Astrophysics & Cosmology
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Lecture – 36
CMBR and Thermal History (Contd…2)

Welcome back, so, let me remind you that we were discussing very high red shift when the temperature of the universe is more than 10 to the power 10 Kelvin.

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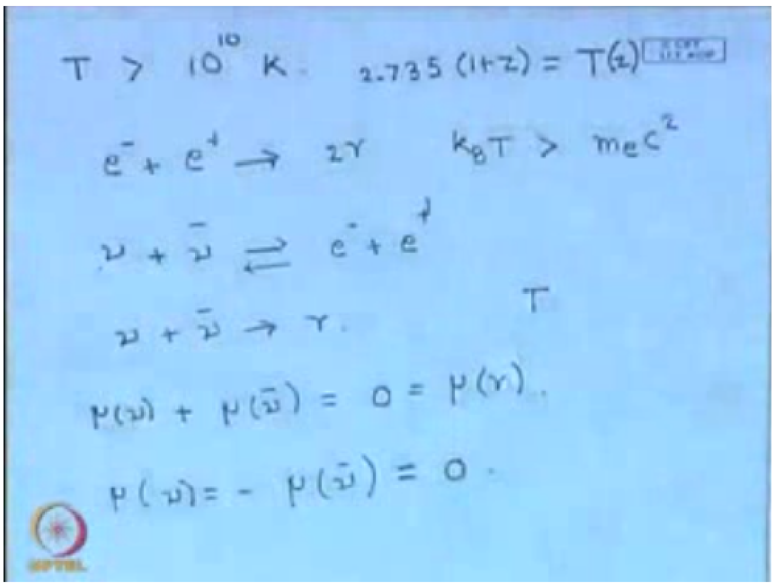

At that epoch we have the electrons and the positrons, the neutrinos and antineutrinos and the photons, all in thermal equilibrium. So, let me draw a picture of this.

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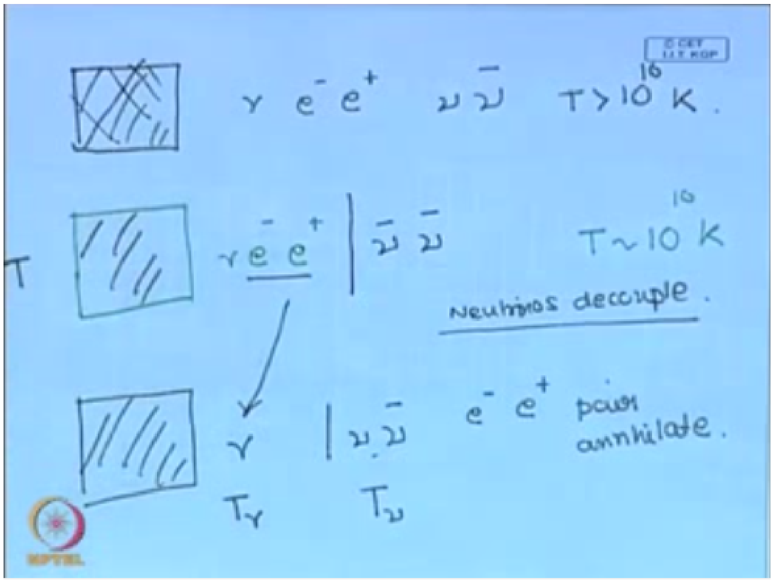

So, we have a part of the universe over here where we have all of these. So, we have the photon, electron, positron, neutrino and antineutrino, all in thermal equilibrium and they are described by the respective occupation numbers and we can work out what the contribution to the energy density from each of these is. This is at a temperature more than 10 to the power 10 Kelvin, okay.

Then what happens as the universe expands, the universe cools and at some temperature around 10 to the 10 K, the first thing that happens is that the 2 neutrinos go out of thermal equilibrium. The neutrinos are weakly interacting particles and for equilibrium to be there all of these particles have to interact with each other, but the neutrinos they are weakly interacting, so the neutrinos go out of equilibrium.

So, at some temperature of the order 10 to the power 10 K, we still have the electron, but the neutrinos are out of thermal equilibrium. So, the neutrinos do not interact with the rest of the material and their occupation number is now frozen. So, subsequent to this all that happens is that the occupation number remains frozen. The frequently is changed due to the expansion of the universe.

So, the universe as far as the neutrinos are concerned, the universe is transparent somewhere over here at much high. The neutrinos become transport somewhere here 10 to the power 10 Kelvin. The universe is not transparent to the photons, okay, but that becomes transparent to the neutrinos and once the universe becomes transparent to the neutrinos, we know how the thing evolves.

