Astrophysics & Cosmology

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Lecture - 35
CMBR and Thermal History (Contd...1)

Welcome, let me remind you that we were discussing the cosmic microwave background radiation and I started off by telling you that the universe around us is transparent.

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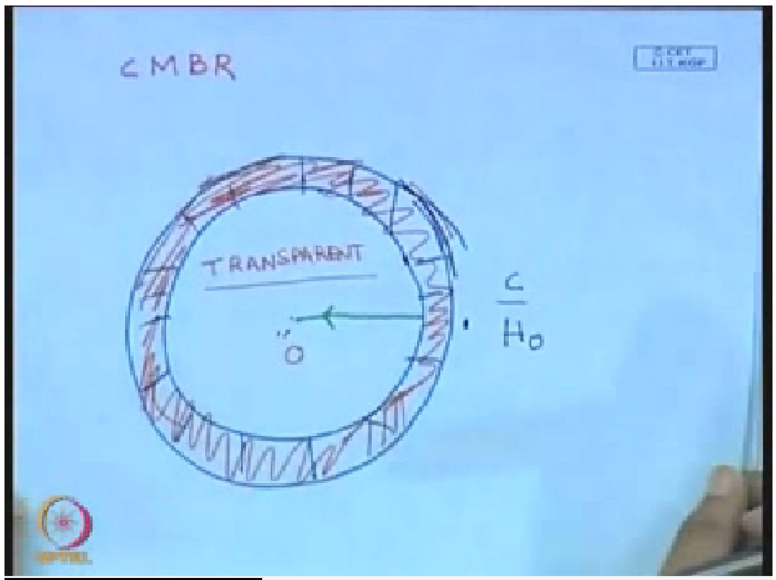

And we know from observation at least that it is transparent to quite a large redshift to distances comparable to the Hubble length scale and beyond those distances at some high redshift, much > 1, the universe, the radiation is sufficiently hot and the matter is sufficiently dense, so that the matter is completely ionized and the radiation and the matter interact strongly, are tightly coupled and they come to thermal equilibrium. So we were then discussing how to describe such a radiation in an expanding universe.

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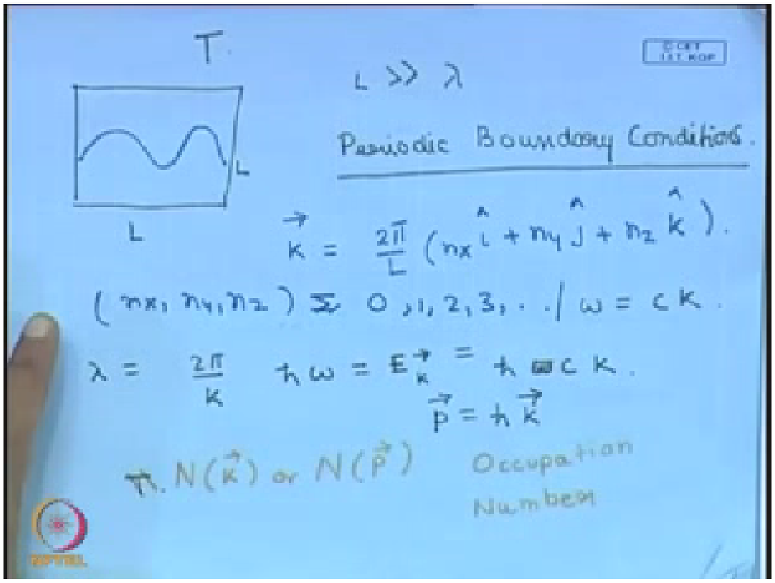

So to do this, we consider the volume L cube and we decomposed the electromagnetic waves inside this, we assume periodic boundary conditions and we decompose the electromagnetic waves inside this into modes and any mode which is represented by a wave vector is labeled by 3 integers, an X, Y and Z. Corresponding to every set of integers, there is a mode, okay and the wave vector, we know from quantum mechanics that particles are represented as waves.

And corresponding to a wave vector k, the particle momentum, we can also calculate the particle momentum which is h cross*k. So the particle in a mode k has a momentum p which is h cross k and energy, if the particle is relativistic, we know that the energy is h cross omega where omega is c*k and if the particle is not relativistic, we also discussed how to calculate this, okay.

Then we introduced the occupation number n which is a function of k or equivalent here function of p because k and p are equivalent and the occupation number tells us the number of particles that are there in a particular mode, okay. So in my box, I have decomposed electromagnetic waves into modes and the occupation number tells us how many particles are there in each mode.

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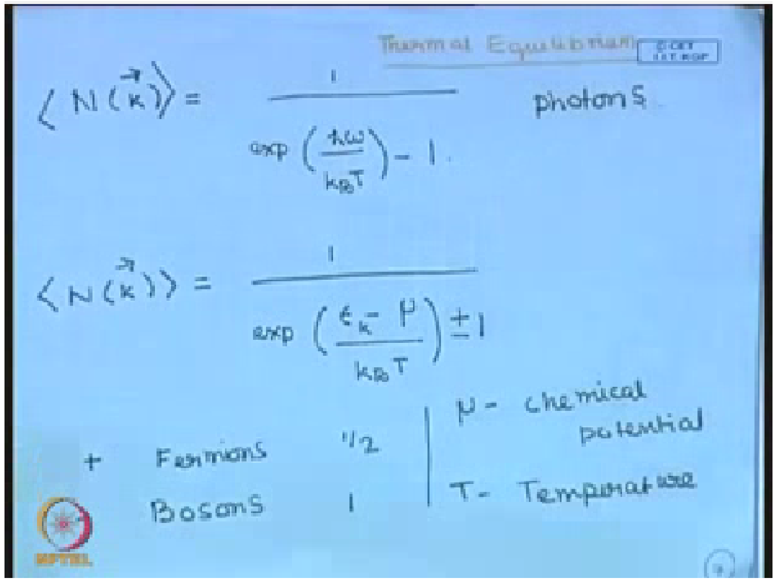

Then I also told you that if the particles in the box are in thermal equilibrium, the occupation number is completely determined if I tell you the chemical potential and the temperature, okay. So the thermal state is completely determined if I tell you the chemical potential and the temperature and the occupation number is given like this. It is very general for Fermions, Bosons everything. Photons are particular case which are Bosons and the chemical potential is 0. So this is occupation number, if it is in thermal equilibrium.

