Astrophysics & Cosmology
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Lecture - 37
Thermal History, Expansion Rate and Neutrino Mass

Good morning and welcome to today's lecture.

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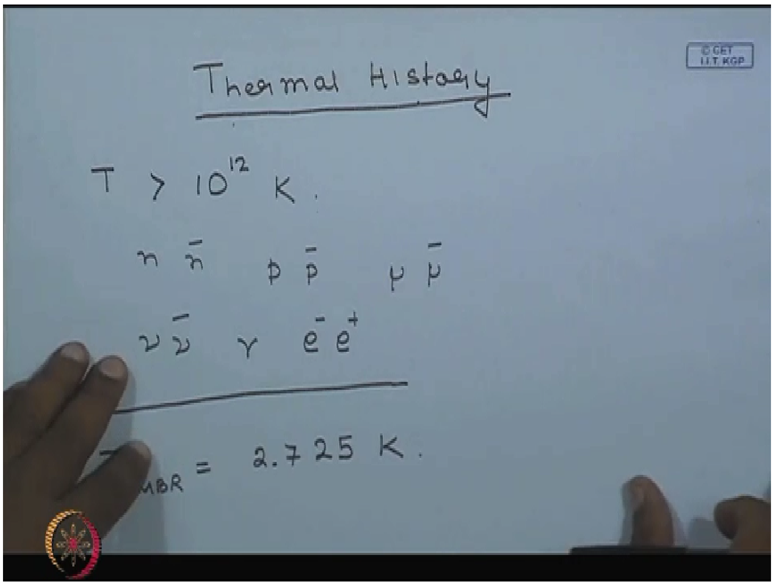

Let me remind you that we have been discussing the thermal history of the universe and at temperature let me just go back a little so at temperature>10 to the power 12 Kelvin we have the neutrons and anti-neutrons, protons, anti-protons, muons, anti-muons and we have the neutrino, anti-neutrinos, photons, electrons and positrons all in thermal equilibrium at these very high temperature 10 to the power 12 Kelvin.

To put things in context the temperature of the universe, the temperature of the CMBR at present is 2.0725 Kelvin. So we are discussing really the early universe where it is extremely hot and we have all of these species in thermal equilibrium that this is just to remind you. Well we are actually not interested in this epoch. If you go to lower temperature the neutron and anti-neutron, proton and anti-proton, the muons they all recombine once the temperature falls below the rest masses the respective rest masses they all annihilate.

And we are here interested going to start from an epoch that is where the discussion has been mainly.

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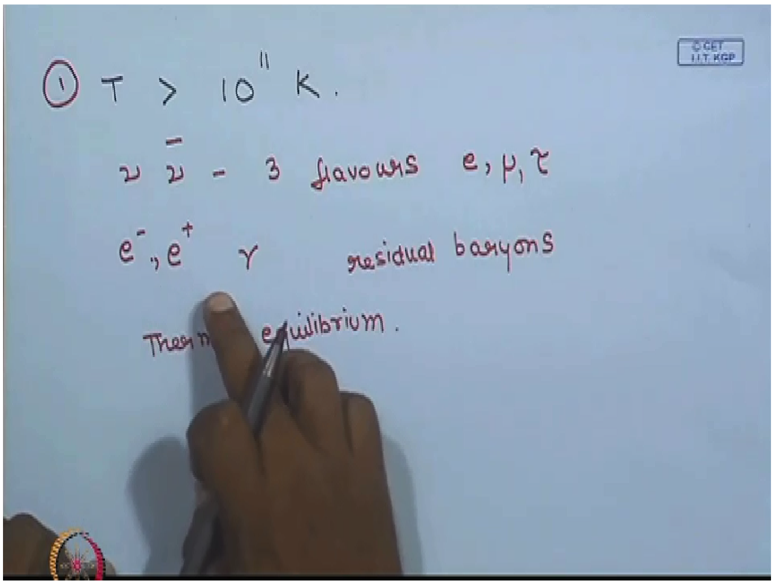

The temperature scale of around ground 10 to the power 11 Kelvin, but lower than the temperature mentioned earlier. So this is where we start. At these temperatures we have got the neutrinos of which there are 3 kinds, 3 flavours. Electron, the mu neutrino and the tau neutrino all three of them. And we have the electron and the positron and the photons all of these in thermal equilibrium and we also have some residual baryons in small quantities all at thermal equilibrium.

So this is the epoch 1 that we are going to discuss today and we have discussed earlier also. So this is the first phase of the universe that we are going to discuss today and we have the second phase. So the universe is expanding and as the universe expands it cools and subsequent of this the temperature this is phase 2.

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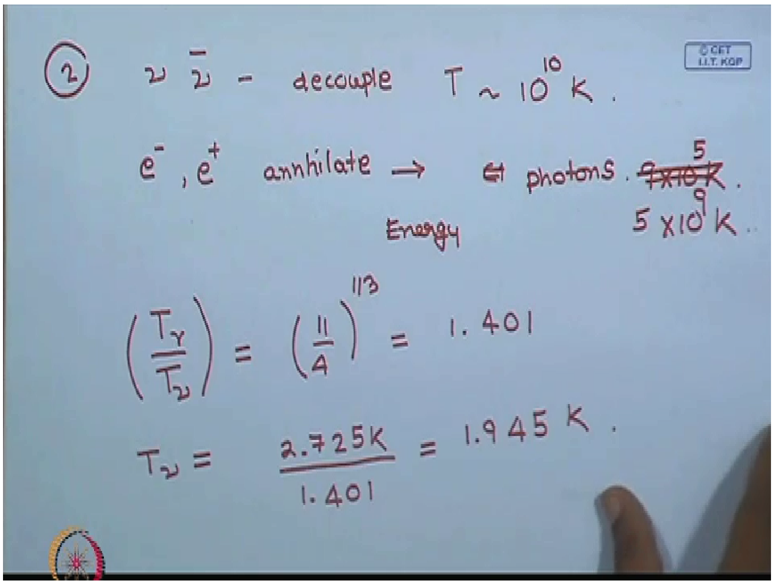

Here as the universe cools the neutrinos decouple. So nu and nu bar they decouple from the rest of the material. They stop interacting with the rest. These are weakly interacting particles and they stop interacting with the rest of the particles at a temperature T of around 10 to the power 10 Kelvin. And subsequent to this the temperature falls below the rest mass of the electron positron pair.

