Astrophysics & Cosmology
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Lecture – 34
CMBR and Thermal History

Welcome. Today we are going to discuss the Cosmic Microwave Background Radiation, CMBR. We have already learnt about its discovery in 1960s. This is the radiation which really Isotropic and it was discovered in 1960s and its spectrum was very precisely measured in 1990s.

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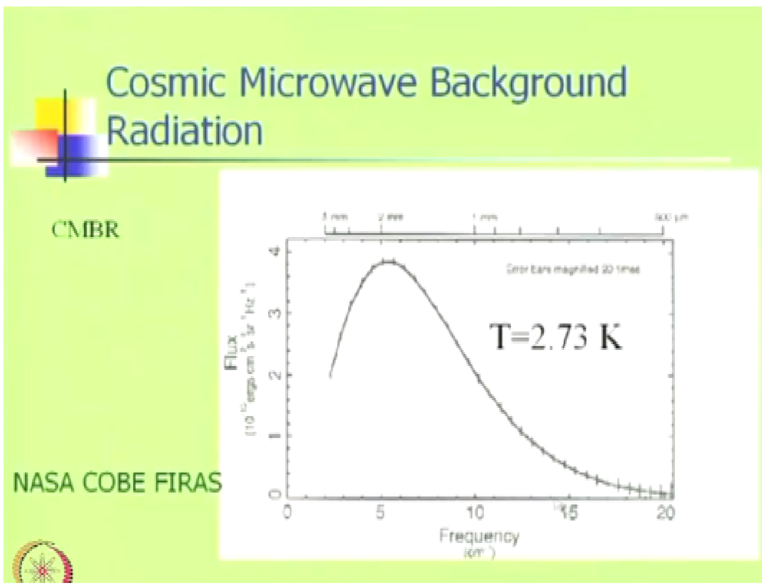

And the spectrum was found to be very well fitted by a Black-body spectrum of temperature 2.73 Kelvin. This was a satellite experiment, a satellite was called COBE and the experiment is called FIRAS on satellite. Satellite was launched by NASA in the 90s. And this experiment established that this radiation which we see coming from all direction is indeed a Black-body spectrum has a Black-body spectrum with temperature 2.73 Kelvin.

This is possibly the best Black-body spectrum that has ever been measured till date anywhere including experiment on earth. And the error bars here have been magnified to 100 times so that they can been seen. They are actually; they are one sigma error which are actually 100 times smaller. So, it is a Black-body radiation, the peak is in the millimetre and the bulk of the radiation is in the millimetre and centimetre range of the spectrum. And, it is quite Isotropic, absolutely Isotropic, okay.

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Now if you take a closer look, so if you remove an Isotropic component, so if you remove a component of the Black-body spectrum, so if you remove the radiation corresponding to a Black-body spectrum at 2.73 Kelvin at from all directions, what you have left looks like this. So there is, the next thing that you see, once you remove this monopole component so the Isotropic component is a Dipole which is quite visible over here.

You see this part is much frontier and this part is much darker. Okay, so this is a Dipole pattern. The thing that you see in the centre; passing through the centre is-- the image is the imprint of our own galaxy. After all we are seeing the Cosmic Microwave Background Radiation through our own galaxy which leaves the imprint, this can also be removed by doing observations at over a large number of frequencies.

Anyway the main point here is that after we remove the Isotropic component the next thing that you see is a dipole and the temperature variation of this dipole is point 0.00335 Kelvin, so it was around 1% fluctuation. It is a 1% fluctuation. So it is Isotropic .1%, not even 1%, sorry this is a . 1% fluctuation in the temperature. And there is a hot one direction in which on the sky in which the temperature is this much hotter and another direction in the sky where the temperature is this much colder. Okay.

And this dipole pattern in the Cosmic Microwave Background Radiation is interpreted in terms of our motion of our solar system. So the, if the solar system is moving in, this is the solar system let us say.

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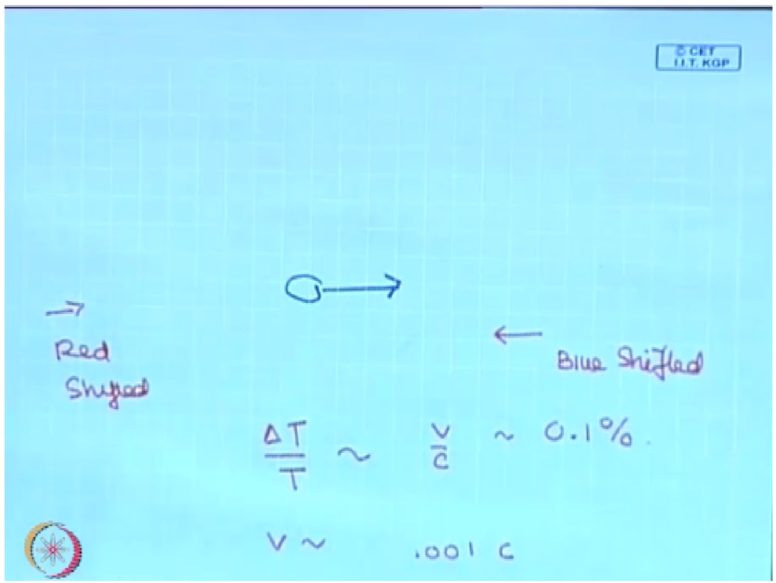

And if the solar system is moving in some direction then the photons from this direction with reference, we are assuming that there is a frame of reference in which the same here is Isotropic. And if the solar system is moving with respect to that then photons from this direction will be Blue Shifted and the photons from this direction will be Red Shifted due to the dark Red shift and the shift is V/C which is also proportional equal. Okay.

So the fractional variation in the temperature of the CMBR is directly proportional to V/C. So this is of the order of 0.1% which straight away tells us that V is of the order of .001*C which is —and so this dipole can be interpreted in terms of our motion with reference to a Cosmological frame of reference where the microwave background radiation will appear Isotropic. Okay. Now if you look at, if you remove the dipole component also, so if you transform everything to a frame of reference where the CMBR the dipole is not there.

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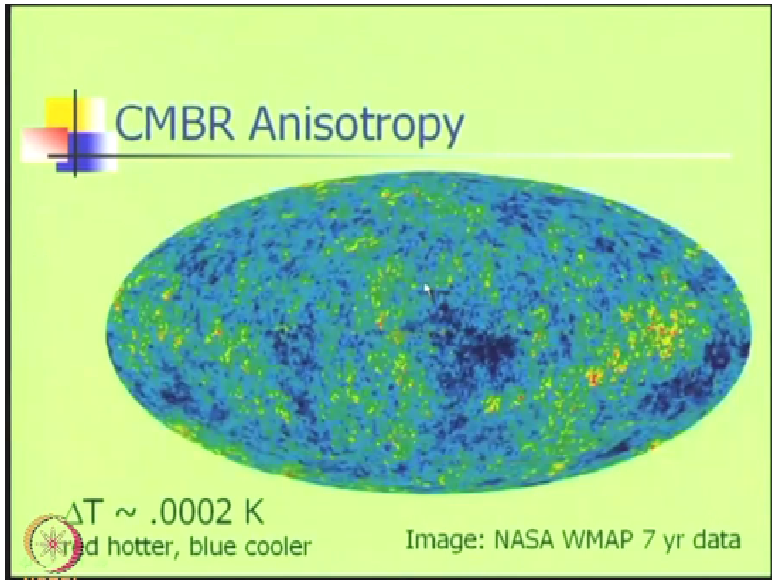

The CMBR still has some small Anisotropy left. So this is a picture of the fluctuation in the Cosmic Microwave Background Radiation after you have removed the main dominant Isotropic part and the dipole as well as the contribution due to our galaxy. Okay. So this is the picture of the sky have the CMBR Anisotropy an Anisotropies in the Cosmic Microwave Background Radiation on the sky after all those things have been removed. The dominant component and the dipole both have been removed.

So the fluctuations that remain you see are even smaller, they are of the order of 2*10 to the power -4 Kelvin. And this is an image made by NASA satellite again called WMAP (()) (06:31) Microwave Anisotropy probe the satellite is still carrying out observation and this image shows you, this image shows you the result of 7 years of the observation. Okay. And in this image the redder parts are hotter and the bluer parts are cooler. Okay. So it is the Cosmic Microwave Background Radiation is Isotropic to a very large degree. Okay.

It is Isotropic to a very large degree but there are minute-fluctuation in this radiation. And these minute fluctuations are very important for our understanding of the universe; let me explain this to you without going into detail.

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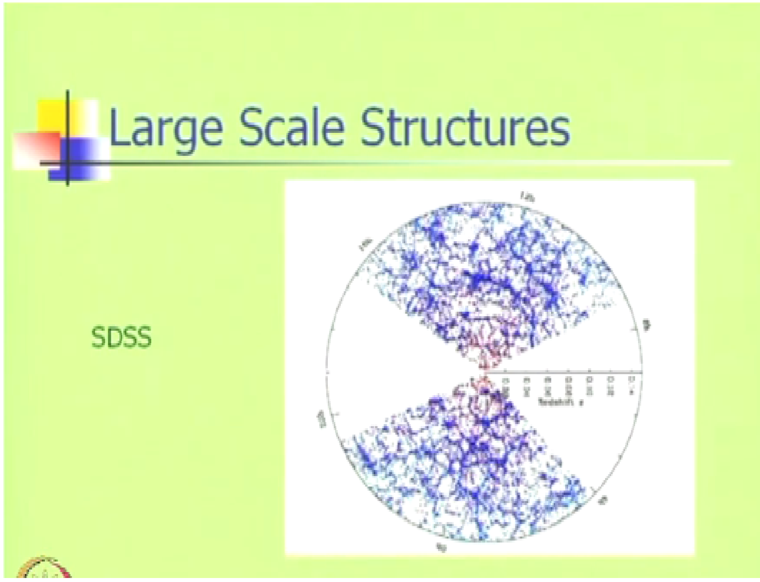

So if you look at the distribution of galaxies in our universe for example. I have told you that we expect the universe to be homogenous Anisotropy, at least on large scales. Now this image shows you that distribution of galaxies in a part of the universe each point here is a galaxy. Okay. The distance was measured using Red shifts and the Red shifts are plotted along this axis. So the largest Red shift that you have here is around 0.1, okay.