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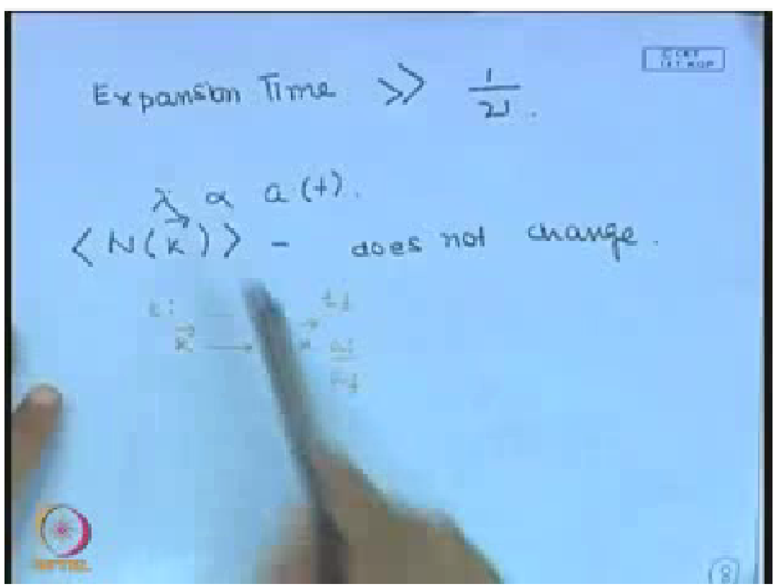

Now the expansion of the universe occurs on a time scale which is much larger than the time period of the oscillation. So we can think of the expansion as not changing the number of particles in each mode but just causing the wavelength or the frequency of the mode to change slowly, the expansion is a slow process, so the frequency or the wavelength changes slowly and the wavelength changes slowly proportional to a.

So the rate at which increases is much slower than the rate at which the wave oscillates, okay. So the expansion causes the frequency to go down or the wavelength to increase. Equivalently, the wave vector gets scaled like this or the momentum also gets scaled in the same way due to the expansion, okay. So we learn how particle momentum evolved under the expansion of the universe, okay.

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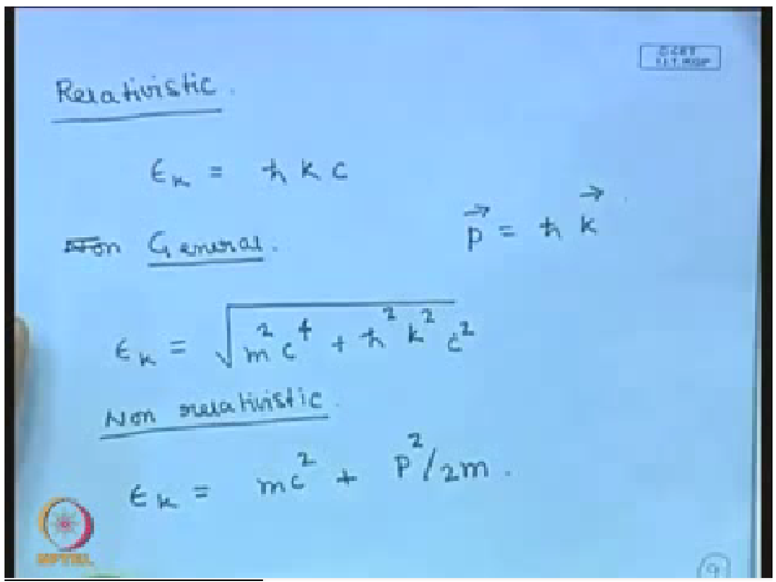

And the energy and momentum have different relations depending on whether the particle is relativistic or not relativistic and then we worked out what happens to the occupation number under the expansion of the universe, okay.

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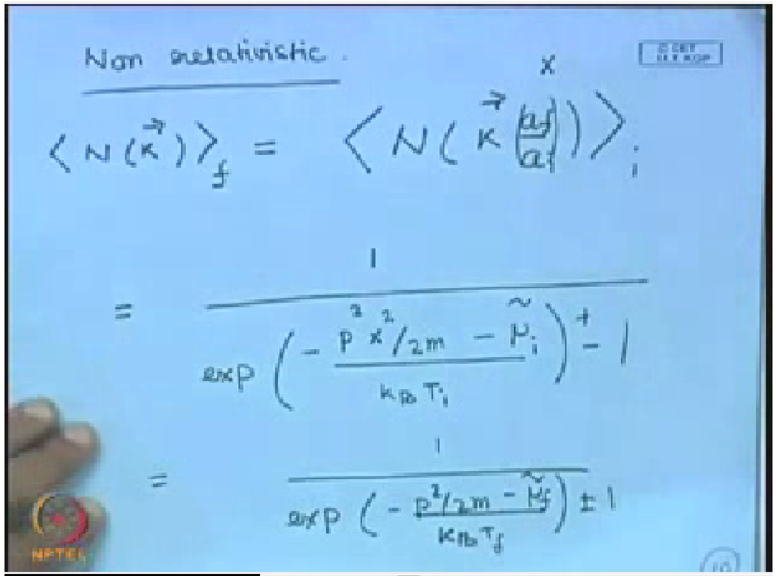

So essentially if I want to calculate the final occupation number of a mode k, it is = the initial occupation number of a different mode where the mode is scaled appropriately, okay.

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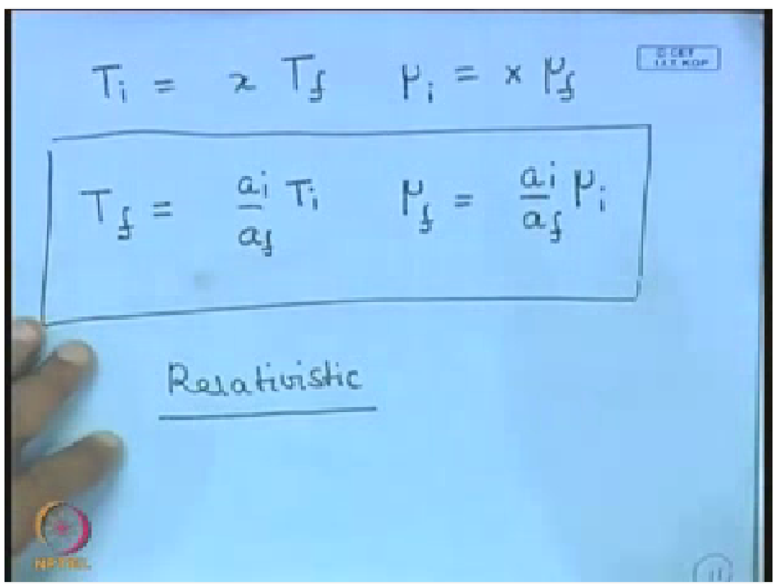

And based on this, we worked out how the temperature, so the entire occupation number is, the thermal state is described by a temperature and chemical potential, so we worked out how the temperature and chemical potential scale for relativistic particles.

