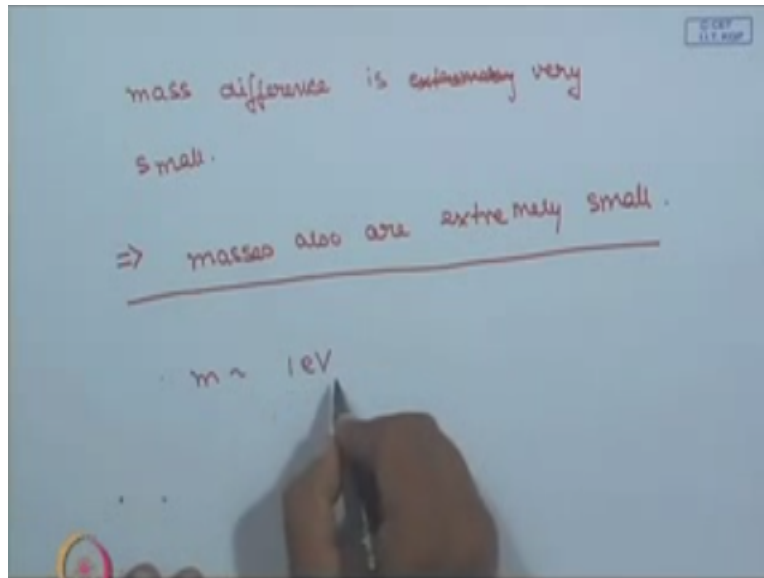
neutrino oscillations is that as the neutrino propagate, if you start off with electron neutrinos it will slowly become mu neutrino and may be tau neutrino and then again, come back to electron neutrino. So, the neutrino oscillates between these 3 flavours and the measurement of neutrino oscillations.

These oscillations have been measured from neutrinos coming from the sun, neutrinos coming from nuclear reactors and also neutrinos coming from cosmic ray showers and the observation of these oscillations impose limits on the mass difference between the mass states of the neutrinos which have well defined mass between neutrinos states with well-defined mass.

These states with well-defined mass do not correspond to electron mu and tau neutrino which is why we have oscillations between these as the neutrino oscillates. So, let us give it a name. Let us call it ΔM^2 . So, these neutrino oscillations tell us that the mass difference between these states has the values. So, there are 2 values, 2 limits; one is 8.0×10^{-4} . These are experimental limits, error parts, and rather $+0.4$ and -0.3×10^{-5} .

So, this is one limit and the other limit that the mass difference should lie in the range -1.9×10^{-3} to 3×10^{-3} . These are all in electron volts square. So, there are 3 different states which have definite mass and the difference in the mass square between these 3 states, one of them is known to be 8 electron volts square, other one is in this range.

(Refer Slide Time: 23:00)

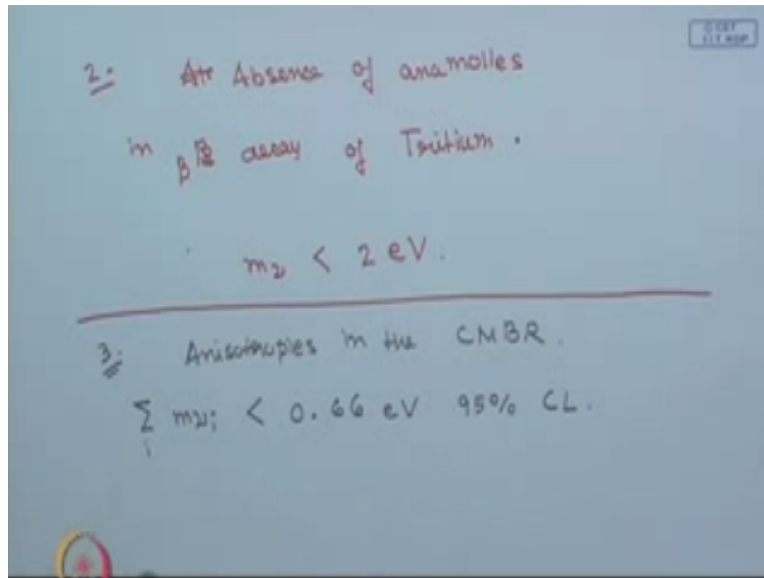


So, these limits tell us that the mass difference between the neutrinos is extremely small and there is no reason the mass difference between different neutrino states is extremely small and one believes that the masses themselves also are extremely small < 1 electron volt. So, of the order of may be if at all there are masses, these put limits that they should be quite small $<$ electron volts.