So it is roughly 10% of the horizon size. Okay, 10% of the horizon size, few 100 mega particles, horizon is few 100 mega particles. And the other direction is the angle on the sky. So you have the distance from red shift and you have angle on the sky. So what do you see is that it really does not look homogenous anisotropy at least, right? So the point is that there are large scale structures in the galaxy distribution in an universe. Okay.

They are not perfectly so there is a Len scale, the len scale is somewhere around 100 mega power 6 of the order of 100 MPC, may be somewhat less maybe somewhat more. Beyond which the universe is homogenous. Okay, but on smaller scale there are considerable amount of structure. At this—the study of the structure is a very interesting topic itself which we shall not going to in this course.

These are refers to a large scale structure, and this is showing you the data from the Sloan Digital Sky Survey, SDSS. Okay. So the question is how do these things arise a universe which is largely Homogenous Anisotropy.

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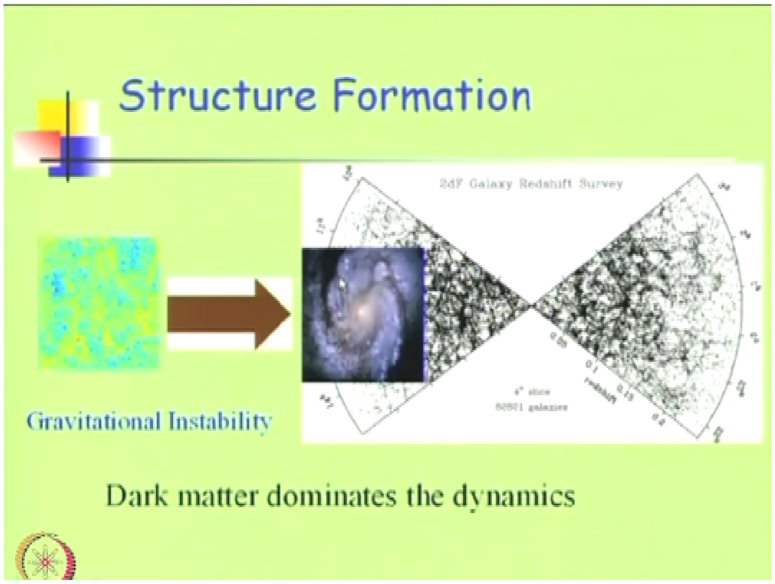

So the current belief is the belief is also quite well established and well accepted is that there were initially in the universe some small fluctuations in the density, it was not perfectly homogenous anisotropic, there were small fluctuation in the density. And these small these fluctuation in the density grow by the process of Gravitational Instability. So what is the process of gravitational instability?

In the process, the regions where the matter density is slightly more than average attract the matter from other places. And the region where it is slightly below average the matter goes out from there. And in this process, when these fluctuations grow as a consequence it is an instability. And as a consequence of this process you form galaxies the universe in the pas did not have galaxies, nothing; okay matter was very close to homogenous anisotropic.

But you have the small fluctuations and the cosmic microwave background radiation is probing these small fluctuations at a red shift of around 1000, that is the interpretation. Okay so Anisotropy in the Cosmic Microwave Background Radiation are probing these small fluctuations at a red shift of around 1000. And these fluctuations grow by the process of gravitational instability to form galaxies and to form this clustering pattern you see in the galaxy distribution.

This is again is another Red shift survey, here again the distance is from red shift and this is the angle in the sky, this is another Red shift survey called the 2DF Red shift Survey. Okay. So the Anisotropy in the Cosmic Microwave Background Radiation are very important probe of how these structures formed in our universe because they allow you to probe this same fluctuation that grow to give you this at a much earlier Red shift.

And the theory that you should be able to explain that you arrive from here to here which it does. Okay. Another point which I should make is that the entire dynamics of this process is dominated by Dark matter. So it is-- the dark matter which—so of the matter in the universe the barrier of the electrons, protons the protons and the neutrons make a very small contribution, it is mainly dark matter.

So, the fluctuations in the dark matter essentially which dominates the dynamic. The Dark energy it is believed, well dark energy have negative pressure and it is believe that the dark energy is not affected does not participated in this process of structure formation except for driving the expansion of the universe that too beyond a certain-- only at low Red shift not in the at high Red shifts. Okay.

So this is a brief overview of the Cosmic Microwave Background Radiation particularly what are the crucial points in the in it is what are its crucial properties. So it has, it is largely anisotropy; there is a dipole component which were interpret in terms of our motion and there are other anisotropy once you remove the dipole and these anisotropies allow us to probe the large scale structure formation in the universe. Okay.

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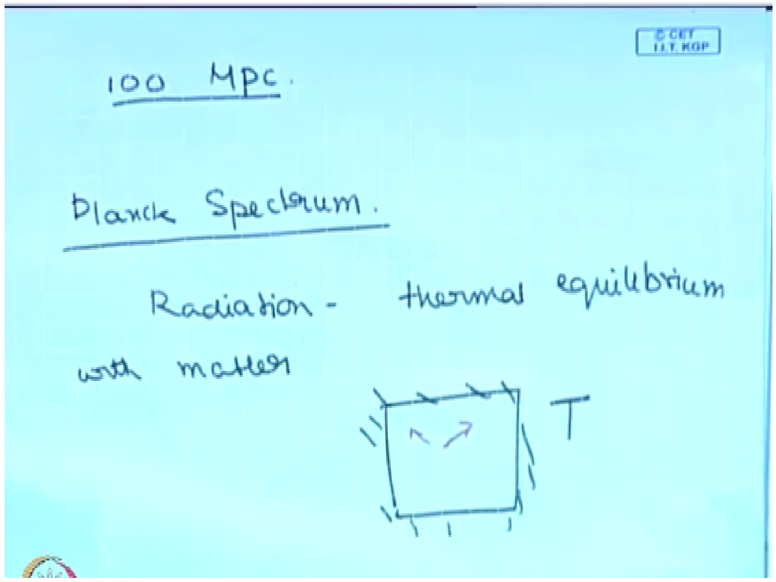

Now the Cosmic Microwave Background Radiation was discovered in the 1960s. And its discovery completely revolutionized our understanding of cosmology. Okay. So let me explain this to you. Okay. So we are just imagine and observer sitting over here and we are receiving this radiation which is in which is a black-body spectrum. Now if a radiation is a black-body has a black-body spectrum we know that it originates then radiation a black-body spectrum comes about when radiation is in thermal equilibrium with matter at some temperature T, right. So this is something that we know.

So the Planck spectrum corresponds to radiation in thermal equilibrium with matter. That is the picture that we have. So we imagine a Cavetti at a temperature T and if the radiation inside; if the radiation inside comes to equilibrium with the walls of the-- with the material inside the Cavetti then the radiation has a b blank spectrum or black-body spectrum. Okay.

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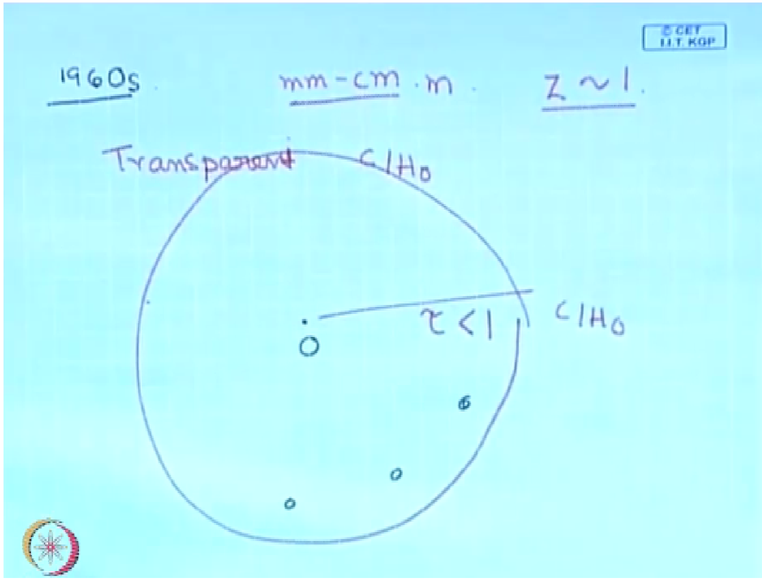

Now this is observer, we are sitting here and we can see inside, so the if you look in the millimetre or centimetre parts of the wavelength or even meter for that matters, parts of the wavelength of the spectrum we can see objects all the way out to a Red shift of order unity and even larger, such objects are visible, so we can identify the object universe till Red shift of order unity, which essentially tells us that the universe is transparent;

Optical depth to nearly the Hubble to the horizons C/H naught, where you have Len scales of the order of C/H naught when you reach a sheet of Red shift of 1. So, the universe the optical depth all the way to a Red shift of (Len) scale of order C/H naught is less than 1, because we can see objects over here. If the optical depth was more than 1 the light from those objects would be obscured.

So we can see astronomical objects all the way out here which tells us at the universe is transparent all the way to this distance. So the CMBR photons that we are receiving the dark the fact that the optical depth is 1tells us that the CMBR photons that we are receiving did not interact with matter within a len scale of the order of C/H naught, there was no interaction. If there was no interaction how did it come to equilibrium? So this basically tells us that the CMBR that we are seeing did not originate in the nearby universe, the photons.