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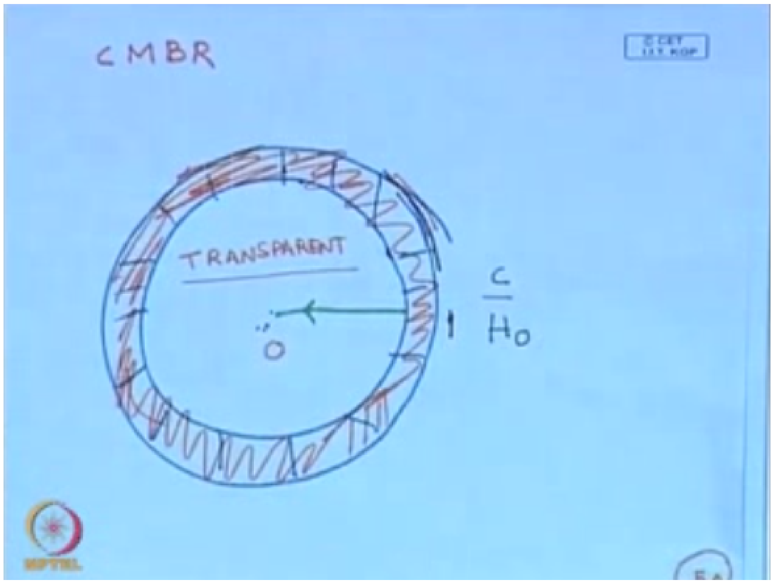

Okay, the distribution function is unchanged. Okay, so here the neutrinos, so basically what happens here is neutrinos decouple because their scattering cross-section are much smaller. They are weakly interacting particles. Okay, now once the neutrinos have decoupled, again this thing has now 2 kinds of things, there are the photons which have adequate energy to maintain the electron-positron pairs in thermal equilibrium.

As the universe cools slightly more at a slightly lower temperature what happens, the electrons and positrons also recombine. So, the same thing you only have the photons and the electrons and the neutrinos and antineutrinos. The electrons and positrons they recombine, the temperature of the photons CMBR is not hot enough to maintain the electron positron pairs and falls below the rest mass of the electron positron and the electrons and positrons they annihilate.

So, this is called pair annihilation, electron positron pair annihilate and you are left with just the photons. There is a small residual amount of electrons and baryons left which scatters the photons and keeps them still in thermal equilibrium. Okay, small amount of baryons which is adequate to scatter the photons and maintain them in thermal equilibrium. The neutrinos have decoupled, okay.

So, you see that the excess energy, the energy which was earlier there in this has now gone into the photons. Okay, electron positron annihilation has occurred and the energy which was there in the electrons and positrons here has gone into the photon, okay. So, you see that in this process, something happens to the CMBR which does not occur to the neutrinos. The neutrinos have decoupled.

So, this energy which was there in the electrons and positrons does not get distributed amongst the photons and the neutrinos. The neutrino is decoupled. Okay, let us see what happens over here. So, the temperature, let us assume that this happens at a temperature T. So, the electrons, all of these species were in thermal equilibrium. Even when they go out of equilibrium, the temperature of all them scaling exactly the same way as inverse of the scale factor.

So, even here, the photons, electron, positron and the neutrinos have the same temperature. Once you have the electron positron annihilation, the temperature of the photon will become different. So, let us assume that the photons here have a temperature T gamma and the neutrinos have a temperature T and u. The conserved quantity is the entropy. The expansion of the universe is adiabatic. So, the conserved quantity is the entropy.

So, let us now analyze how these temperature are related. Okay, the entropy, per unit volume. So, let us ask what is the entropy of this. The entropy of this will have 2 parts; one part will be in the electron, positron, and photon gas which is interacting.

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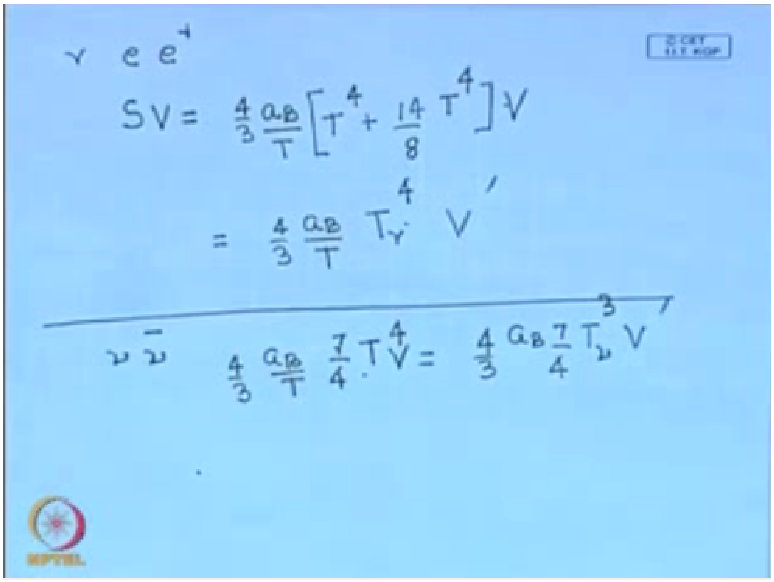

So, to start with the entropy of this. Entropy we have already calculated, it is 4 third internal energy/T. So, the entropy of this to start with is going to be S is equal S and V which is 4 third AB/T and we have 3 kinds of particles; we have the photons, for photons it is T to the power 4 plus we have the electrons and the positrons which are fermions, right fermion. For each kind of fermion, the internal energy is 7/8th that of the photon. So, this will be 14/8.

So, this is the entropy of the volume. In this volume, V is the volume corresponding to this due to the photons and electron and positrons, okay. This will be equal to the entropy in the photons alone at the later instance after the recombination. So, after the recombination, this will be equal to 4/3rd AB/T, T photon to the power 4 x V prime where V prime this volume also would have changed due to the expansion of the universe. This is V and this is V prime.

Okay, so we are here analyzing this part which initially had photons, electrons, and positrons; after the annihilation, it has only photons. The conserved quantity in adiabatic expansion is the entropy, so the entropy is going to be same. Okay, before the annihilation and after the annihilation. Let us now also consider the neutrinos. The entropy of the neutrinos again you can calculate in the same way, it is 4/3rd AB/T. So, we have 7/8.