So, this is interpreted as telling us that the neutrino masses also are extremely small. The other possibility is that the masses are not small but they have an extremely small mass difference. So, even if one neutrino has a mass, let us say, 1 EV or so then the other one will have a mass which is very close to this because the difference in the mass square is of this order into the power -5 . So, if one of the neutrino has a mass, then all 3 of them must have nearly equal masses, that's what it tells us.

The most natural interpretation is that all of them have 0 mass or extremely small mass. So, this is one of the most stringent kind of inputs that we have on the neutrino masses.

(Refer Slide Time: 24:46)

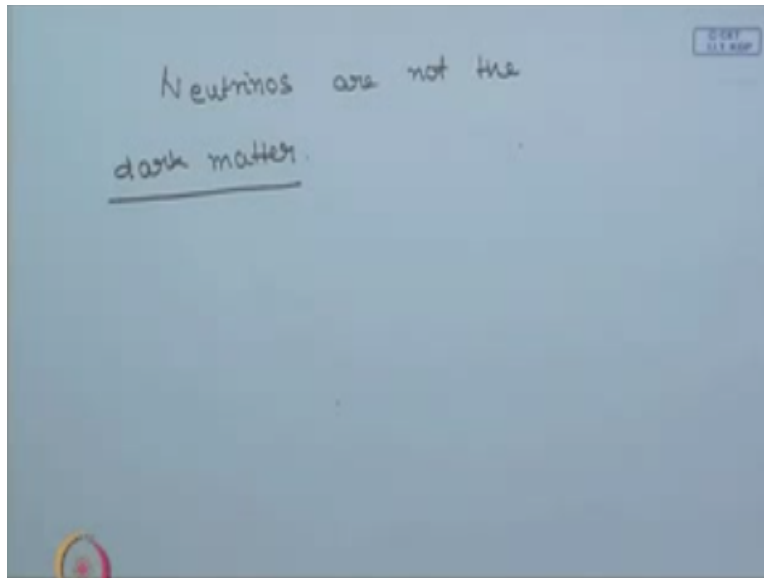


Then, the second input which I can tell you about is that there are absence of anomalies in the beta decay of Tritium. This imposes the limit that the neutrino mass should be < 2 electron volts. The third limit which I can tell you about is of also again cosmological. The 2 which we just talked about are first one. This also involves some amount of astrophysics, the solar neutrinos and the cosmic ray showers. This is purely terrestrial.

The third is from the anisotropies in the cosmic microwave background radiation (CMBR) and this imposes the limit that the sum of the neutrino masses. So, we know that there are 3 flavours. So, the sum of the neutrino masses should be < 0.66 electron volts and this is at the 95% confidence level. So, at 95% confidence, the sum of the neutrino masses should be < 0.66 electron volts.

We just saw arguments why if the neutrinos are massive then all 3 of them must have nearly equal mass because the mass differences have been measured and they are extremely small. So, this imposes a limit that the mass of a single species should be < 0.22 electron volts. So, all of these together seem to indicate that the neutrinos that we have been discussing are not the dark matter.

(Refer Slide Time: 27:40)



So, they do not contribute very significantly to the dark matter. So, the current mass limits and various other observation limits seem to indicate that these neutrinos that we have been talking about the dark matter that we see are not neutrinos, not the kind of neutrinos which we have been talking about.