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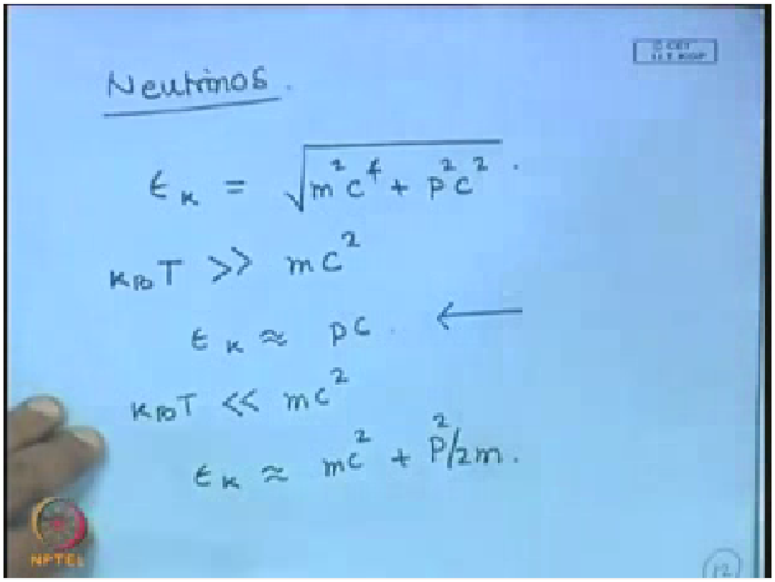

We also worked out how they scale for non-relativistic particles, so this is what happens if the particles are non-relativistic, okay.

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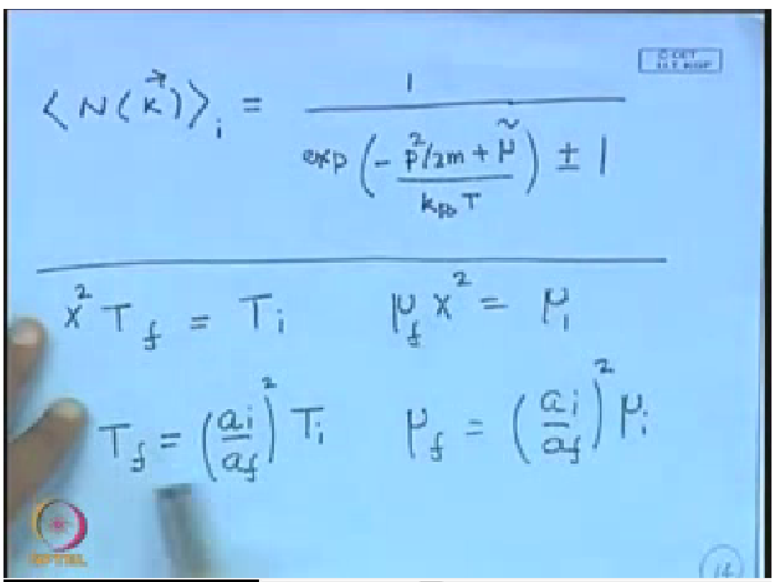

So the temperature which describes the distribution and the chemical potential, scale proportional to 1/a for relativistic particles and the scale as 1/a square for non-relativistic particles, okay. This is what we had done in the previous class.

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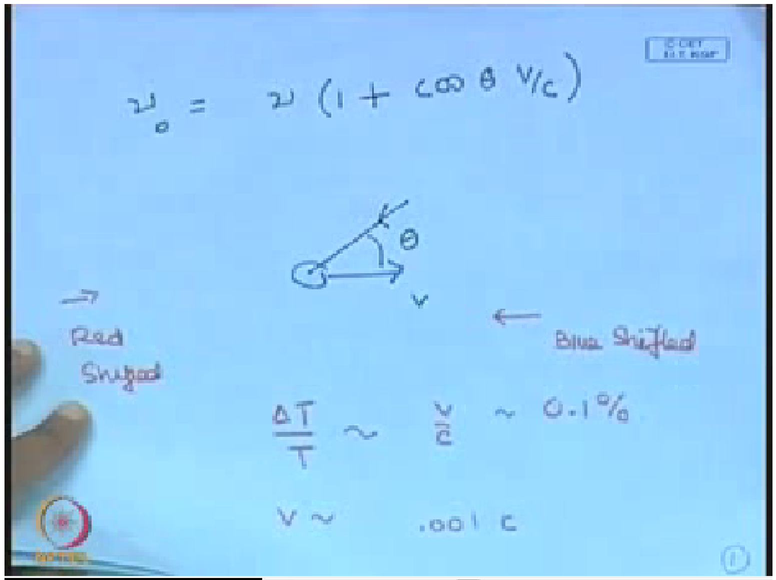

Let me also mention one more thing, the same argument can also be used to calculate the effect of our motion on the cosmic microwave background radiation, because our motion will cause the frequency, observed frequency for every mode to be slightly different because of the Doppler shift and the frequencies will get modified. So the observed frequency will be the frequency in the frame where the CMBR is isotropic. This will get multiplied by a factor 1+cos theta*V/C, okay.

Where theta is, this is our direction of power motion and theta is the angle with reference to this. So if I see a photon coming from this direction and if it has a frequency mu in the frame where the CMBR is isotropic, then I will observe it at a different frequency given by this. Similarly, the way vector will also get changed, okay and we know that the occupation number of that mode will remain unchanged.

The occupation number will not change, that is under this transformation, so if I want to calculate the occupation number of particle k, so frequency in a particular direction, all that I have to do is go back and look at the different frequency of the blackbody spectrum. So from this, you can work out how the temperature of the CMBR, you can see that in the moving frame of reference, the spectrum of the radiation will still be a blackbody spectrum, only that the temperature will now be dependent, different for different directions.

I hope the problem is clear. So imagine that in a frame of reference, the CMBR is the same in all directions. I go to a frame which is moving, each frequency gets modified depending on the direction I look at. So we can now show, using the same arguments, that the temperature is now direction dependent. Okay, I leave this to you for you to do as an exercise, okay.

So what we have been discussing till now has been how the chemical potential and the temperature of the distribution evolved as the universe expands. Let us now see how to use the distribution function, how to use the occupation numbers.