So the electrons and the positrons they annihilate dumping whatever excess energy they had into the CMBR into the photons. So there is extra energy that goes into the photons at around 9 *10 to the power 5 Kelvin we have studied this and as a consequence of this what happens is that the photon energy T gamma is more than the neutrino energy. The neutrinos have decoupled.

So as the universe expands the neutrino free stream and the temperature of the neutrino just scales inversely with the expansion of the universe, but the photons do the same thing till 5 *10 to the power 9 Kelvin somewhere around here where the electron positrons annihilates dumping extra energy into the photons. So the photon temperature goes up relative to the neutrino temperature at this epoch.

And this ratio we have work this out in earlier class it is 11/4 to the power 1/3 and this ratio is 1.401. We have worked out all of these things in the last class. So the neutrino temperature after this after 5 *10 to the power 9 Kelvin is less than the photon temperature and the neutrino temperature at present is 2.725 Kelvin/1.401 which turns out to be 1.945 Kelvin. And this continues.

So the neutrino at present we have a photon background which we see as the cosmic microwave background radiation at a temperature of 2.725 Kelvin and we also have a neutrino background at 1.945 Kelvin. So this is the situation after once the electron position annihilates we are left with a neutrino background and a photon background both of which more or less just the temperature of both of which are different.

And it continues to scale inversely as the scale factor till the present and at present we have both these backgrounds the photon background and the neutrino background. So we have discussed all of these things in somewhat great detail in previous classes. Today, let us ask the question how does the universe expand during these 2 different epoch. One is at temperature before the electron positron pair annihilates.

So at temperature more than 5 *10 to the power 9 Kelvin and second how does the universe what is the expansion rate of the universe how does it expand at temperature below this. This is the question that we are going to work out in today's class to start with. So let us embark upon this. So let us set out with this phase. So we are going to set out with this phase where we have all of this species in thermal equilibrium.

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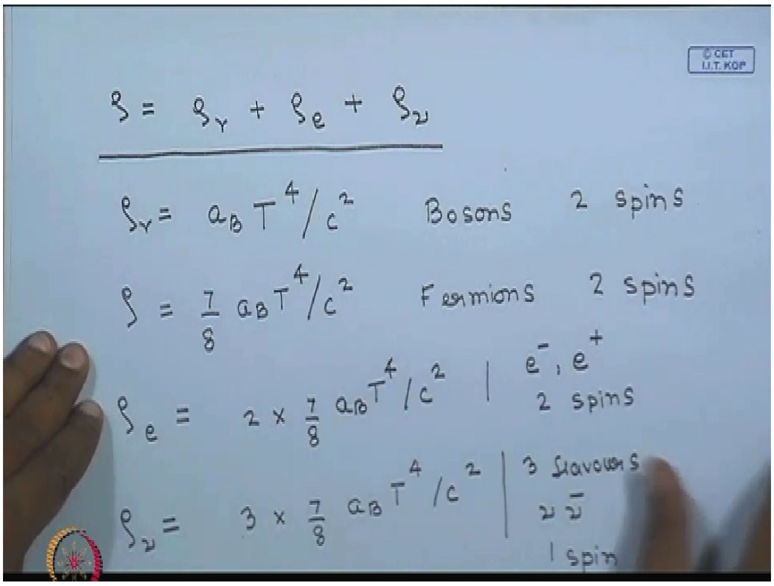

So the total energy density of the universe rho total energy density rho = the energy density of the photons + the energy density of the electron positron pairs + the energy density of all the neutrino 3 different flavours of neutrinos. So this is what we would like to calculate and we have already discuss this to some extent. Let me go through again. So let us first start off with a photon.

So we know that the energy density of the photon rho gamma =Stefan-Boltzmann constant AB T gamma they are all at the same temperature so there is no need to distinguish between the different temperatures. So the photon energy density is the Stefan-Boltzmann constant *temperature to the power 4 and to convert this into energy density we need to divide it by C square.

And so this is the energy density of boson essentially with 2 spins. Photons we know have 2 spins. So the energy density of species of bosons with 0 chemical potential 2 spins is given by this which is exactly what these photons are. Similarly if you have fermions let me remind you that if you have fermions so then these are boson then if you have fermions 2 spins then the energy density is slightly different.

The energy density is down by a factor of 7/8. So this is for fermions in thermal equilibrium at a temperature T the energy density of fermions. So this is the mass density and assuming that the fermions have 2 different spins. So we have the electrons. Electrons we know have 2 different spins. So for the electrons the energy density is given by this much. For the positrons also the energy density is given by the same thing.

Both electrons and positrons have 2 spins each. So we have 2 times 7/8 AB T to the power 4/C square for the electrons and positrons. Let us now take up the neutrinos. The neutrinos only have a single spin. So neutrinos have only 1 spin, but there are the neutrinos and the anti-neutrinos. So that is a factor of that compensates for their being only a single spin and we have 3 flavours of neutrinos.