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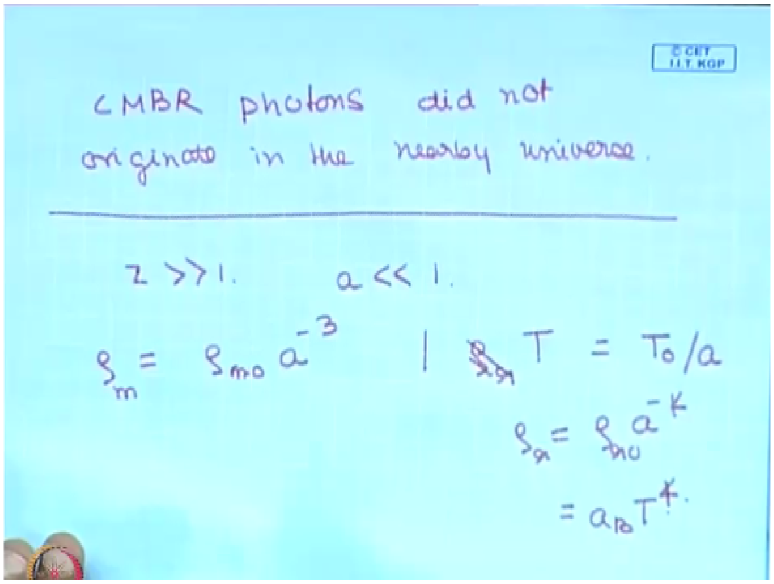

So the CMBR Photons that we are seeing, because these photons have a propagating freely for nearly the horizon, nearly not exactly but len scale compare to the horizon all the way out to Red shift unity and larger okay 3, 4, 5, 6 till where you can discrete objects. Okay. So where did the CMBR photons originate? Where were these photons in thermal equilibrium with matter? So the picture is that if you go back to high Red shifts.

if you go to Z much > 1 or A much scale factor much < 1; we have seen that the density of matter at those Red shifts will be = the rho naught, rho matter naught*A to the power -3 because rho*a cube is a constant for matter, okay. So the matter density and the high Red shift universe much larger. The radiation density, okay or the temperature of the radiation, we have briefly discussed.

We shall discuss it again, also scales as the present temperature by A. So in the high Red shift universe the density of the material was much larger, the temperature of the CMBR if you just take this temperature is proportional-- temperature into scale factor is the constant which we have seen by looking at the energy density, we know that the energy density scales as rho r naught a -4 this we have worked out and we also know that this is = AB T to the power 4, right.

So from these arguments you can describe clear that the temperature into the scale factor is constant.

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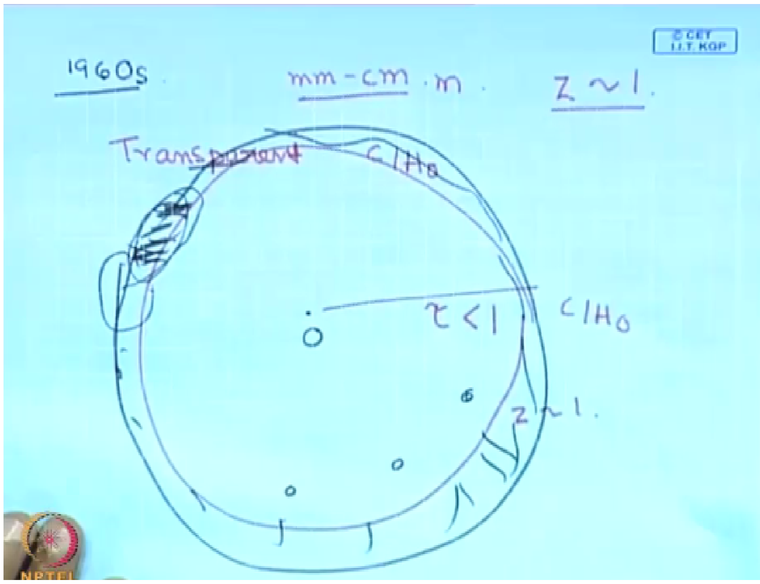

So in the early universe in these parts in the early universe, the temperature of the CMBR was much hotter and the density was much larger, so sufficiently back in the past the CMBR the universe was hot and dense and the CMBR we are in the thermal equilibrium with the matter at some high Red shift. Okay. So this is the basic feature. So the CMBR was in equilibrium with matter at some high Red shift, where the density was much more and temperature was much higher.

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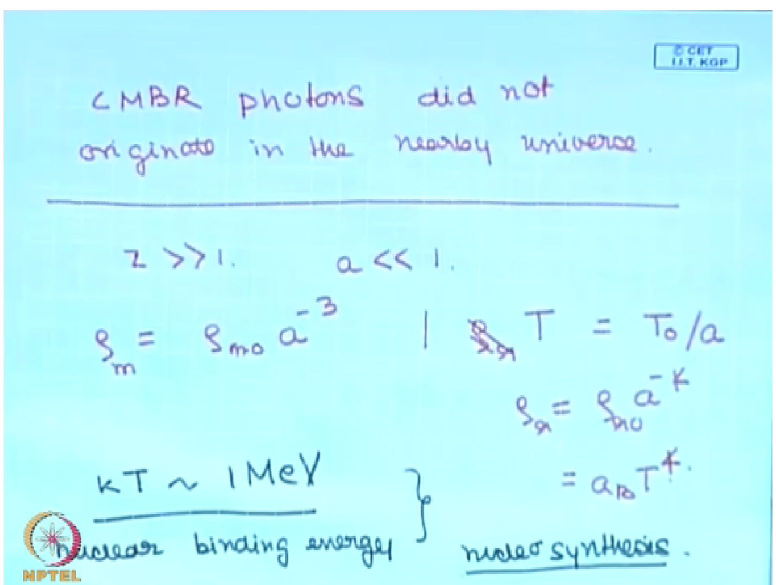

Further, if you go back sufficiently into the past the temperature will at some stage V of the order of 1 MeV, we know that-- so I have told you the temperature increases as inverse of the scale factor. So if you go sufficiently back in the past the temperature will become of the order of 1 MeV, this is adequate this is comparable to nuclear binding energy, so all the nuclear I will get this associate, okay. And, so essentially you have the nuclear reactions taking place that goes high Red shifts.

And so you can explain the abundance of light elements of abundances assuming that they were found in this high density and high temperature material that was there in the universe high Red shift, okay. So you can also have nuclear synthesis of light elements, okay. The heavier elements were made in start and we have already learned about that. And the nuclear synthesis the abundances of light elements are found to be more or less the same all through the universe. So this is the natural explanation for that.

Now what is the significant, why is the CMBR is so important. Now that we have this basic picture, let us go back to our discussion of the CMBR. So I have told you, that the CMBR was discovered in 1960s.

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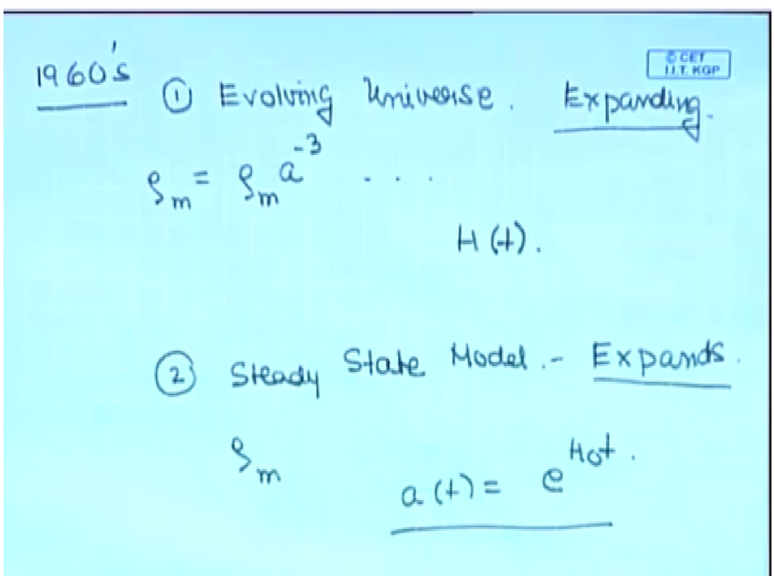

And at that time there were 2 models which were competing in Cosmology. One was the Evolving Universe which is the model that we have been discussing. So in this model things change the scale factor. So for example, the matter density = the present matter density A to the power -3 etcetera. Everything evolves with the expansion of the universe. Okay. There was another model at that time which was also competing with this model which is the Steady State Model.

In the Steady State Model, there is no similarity in the past. The density of the universe remains a constant. So the universe expands in this model also that is an observed fact. Both of these model have an expanding universe, but in this steady state model you have to have matter been created so that the density remains constant. Okay. So nothing evolves in this model. On the average the universe always looks the same as it does now.

And the scale factor has an exponential will have a exponential expansion. There is no similarity if you have an exponential expansion the Hubble parameter also remains constant. Here the herbal parameter is a function of time. It was larger in the past. Okay, in the Steady State model you have an exponential expansion-- it is like our cosmological constant dominated module where you have an exponential expansion, the Hubble parameter remains fixed. The universe looks the same all the time.