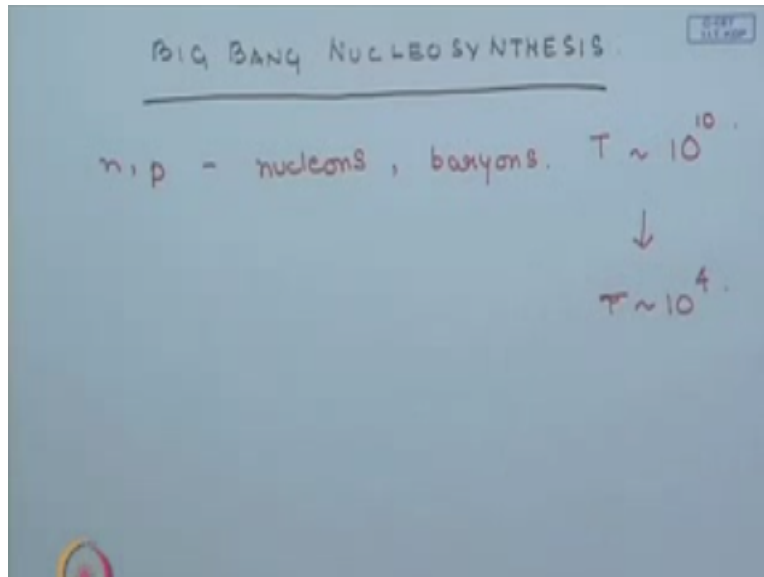
There is another mass window which corresponds to very extremely high mass neutrinos which could be dark matter candidate but that does not fall within the preview of these 3 kind of neutrinos which we have been talking about. So let me bring this topic to a close over here. So, till now what we have been discussing is the possibility that the neutrinos have mass and we saw that astrophysical observations provide quite stringent limits on the neutrino mass.

Which also indicate that the kinds of neutrinos which we have been talking about which decouple from the CMBR at a temperature of around 10^9 K which were in thermal equilibrium before that. These neutrinos are most probably not the dark matter that we see around us, okay. So, the question what is the dark matter really made up of is still an open question and let us now move on to something else.

So, till now our discussion of the thermal history has mainly focused on the photons and then we considered the electron positron annihilation and the neutrinos. Let us now also discuss what happens to the small amount of baryons nucleons that is present and this brings us to be very

interesting topic which is the topic of Big Bang Nucleosynthesis.

(Refer Slide Time: 29:40)

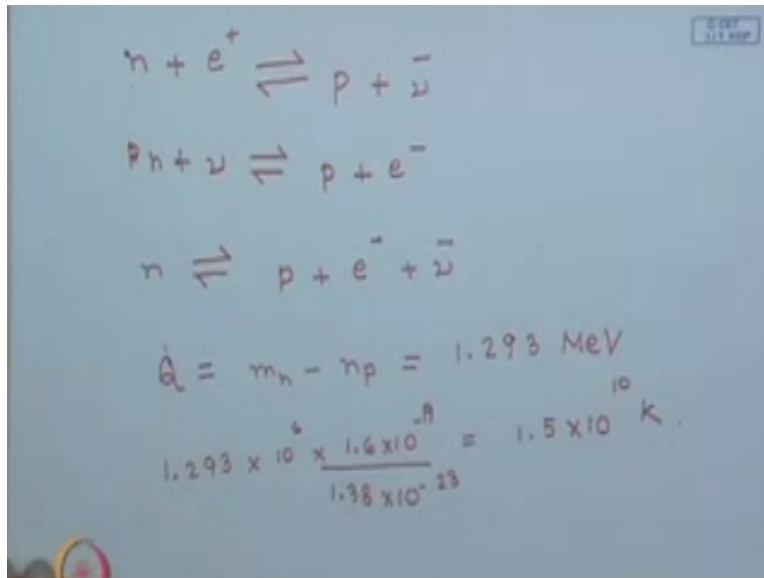


So what we are discussing now is the small amount of neutrons and protons which we are the strongly interacting particles, the nucleons, we shall sometimes refer to them as nucleons and sometimes as baryons which are present in the universe at temperatures of around 10 to the power 10. So, we have seen that 10 to the power 10 to 10 to the power 4 and even higher than 10 to the power 10.

These particles do not play a very significant role in the dynamics of the universe. The dynamics of the universe is mainly governed by the neutrinos and the photons which are relativistic particles. These particles have become non-relativistic and they do not play a very important role in the dynamics of the universe. We have essentially ignored these particles till now and we have worked out the equations that govern the dynamics of the universe in this epoch.

We shall come back to this shortly. Now, let us ask the question what happens to the neutrons and protons and the nucleons in this epoch. Now, in this we know that there are several reactions which can convert neutrons and protons. So, these reactions, let me outline these reactions. These reactions, let me move on to another sheet of papers.