So the contribution from the neutrinos as you mean they are massless is 3 *7/8 AB T to the power 4/C square. So we have worked out all the contributions. The electrons we have electron and positron which is why we have a factor of 2 here both of which have 2 spins. For neutrinos there are 3 flavours then we have the neutrino and the anti-neutrino, but only 1 spin which is why we just have a factor of 3 over here.

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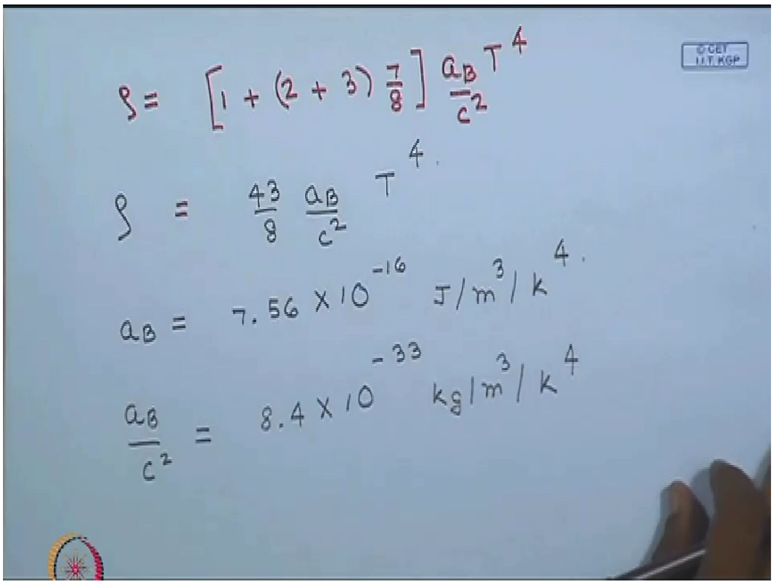

Let us calculate the total energy density. The total energy density rho = 1+2+3, 7/8 Stefan-Boltzmann constant T to the power 4/C square. And this =43/8. So this is 5 times 735+8 that is 43/8. So the total energy density is 43/8 times the Stefan-Boltzmann constant by C square T to the power 4. So that is the total energy density during this epoch at temperatures > around 5 times 10 to the power 9 Kelvin.

So with this let me also give you the value of the Stefan-Boltzmann constant. The Stefan-Boltzmann constant AB has a value 7.56 * 10 to the power -16 joules per meter cube per Kelvin to the power 4. What we require here is not the energy density, but the mass density which can be calculated if we use the Stefan-Boltzmann constant by C square. So we have to divide this by C square 9 times 10 to the power 16 that is what you have to divide with.

And what you get is then 8.4*10 to the power -33 kg per meter cube per Kelvin to the power 4. So this is what we will use to calculate AB/C square the energy density in terms of the temperature and it will give us the energy density in units of kg per meter cube. So we will use this now to calculate the expansion rate of our universe in this epoch. So let us now put this into the expression for the dynamics of the universe.

So this will take us a few lectures back if you recollect we had worked out the dynamics of the universe and I had told you that in the early universe the curvature can be neglected and during these epoch we have seen that the universe is radiation dominated. So we can dominated by relativistic particles. All of these are relativistic particle only the residual baryons are non relativistic particle and the dark matter possibly, but these do not make any significant contribution in this epoch. It is only the relativistic particle which contributes.

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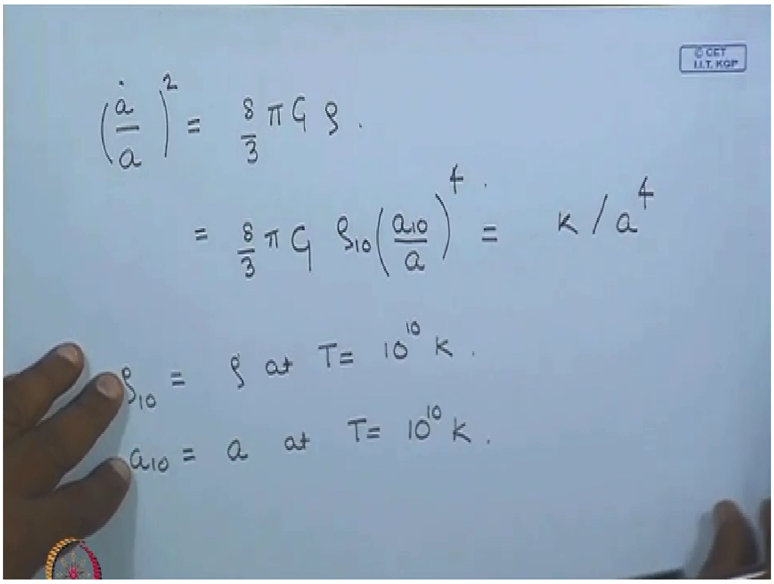

So we can straight away write down the equation for the expansion of the universe and this equation is that the Hubble parameters A dot/A square =8/3pi G rho. We have worked out this equation. The rho here is due to these relativistic particles that we have just considered and there could be curvature, but curvature I have told you earlier also that curvature does not contribute in the early universe.

So this is the equation that we need to solve to work out the dynamics of the universe and we also know that the temperature of the universe of the fluid scales. So the temperature scales proportional to 1/A. We have worked this out for relativistic particle, massless particles. So this is we are going to use this also. So the energy density of the universe can be written in the following way.

So we can write this as 8/3 pi G * rho 10 A 10/A to the power 4. Let me tell you that rho 10 is the energy density of the universe at T =10 to the power. So this is the energy density, this is the scale factor at T + 10 to the power 10 Kelvin. So all that we have done is we have used the fact that the temperature scale inversely as A and the density scales as T to the power 4. So that is all that we have done rho scale as T to the power 4 and T scales as 1/A.