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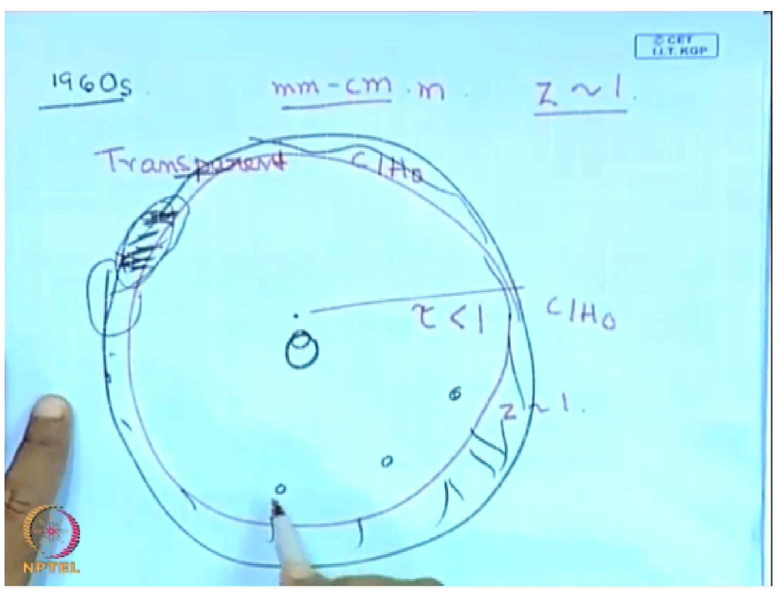

Now you see if the universe looks the same all the time, it is transparent, you know it is transparent arounded, let me transparent throughout, it is not possible to, this was refers to CMBR was discovered. Now if you have the, the universe we know is transparent the observe—we are observer here we see that the universe is transparent. So in the steady state model, the universe will be transparent throughout and the density will not be higher in the past nor it will reduce in the future.

So in such a model it is very difficult if not nearly impossible to produce to see the cosmic microwave background radiation in a very natural way. Okay, the one method which was tried out like the scattering of re-emission of star light but these methods did not worked.

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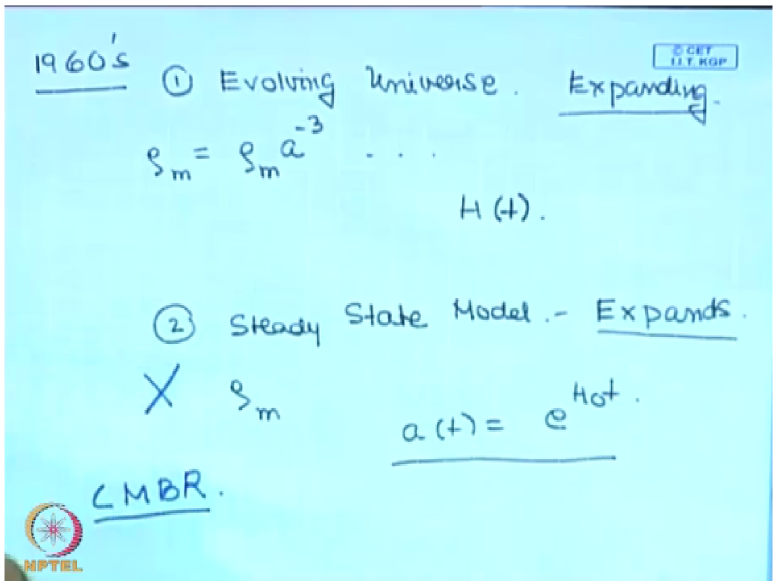

So this model was more or less given up after the discovery of the Cosmic Microwave Background Radiation. Okay, because the universe remains transparent all the time. And the matter will never come to equilibrium with radiation. Okay. So let me again remind you of the basic picture that we have. The basic picture that we have is that,,,

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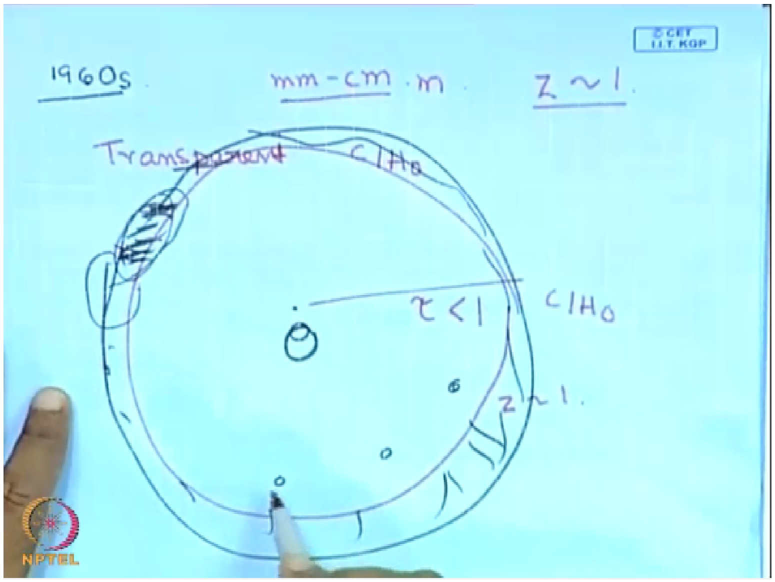

The universe around is transparent now, the CMBR photons have been propagating freely. And more or less till the till a distance of order C/H naught the photons have been propagating freely, somewhere over there at much higher red much higher than the 1, the universe is hot the temperature the CMBR is adequately hot and the density is adequately high so that the matter at radiation are in thermal equilibrium. And if you go back further in the past...

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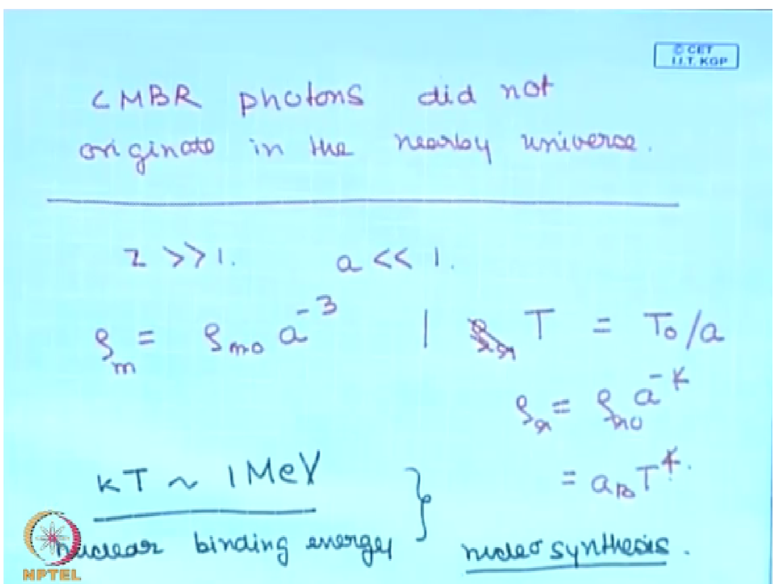

The temperature is higher and you have matter being photon the nuclear is getting associated photon associated and there was no nuclear before that; the temperature is higher than the nuclear binding energies. Okay. Now, with this background let us.

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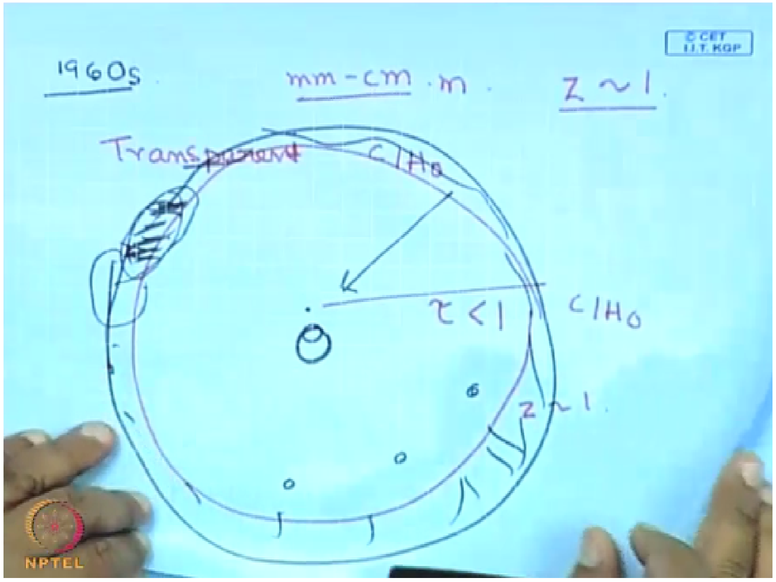

So the first thing that we shall study is the how does the CMBR spectrum evolve? How do the CMBR photon evolve in this regime where they are not interacting with matter. How do the CMBR photon is evolve, how does the photon distribution evolve?

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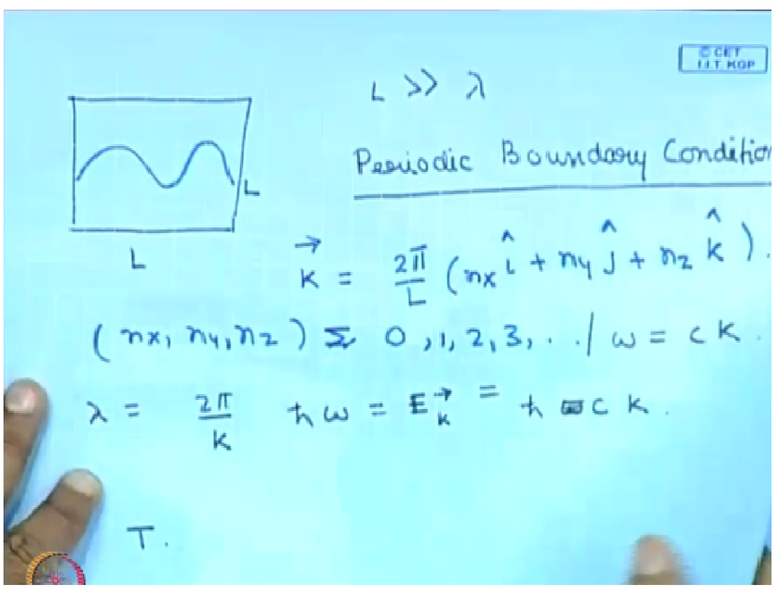

So to understand this, let us consider a region of the universe of Len scale L of box in all sides. I am just drawing 2 of them. Okay. And the Len L is assumed is much larger than the wavelength of the radiation that we are dealing with, but is small compare to the curvature len scale of the universe. Okay, smaller than the curvature of the universe. So let us now, so this is the box and we will assume periodic boundary conditions.