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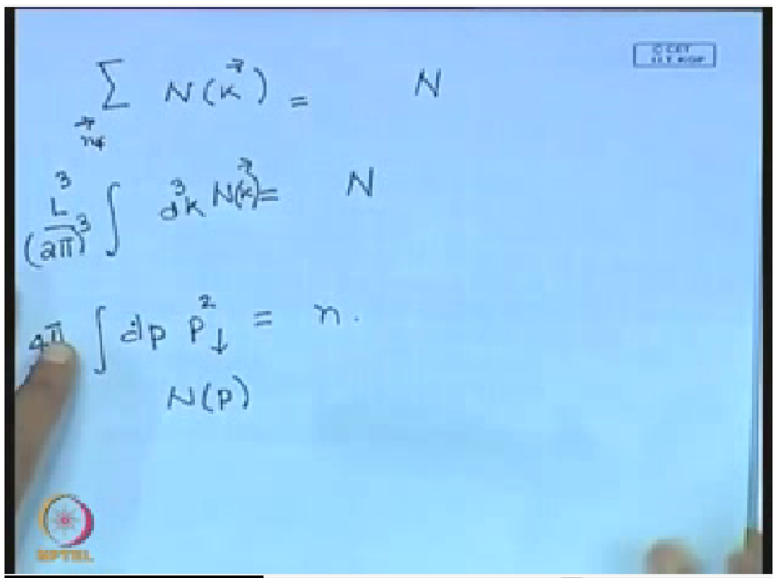

So the occupation number N K or NP, if I sum this over all modes, so then the modes are discrete, they are labeled by nx, set of vectors n, nx, ny, nz, which are integers, so each case is labeled by a set of integers. I sum over all these integers. This will give me the total number of particles in my box, okay. So this should give me the total number of particles in my box and in the limit where the box size goes to infinity, this sum I can replace by an integral d cube, so K is 2 pi, so I can replace this by an integral, d cube K L cube by 2 pi cube*N.

This should be = total number of particles. So in the continuum limit, the spacing between the K has become, L goes to infinity, L becomes extremely large, so the spacing becomes small and I can write the same sum as an integral like this, okay and since this is spherically symmetric, the occupation number is a spherically symmetric function of K, I can write this as a integral multiplying this with h cross, I can write it as dp*p square 4 pi, 4 pi comes from the solid angle integral in this d cube K, I am writing it in spherical polar coordinates.

There is a 2 pi here, so I have multiplied it with h cross cube to convert it into P. So I have to divide by h cross cube and h cross cube*2 pi cube will give me h cube. This is = the number density of particles, okay. Sorry into the occupation number which I keep on forgetting. So there will be occupation number as a function of P over here, okay. So I get this relation. So let us take, for example, photons in thermal equilibrium at a temperature T.

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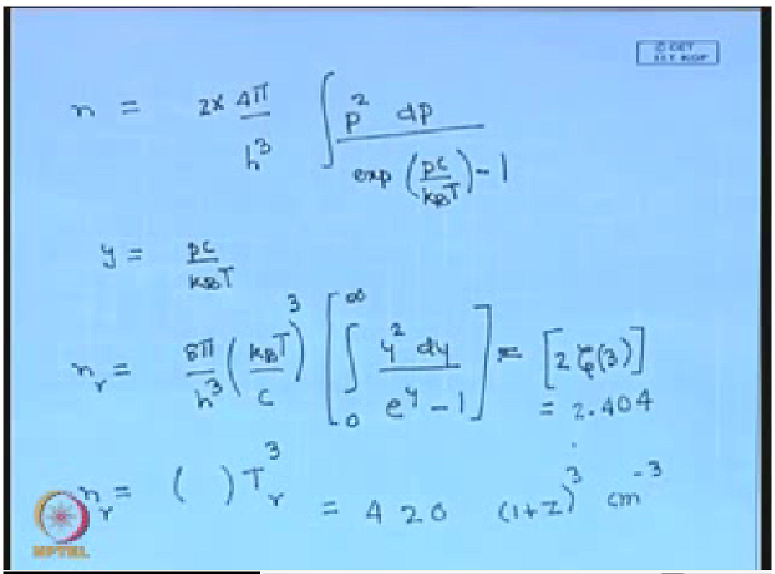

So for photons at thermal equilibrium at temperature T, what this tells us is that the number density of photons = 4 pi h cube and I have the integral, p square dp divided by exponential pc/kBT-1. Chemical potential for photons, photons can be destroyed and created. There is no conservation number, so the chemical potential is 0 and there is a factor of 2 here because photons have 2 polarizations, both of which are like independent particles, so they will both contribute to the number density. So I will have a factor of 2, okay.

So this allows me to calculate the number density of particles from the occupation number and the way you handle this, do this, is that you introduce a variable y which is pc/kBT and you write this integral in terms of y. So what you have is that the number density = 8pi/h cube and I will write this integral as y square dy e to the power y-1 0 to infinity and I have to put extra factors because I am converting this p to y.

So I will have kBT/C cube, okay. This quantity in the square brackets over here, so the quantity in the square brackets only, okay, so this is =, the quantity in the square brackets is twice the Riemann zeta function evaluated at 3 which is known to have a value 2.404, okay. So what do we find from here, we find that the number density of particles, of photons, so this is photons, so number density of photons, gamma, is some constant which I will not write down the value but the constant involves 8 pi h cube, the Boltzmann constant, the speed of light and 2.404.

So which are all known, okay. So we can determine this and it is proportional to T cube, okay, photon cube and the temperature we have seen scales as inverse of the scale factor with the expansion of the universe. So we can write this as 1+z cube into the present value, put in the present value of the temperature, 2.735. So if you put in the present value of the temperature and put in all these constant 8 pi h cube kB C cube*this number which is 2.404, what do you get is that this number density of photons is 420 1+z cube centimeter -3.

So there are 420 CMBR photons in a centimeter cube of volume present for every centimeter cube of volume in this room or anywhere in the universe, okay, 420 CMBR photons at redshift 0. If you go to higher redshifts, the scale factor is smaller, so this number goes up as cube, 1+z cube, okay. So this is how you can calculate the number density of particles if you know that your particles are in thermal equilibrium at a temperature T and chemical potential 0.

You can also do the same thing for any arbitrary chemical potential, okay. Now this is, if they are Bosons, you see for Bosons, you have a - sign here. If you had particles which were Fermions instead of being Bosons, so let me write the same thing for Fermions.