So I have written the density in this way in terms of the density at the temperature 10 to the power 10 Kelvin and the scale factor when the temperature is 10 to the power 10 Kelvin. In terms of this equation and this can be written as K * A to the power K/A to the power where

K is the constant 8/3 pi G rho 10 * A 10 to the power 4. So this is the equation that we have to solve and solving this equation is quite straight forward. So let us solve this equation.

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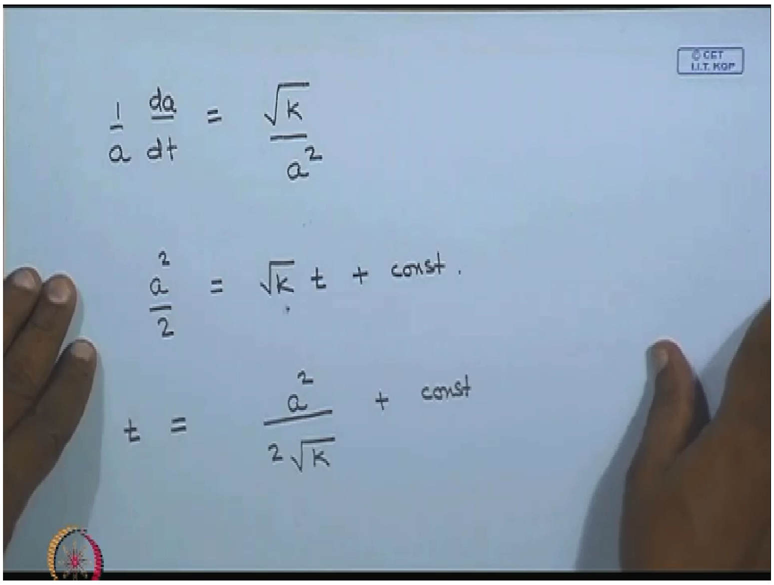

So what we have now is that DA/DT 1/A = square root of K/A square. So we are solving this equation this is equal to this. So I have taken square root of this and this is what we get and we can integrate this straight away. So what we get if I integrate this is that A square/2. So I will take this A square over here I will have ADA which will give me A square/2 = square root of K * T+A constant and I can straight away invert this.

So what I will get is that the time = A square/2 root K + another constant the different constant this is not the same as this, but this is what we have. So this gives us the age of the universe in terms of the scale factor, but what we would like now is to write this in terms of the density. So let us put back the expression for K.

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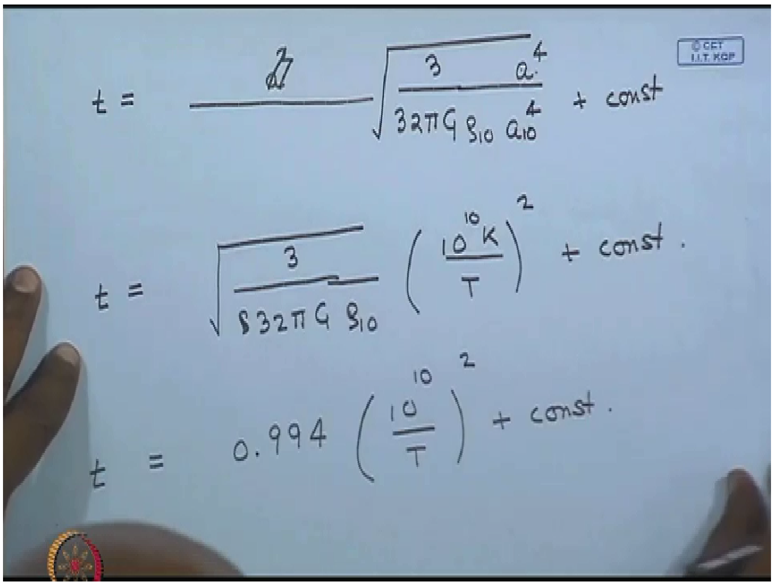

And what we have here is that the age of the time T the age of the universe T = A square. Let us put back this factor square root of K where K let me remind you that we have used K to express 8/3 pi G. So I have a factor of 2 over here. So what we have here is square root of 3/32 pi G. This 2 has gone in. So we have 32 pi G rho 10 and A 10 to the power 4. So what we can do is we can take this inside and write it as A4 and the ratio A4/A10 4. We know that the scale factor is inversely proportional to the temperature + a constant.

So this allows us to write the age of the universe as 3/32 pi G the density at the temperature 10 to the power 10 Kelvin and this ratio A/A 10 to the power 4 and then the square root of that. So we know that the scale factor is inversely proportional to the temperature. So we can write this as 10 to the power 10 Kelvin by the temperature square + a constant. So this is the solution in the relativistic epoch we see that the temperature, the time the age of the universe is inversely related to the square of the temperature.

So as the universe expands as time evolves the universe gets cooler and cooler that is what we see and we can work out this factor over here. So for the particular phase that we are dealing with where we have the neutrinos and the electron positron pairs and the photon all of these in thermal equilibrium the density at 10 to the power 10 Kelvin is going to be 43/8 this factor * 10 to the power 10 to the power 4.

So we can work out this and if you work this out you have to just put in the number all the numbers are given over here. So AB/C square is given over here. You have to put 10 to the power 10 Kelvin for the temperature we have this factor here and so putting this in this expression what you get finally is that for phase 1 this is = the age of the universe = 0.994*10 to the 10/T square + constant.

So this is the expansion rate the time since the Big Bang as a function of the temperature of the universe. Let us just see what we can learn from this. This constant appear here to account for the fact that the equation of state of the universe the relation between the density and the temperature could have been different in the past, but it is not very important as we shall see now.

Let us just calculate the time it takes for the universe to cool from 10 to the power 12 to 10 to the power 10 Kelvin.