So the electromagnetic waves insides can be broken up into modes, so the electro electric field in magnetic field inside can be broken up into modes and these modes have to be periodic, this does not look periodic, but so it basically restrict the wave numbers that are allowed K to be 2 pi by L*nx i + ny j + nz K, where nx ny nz are integers. So basically they can take integer value 0, 1, 2, 3 etcetera, right.

Because any other value of K will not be commensurate with the periodic boundary conditions, the wave have to repeat after this. So there are only a discrete number of modes which are allowed inside of for the electromagnetic wave which are allowed inside and they can be labelled by wave vector K which are enumerator by nx, ny, nz which can be integers. Okay. And the energy, for electromagnetic wave.

We know they massless so the energy, before that the wavelength is 2 pi by lambda, 2 pi by K and the energy is H cross omega E for a particular corresponding to a particular mode and this is-- and the dispersion relation we know for massless for electromagnetic wave since they are massless, the dispersion relation is that omega = CK. So this is the energy is H cross omega H cross C*K.

So corresponding to each mode the so photons will occur only with certain K vectors and the one photon of a particular K vector will have an energy EK which is H cross*CK. Okay, and the K's are labelled by nx, ny, nz which can be only be integers. Right, so now in thermal equilibrium-- this is the situation and we know that in thermal equilibrium, so if the photon inside are thermal equilibrium at a temperature T, then the photon occupation number the occupation number of each mode.

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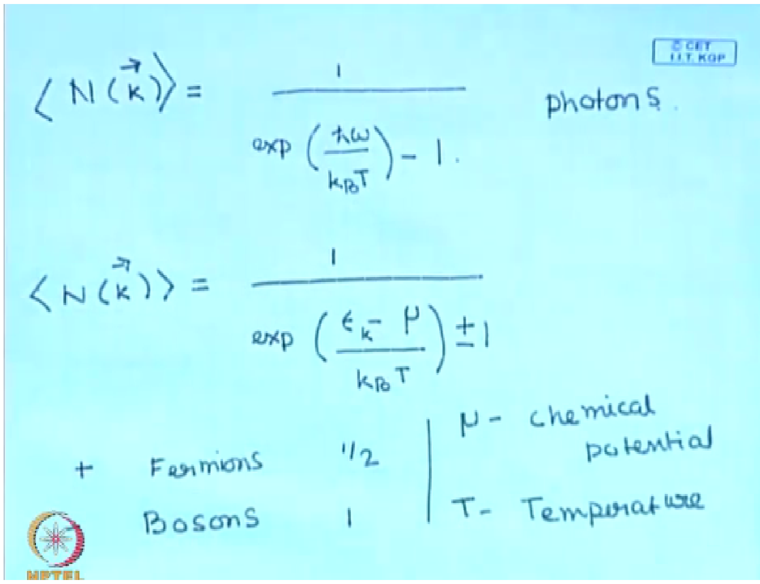

So this tells us how many particles are there in each mode, or how many photons are there in each mode. So for a Boson gas it is given by 1/exponential H cross omega, or, okay H cross omega by KBT -1. This gives me a number of particles in a particular mode, on the average. So if I-- this is a statistical thing, so this is mean number of particle in the particular board. So the electromagnetic wave inside this wave inside this Cavetti can be decomposed into set of discrete modes.

And we can ask the question how many particles are there corresponding to each mode. And the mean number of particles corresponding to each mode, we cannot predict exactly how many there will b because this is the statistical thing it is an equilibrium with a temperature T. What we can predict is that the mean number of particles is divided by this. Okay, this is for photons Black-body so photons in thermal equilibrium.

In general, let me write down general expression, in general the occupation number of any mode is given by 1 by exponential, the energy of that mode Epsilon, - the chemical, this is the energy of the particular mode, - the chemical potential which describe the entire system divided by KBT +- 1. Okay + is for Fermions and - is for Bosons. So Fermions are particles where you have half integer spin, half for example.

And these are particle that has been 1, 0, etcetera. Okay, new is the chemical potential, which determines a total number of particles, which is determined by the total number of particles e is the temperature. So in thermal equilibrium, so if the particle here I could have Bosons and Fermions whatever in Thermal Equilibrium the entire distribution is described by 2 things, the chemical potential and the temperature. Okay, Chemical potential and temperature. And the distribution is different if there are fermions, if there are bosons. Okay.

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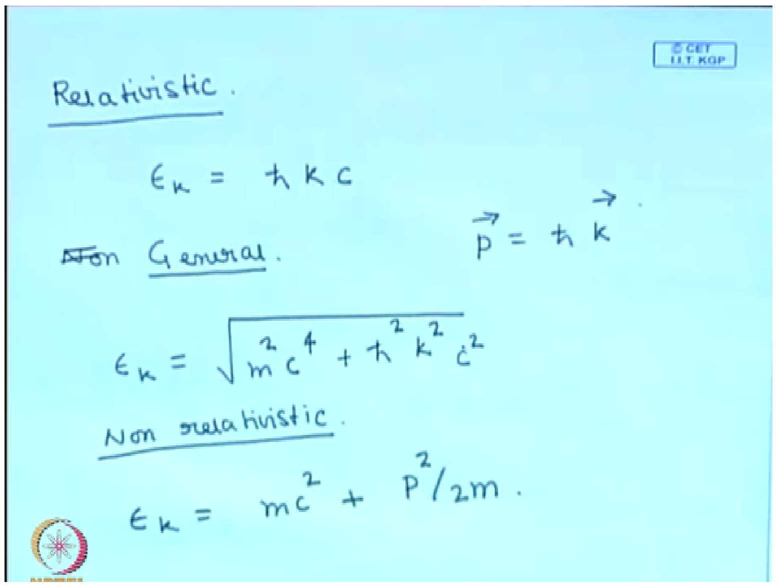

Further, if the particle are Relativistic, when we already written down that Epsilon K = H cross*K, into C omega basically, whereas if they are non-relativistic or in general let me write down in general, so they are massless particle that is this, in general the relation is that Epsilon K the energy of a particle is m square C 4 + T square and C square. And we know that P the momentum of a particle is H cross*K the wave vector, elementary contour mechanics. Okay.

So this is this + H cross square K square*C square, or you can write it in terms of P also whichever is convenient P square, C square. Fine. And if you look at the non-relativistic limit, then this term is much small compare to this, so you can do a tailored series expansion, and what you have is that the energy is mc square + P square by 2m, where P has to be calculated this way. Okay.

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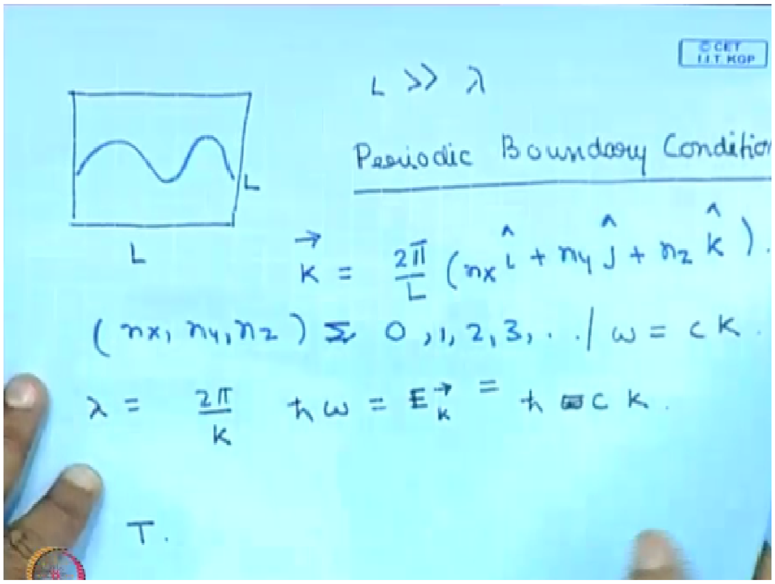

So in thermal equilibrium the entire thing is described by this by distribution function by the occupation number of each mode and this is a situation that we are dealing with. Now let us see what happens, so suppose let us assume that my system my particles are in thermal equilibrium to start with. At some time, they are in thermal equilibrium. Okay.