Let us take one kind of neutrinos. Neutrinos do not have 2 polarisation, they only have single polarisation but there is a neutrino and the antineutrino, so again we will have 14/8. 14/8 can be written as 7/4. So, this is 7/4 T to the power 4 x V. So, before this annihilation, they are at the same temperature. After the annihilation, this will be equal to 4/3rd AB 7/4 T neutrino cube x V prime. Okay, this is after the annihilation.

The annihilation process does not affect the neutrinos because they are decoupled. They do not know that the electron positron pairs have annihilated. The energy does not get transferred to the neutrinos. So, entropy in the neutrinos is going to be conserved, whereas the entropy in the electron positron pair has gone into the photons. Okay, the total entropy is conserved. So, we have these 2 relations.

This is before annihilation, before annihilation, after annihilation and after annihilation. So, from this we can just straight away eliminate these things and what do we get. So, let us look at the left-hand side of this equation.

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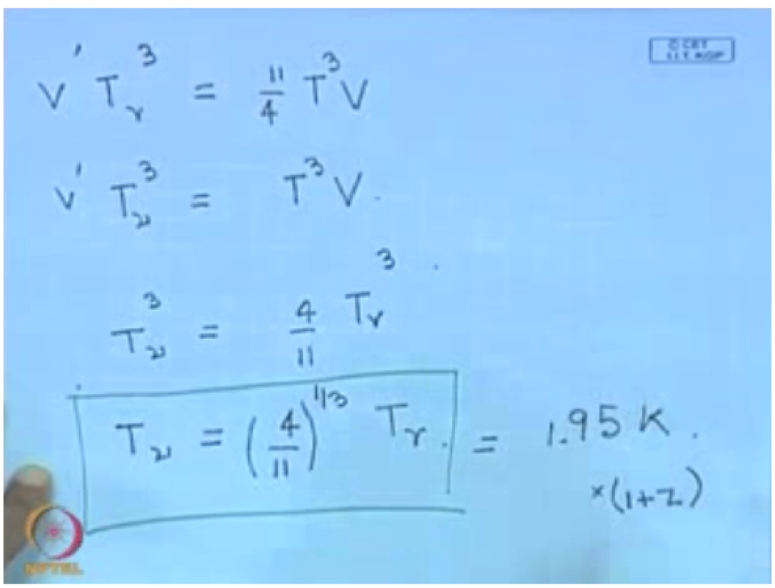

The left-hand side of this equation tells us that T gamma cube=T gamma cube x V prime is equal to, so this is 7/4, so this is 11/4 T cube, okay. 11/4, 7/4+1 is 11/4, so 11/4 T cube V. For the neutrinos, V prime. So, what we see from this is that the neutrino temperature is equal to. So, this is the final result. So, what we have here is, let me recapitulate what we have learned. So, at temperatures more than 10 to the power 10 K, the CMBR photons are adequately hot.

The universe is adequately hot, so that all of these things are in thermal equilibrium. Then, the neutrinos go out of thermal equilibrium because their interaction rates are very weak. So, they just freeze stream and then what happens is that the electron positron pairs annihilate and the electron positron annihilation pumps in extra energy into the photons and though the neutrinos and the photon started off with the same temperatures, the photon temperature gets jacked up by a factor which is 11/4 to the power 1/3rd relative to the neutrinos temperature, okay.

So, now after that both of these evolve in exactly the same day subsequent to this, the scale as 1/A. So, the present universe has a photon background with a thermal spectrum at 2.736 K. In addition to this, there is also thermal neutrino background whose present temperature=1.95 K which is a relic of the hot past of the universe when the 2 were in thermal equilibrium, okay. So, the fact that you see CMBR (Cosmic Microwave Background Radiation) at 2.735 K also implies that there is a neutrino background with temperature 1.95 K.

This neutrino background is very difficult to detect and it has not been detected. It is unlikely that it will be detected because the neutrinos are very weakly interacting particles.

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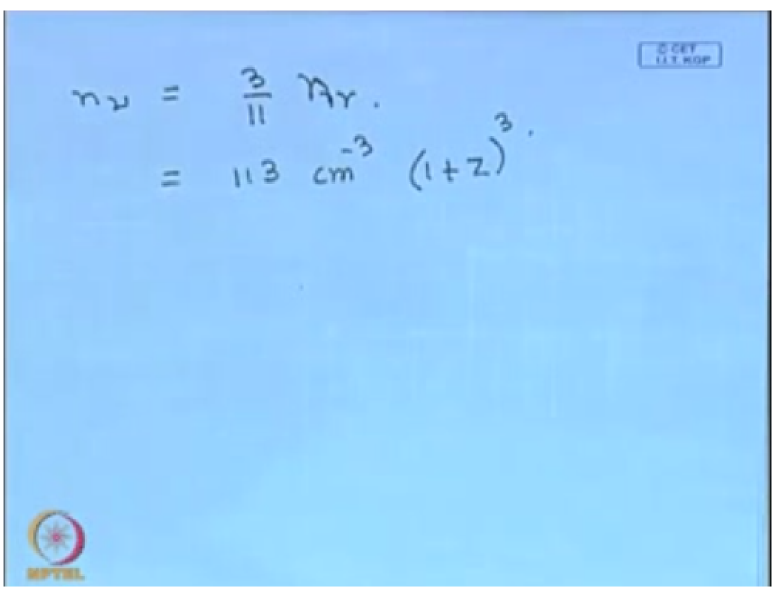

So, the number density of neutrinos corresponding to this, you can calculate that, this comes out to be 3/11 times the number density of photons for this particular case because the photo temperature is also higher, so this and this comes out to be 113 cm-3, okay. This also will have a factor of 1+Z which I have not written.