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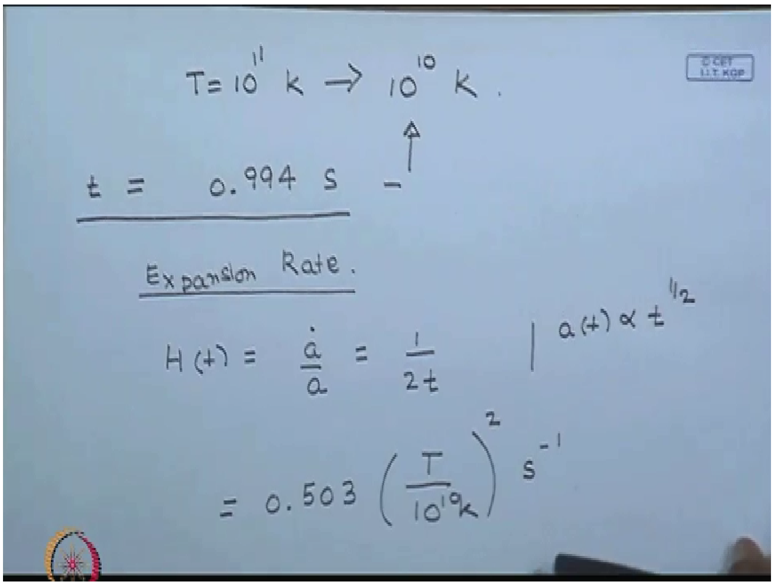

So let us calculate the time it takes the universe to cool from T =10 to the power or let us say 10 to the power 11 Kelvin to 10 to the power 10 Kelvin and we know that this equation will hold in this entire range. So the time it takes is 0.994 seconds. The 10 to the power 11 factors if you put here and subtract it out it will not make a bid difference. So essentially what it tells us is that the time from the Big Bang is also of this order.

So this is the time since the Big Bank roughly when the universe is at a temperature of 10 to the power 10 Kelvin age of the universe when the universe is 10 to the power 10 Kelvin. Another quantity which is of interest which you can calculate from this. Let us do the exercise is to calculate the expansion rate of the universe. The expansion rate we have seen is quantified by the Hubble parameters HT which is A dot/A.

And in this model we have seen that A is proportional to T to the power half. So A is proportional to T to the power half. So A dot by A is going to be ½ T. We have worked this out earlier also that for a relativistic model where A of T is proportional to T to the power half. A is proportional to T to the power half. So the Hubble parameter just differentiate this you will get half/T and then you divide by A again you will get 1/2t.

So the Hubble parameter scales as 1/2T and we have just worked out how the time evolves with the temperature. So all that you have to do is you have to plug this in here this expression for T over here and if you do this what you get is that this turns out to be 0.503 T/10 to the power 10 Kelvin square (()) (31:10). This is in second we should mention this here.

So we have worked out the expansion rate of the universe and what we see is that the expansion rate slows down as the universe cools which feels what we expect.

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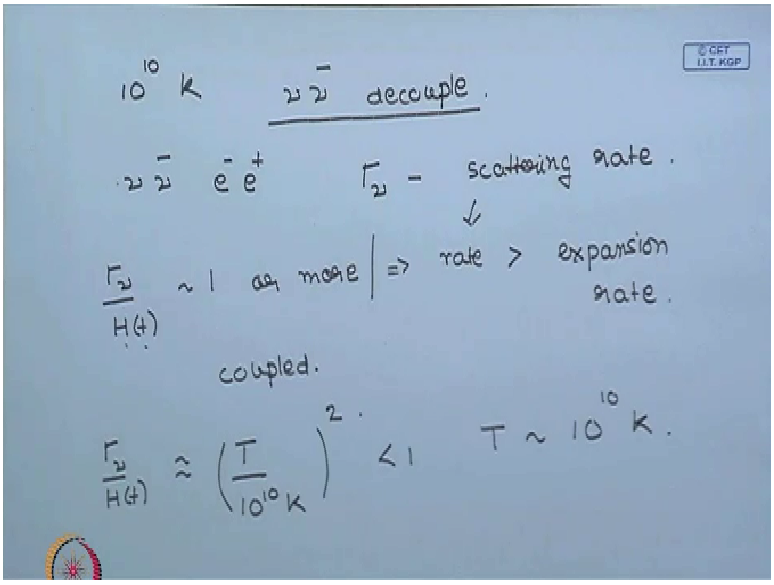

Let me move on to discussing what is the implication of this expansion rate of the universe. So I have told you some time ago that when the universe reaches a temperature of around 9* 10 to the power 10 Kelvin the neutrinos they decouple. Question is how do we quantitatively determine whether particular species or a particular reaction is in equilibrium or not that is the question.

So when neutrinos decouples essentially the scattering between the neutrinos and the electrons the scattering between the neutrinos and the electrons the rate of this equation. So let us write gamma nu this is the scattering rate for the neutrinos. If this is the scattering rate for the neutrinos then gamma nu/ht is the quantity which one has to look at. If this number is of the order unity or more it essentially implies that the reaction rate or the rate at which the scattering is taking place is >the expansion rate which implies that these are coupled for the neutrinos.

And the same argument let me mention here that the same kind of argument also holds for any other reaction taking place in the universe. If you want to ask the question is the reaction are the different reactants taking part in this reaction in thermal equilibrium. If thermal equilibrium is to hold then the reaction rate should be more than the expansion rate of the universe. And we just calculated the expansion rate of the rate in the epoch of around 10 to the power 10 Kelvin. So if the neutrinos are to be in thermal equilibrium with the electrons and the photons.