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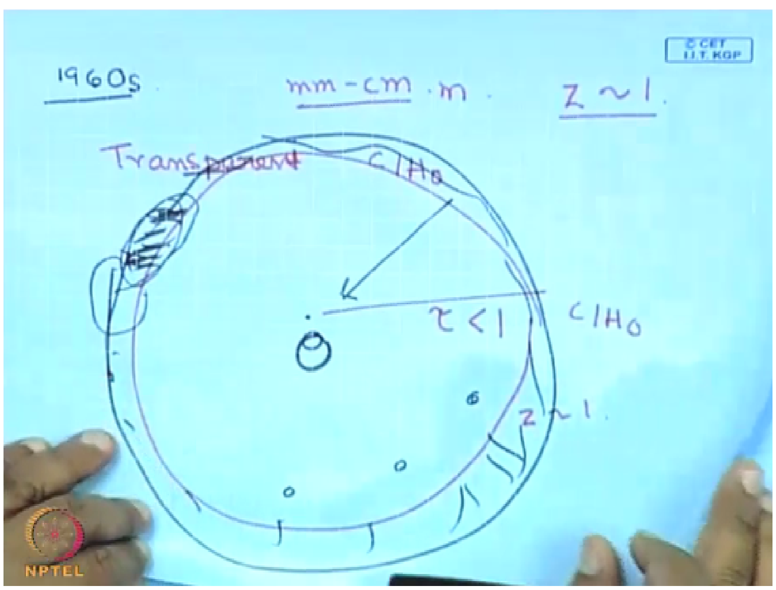

So at some epoch somewhere here my system, my entire thing set of particles are in thermal equilibrium at some temperature T, after which they just evolved freely. So, what happens when they evolve freely. This is in Thermal equilibrium. Okay, what happens when the entire thing is freely, let us just ask that. So if the system evolves freely, the all that-- so we are assuming that the expansion of the universe,

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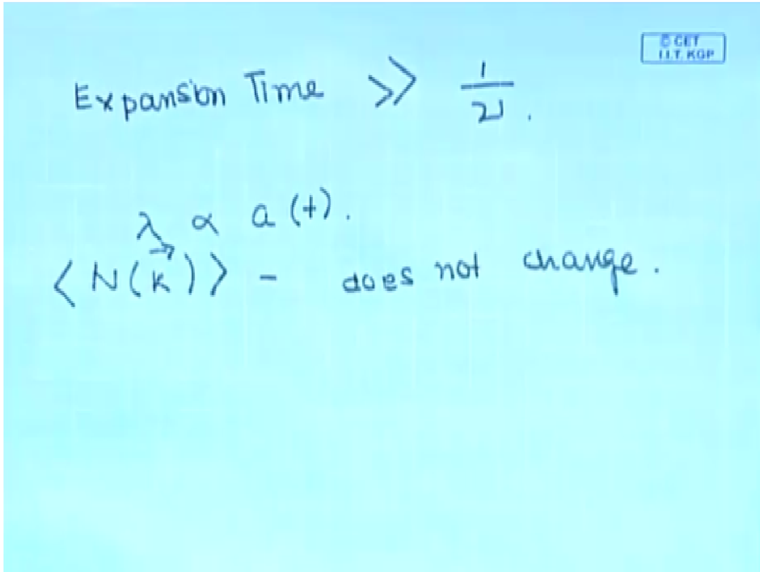

The evaluation of Expansion is on a time scale which is much larger than the inverse of the frequency of the radiation, of the wave. Okay. So we can assume that the expansion all that the expansion does is that it does not it so this box that we are dealing with will basically expand due to the expansion of the universe. And we are assuming that the expansion is adequately slow and the oscillation that take place in the electric field and by whatever in the wave are much faster.

Under this assumption, you can assume that all that happens is that the wavelength will get stretched proportional to A of T, this we have seen. The wavelength of each mode gets stretched proportional to A of T. The number occupation number, so the number of particles in each mode in a particular mode does not change. So all that happens due to the expansion of the universes but the value of K corresponding to a mode will get different but the number of particles will not change.

Okay, so there is a particular mode we are looking at the K value corresponding to that mode will change because of the expansion of universe the wavelength will change so the value of K will change but the number of particles occupying that mode the occupation number that value will not change. Okay. So given this let us see what happens to the distribution function occupation number as the universe expands.

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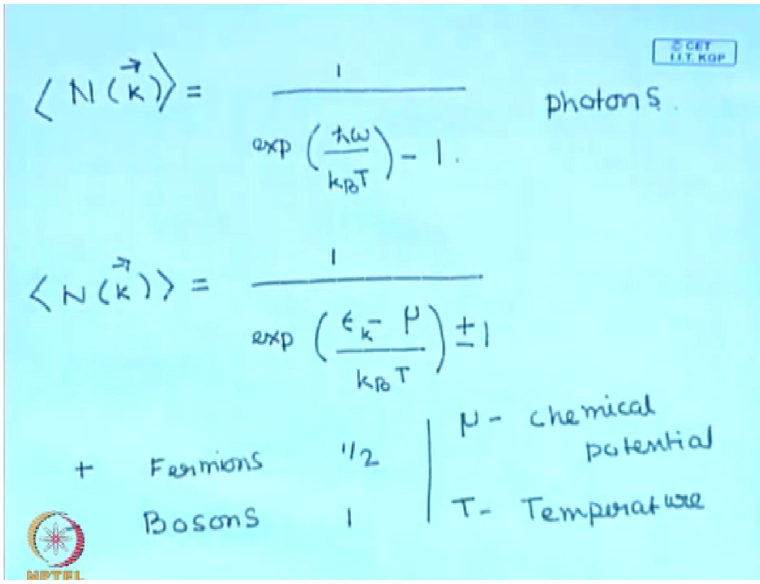

So this is, let us say the starting occupation number and let us assume that it is relativistic. So from a relativistic distribution.

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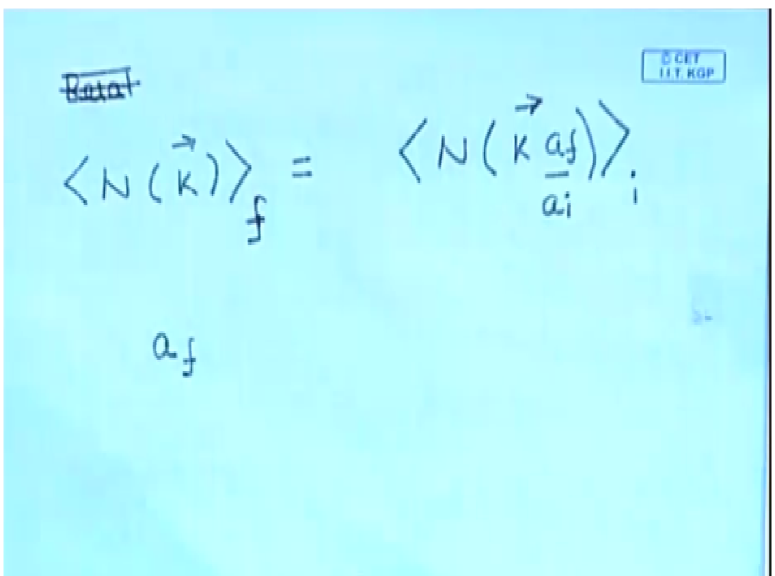

Okay, before we start so the basic assumption as follows that the occupation number of some mode K initially, this is initial will be equal to or let me put it other way around the final occupation number if a mode K if I want to calculate this at-- in terms of the occupation number at some so-- this is-- the value of the scale factor here is af, I want to calculate the occupation number of a mode at this epoch.

I have told you that now occupation the value of-- number of particles in this mode will not change, all that will happen is I have to now value of K will change. So this will be equal to the occupation number of a different mode. And the mode that I have to look at-- so the wavelength scales proportional to A so the value of K will scale inversely right so I have to look at a-- if I go into the past I have to look at a value which has a higher K value.

Because this K value this mode would have a higher wave number in the past. So I have to look at the mode which is afk*af by ai at the initial. So the final distribution function will be the initial distribution function evaluated at a different for a mode K value then the different K value is the K value I have (()) (42:08) here scaled appropriately like this.

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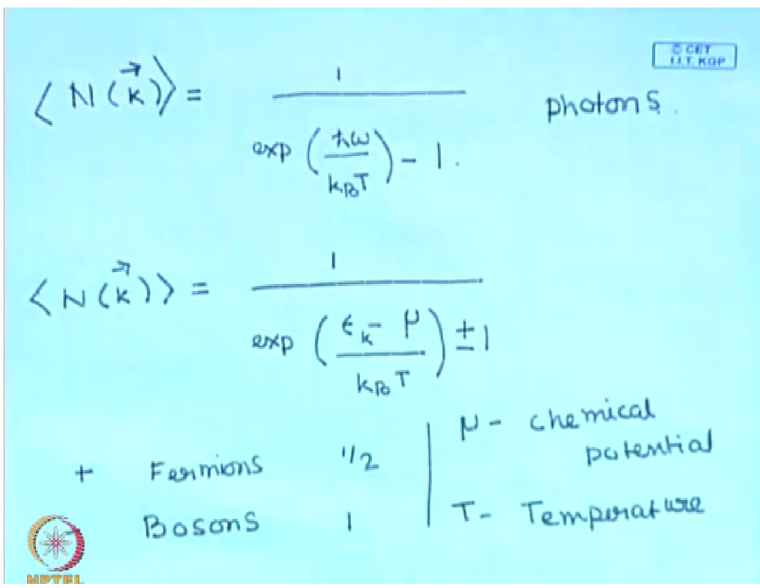

Okay, so let us assume that we have a distribution function like this to start with for a relativistic particles and we know for a let us assume that we know that distribution function is initially has a temperature TI and chemical potential Mu I. So we want to calculate the final distribution function. Okay. So the initial distribution function is for relativistic particles. It has a initial chemical potential Mu I initial temperature T I. Okay. So what we would like to do is that we would like to calculate final.

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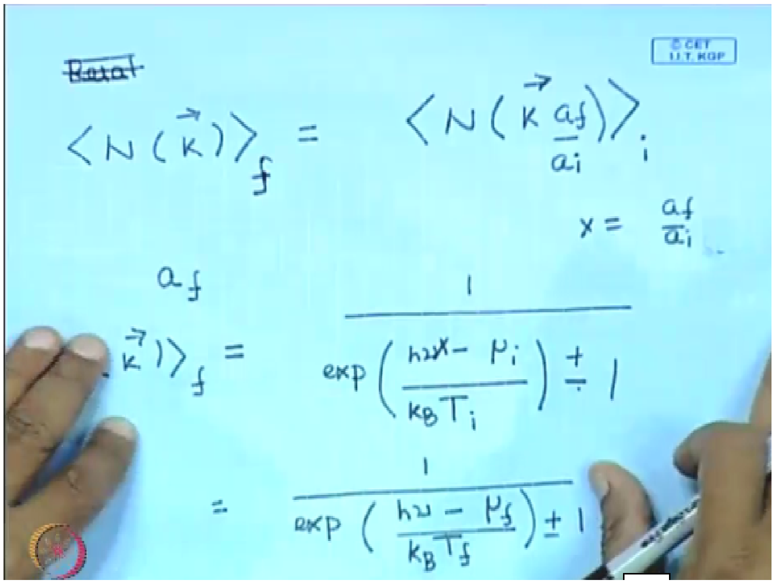

This will be =, so I should calculate the initial distribution function. This is equal to the initial distribution function which is given by 1/exponential, this will be E, E is we know H new - the-- sorry new but the New that you have to calculate here is not corresponding to this mode K but it is corresponding to this mode K scaled with af/ai. So I have to put a factor of, let us call it factor x, so I have to put a factor x over here, where x = af/ai. Okay.