(Refer Slide Time: 32:10)



So, these reactions are neutron. The neutron can interact with positron to form a proton and antineutrino. This is the first reaction. The second reaction is that neutron can interact with neutrino to form a proton and an electron and the third reaction is that the neutron can go over to a proton and electron and antineutrino. So, all of these 3 reactions essentially can convert a neutron to a proton and a proton to a neutron.

At sufficiently high temperatures and densities, these reactions are in thermal equilibrium. Now at the temperatures that we are interested in of the order of 10^{10} K, the neutrons and protons are essentially at rest. The temperature is smaller than the rest mass of the neutrons and protons. They are essentially at rest and the mass difference between these 2 is essentially the difference, the neutron has a larger mass than the proton.

This has a value of 1.28293 MeV and it is also useful to write this in terms of a temperature. So, what we can do is 1.293×10^6 times the charge of the electron which is 1.6×10^{-19} and divided by the Boltzmann constant which is 1.38×10^{-23} and it gives us the temperature scale which is a 1.5×10^4 K. So, the energy difference between the mass difference between the neutron and the proton.

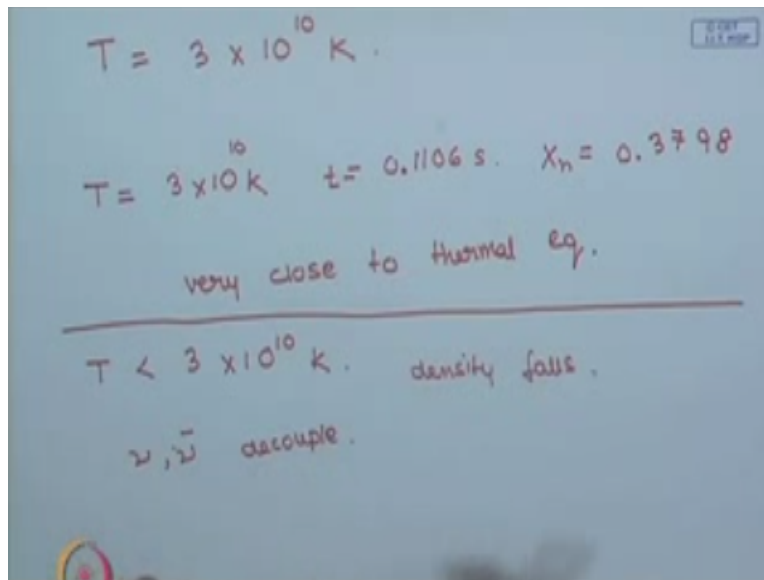
The corresponding temperature corresponding to this scale is of the order of 1.5×10^{10} K which is somewhere in the range that we are talking about. Now, if these reactions are in

thermal equilibrium, then the ratio in thermal equilibrium. In thermal equilibrium, the ratio $X_N/X_P = E$ to the power - the mass energy difference between the neutron and the proton.

We have seen 1.293 mega electron volts divided by $k_B T$. So, now here let me just tells you what X_N and X_P are. X_N for example is the fraction of nucleons in neutrons. Similarly, X_P is the fraction of nucleons and protons and at sufficiently high temperatures of the order 10^{10} to the power 10, these are only 2 kinds of particles that we have and the total sum is 1 which is why we can write it in this way and in thermal equilibrium we can solve and it gives us that X_N the neutron fraction is $1/1 + \text{exponential } Q/k_B T$.

Now, we have discussed already that to determine if these reactions are in thermal equilibrium or not, we have to look at the reaction rates and compare them with the rate of expansion, the Hubble parameter at this epoch and we have already calculated the Hubble parameter. If you remember, we can use this to calculate the Hubble parameter which have done in the last class during the epoch where the temperature is 10^{10} to the power 10.