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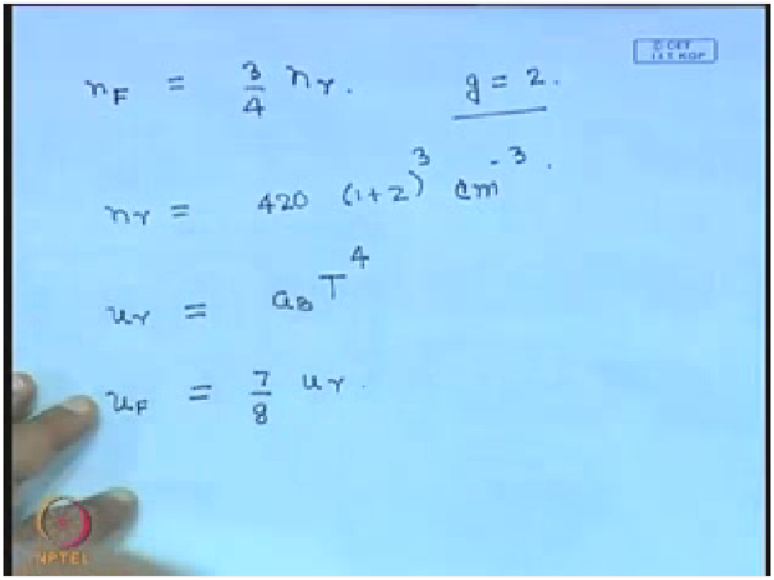

So for Fermions, the number of particles, if you Fermions with 2 polarizations, okay, then the number density of particles would be 3/4 the number density of particles for photons, okay. Assuming that there are 2 polarizations, g=2, okay. So for Fermions, all that you have to do is put a+ sign here and repeat the same thing. Fermions whose chemical potential is 0. So let me also put this here n gamma=420 particles 1+z cube centimeter -3, okay.

Now you can repeat the same exercise and calculate the energy of the system.

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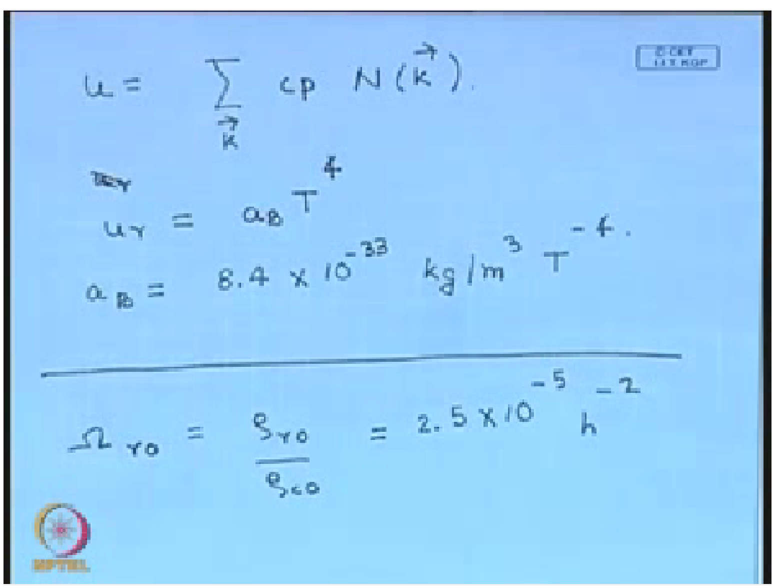

How will you calculate the energy of the system, if the particles are relativistic, what you will do is, you will use this, so the sum over all modes which are labeled by k, which are labeled by integers and you will have to take the energy of each mode. So the energy of each mode is c into the corresponding p, so each K value can be converted to a p value, right, c*p for a relativistic particle is the energy, we are doing it for relativistic particles into the occupation number.

This will give me the total energy. So you could repeat the same thing, write it as an integral over d cube K d cube p, etc. and what you are led to is that you will have the energy density of photons. We have done this exercise earlier if you remember when we were considering blackbody radiation. We did the integral, I told you how to do the integral. You will be multiplying the whole thing by a factor of c*p.

So you will get a factor of p extra over here, you will have p cube in the integral. You will have a factor of c also outside. So there will be an extra factor of c and there will be a factor of p cube here. If you now replace it using y, then you will have kBT/C to the power 4 because you have 3 p for that situation, okay and we have discussed this, so the energy density for photons comes out to be ab.

This Stefan-Boltzmann constant T to the power 4 which you get by summing up the occupation number of over the different states and aB, we have already discussed the value too, I have already told you what the value is in units of joules per, in the SI units. Let me now write down the value in a slightly different unit which is useful over here. So the Stefan-Boltzmann constant can also be expressed as 8.4*10 to the power -33 kg per meter cube T to the power -4.

So what we have done is that we have taken the energy density and divided it by c square and converted it into mass units, okay. Energy/C square converts it into mass units. So this gives me the energy density in mass units, kg per meter cube, okay. So we can use this to calculate the contribution from the CMBR to be present density of the universe and this contribution is parameterized by using omega gamma0, which is the ratio of the present density in the CMBR to the critical density which we have already seen, is 3h0 square by 8 pi g.

So you have to just put in this value over here, put in the value of the temperature 2.735 Kelvin and what you will get is that this comes out to be 2.5*10 to the power -5 h -2, h -2 comes because of the h0 not square here. So what we see is that at present the CMBR makes a very small contribution to the total density of the universe, omega matter is somewhere around 0.3 I told you. Lambda is around 0.7.

So compared with these, the cosmic microwave background radiation makes an extremely small contribution to the overall density of the universe, it is of the order of 10 to the power -5, 10 to the power -5 times smaller, okay, but there in mind that with increasing redshift, this contribution increases proportional to T to the power 4 or 1+z to the power 4. So in the past, the universe was radiation dominated. Okay this is something we shall come to in the future.

Now let us go back to the picture that we have. So in the picture that we have, the universe is transparent now. So with the expansion of the universe, the temperature just falls as 1/a. So if you go back into the past, the temperature increases as 1/a or 1+z proportional to 1+z and somewhere over here, the matter and the radiation interacts strongly. Now the thermal nature of the radiation, the blackbody spectrum, we have seen will not change all the way up to here because that is what we just saw, all that will happen is that the temperature will change.

The nature of the spectrum will remain same, the temperature will calculate C, temperature is the only thing that will change. Now the question arises that the matter and the radiation here may have different temperatures. So when the radiation interacts with the matter, I just trace it backwards, when the radiation interacts with the matter at this place where they both become sufficiently hot, maybe the thermal nature will be changed.

Because you have a temperature, the CMBR photons at a certain temperature, matter may have a different temperature, whatever, so how much is the effect of the matter on the CMBR. Now it turns out, let us just estimate this. So to estimate this, let us calculate the heat capacity of the CMBR.