So I have to evaluate this at not at the same frequency corresponding to this wave vector but at a different frequency which is x time this value. New is proportional to K so New also will get scaled with x - the initial chemical potential divided by the initial temperature +-- 1. This is for relativistic particles. Okay, so this is the final distribution function. So you can see that we can write the distribution function as 1/exponential H new - Mu F by TF - +-1.

So I am redefining this as H new – Mu F by KB, so there should be a KB Tf + -1. So how should you define Mu f in terms of Mu i, so Mu f should be = x*Mu i and Tf should be x into rather Ti should be x, right Ti should be x*

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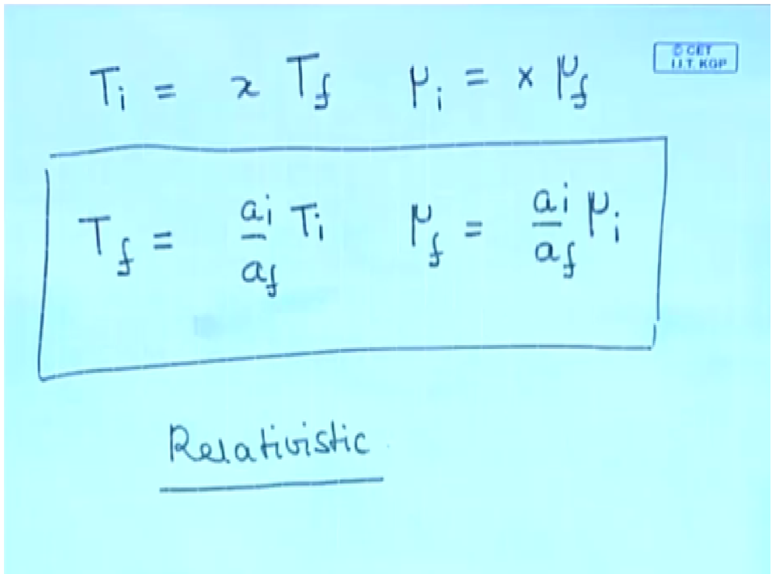

So if you can identify if you identify Ti to be x*Tf and Mu i to be x* Mu x, so with this identification, the x will cancel out throughout and you will left to do this. Okay. So what we see is that final temperature, Tf =, so for relativistic particles what we saw is that if the particles evolve freely without interacting with anything then due to the expansion of universe all that happens is that the wavelength corresponding to this particles scales proportional to A or the momentum scales us 1/A.

Because the momentum is inversely proportional to wavelength okay. Due to the expansion of the universe momentum scale is 1/A. And if I thermal distribution function, if I start with the thermal distribution function at some epoch then it remains the thermal distribution function only the temperature and chemical potential get scale like this due to the expansion of universe, that is the (()) (47:18). So they will scale proportional to A. Okay universally proportional.

So Tf *af remains a constant. So if the universe expands the temperature of the black-body radiation for example they are relativistic particles will fall as inversely with the scale factor. Okay.

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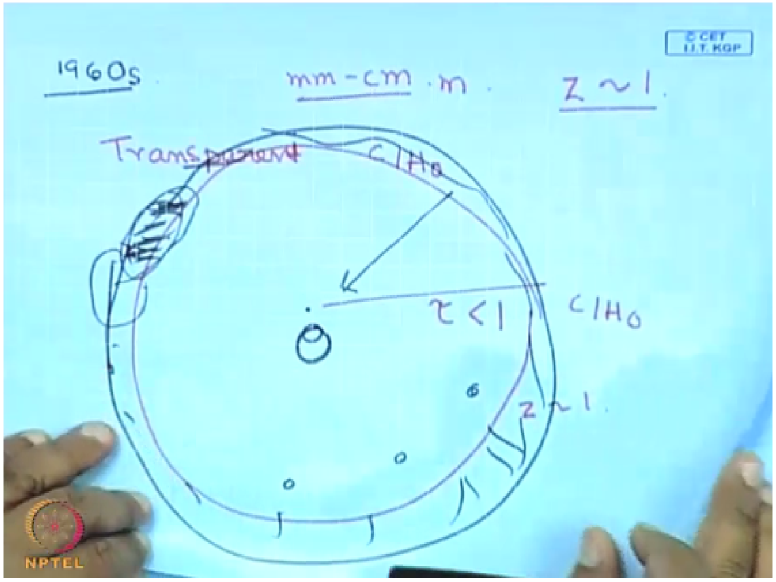

So in the epoch when the radiation does not interact with matter you only have the expansion of the universe, in this epoch the temperature the entire spectrum of the Black-Body radiation it remains a black-body spectrum all that happens is that the temperatures scales as inverse of the scale factor and if there is a temperature chemical potential for black-body radiation it is 0 but if it is some other kind of particle fermions etcetera the chemical potential also will scale same as the temperature.

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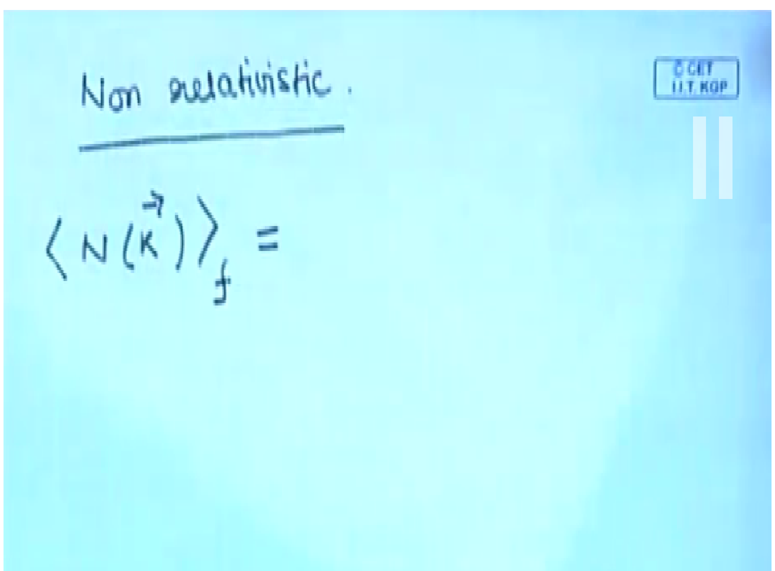

Okay, now we can repeat the same exercise for non-relativistic particles. So we have to do the same thing again. So we want to calculate the occupation number, the final occupation number for a non-relativistic particles. We know that the initial occupation numbers are described like this. Okay, so let me write down the initial occupation number first.

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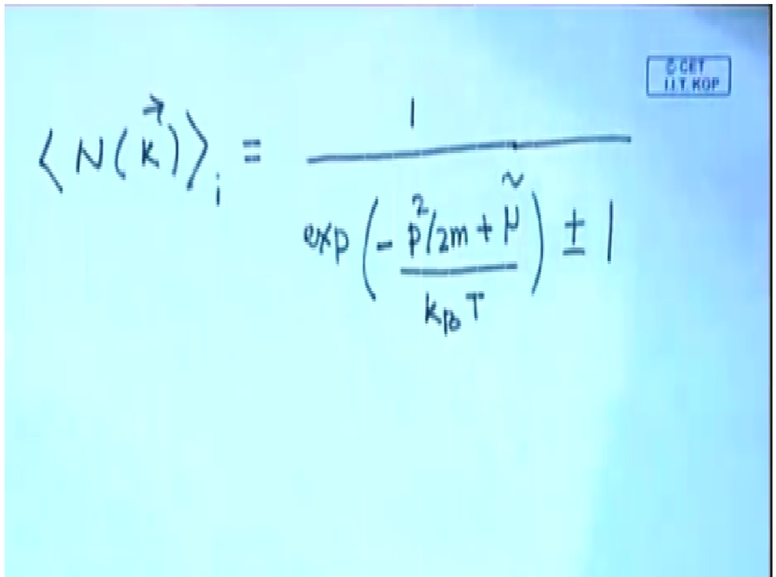

So the initial occupation number let me write it here. Now for non-relativistic particles we have seen that the energy is P square/2m+mc square. This mc square term can be observed from inside the chemical potential here to redefine the chemical potential. Okay. So what we have here then is –p square/2m+Mu Tilde / KBT + - 1, where the mc squared term I have put inside Mu tilde, okay. Now, so this is the initial distribution function.