So, this is the number density of neutrinos and such small neutrino number density cannot be detected by and large because the neutrinos are weakly interacting particles and need to be detected here. The probability of detection would be extremely small, okay. Though the number densities are comparable to those of the photons, they have a much weaker interaction, okay. So, assuming that the neutrinos of relativistic.

So, we know of 2 distinct background of relativistic particles, one is the photons and other is the neutrinos.

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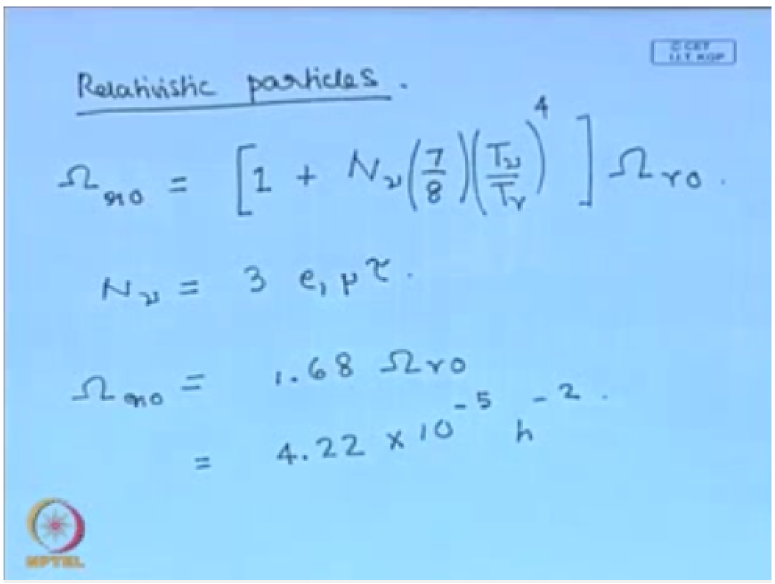

So, we can now estimate the total density in relativistic particles at present and this will be equal to 1 plus the number of neutrino families, number of flavours of families of neutrinos x 7/8. So, given a neutrino of temperature T and contribution to the energy density is 7/8 times smaller x the neutrino by the photon to the power 4 which this ratio we have just calculated, it is 4/11 x omega gamma0. This ratio we just calculated it is 4/11.

So, this is the total energy density of relativistic particles. In terms of the energy dense contribution to the energy density from the photons which we have calculated right in the beginning of today's class. So, if you assume that the number of neutrino family is 3, so you have the new neutrino, the electron neutrino and the Tau neutrino. These are the 3 kinds which are known to exit at present.

So, if you put in 3 here for N then this comes out to be 1.68 omega gamma0 and this is 4.22 x 10 to the power -5 h-2. Okay, so this is the total contribution from relativistic particles to the present energy matter density of the universe. Okay, with this let us now go back to our understanding of the dynamics of the universe.

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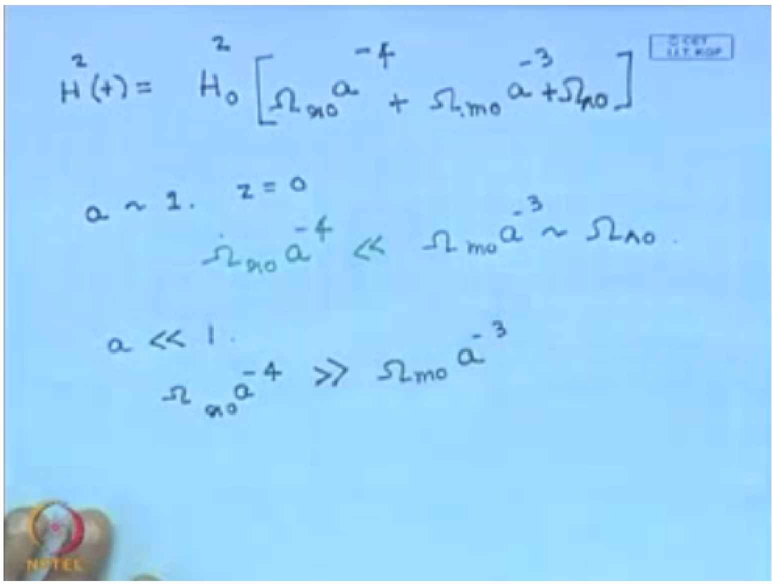

So, if you consider the dynamics of the universe, let me remind you again that the basic equation that we wrote down was H square T=H0 square x so I will omega relativistic particles and with the expansion of the universe, the contribution from the relativistic particles goes down as A to the power -4+ omega matter 0 A-3+ other term. So, I could have curvature and cosmological constant which will have larger power (()) (22:16).

So, if I go to higher and higher redshifts, we see that there will be a transition. The dynamics of the universe will forget about curvature and cosmological constant. Okay, you can put in a cosmological constant also. Let us assume that the curvature does not exit 0, okay. Now at scale factor of order unity, the nearby universe that is Z shift 0, the first 2 terms the contributions from radiation is going to be much smaller than the other terms.