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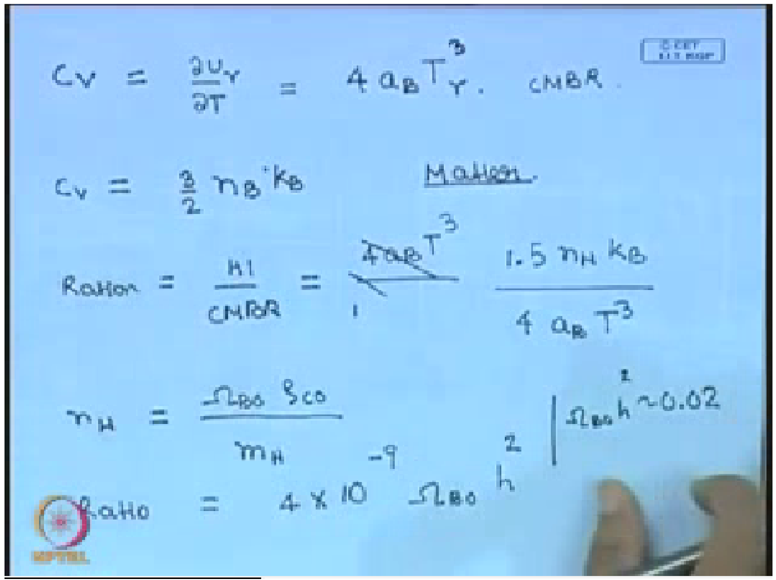

The heat capacity of the CMBR at constant volume, let us take a unit volume of the CMBR and the heat capacity at a constant volume. So all that we have to do is that we have to differentiate this with respect to T that will give us the heat capacity del U del T and this is 4aBT gamma cube. This is for CMBR. Let us now look at the heat capacity of the matter. So let us assume that the universe is made up of hydrogen, simple assumption.

So for hydrogen atoms, the heat capacity at constant volume is 3/2, the number density of hydrogen nH or number density of baryons, of baryons we are assuming that the matter is entirely hydrogen, so all the baryons are in the form of hydrogen atom, into T. So that is the internal energies in kBT 3/2nkBT, so if I differentiate with T, I will get 3/2nkB, that is the heat capacity of an idea gas.

So let us now compare the ratio of these 2 heat capacities. So this is for the matter assuming that it is all hydrogen. Dark matter does not interact with the CMBR. Let us assume that the baryonic matter. Baryon, I refer to protons and neutrons, assuming all of it is in hydrogen, okay. So the ratio of these 2, so for hydrogen and CMBR, this ratio =, so 4aBT cube and here I have, sorry other way round 1.5nHKB/4aBT cube, that is the ratio that we want to see and the number density of hydrogen can be calculated.

So we know omega baryons0, omega baryon is the contribution to the total density of the universe at present from baryons. By baryons, we refer to protons and neutrons, okay. Dark matter obviously does not contribute to this. So the heat capacity of the hydrogen of the matter that interacts with radiation is omega baryon into, so this will be omega baryon into rho critical 0, the present value of the critical density divided by the mass of a hydrogen atom which is known, the mass of a proton, okay.

This is how we can calculate the number density of hydrogen, this is the Boltzmann constant whose value is known and you put in this value here and if you put in these, then the ratio comes out to be = 4*10 to the power -9 omega baryons 0 h square. H square is there because I have a rho critical here, okay and omega baryon 0 h square has a value of around 0.02, even if we resume it to be 1.

Let us forget about, there is no dark matter, the maximum value this can have is of the order of unity because the maximum density the universe can have is of the order around critical density, okay. Even if you take a value 1, now presently observed values is around0.02. If you take a value around 1, this ratio comes out to be extremely small, okay. So what we learn from this is that though the density of the universe is here, is largely from matter, the density from the CMBR is extremely small.

The heat capacity of the CMBR is much higher than the heat capacity of the matter, okay. And this ratio does not change with the redshift because this scales as T cube which scales as a-3. This also scale as a-3 because the number density of particles scales as a-3.

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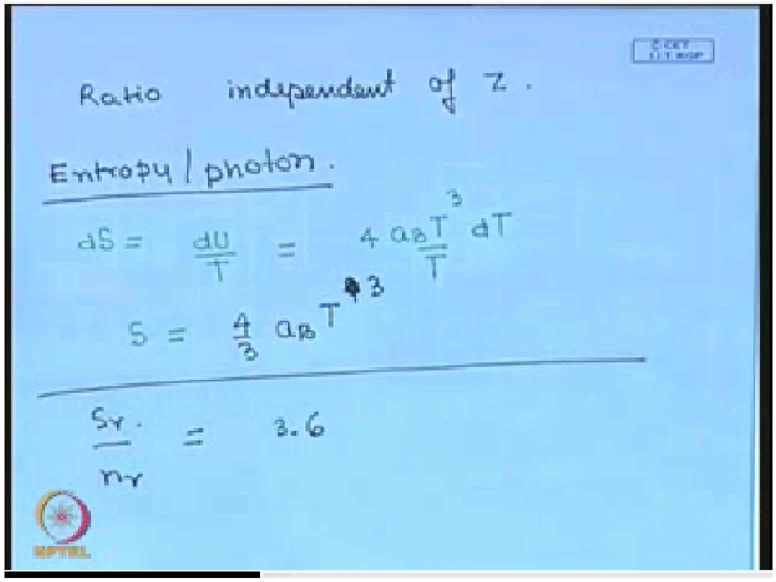

So this ratio is independent of z. So the ratio remains unchanged all the way over here and when the CMBR and the matter interact, come to thermal equilibrium, the CMBR loses a very small fraction of its heat to the matter or gains very little heat from the matter, whichever, which matter is hotter or cooler. So it is quite clear that the thermal property of the CMBR will not be affected much even when if it were to interact with the matter, okay because the matter does not contain much heat, okay.

Now if you repeat the same exercise, calculate the energy density, if you were to calculate the energy density using Fermions instead of Bosons, then the difference would be again, so for Fermions with 0 chemical potential, the difference would be that the occupation number would have a+ sign and if the occupation number had a+ sign, the difference would be, that okay, so we know that energy density of Bosons is aBT to the power 4 and if you repeat the same thing for Fermions.

So for Fermions what you would have to do is, you would have to put a+ sign here and repeat the same exercise. If you repeat the same exercise, you find that for Fermions, this ratio comes out to be 7/8, it comes out to be 7/8, energy density comes out to be 7/8 of the photons, okay. Assuming that again there are 2 polarizations for the Fermions. Okay. Now let us use this, so we have worked out how the number density of particles for Fermions and Bosons and all of these things.