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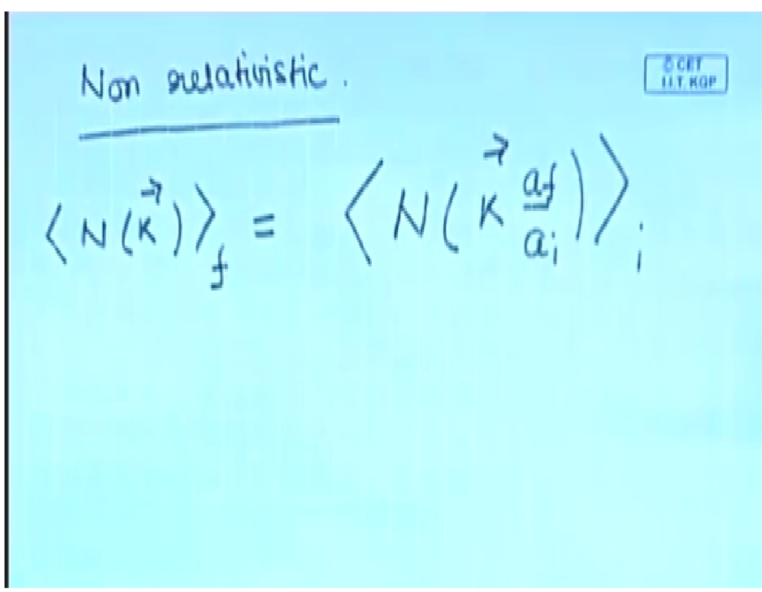

And we have seen that the due to the expansion of the universe what will happen is that this will be = the initial distribution function scaled by af/ai, okay.

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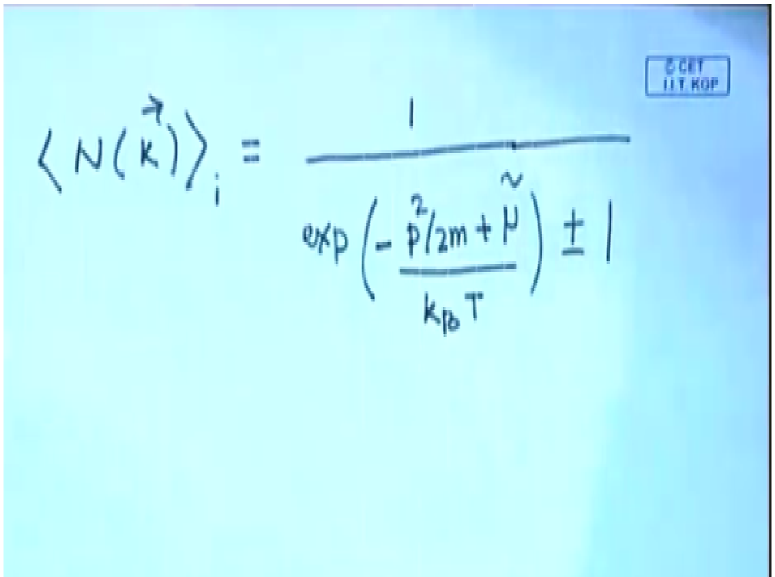

So, p and k are proportional, so what you see is that now this will be = again.

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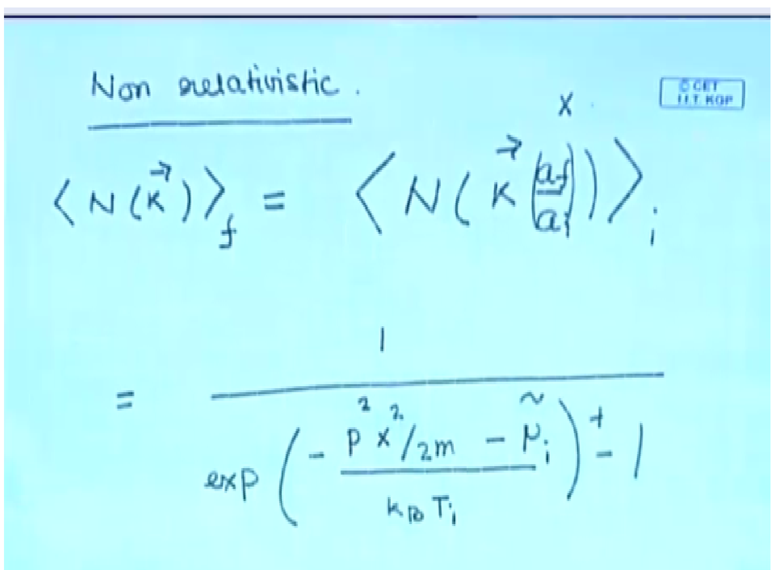

Calling this factor x, this will be = 1/ exponential –p square x square/2m-Mu Tilde/KB T initial Mu initial + -1.

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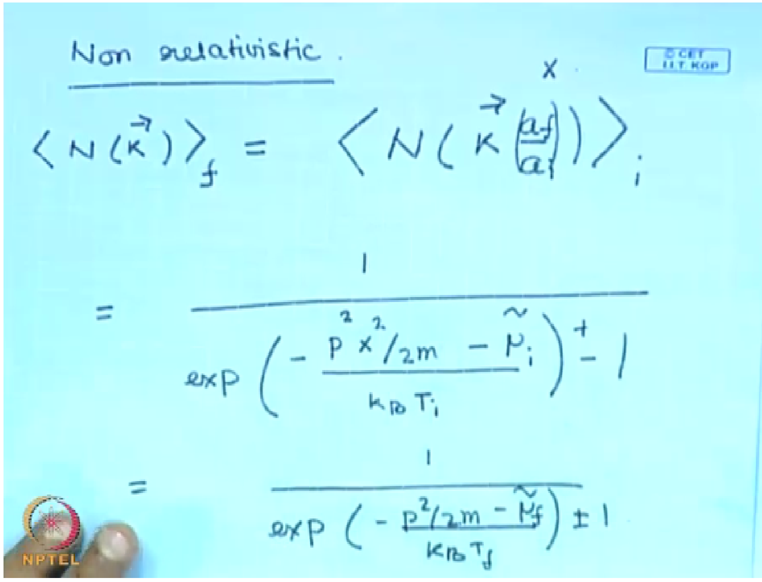

So if you define Tf = x square, here you have to define tf to be x square*T initial, so Tf*x square should be T initial Mu f*x square should be Mu initial with this you recover the same distribution with new temperature and new chemical potential. Okay.

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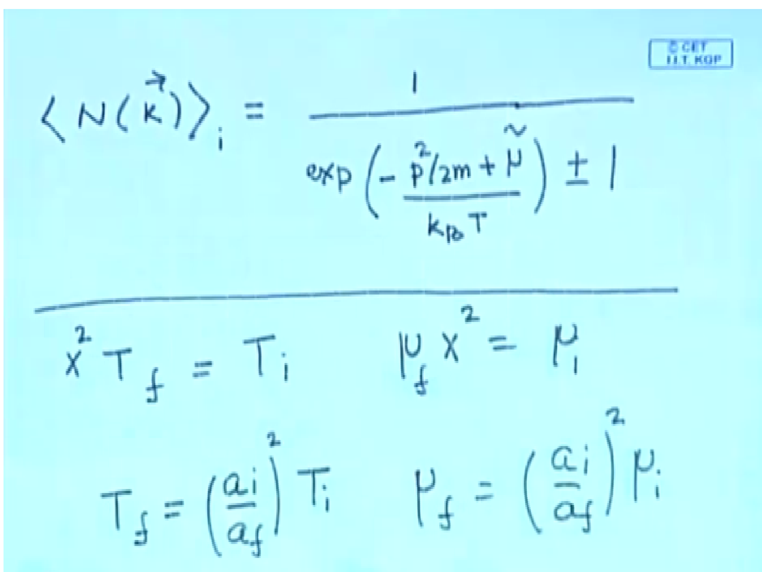

Whereas the new chemical potential Tf and the new chemical potential are related to the old ones like this or what we see is that the temperature now scales as, so the scaling is different for non-relativistic particles as from relativistic particles. Okay.

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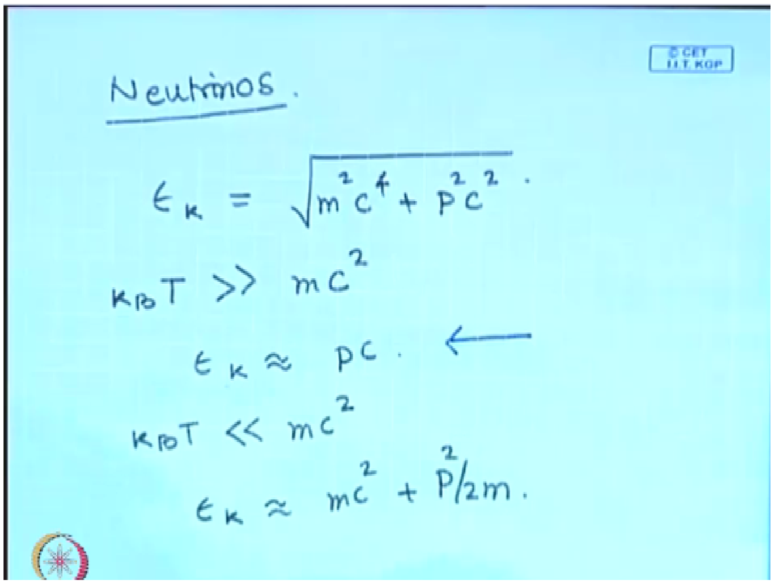

Now a very interesting case; so Neutrinos for Neutrinos or any such particle the energy. If the temperature is much larger than mc square then we can treat them as relativistic particles. If temperature is much smaller than mc square they behave like non-relativistic particles. Okay, so for both the situations the—the distribution will behave differently with the expansion of the universe.

Now, if the neutrinos—the other interesting thing is that if the neutrinos go out of thermal equilibrium in a regime where the temperature is much higher than the mass. Suppose the neutrinos go out of thermal equilibrium in a regime where the temperature is much higher than the mass. Okay. Then the distribution function gets fixed the occupation number does not change again due to the expansion of the universe, all that change is that the scaling of each mode. Okay.

So let me bring today's discussion over here and about how the distribution function changes during the free evolution of the universe and then in the next lecture we shall resume our discussion about the thermal history of the universe based on this. "Professor to Student conversation starts" Are there any questions? Would you follow this or ...? "Professor to Student conversation ends".