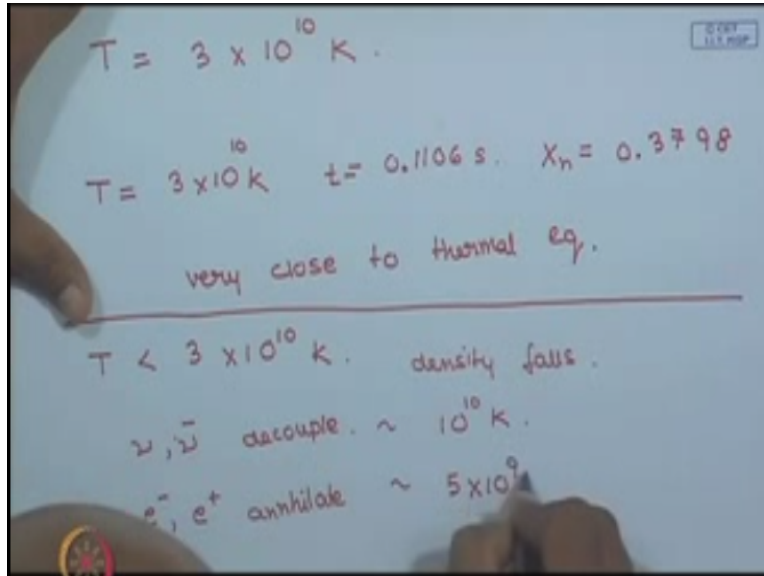
(Refer Slide Time: 39:14)



So, these are all weak interaction, so the reaction rates can be calculated if one knows how to calculate the weak interaction reaction rates and these detailed calculations they show that these reactions remain in thermal equilibrium till temperature for 3 times 10^{10} K. So, these calculations of weak interaction reaction rates, they essentially tell us that this is formula

the neutron fraction can be calculated using this formula till the universe has a temperature of 3×10^{10} K.

(Refer Slide Time: 39:54)



The corresponding age of the universe can be calculated using this and to just give you the feel for the numbers, a detailed calculation of the reaction rates tells us that at a temperature of 3×10^{10} K, numerical calculations, the age of the universe is 0.1106 seconds and the neutron fraction $X_n = 0.3798$ which is very close to thermal equilibrium value which you would get if you were to substitute 3×10^{10} over here.

So, at 3×10^{10} K, the reaction rates are sufficiently high for thermal equilibrium to hold but at temperature below this the $T < 3 \times 10^{10}$ K what happens is that at around the densities fall further, the neutrinos decouple. So, once the neutrinos decouple, they do not interact much with anything else. After that the neutrinos cross section falls and these reactions where protons get converted into neutron or the neutron scatters with the neutrino to form a proton and an electron these no longer take place.

So this act decouples at around 10^{10} K and we also remember that the electron positron annihilate at around 5×10^9 K. So, further the electron density also falls and the electron positron density falls. So these reactions also gets suppressed. So, these

reactions all stop. By and large, the only reaction that continues to happen beyond this is this reaction where the neutrinos get converted into protons.

(Refer Slide Time: 43:16)

$$n \rightarrow p + e^- + \bar{\nu}$$

$$T < 3 \times 10^{10} \text{ K}$$

$$\tau_n = (885.7 \pm 0.9) \text{ s}$$

$$X_n = 0.1609 \times \exp(-t/\tau_n)$$

$$T = 10^9$$

$$X_n = 0.1333$$

very close to

$$t = 168.15$$

So let me write it down, the reaction that continues to occur is that the neutrinos they go over to proton + an electron + the antineutrino. So, we know that the neutron is an unstable particle, a free neutron is unstable particle. So, essentially once it goes out of equilibrium the only reaction that proceeds is that the free neutrons they decay to form the proton, electron and the antineutrino.

This is all that occurs once the reaction goes out of equilibrium. So, once you this 3×10^{10} K essentially the neutrons all decay and get converted into protons. So, this is the basic thing, and this reaction has a lifetime which I denote here by τ_n of 885.7 ± 0.9 seconds. So, this is the lifetime of this neutron once it goes out of thermal equilibrium. So, this free neutron essentially decays with this lifetime.

The neutron fraction X_n in this epoch can be well described by this $0.1609 \times \exp(-t/\tau_n)$. So, this is what happens. So, once the thermal equilibrium is gone, the neutrons decay to form protons and neutrons fraction falls exponentially with the mean lifetime given by this and just to give you a feel for the numbers, remember that we had a neutron fraction of around 0.38 at 3×10^{10}

to the power 10 when thermal equilibrium was last there by a temperature of 10 to the power 9 the neutron fraction falls to 0.133 which again is very close.