So, we can as well forget about this because this we have seen is of the order of 10 to the power -5. The contribution from the relativistic particles which is often referred to as radiation is of the order of 10 to the power -5. Whereas omega matter and omega lambda are of order unity. So, when the scale factor is of order unity, this can be ignored.

So, for the part of the universe where we see galaxies and other sources, supernova etc., we do not have to bother about the contribution of the radiation to the dynamics, it is negligible. But if you go to higher and higher redshifts, then there will be an epoch when this term will by far outweigh these 2 terms. Okay, there will be a transition where the radiation relativistic particles dominate the universe.

So, we shall have a transition from a matter-dominated universe to a radiation-relativistic particle dominated universe. So, let us estimate where this transition occurs. So, at A, if A is much smaller than 1, it is the first term omega 0 A-4, because A it has A to the power -4 is going to be much >omega matter 0 A-3 and we can forget about the matter and the cosmological constant.

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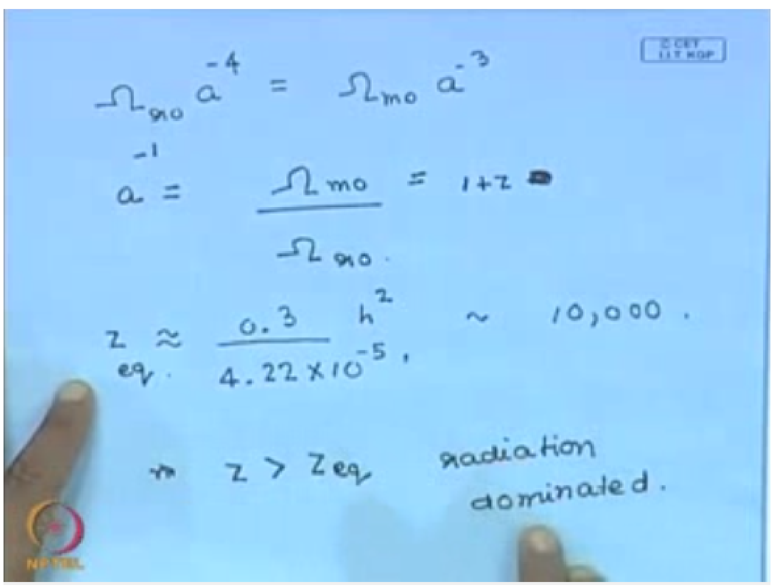

So, let us see where there will be a transition between these 2. So, the transition will occur at the epoch where omega R0 A-4=omega matter 0 A-3 or A=or 1+Z, sorry other way around which is also equal to 1+Z. Okay, so we know the values we have now values for this. So, let us put this in, so this will give us that this will occur at redshift of approximately 4 point. So, this is 0.3 that is the omega matter divided by 4.22 x 10 to the power -5.

I can take this 8 square on top, okay. So, this is of the order of tens of thousands and a redshift of around 10000, the universe has a transition to a radiation dominated. So, this is called Z equality. So, the redshifts > the redshift of matter radiation equality, the universe is dominated by radiation and we do not have to bother about the matter part or the cosmological constant. So, the early universe is radiation dominated, dominated by relativistic particles.

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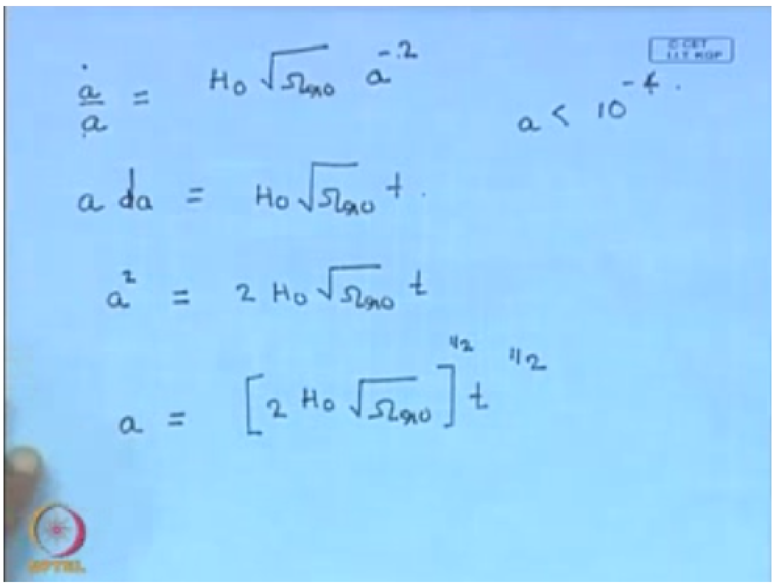

Let us also workout the solution at those times. So, the evolution of the scale factor at those epochs is given by A./A=H0 square root of omega R0, A-2. So, I can write this as DA/DT and if I take this on to the other side, I will get A square. One of the A will cancel out. So, I have ADA=H0 or A square=2 H0. So, the scale factor goes ST to the power 1/2 during the radiation dominated epoch.

So, at a redshift of more than around 10,000 or scale factor at A value is < 10 to the power -4 somewhere over there. This is the evolution of the universe. Okay, this is how it evolved and we also know that the temperature and the scale factor are inversely related, so we can straightaway relate this to the temperature. So, the temperature and the age are also related.