What values they have for the CMBR and let us now look at some more quantities for the CMBR. So let us look at the entropy. The next interesting thing to look at is the entropy per photon of the CMBR. How much is the entropy per photon of the CMBR. Now the entropy does not depend on the details of the process, if you have a reversible process. So the expansion of the universe is a reversible process.

So the entropy, we can calculate the entropy as the dS=dU/T, okay and this comes out to be, so we have worked out what dU is, it is 4aBT cube dT/T and if you do the integral, you find that the entropy per unit volume, entropy per unit volume turns out to be 4/3aBT to the power 3 because there is a 3 here 2 and then if I integrate, now I get 4/3aBT to the power 3, that is the entropy per unit volume for the CMBR.

No if I look at the ratio of the entropy of the photon per unit volume to the number of photons per unit volume, both of these we have seen scale proportional to T cube. So this ratio is independent of redshift. With redshift, the T dependence cancels out, the scale factor dependence cancels out and the T dependence, so this ratio turns out to be 3.6, right. We have already learnt what aB is, so you can calculate this ratio, it does not change with redshift, it is a constant ratio, it is constant for the blackbody spectrum actually.

And it has a value 3.6, okay. The temperature dependence cancels out. So this is a number, the entropy per photon has a value, 3.6.
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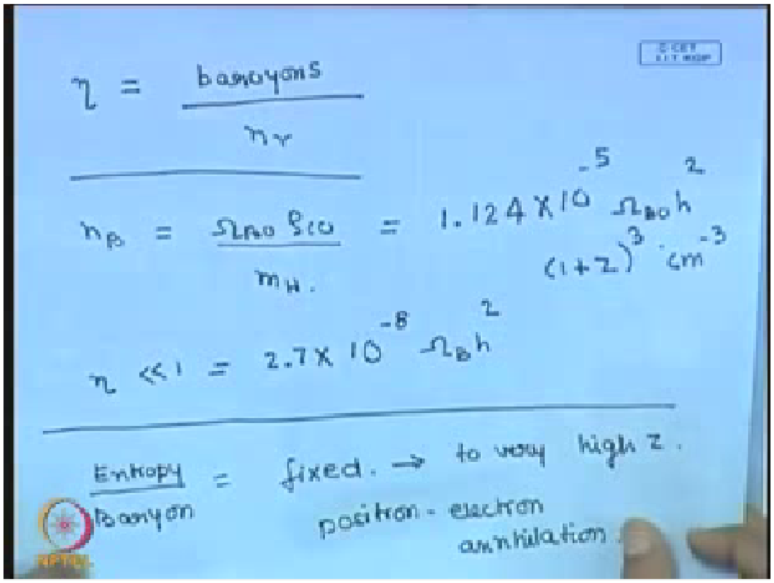

Now if you calculate the entropy per Baryon, so the ratio of eta, the ratio of photons of Baryons to photons number density of Baryons to photons. So this is a number density of Baryons to photons. Let us calculate this ratio. So we already know how to calculate the number density of Baryons, omega baryons 0 rho critical 0 by mH, the mass of a hydrogen atom, mass of a proton.

This turns out, you put in the values, this turns out to be 1.24, so this is the number density of Baryons per centimeter cube. This is number density of Baryons in the universe. So it is quite clear that this is much smaller than the number density of the CMBR photons in the universe. The CMBR photon, we have seen there are 420 of them per centimeter cube, the number of Baryons even if all the metal in the universe, omega is Baryon.

So if the total density of the universe were Baryons, the number density of Baryons would be much smaller, 10 to the power -7 times smaller at least, okay. So the matter that we see around us are all made up of Baryons, this paper, pen, hands, everything, protons and neutrons. So the mean density in the universe is of the order of 10 to the power -7 because I have told you this is of the order of 0.02.

So the mean density of the universe that you expect in the universe is of the order of 10 to the power -7 per centimeter cube. So obviously these are not representative parts of the universe, they are much denser, okay. Whereas CMBR photons, there are 420 of them in 1 centimeter cube. So this ratio eta is extremely small and it has a value which is 2.7*10 to the power -8 omega Baryon h square, okay, which is independent of redshift because both of them scale as 1+z cube, okay.

So you see what we learn from this, the entropy of the universe is largely in the photon's, the heat contained in the Baryons is extremely small in the universe at present or at any epoch, the heat contained in the Baryons is extremely small, the heat entropy is extremely small in the Baryons. It is mainly in the photons and the entropy per baryon, see the ratio of entropy per photon is fixed and the ratio of number of Baryons to number of number density photons also is fixed.

So the entropy per Baryon is also fixed independent of redshift all the way to very high redshifts, okay. Not over all redshifts, this ratio is fixed till the epoch of positron electron annihilation. So the entropy per Baryon is fixed, it has a fixed value and the fixed value is approximately this because this ratio is of the order unity, okay, so it has a very large value, inverse of this, not this, it is the inverse of this, okay.

So the entropy per Baryon is a very large value and it is mainly coming from the CMBR photons and it remains fixed all the way to a very large redshift till the positron electron annihilation, okay. So till now we have discussed mainly this part of the evolution. What we see is that there are CMBR photons, number density of photons we have seen, we have also seen the entropy, the heat content of these CMBR photons and then if you extrapolate go backwards to higher and higher redshifts, there is an epoch when they will all in equilibrium, okay.

And once we are in equilibrium, the effect of the matter on the CMBR is negligible, so we can just continue with the same temperature, thermal distribution even backwards to higher redshifts.

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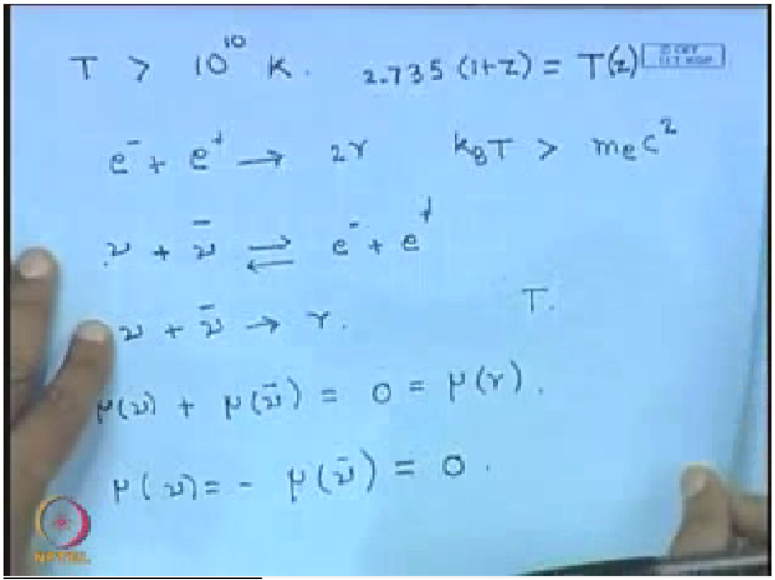

Now let us shift our attention to the redshift, to epoch when the universe had a temperature > 10 to the power 10 Kelvin. Let us jump. So we are considering an epoch at a very high redshift, okay. Temperature of the universe, if I go to higher and higher redshifts, the temperature increases. Let us jump to a temperature, so let us look at the universe at an epoch when the temperature is 10 to the power 10 Kelvin, okay.