Then the neutrinos scattering rate should be more than the expansion rate of the universe or should be of the order of the expansion rate of the universe. So if you calculate the neutrino scattering rate which we shall not go into here it turns out that this ratio gamma nu/HT. We have just calculated HT and found that it is proportional to T square. If you also calculate the neutrino scattering rate which depends on the energy scales and the density of the neutrinos and the electrons positrons.

So it depends on all of these factors and if you calculate that then this ratio turns out to be approximately T/10 to the power 10 Kelvin square and we see that the neutrinos decouple so this ratio becomes less than 1 at temperature of the order of 10 to the power 10 Kelvin. So the point I wish to make over here is that the expansion rate of the universe plays a very important role in deciding whether a particular reaction some reactants some substances particles are in thermal equilibrium or not.

And we have just calculated this in the phase of the universe before the electrons positrons and positrons recombine. Let us now repeat the calculation for the phase after the electron positrons have recombined in the phase after the electron and positrons have recombined.

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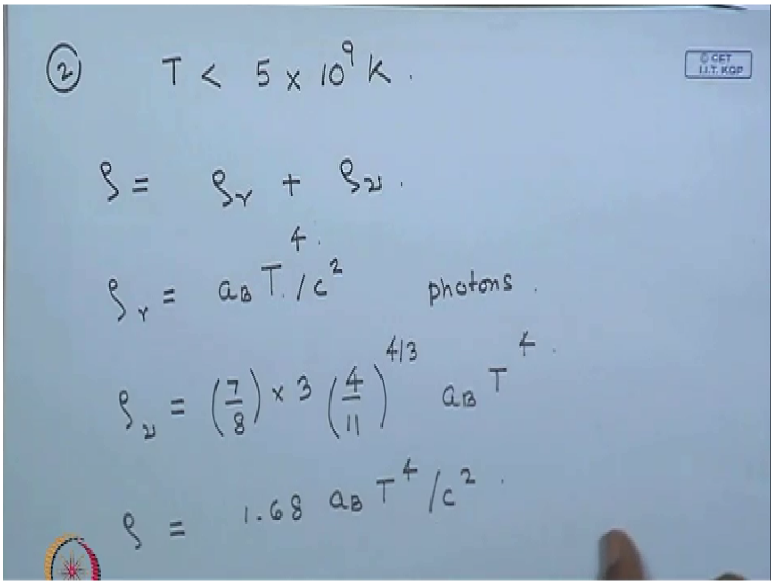

We now have so that is the phase 2 which I had mentioned earlier. In phase 2 what are the particles that we have in phase 2. So in this phase we have the neutrinos and the positron they are totally aloof from the other particles and we have the photons that we have and they have their own different temperature. So let us calculate the energy density in this phase of the universe which is at temperature less than 5 somewhere of this order.

The density = to the density of the photons + the density of the neutrinos and the density of the photons is = AB T to the power T photon. So T photon is the temperature of the universe. So AB T to the power 4/C square. This is the photon whereas the neutrinos we have seen are a factor 7/8 lower there are 3 flavours. So a factor of 3 and the temperature of the neutrino is lower than the temperature of the CMBR by a factor of 4/11.

So we have 4/11 to the power of 4/3 AB T to the power 4. So this is the temperature the energy density the mass density of the neutrinos. So let me remind you again. We have three flavours of neutrinos for fermions there is a factor of 7/8 which is the energy density 7/8 times lower than that of bosons and we have this factor of 11/3 to the power 4/3 because of the extra energy that has gone into this.

So the temperature of the photon has gone up, temperature of the neutrino is less and this is the factor that accounts for that. So putting in all of these together we can calculate the energy density this turns out to be 1.68 * Stefan-Boltzmann constant AB T to the power 4 by C square. We can again use this to calculate the expansion rate of the universe. So remember that the time the age of the universe here can be calculated in terms of the mass density at 10 to the power 10 Kelvin.

So repeating the same exercise using this value instead. So you have to replace the temperature to be 10 to the power 10 Kelvin here.

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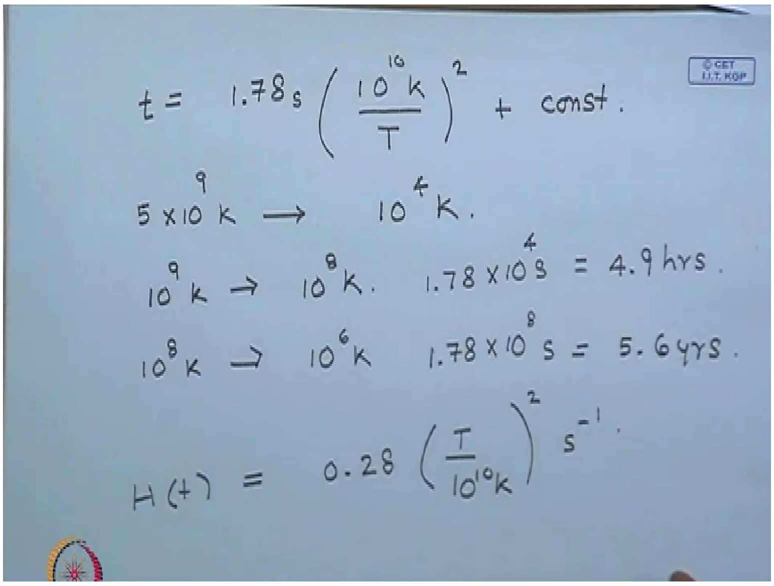

So using this what we get is that in phase 2 the temperature the time the age of the universe =1.78 times this is second times 10 to the power 10 Kelvin/T square + constant and this holds all the way roughly. So this holds from around 5 *10 to the power 9 Kelvin. All the way to around 10 to the power 4 Kelvin where the non-relativistic particles have to be taken into account.