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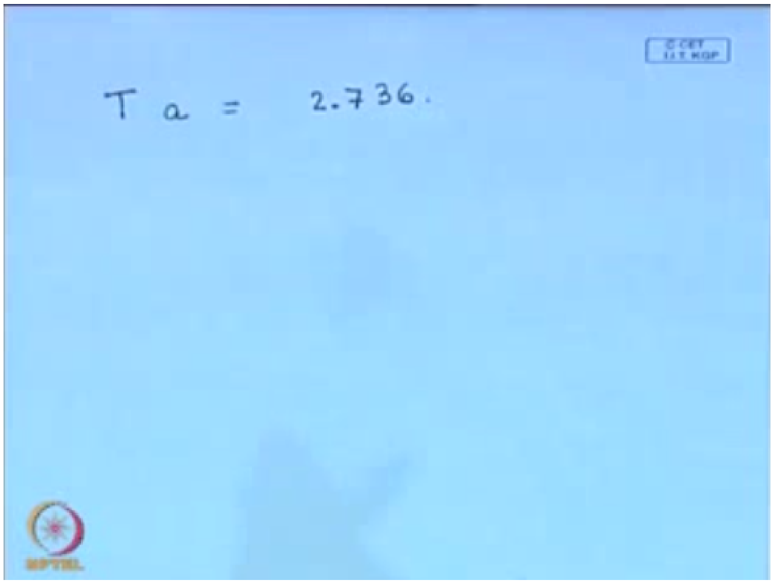

So, the scale factor can be written as we know that TxA=2.736. So, we can now express this as the temperature. So, given the age of the universe, we can directly convert it into the temperature. So, let me stop over here. I shall resume on this. Can I take an extra class. I will need one. It will be good if I can take one extra class. Monday afternoon or Tuesday afternoon.

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Welcome to today's lecture. Let me remind you that we have been discussing the thermal history of the universe.

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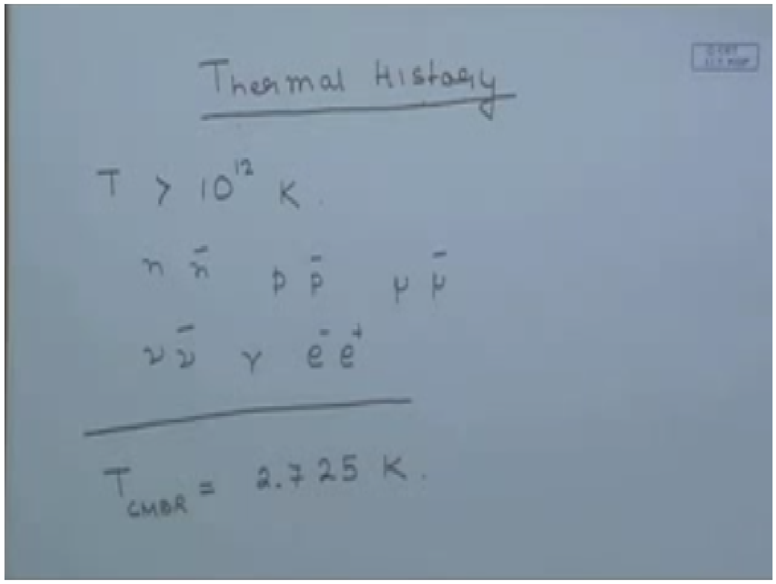

At temperatures, let me just go back a little and so at temperatures are > 10 to the power 12 K, we have the neutrons and antineutrons, protons antiprotons, muons antimuons and we have the neutrinos anti-neutrinos, photons, electrons and positrons all in thermal equilibrium at these very high temperatures 10 to the power 12 K. To put things in context, the temperature of the CMBR at present is 2.725 K.

So, we are discussing really the early universe where it is extremely hot and you have all of these species in thermal equilibrium, this is just to remind you. Well, we are actually not interested in this epoch. If you go to lower temperatures, the neutron and anti-neutron, proton and antiproton, the muons they all recombine once the temperature falls below their respective rest masses, they all annihilate.

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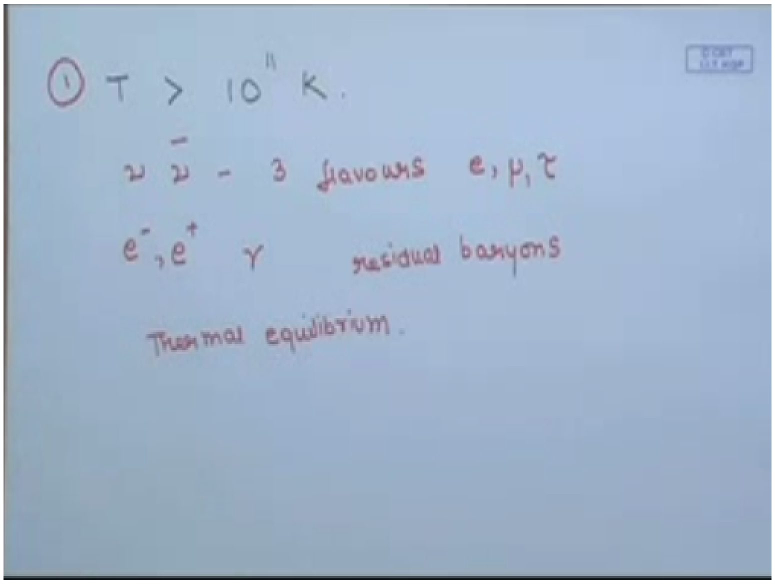

We are here going to start from an epoch that is where the discussion has been mainly. The temperature scale of around > 10 to the power 11 K but lower than the temperatures mentioned earlier. So, this is where we start. At these temperatures, we have got the neutrinos of which there are 3 kinds, 3 flavours, electron, the new neutrino and the tau neutrino; all 3 of them.