At these epochs, so you can estimate the redshift, quite simple, right. We know that the temperature now is 2.735*1+z = the temperature at any z, arbitrary z. So we can estimate at what redshift, this is, obviously you cannot see sources from those redshifts. The redshift also is of the order of 10 to the power 9 or something like that, okay. At these redshifts, the temperature is adequately high for the CMBR to create positron and electron pairs, produce gamma, okay.

So basically the CMBR is hot enough, so the temperature of the CMBR is > m electron C square. So the temperature of the energy kBT, sorry, kB into the temperature of the CMBR, the energy corresponding to that is > the rest mass of the electrons. So the photons in the CMBR have adequate energy to produce electron-positron pairs, okay, at temperatures > 10 to the power 10 Kelvin.

Also at these temperatures, we know that there are neutrinos in the universe. So the neutrinos will also be in equilibrium, will also be produced and they can also produce photons. So you can have the annihilation producing photons, okay. So you have at these temperatures > 10 to the power 10 Kelvin, the CMBR is adequately hot so that it can produce electron-positron pairs, their photons will be in equilibrium with the electron-positron pairs.

Photon is also be in equilibrium with the neutrinos, okay and they will all be at the same temperature, let us say it is T, okay, given over here. So at temperatures higher than this, we have all these 3 things in thermal equilibrium. Now there are 2 things that you required to describe the entire state. One is this and other is the chemical potential. Now if you have a chemical reaction like this, so if this is in thermal equilibrium.

Then we know that the chemical potential of the neutrinos + the chemical potential of the anti-neutrinos, should be = 0, which is the chemical potential of the photons, okay. So in thermal equilibrium, the chemical potential of the reactants = the chemical potential of the product, okay. So the sum of these 2 chemical potential should be = 0 because photons have no chemical potential, they can be produced or destroyed. There is no conservation number of photons, okay.

So this tell us that the chemical potential of the neutrino should be = - the chemical potential of the anti-neutrino, nu bar and we will assume that these are both 0, okay.

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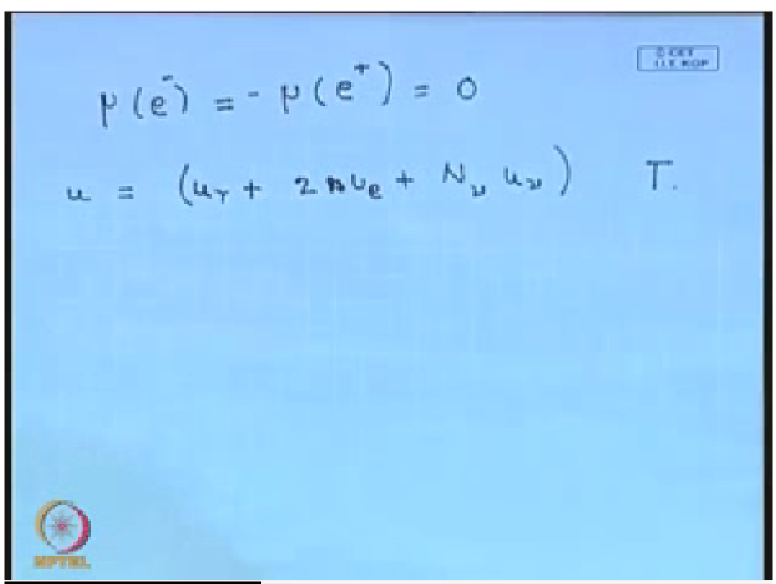

We will also assume that the chemical potential of the electron and the chemical potential of the positron, e- e+, are also 0, okay. So at these temperatures of 10 to the power 10 Kelvin, the number density of photons will be 420*1+z to the power, z will be of the order of 10 to the power 9. So the photons will have an enormous number density, the electrons which are Fermions will have a comparable number density at those temperatures.

At present, we have seen that the electrons whose number density is of the order of the number density of Baryons have a very small number density. So electrons that we have leftover now, they are insignificant basically at that epoch, okay. You see the chemical potential decides the number density. So we can safely assume that corresponding to the present electrons that we have, that the chemical potential is 0.

Strictly speaking, there should be no electrons left, that small fraction, we shall not bother about at those epochs, the small number density corresponding to the number density of Baryons. Universe, we know is neutral, so protons and electron number density has to be same, right. So we will assume that the electron number density that we have now is basically consistent is 0, okay.

We shall ignore that and so we will assume that the electrons and the positrons, so if you assume the electron and positron have the equal and opposite chemical potential, there total number densities will be same but now we see access electron. We will not bother about that, that is what we are doing basically because this will make a very insignificant contribution at these redshifts, okay, where the number density of electrons is comparable to the number density of photons.

So let us now start with the universe at somewhere over here in redshifts > 10. At these epochs, the total energy content of the universe, volume density of energy will have one contribution from the photons, U gamma, + one contribution from the electrons, so electrons, there are 2 kinds of particles electrons and positrons, sorry electrons and positrons, each of them have spin half, so the electron and the positron, they will both contribute.

So there will be a factor of 2 basically and energy density of electrons which is a Fermion because the positrons are also there and then if I have N species of neutrinos, N families of neutrinos, so I will have the number of neutrino families. We now believe that there are 3 families of neutrinos, the electron neutrino, muon neutrino, and tau neutrino, so N will be 3 into the energy density of each neutrino, okay. Each neutrino kind has only 2 polarizations.

So this is what we have at some early epoch, okay, that is the energy density of the universe at some early epoch and it has a temperature T. Let me bring today's discussion to a close here with this right here and let me recapitulate what we have learnt. So what we have learnt in today's lecture is that if you go sufficiently back in the past, when the temperature is more than 10 to the power 10.

The universe is sufficiently hot to produce electron-positron pairs and also adequately hot to produce electron neutrino and anti-neutrinos and they are all in thermal equilibrium at some temperature T. Further we assume that the chemical potential of each of them is 0.