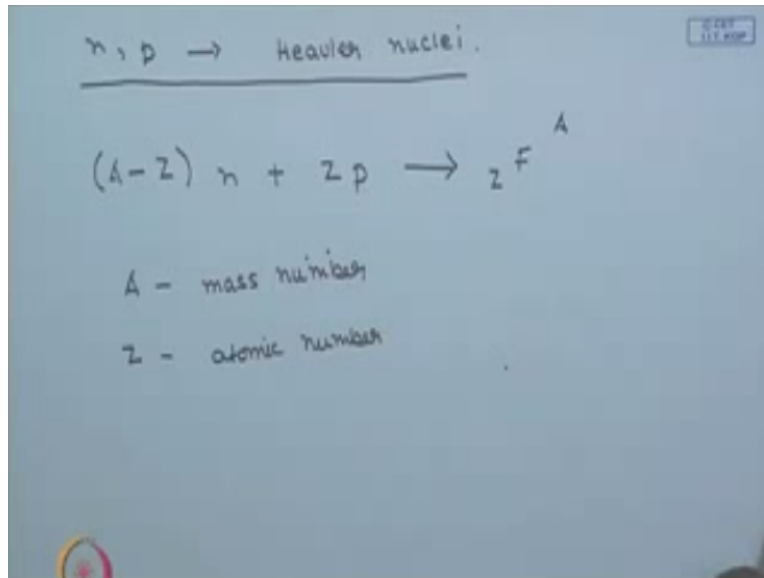
To check this, I should give you the age of the universe over here and the age of the universe here is 168:1seconds at this temperature. So, these are from new detailed numerical calculations of these reactions putting in all the nuclear weak interaction rates etc and it is very close to this exponential decay once the neutrons decouple. So, let me recapitulate what we just learned about the behavior of this of the baryon species.

So, there is an epoch where they are in thermal equilibrium and this is at high temperatures and in this epoch the neutron fraction is well described by this formula where they are in thermal equilibrium and I have also told you the value over here. So, one can use this to calculate the neutron fraction in all the way till 3×10 to the power 10 K.

Beyond that once you cross that value when they are no longer thermal equilibrium all that happens is that neutrons they decay to give protons, electrons and anti-neutrinos and in this regime. So, this is $T < 3 \times 10$ to the power 10 K. This is $T \geq 3 \times 10$ to the power 10 K. Okay, the neutron fraction decays exponentially and it forms the protons. Now, given this background about what happens to your neutrons and protons in this expanding background.

Let us now ask the question and address the issue of the formation of the other elements heavier than these particles during these epochs that we have been discussing. So, the basic idea is a reaction of this kind, first of all the question that we have going to ask is that can the neutrons and positron fuse to form a heavier element, heavier nuclei.

(Refer Slide Time: 50:04)



So essentially in a nutshell, the kind of reaction that we are interested in is like this. So, what happens the kind of process that we are interested in, the question we are asking is that Z protons and A-Z neutrons they fuse they combine to form the nuclei of some heavy element which has got atomic mass number A and atomic number Z. So, this is the question that we are asking. So Z protons and A-Z neutrons they fuse.

They combine to form an element, the nucleus of an element with atomic mass number A and atomic number F. So, this is the process that we are interested in. In the next class we shall consider the possibility that the neutrons and protons fuse together to form heavier elements of atomic mass number A and atomic number Z. So, let me just briefly recapitulate what we have done in today's lecture and stop after that.

So, in today's lecture, we first considered the possibility that neutrinos have mass and after that we worked out the predicted mass density than and we saw just the expansion of the universe puts limits on neutrino mass. After that, we addressed the question and if one of neutrinos does have mass, it will be a very promising dark matter candidate, but after that I showed you that there are very stringent limits now which more or less indicate that the kind of neutrinos which are discussing are not the dark matter candidates.

After that, we shifted our attention to another very interesting issue this is Big Bang Nucleosynthesis. The essential question is that we know for heavy nuclei to be synthesized, we need a very hot and dense environment. The early universe which we have been discussing provides us with such an environment. So, the question we are addressing is what kind of nuclei can be synthesized in the early universe.

We started off with discussing the residual protons and neutrons and we saw that these will be in thermal equilibrium till the universe is hotter than 3×10^{10} K. Once the universe cools below 3×10^{10} K, the neutrons will decay exponentially and I showed you how you can calculate the neutron fraction that is before 3×10^{10} at temperatures hotter than 3×10^{10} and at temperatures cooler than 3×10^{10} .

So let me stop here. In the next class we shall continue our discussion on the Big Bang Nucleosynthesis.