We have the electron and the positrons and the photons all of these in thermal equilibrium and we also have some residual baryons small quantities, all at thermal equilibrium. So, this is the epoch 1 that we are going to discuss today and we have discussed earlier also. So, this is the first phase of the universe that we are going to discuss today and we have the second phase. So, the universe is expanding and as the universe expands, it cools.

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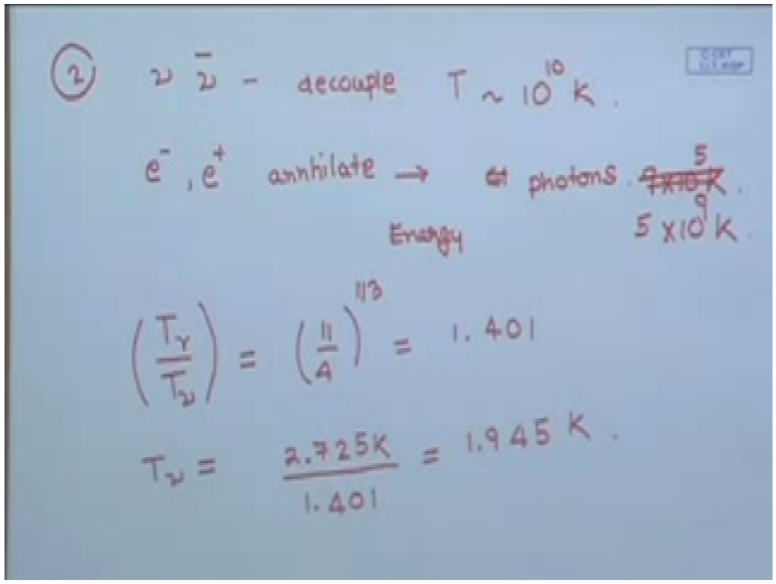

Subsequent to this, the temperature, this is phase 2. Here, as the universe cools, the neutrinos decouple. So, new and new bar they decouple from the rest of the material. They stop interacting with the rest. These are weakly interacting particles and they stop interacting with the rest of the particles at a temperature T of around 10 to the power 10 K and subsequent to this the temperature falls below the rest mass of the electron positron pair.

So, the electrons and the positrons they annihilate dumping whatever excess energy they had into the photons. So, there is extra energy that goes into the photons at around 9 x 10 to the power 5 K. We have studied this and as a consequence of this what happens is that the photon energy T gamma is more than the neutrino energy. The neutrinos have decoupled, as universe expands, the neutrinos free stream.

The temperature of the neutrinos just scales inversely with the expansion of the universe, but the photons also do the same thing till 5 x 10 to the power 9 K somewhere around here where the electron positrons annihilate dumping extra energy into the photons. So, the photon temperature goes up relative to the neutrino temperature at this epoch and this ratio we have worked this out in earlier class. It is 11 x 4 to the power 1/3rd and this ratio is 1.401.

We have worked out all of these things in the last class, so the neutrino temperature after 5 x 10 to the power 9 K is < the photon temperature and the neutrino temperature at present is 2.725

K/1.401 which turns out to be 1.945 K and this continues. So, the neutrino at present, we have a photon background which we see as the cosmic microwave background radiation at a temperature of 2.725 K and we also have neutrino background at 1.945 K.

So, this is the situation. So, once the electrons and positrons annihilate, we are left with neutrino background and photon background both of which more or less the temperature of both of which are different and it continues to scale inversely as a scale factor till the present; and at present, we have both these backgrounds the photon background and the neutrino background. So, we have discussed all of these things is somewhat great detail in previous classes.

Today, let us ask the question, how does the universe expand during these 2 different epochs. One is at temperatures before the electron positron pair annihilate. So, at temperatures more than 5 x 10 to the power 9 K and second how does the universe expand. What is the expansion rate of the universe, how does it expand at temperatures below this. This is the question that we are going to work out in today's class to start with.

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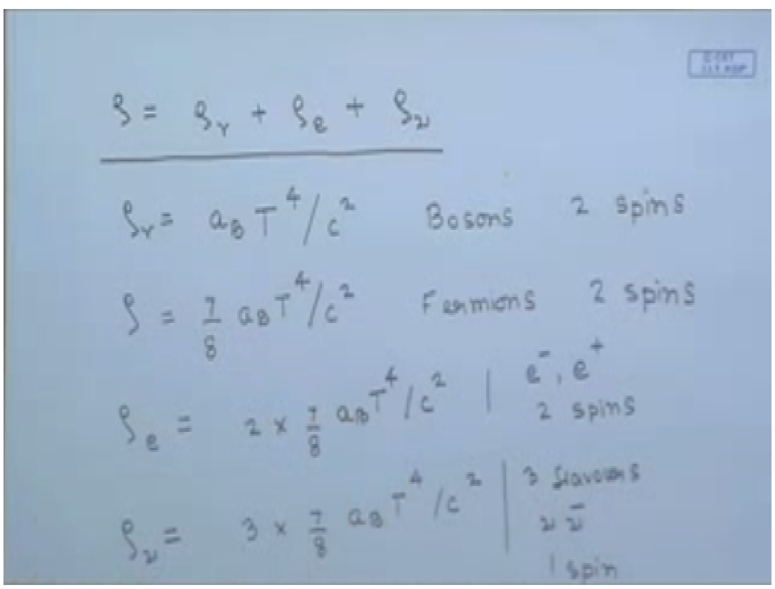