The non-relativistic particles the universe is no longer radiation dominated. So in this entire range you can use this expression roughly and what does it tell us. It tells us that the universe takes the time taken to cool from 10 to the power 9 Kelvin to 10 to the power 8 Kelvin this is 1.78* 10 to the power 4 seconds which is =4.9 hours. So this gives us the time scale of cooling of the universe. So when you talk of a temperature scale 10 to the power 8 Kelvin in the universe the age of the universe is roughly of the order of 5 hours a few hours.

The universe is only a few hours old and it takes. So if you consider the universe cools from 10 to the power 8 Kelvin to 10 to the power 6 Kelvin and if the universe cools to10 to the power 6 Kelvin then the time taken is 1.78 *10 to the power 8 seconds which is =5.6 years. We can also calculate the Hubble parameter and the Hubble parameter exactly the same way as we did earlier and the Hubble parameter turns out to be 0.28 * T/ 10 to the power 10

Kelvin square second inverse.

So the expansion rate slows down once the electron positron pairs have annihilated. So what we have done in a nutshell is that we have worked out how the expansion rate in 2 different phases of the universe how the age of the universe can be directly expressed in terms of the temperate in 2 different phases. One is before the electron positron annihilates and the other is after the electron positron annihilates.

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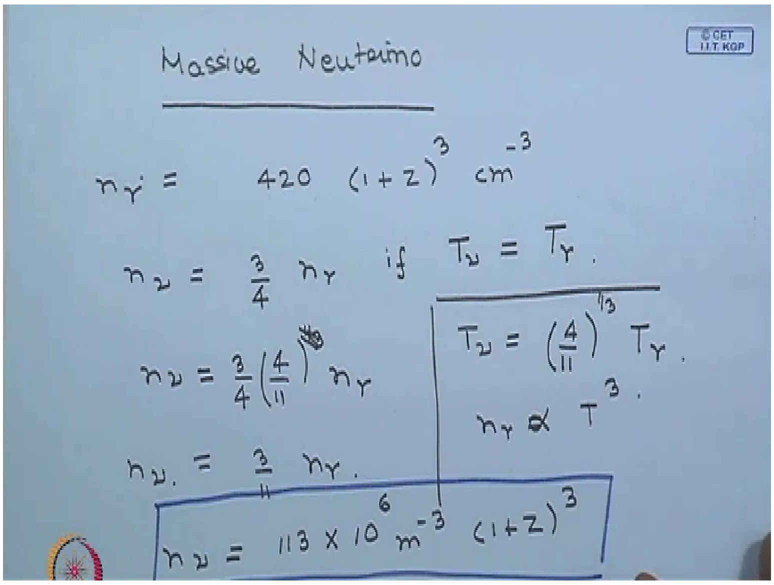

We will now discuss a very interesting possibility and this possibility is that the neutrinos have mass. So the possibility that the neutrinos that we have a massive neutrino. So in earlier on in this course we have seen that we have this CMBR photons and the whole universe is filled with this CMBR photons. And we have worked out that the energy density the number density of these photons is 420 particles * 1 + Z cube centimeter -3.

So we have seen that the entire universe is filled with this cosmic microwave background radiation and the number density of these photons is 420 * 1 +Z cube. Now we have also now seen that in addition to this photon background there is also a neutrino background not only 1, but there are 3 different kinds of neutrinos all of which constitutes the neutrino background.

So we have mu neutrino background a tau neutrino background and then electron neutrino background. Let us first ask the question what is the number density of these neutrinos. So these are neutrinos are fermions and the number density for a fermions is =¾ the number density of boson if the temperature are same. So if the 2 temperature were the same then this would be the relation, but we know that the temperature of the neutrino is the factor 4/11 to the power 1/3.

So we know that T nu is actually = 4/11 to the power 1/3 the temperature of the CMBR and we also know that the number density is proportional to T cube. So finally what we can say is that this will be = so there will be factor of ¾ and there will be a factor of 4/11 to the power 1/3 this is the ratio of the neutrino number density for each flavour with respect to the photon number density. So this turns out the number density of neutrinos. So this 4 cancel out/

So this is to the power 1/3 and so this power actually goes away. So finally what we have 3/11 times the photon number density and then this basically implies that we have a neutrino number density the number density of neutrino is 113 *10 to the power 6 neutrinos per meter cube * 1+Z cube. So each flavour of neutrino has a number density which is given by this. Now let us consider the possibility that one of the various neutrinos.

One of the 3 flavours of neutrinos let us consider the possibility that one of them is massive so that is the possibility that we are going to consider.

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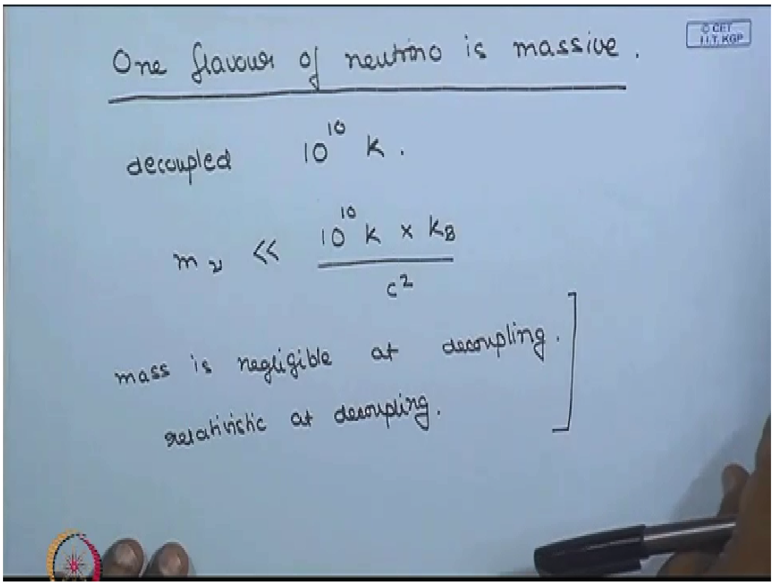

So let us consider that one of the neutrinos let me write it again. So 1 flavour of neutrino is massive. So this is the possibility that we are going to discuss. It is also possible to discuss the possibility where all 3 of the flavours are massive or 2 of them are massive. Let us just take up this particular discussion and we will assume that. So we have seen that the neutrinos decouple at a temperature of around 10 to the power 10 Kelvin.