So, let us embark upon this. So, let us set out with this phase where we have all of the species in thermal equilibrium, so the total energy density row of the universe is equal to the energy density of the photons plus the energy density of the electron positron pairs plus the energy density of 3 different flavours of neutrinos. So, this is what we would like to calculate and we have already discussed this to some extent. Let me go through this again.

So, let us first start of with the photons. So, we know that the energy density of the photon row gamma is equal to the Stefan Boltzmann Constant AB, T gamma, they are all at the same temperature, so there is no need to distinguish between the different temperatures. So, the photon energy density is Stefan Boltzmann Constant * temperature to the power 4 and to convert this into energy density, we need to divide it by C square.

So, this is the energy density of bosons essentially with 2 spins. Photons we know have 2 spins, so the energy density of species of bosons with 0 chemical potential 2 spins is given by this which is exactly what these photons are. Similarly, if you have fermions, let me remind you that if you have fermions, 2 spins, then the energy density is slightly different. The energy density is down by a factor off 7/8.

So, this is for fermions in thermal equilibrium at a temperature T, the energy density of fermions. So, this is mass density and assuming that the fermions have 2 different spins. So, for the electrons, the energy density is given by this much. For the positrons also the energy density is given by the same thing, both electrons and positrons have 2 spins each. So, we have 2 x 7/8 AB T to the power 4/C square for the electrons and positrons.

Let us now take up the neutrinos. The neutrinos only have a single spin but there are the neutrinos and the anti-neutrinos, so that compensates for there being only a single spin and we have 3 flavours of neutrinos. So, the contribution from the neutrinos assuming they are massless is 3 x 7/8 AB T to the power 4/C square.

So, we have worked out all the contributions the electrons we have a electron and positron which is why we have a factor of 2 here both of which have 2 spins. For neutrinos, there are 3 flavours. Then, we have the neutrino and the anti-neutrino but only one spin which is why we just have a factor of 3 over here.

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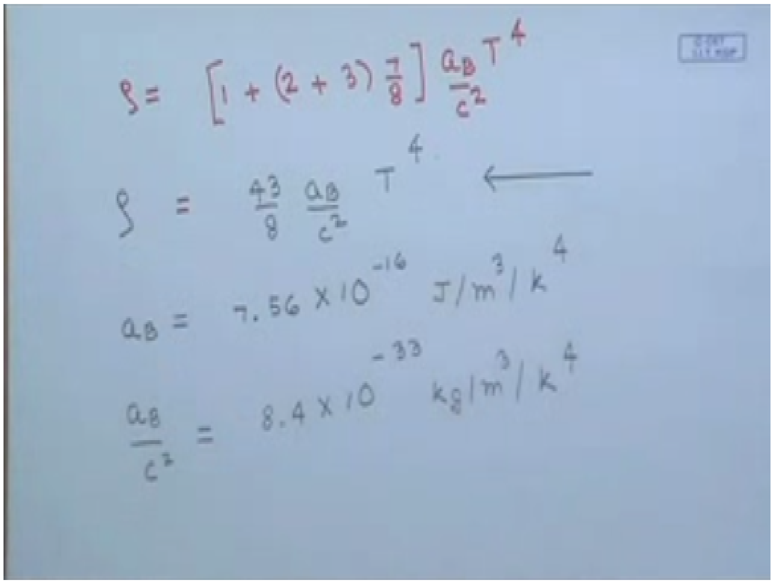

So, the total energy density row is equal to 1+2+3 7 x 8. Stefan Boltzmann constant T to the power 4/C square and this is equal to 43/8. So, this is 5x7, 35+8 that is 43/8. So, the total energy density is 43/8xstefan Boltzmann constant/C square T to the power 4. So, that is the total energy density during this epoch at temperatures > 10 to the power 10 around 5 times 10 to the power 9 K.

Okay, with this let me also give you the value of the Stefan Boltzmann Constant. The Stefan Boltzmann Constant has a value of 7.56 x 10 to the power -16 Joules per meter cube per Kelvin to the power 4. What we require here is not the energy density but the mass density which can be calculated if we use A the Stefan Boltzmann Constant/C square. So, you have to divide this by C square 9 times 10 to the power 16 that is what you have to divide with.

What you get is then 8.4 x 10 to the power -33 kg per meter cube per Kelvin to the power 4. So, this a stiffened boson constant policy scholar under soul of advisors versus Square wine pairings are in port 16 that suit you to divide with and what you get is the 8.4*10 to the power -33 KG per meter cube per Kelvin to the power 4. So, this is what we will use to calculate AB/C square the energy density in terms of the temperature and to give us the energy density in units of KG per meter cube.

So, we will use this now to calculate the expansion rate of our universe in this epoch so, let us now put this into the expression for the dynamics of the universe. So, this will take us a few lectures back. If you recollect, we had walked out.