So we will assume that the neutrino mass is much less than 10 to the power 10 Kelvin * KB/C square. So the neutrino mass we will assume is much than the temperature scale that the mass scale when the neutrinos decouple. So the mass is negligible. So the effect of the mass negligible at decoupling. So at decoupling the neutrino can be essentially treated as being complete relativistic particles massless.

So the neutrino is relativistic at decoupling and it does not affect the physics or the dynamics of the decoupling or the expansion of the universe at all at that epoch. So this is the assumption that we are going to make, but then the mass is negligible during the decoupling. It is the neutrino is relativistic.

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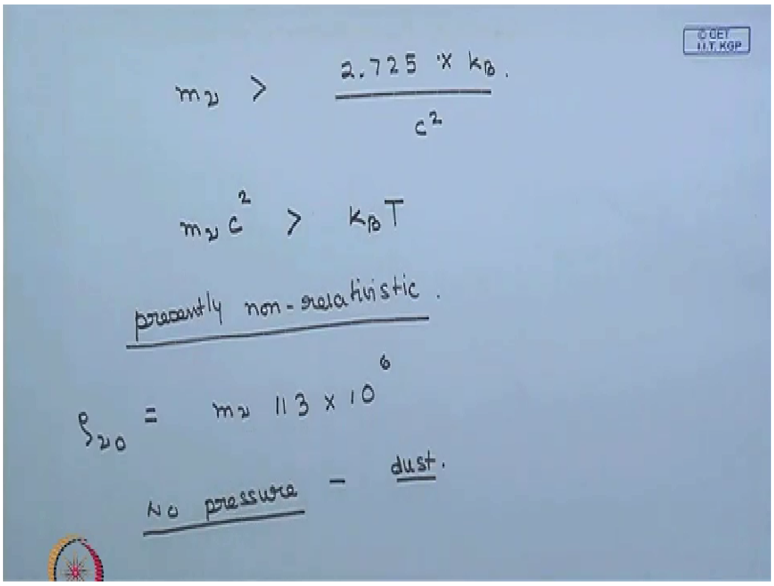

But we will assume that the mass is such nu is such that it is more than the present temperature of the CMBR. So we will assume that the mass is more than the present temperature of the CMBR. So mass is more than 2.725 * KB/C square. So somewhere in the past the particle became non-relativistic. So somewhere in the past the mass of the neutrino * C square exceeded was greater than the KB * T and the particle became relativistic is now presently relativistic.

So in such a situation what happens the entire universe now is we have a particle of non-relativistic particle of mass m nu which is filling the entire universe and it has a mass number density which we have just calculated. So the density matter density from these neutrinos at present is going to be m nu that is the mass of each particle into the number density of particles that is the matter density from these neutrinos and we have just worked out what this is 113 * 10 to the power 6.

So at present if one of the neutrino is massive such that it was relativistic the mass is not so large that it was sufficiently small so that it was relativistic during decoupling but it became massive later on then the universe at present is going to be filled with massive relativistic particles whose density is the mass * number density of particles. These particles are going to be non-relativistic so they will have no pressure, they will not have any significant pressure.

And they behave essentially like dust the familiar matter the pressure less dust. So if one of the neutrino is massive the consequence is that the entire universe they will be a matter of component in the universe which has a density which is given by this.

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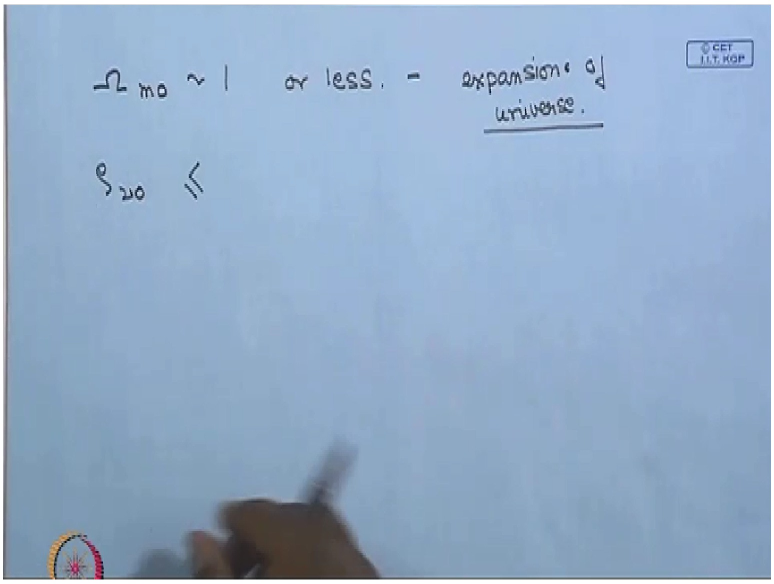

Now there are observations which indicate that the present value of the density of the universe which quantified by the density parameter is not very much more than 1. It is of the order unity or less. Let us not go into the details of this. So whatever density you have from these neutrinos it had better be < or = the mass density of the universe which is inferred from other observations.

So this is observations of the expansion of the universe and observations of large scale structure. It seems we are running out of time now so I will stop today's lecture here and resume on this topic on the consequence of there being a massive neutrino in the next class.